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Interval-Based Identification of Characteristic Curves

European Control Conference (ECC) 2026, Reykjavík, Iceland

Marit Lahme, July 7, 2026

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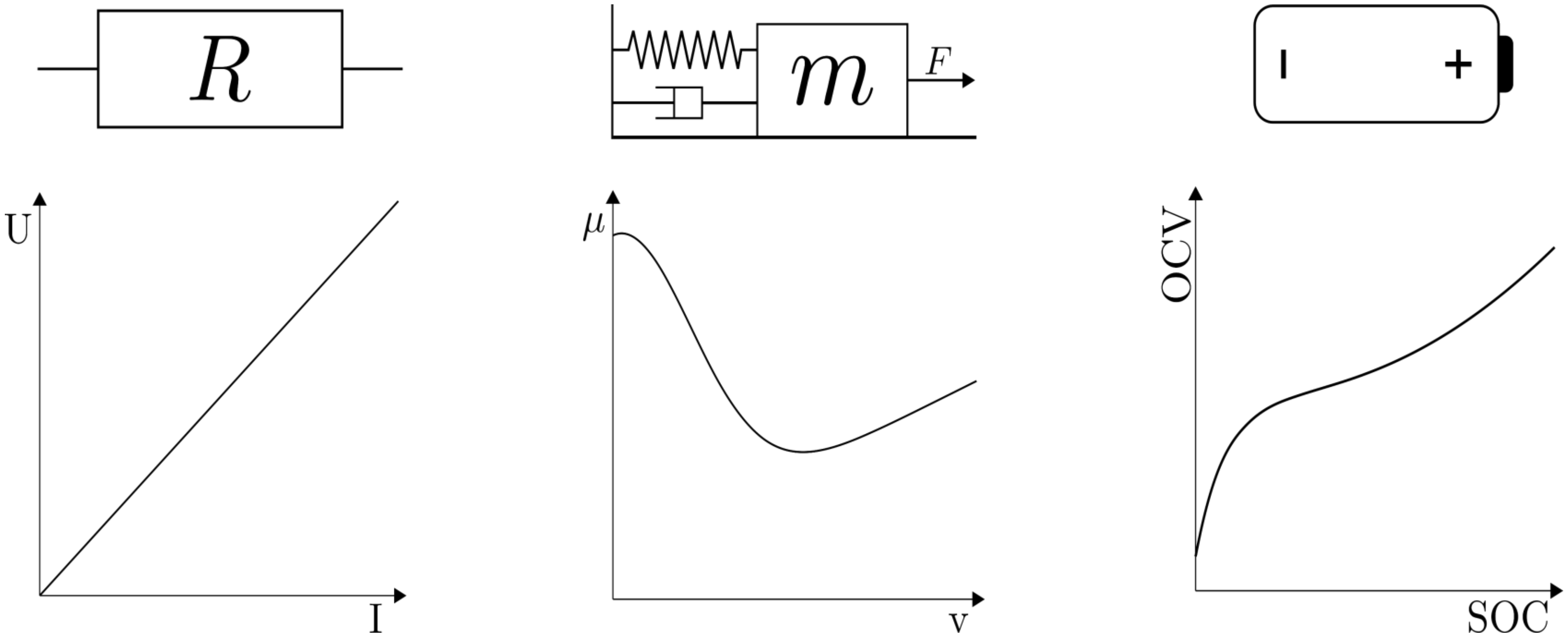
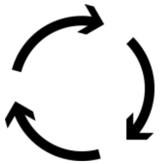
Objective



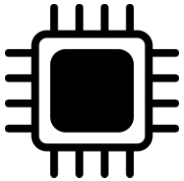
Objective

Guaranteed uncertainty bounds

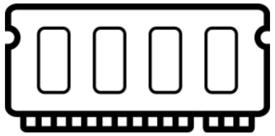
Without knowledge about structure



During system operation



Reasonable computational effort



Reasonable memory requirements

Modeling Approaches

White Box

Model structure can be derived from physical laws

Examples:
- First principle modeling
- Parameter estimation

Grey Box

Model structure is partially known

Examples:
- First principle modeling
- Parameter estimation
- Physics informed neural networks

Black Box

Model structure is unknown and learned from data

Examples:
- Maximum likelihood methods
- Gaussian Process modeling
- Neural networks

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Modeling Approaches

White Box

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Examples:
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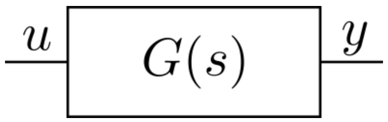
Grey Box

Model structure is partially known

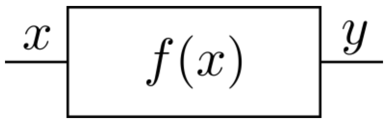


$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

Examples:
- First principle modeling
- Parameter estimation
- Physics informed neural networks

Black Box

Model structure is unknown and learned from data



$$y = f(x)$$

Examples:
- Maximum likelihood methods
- Gaussian Process modeling
- Neural networks

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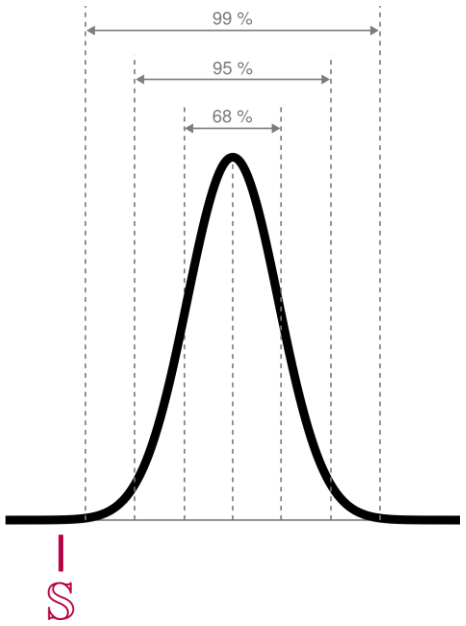
Guaranteed Uncertainty Bounds

Guaranteed uncertainty bounds

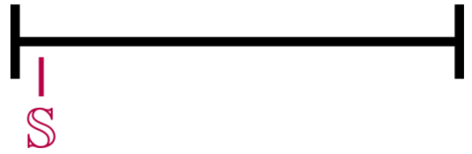
Stochastic



Not guaranteed

Set-based

$$\mathbb{S} \in [a] = [\underline{a}, \overline{a}]$$

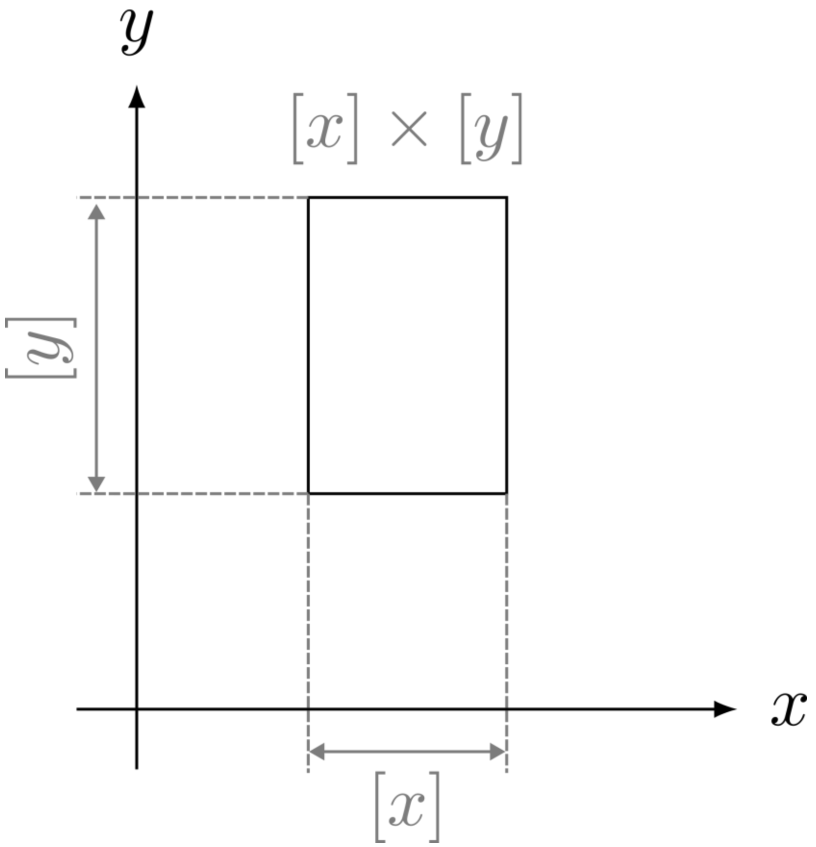


Guaranteed

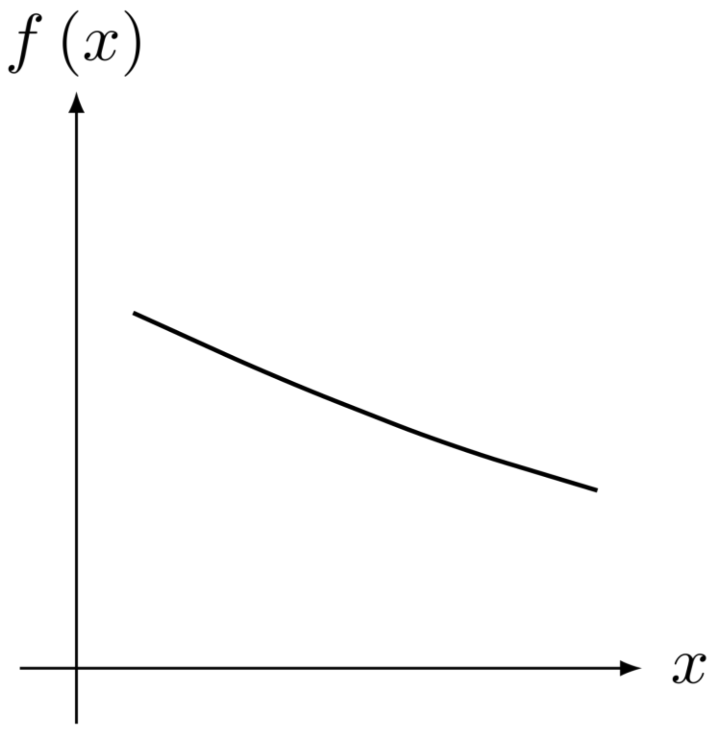
Guaranteed Uncertainty Bounds

$$[x] = [\underline{x}, \overline{x}]$$

$$[y] = [\underline{y}, \overline{y}]$$

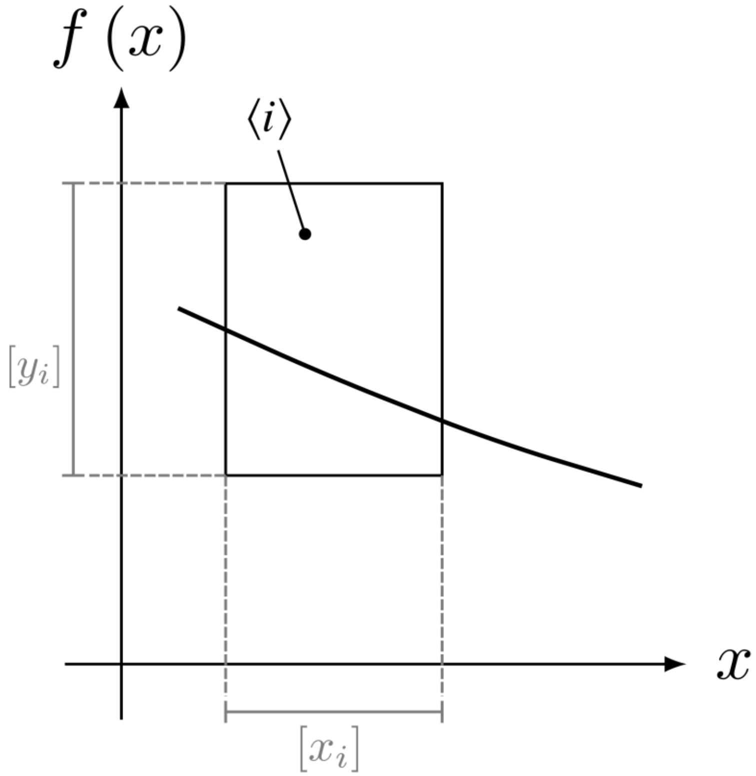


Identification



$$y = f(x)$$

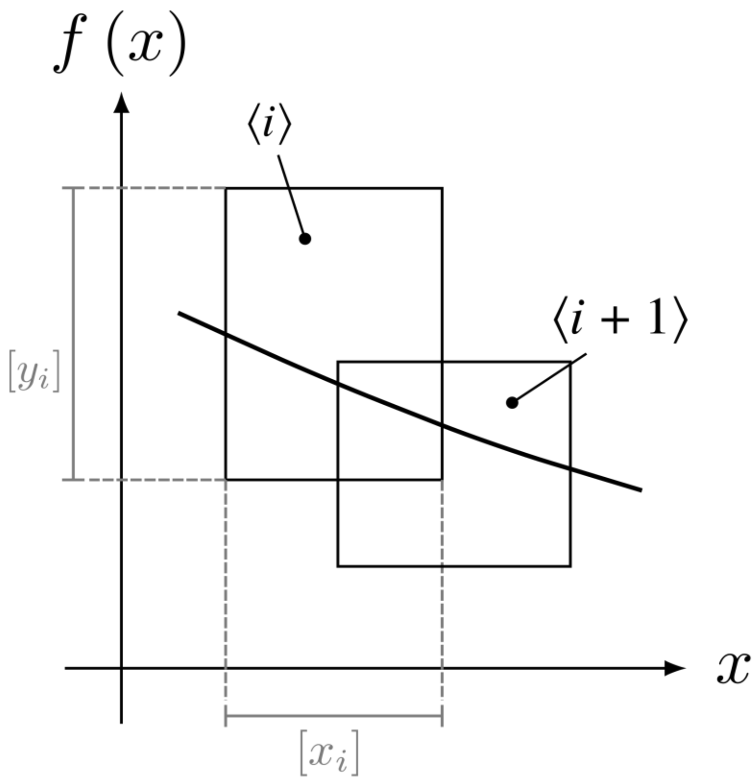
Identification



$$y = f(x)$$

$$\forall x \in [x_i],\ \exists y \in [y_i],\ s.t.\ y = f(x)$$

Identification



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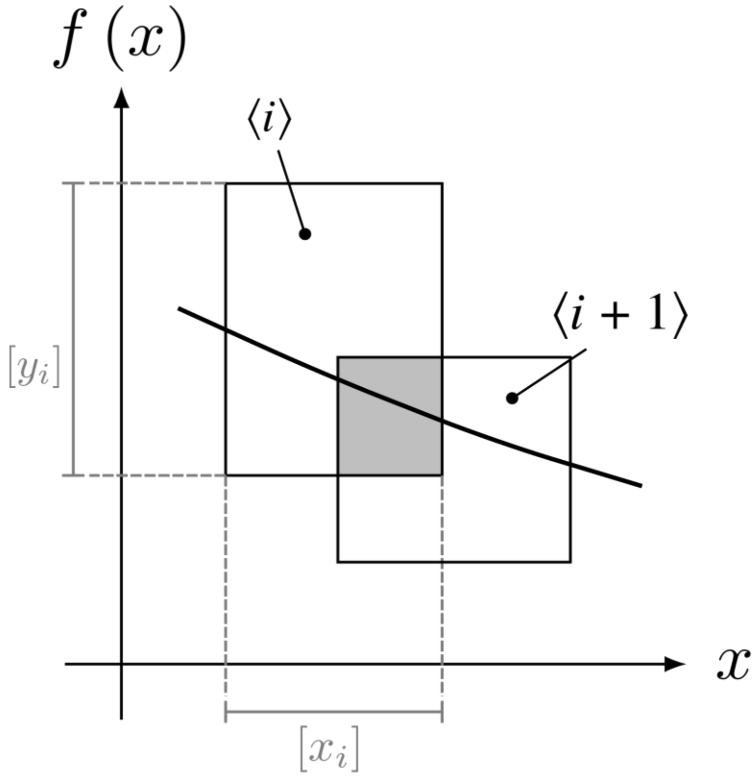

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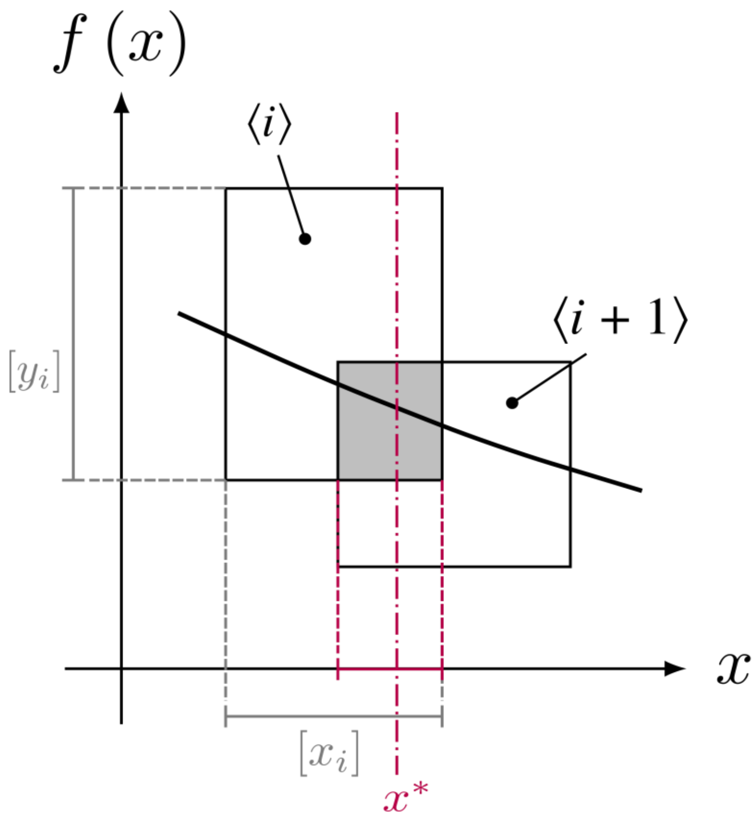

$$y = f(x)$$

$$\forall x \in [x_i]\,,\ \exists y \in [y_i]\,,\ s.t.\ y = f\left(x\right)$$

$$\forall x^* \in \left[\max\left\{\underline{x}_i, \underline{x}_{i+1}\right\}\ ;\ \min\left\{\overline{x}_i, \overline{x}_{i+1}\right\}\right],$$
$$\left(f(x^*) \in [y_i]\right) \wedge \left(f(x^*) \in [y_{i+1}]\right)$$

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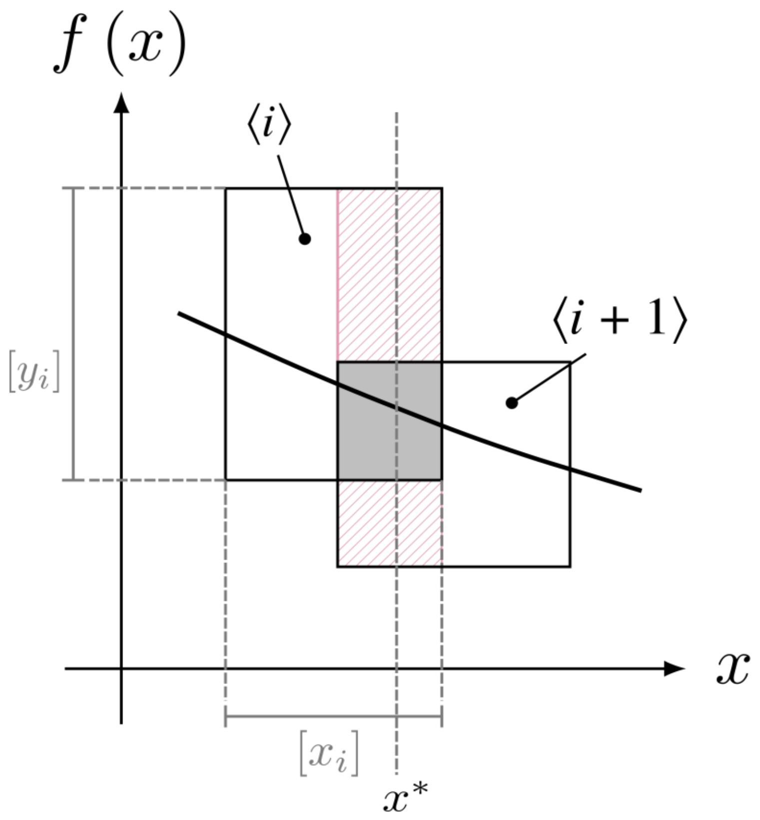

$$y = f(x)$$

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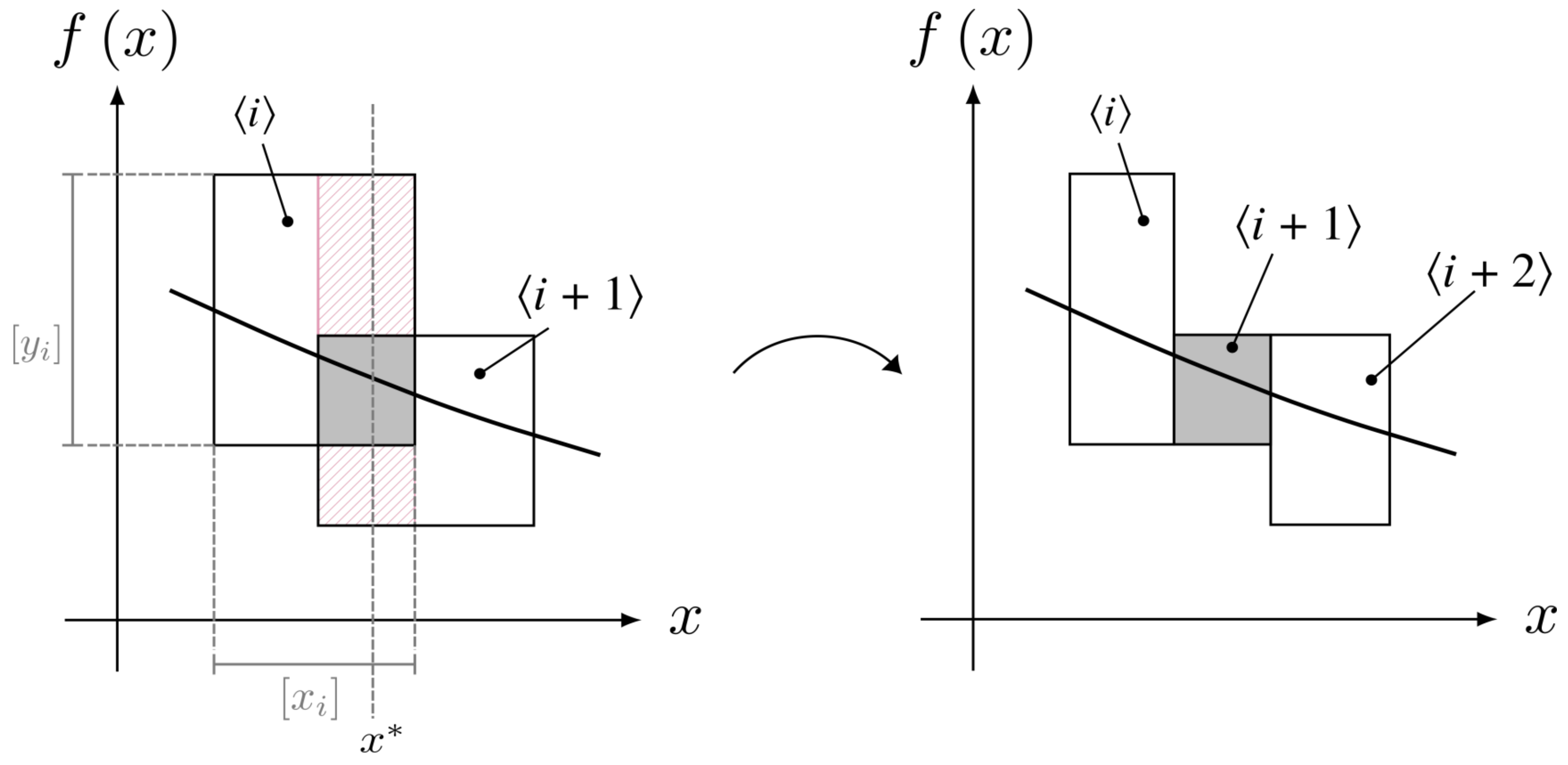
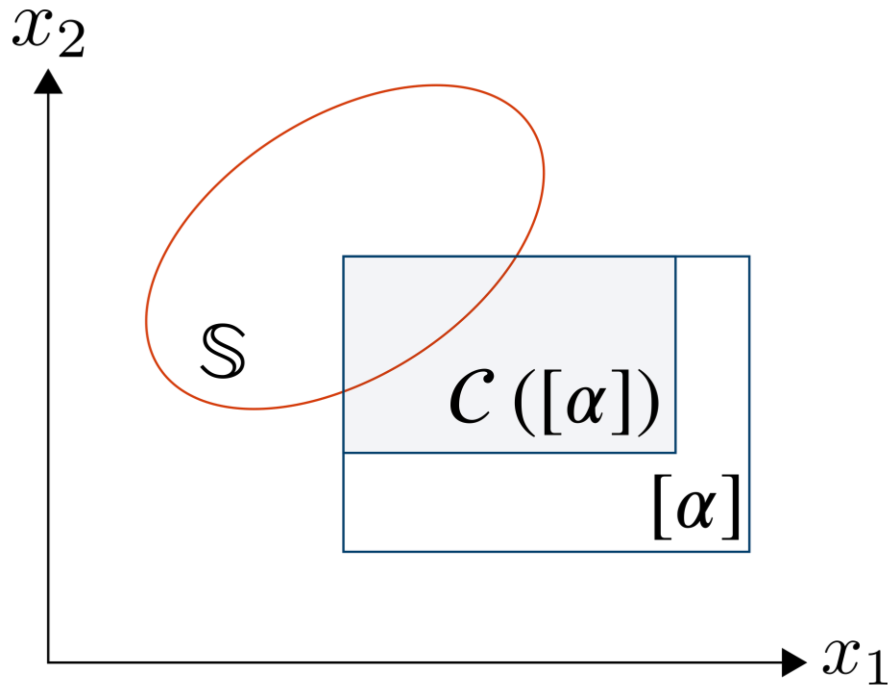
$$(f(x^*) \in [y_i]) \wedge (f(x^*) \in [y_{i+1}])$$
$$\Downarrow$$
$$f(x^*) \in ([y_i] \cap [y_{i+1}])$$

Identification

$\mathcal{C}_{Int}$



Identification

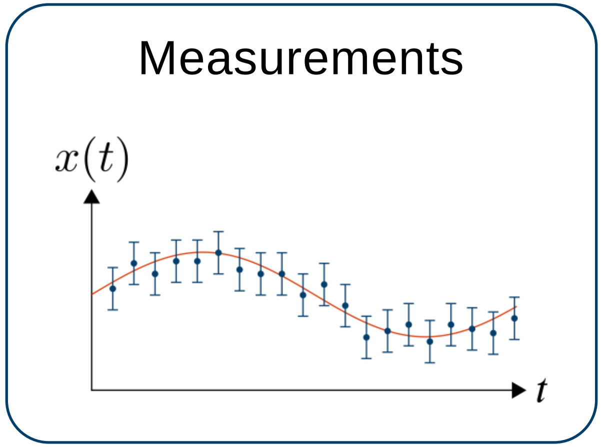


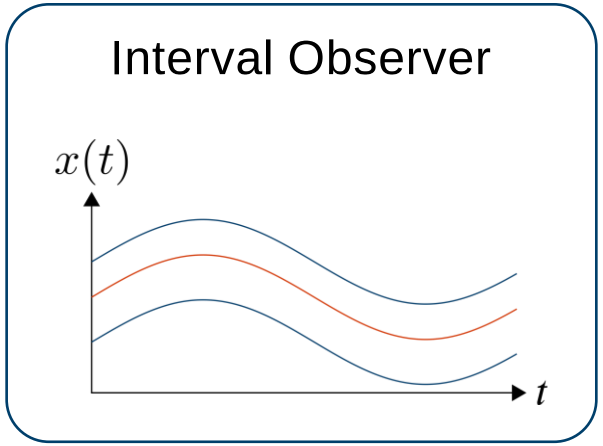
Contractor

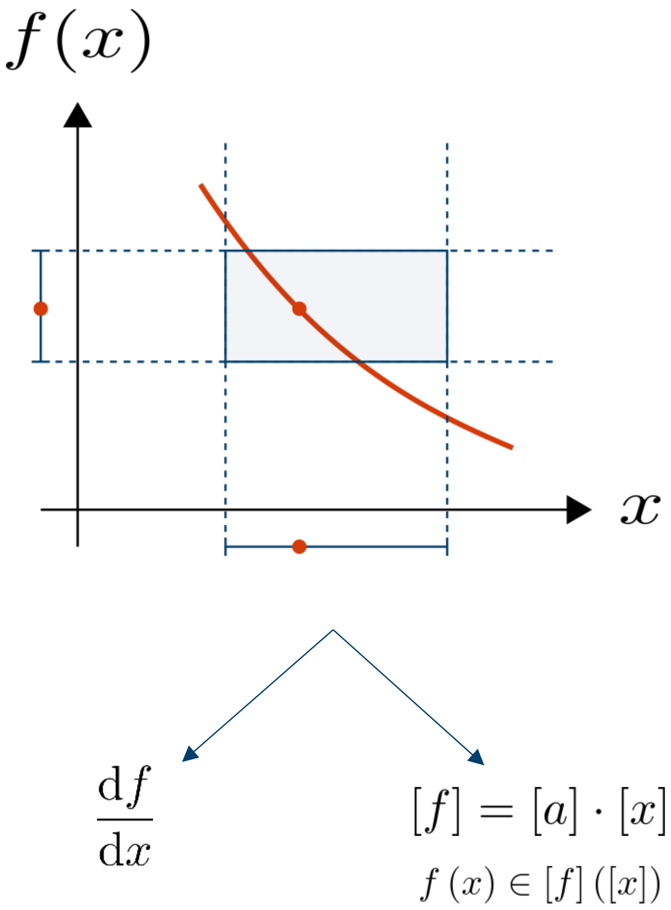
$$\forall\,[\alpha]\,,\; \mathcal{C}([\alpha]) \subset [\alpha]$$

$$\forall\,[\alpha]\,,\; \mathcal{C}([\alpha]) \cap \mathbb{S} = [\alpha] \cap \mathbb{S}$$

Identification







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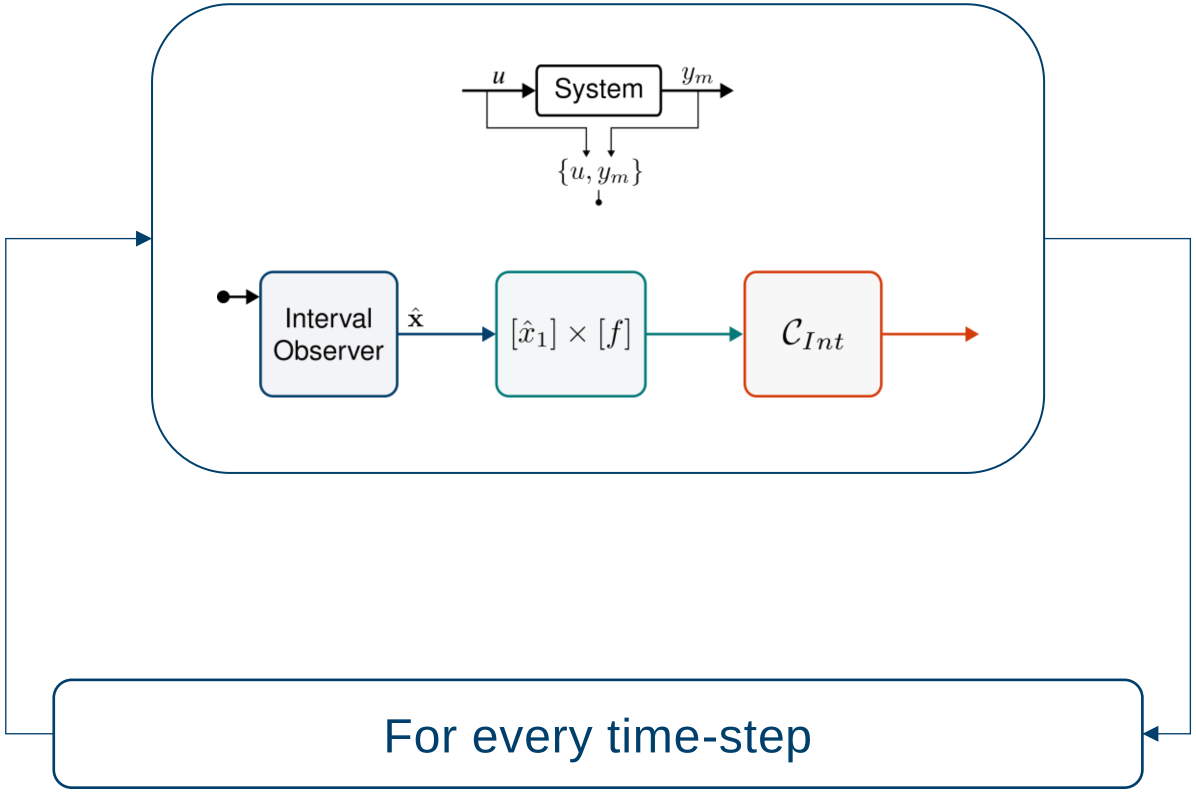
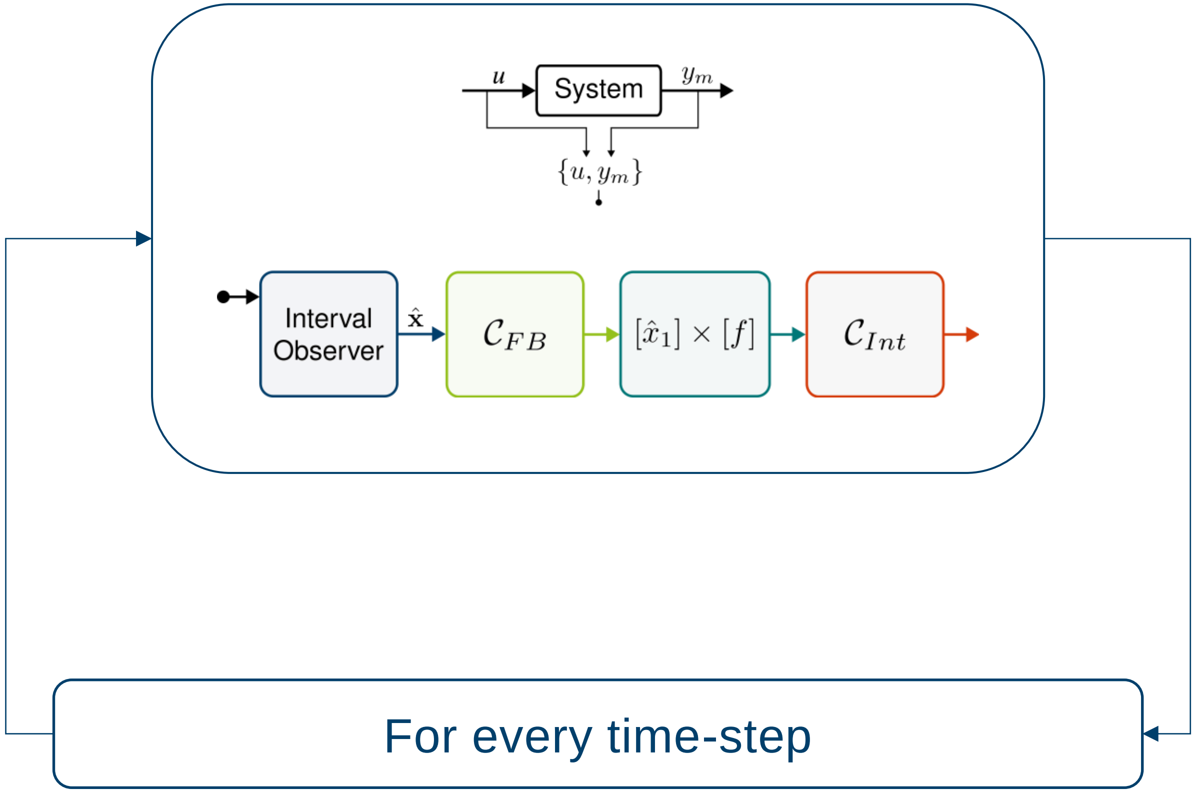


Identification



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Forward-Backward-Contractor

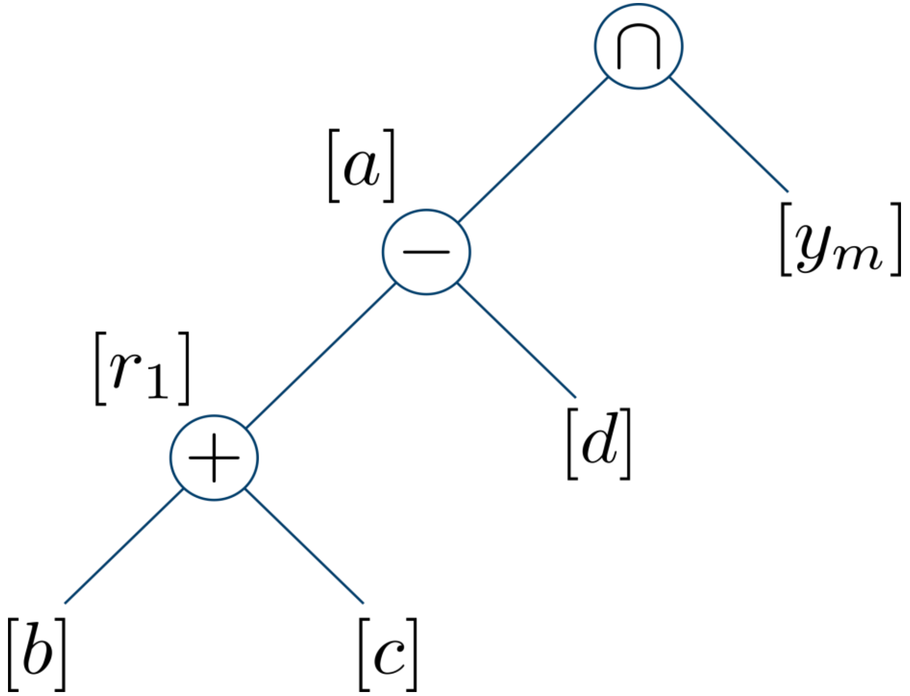
$\mathcal{C}_{FB}$

Constraint:

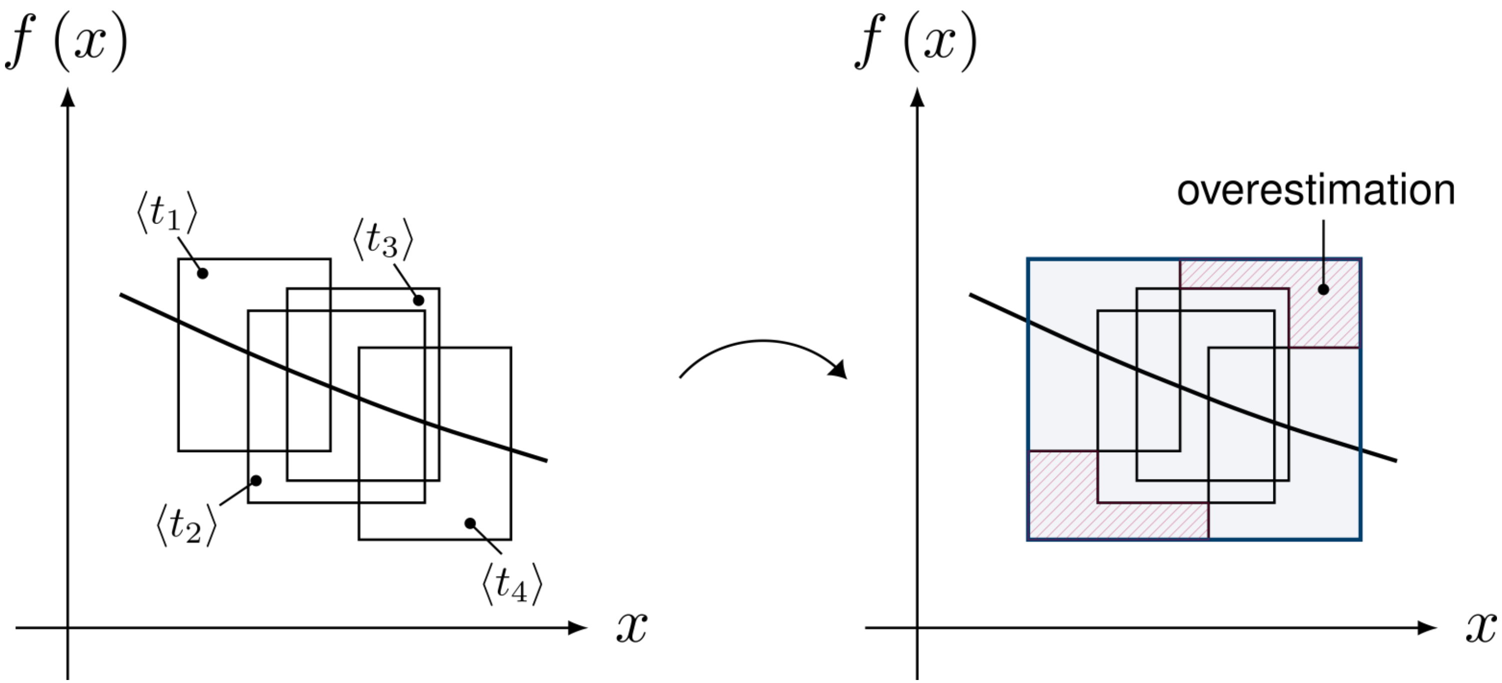
$a = b + c - d$

$a \in [a]\,, b \in [b]$
$c \in [c]\,, d \in [d]$

New information:
$y_m = a$



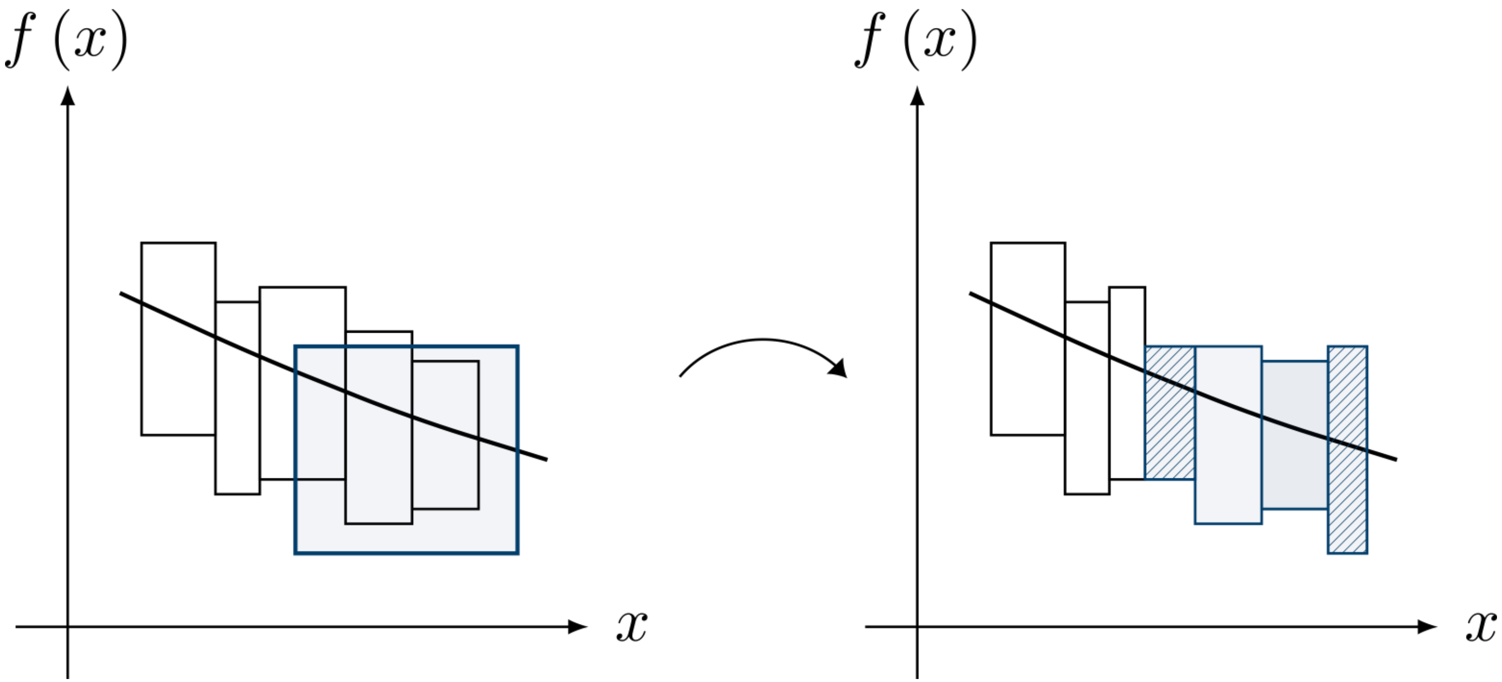
Computational Effort



overestimation $\leq$ threshold

number of boxes $\leq$ threshold

Computational Effort



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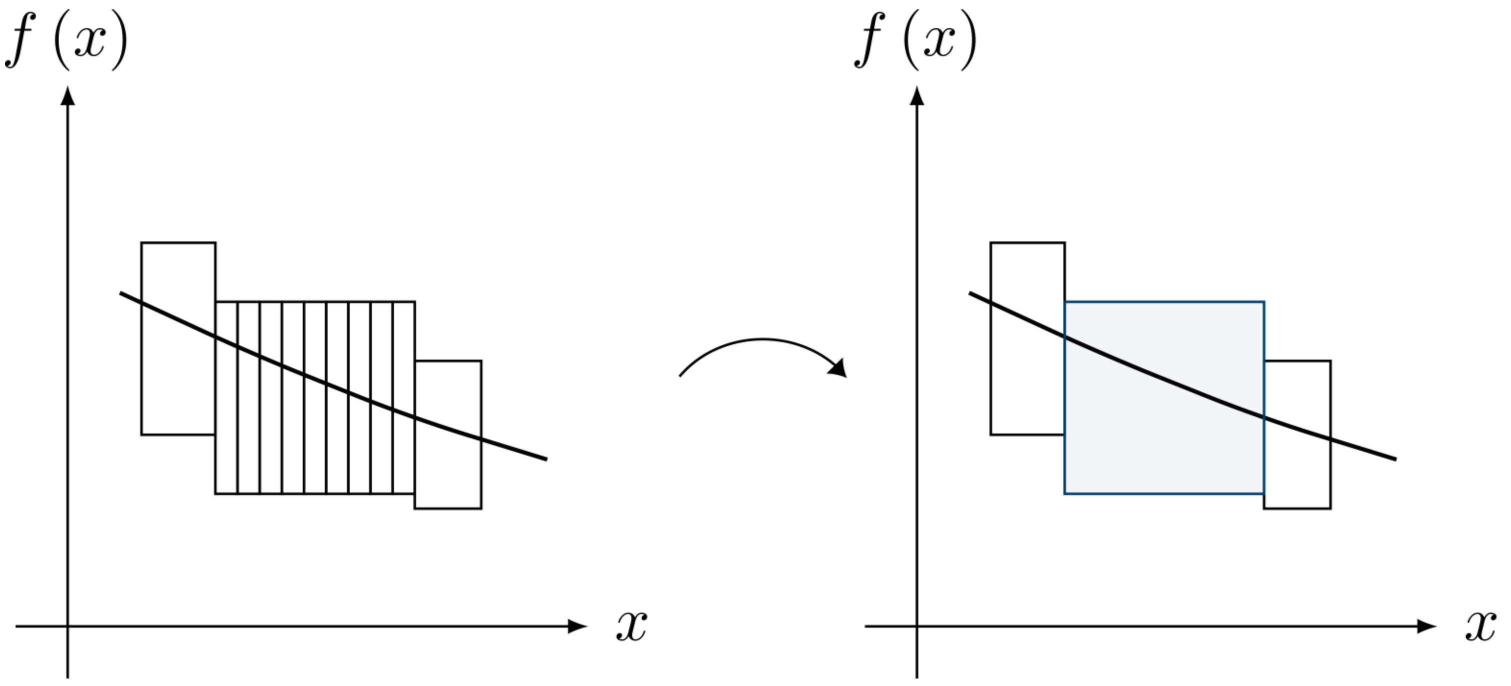
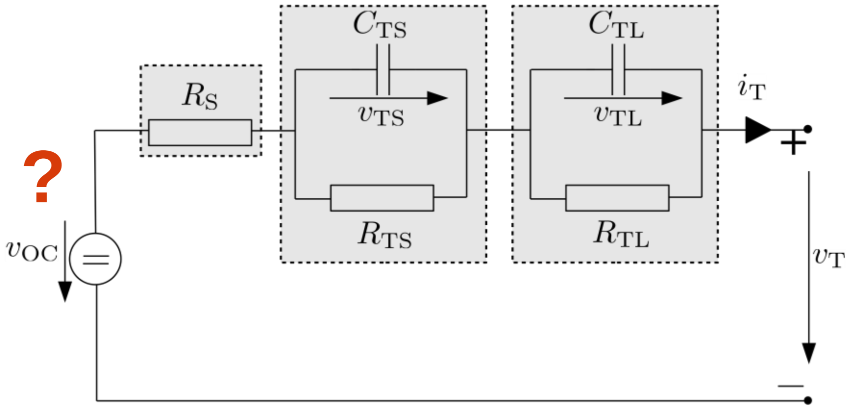
Computational Effort



Example: Lithium-Ion Battery



x_1 : State of charge

$x_2 : v_\mathrm{TS}$

$x_3 : v_\mathrm{TL}$

$u : i_\mathrm{T}$

$y_m : v_\mathrm{T}$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -\frac{1}{C_\mathrm{TS}(x_1)R_\mathrm{TS}(x_1)} & 0 \\ 0 & 0 & -\frac{1}{C_\mathrm{TL}(x_1)R_\mathrm{TL}(x_1)} \end{bmatrix} \mathbf{x} + \begin{bmatrix} -\frac{1}{C_\mathrm{Bat}} \\ \frac{1}{C_\mathrm{TS}(x_1)} \\ \frac{1}{C_\mathrm{TL}(x_1)} \end{bmatrix} u$$

$$y = \begin{bmatrix} \eta(x_1) & -1 & -1 \end{bmatrix} \mathbf{x}$$

$$\boxed{v_\mathrm{OC} = \eta(x_1)\, x_1}$$

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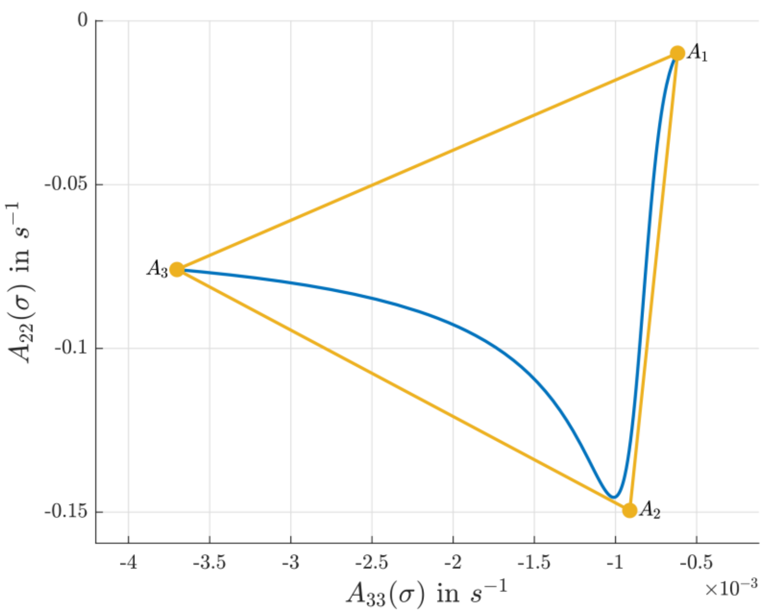

$$\mathbf{A}([\sigma(t)]) \in \left\{ \mathbf{A}(\zeta) \middle| \mathbf{A}(\zeta) = \sum_{\iota=1}^{3} \zeta_\iota \cdot \mathbf{A}_\iota \; ; \; \sum_{\iota=1}^{3} \zeta_\iota = 1 \; ; \; \zeta_\iota \geq 0 \right\}$$

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Example: Lithium-Ion Battery

$$y = \begin{bmatrix} \eta(x_1) & -1 & -1 \end{bmatrix} \mathbf{x}$$

$$\eta(x_1) \in [c_{11}]$$

	True	Estimate
$[c_{11}]$	$[1.54, 23.59]$	$[-50, 50]$

Example: Lithium-Ion Battery

$$
\begin{aligned}
\overline{\zeta}_{k+1} &= \mathbf{T}\mathbf{A}_{\mathrm{d}}\hat{\overline{\mathbf{x}}}_k + \mathbf{T}\mathbf{B}_{\mathrm{d}}\mathbf{u}_k + \mathbf{L}(y_k - \mathbf{C}\hat{\overline{\mathbf{x}}}_k) + \overline{\boldsymbol{\Delta}}_k \\
\hat{\overline{\mathbf{x}}}_k &= \overline{\zeta}_k + \mathbf{N}y_k
\end{aligned}
$$

$$
\begin{aligned}
\underline{\zeta}_{k+1} &= \mathbf{T}\mathbf{A}_{\mathrm{d}}\hat{\underline{\mathbf{x}}}_k + \mathbf{T}\mathbf{B}_{\mathrm{d}}\mathbf{u}_k + \mathbf{L}(y_k - \mathbf{C}\hat{\underline{\mathbf{x}}}_k) + \underline{\boldsymbol{\Delta}}_k \\
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$$

Z. Wang, C.-C. Lim, and Y. Shen, “Interval Observer Design For Uncertain Discrete-Time Linear Systems,” *Systems & Control Letters*, vol. 116, pp. 41–46, 2018.

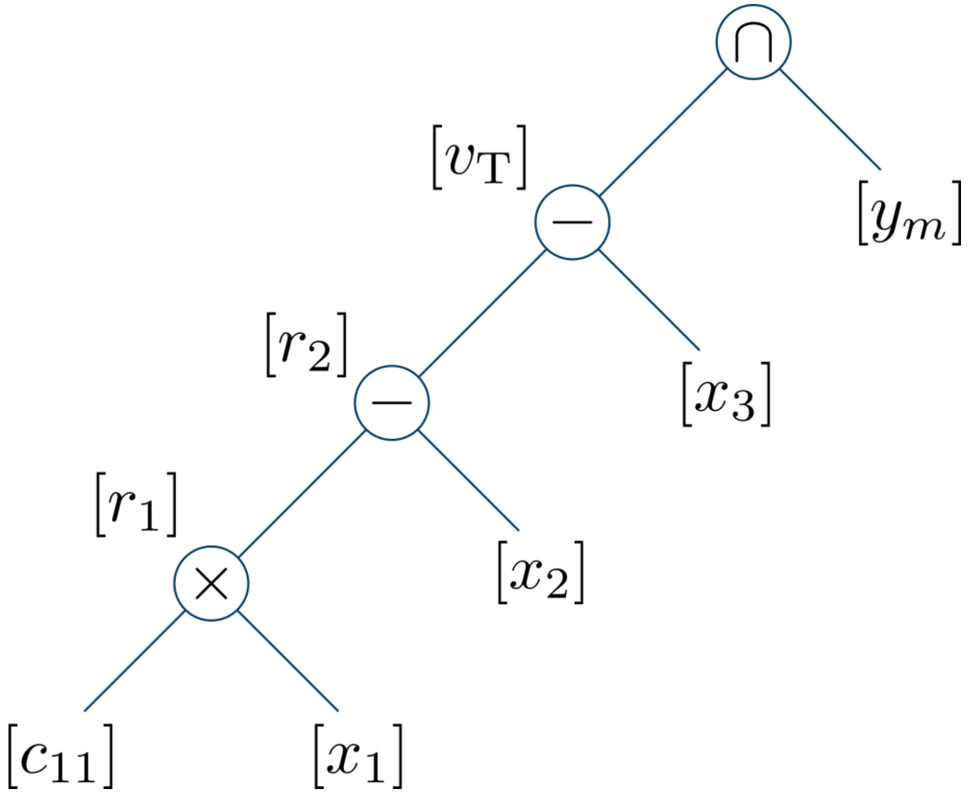
Example: Lithium-Ion Battery

Constraint:

$$[v_{\mathrm{T}}] = [c_{11}]\,[x_1] - [x_2] - [x_3]$$

New information:

$$y_m = v_{\mathrm{T}}$$



Example: Lithium-Ion Battery

$$[\hat{x}_1] = \left[\underline{\hat{x}}_1, \overline{\hat{x}}_1\right]$$

$$[f] = \left[\underline{[c_{11}]\,[\hat{x}_1]}, \overline{[c_{11}]\,[\hat{x}_1]}\right]$$

$$[\hat{x}_1] \times [f]$$

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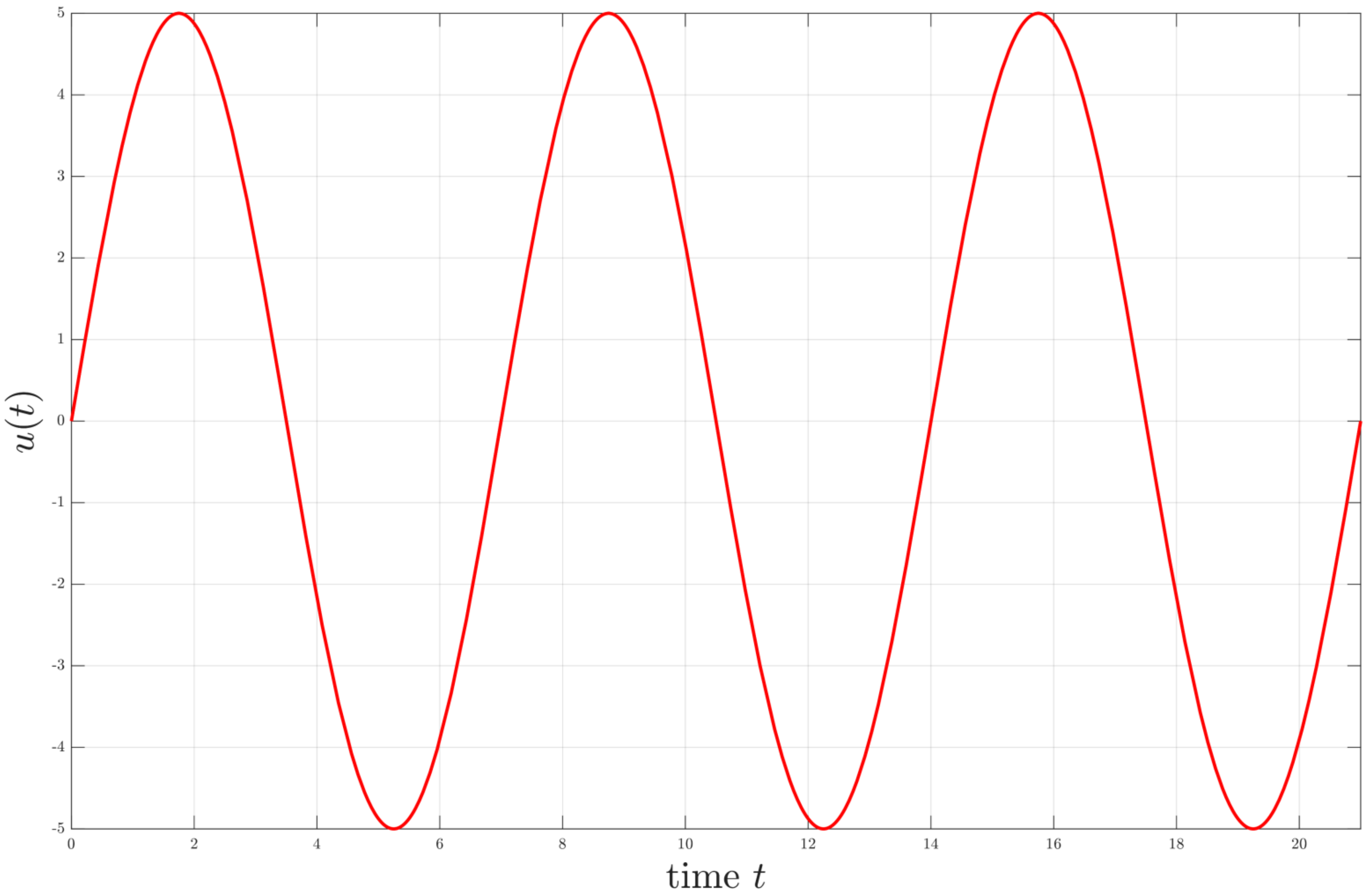
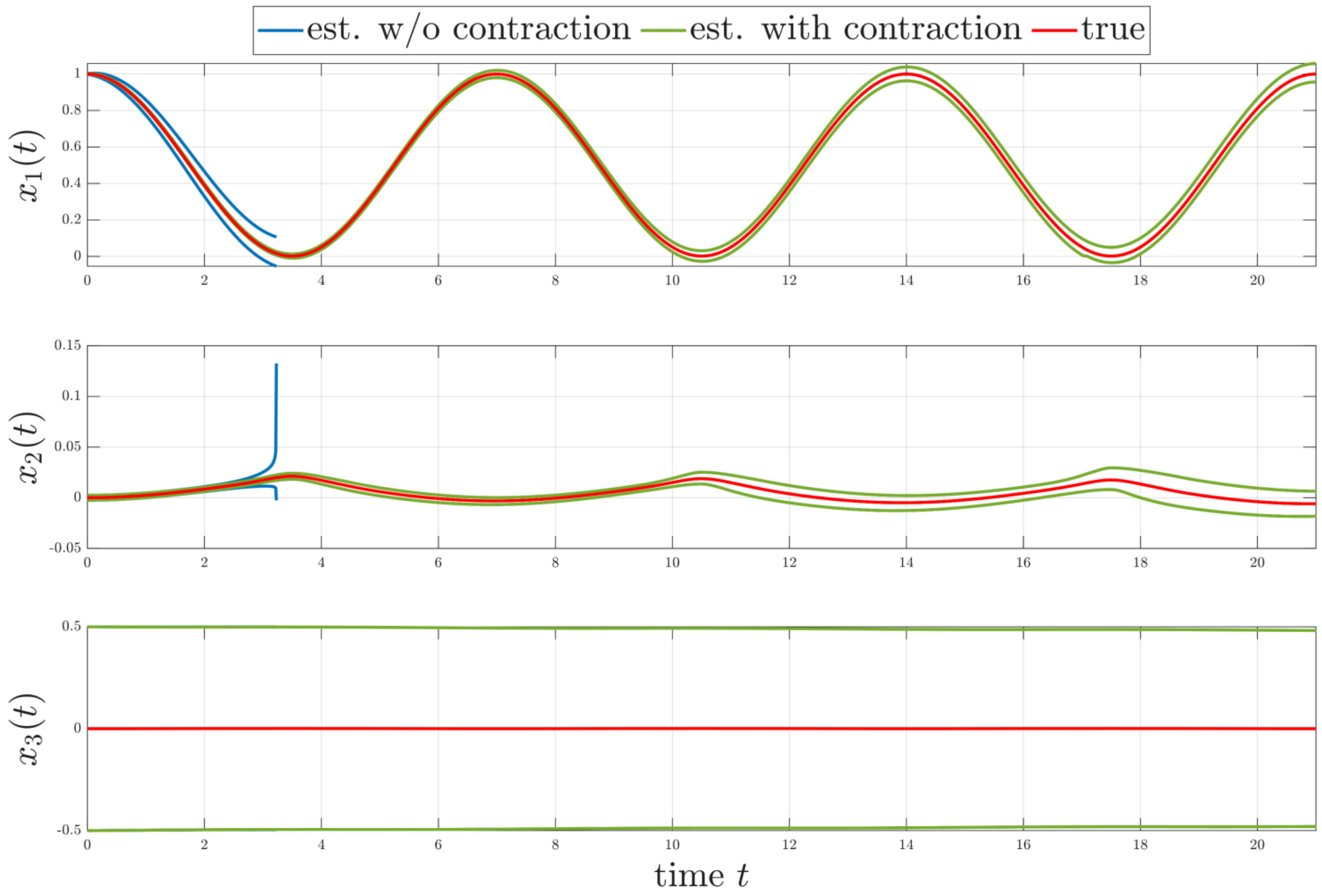
Example: Lithium-Ion Battery



Example: Lithium-Ion Battery



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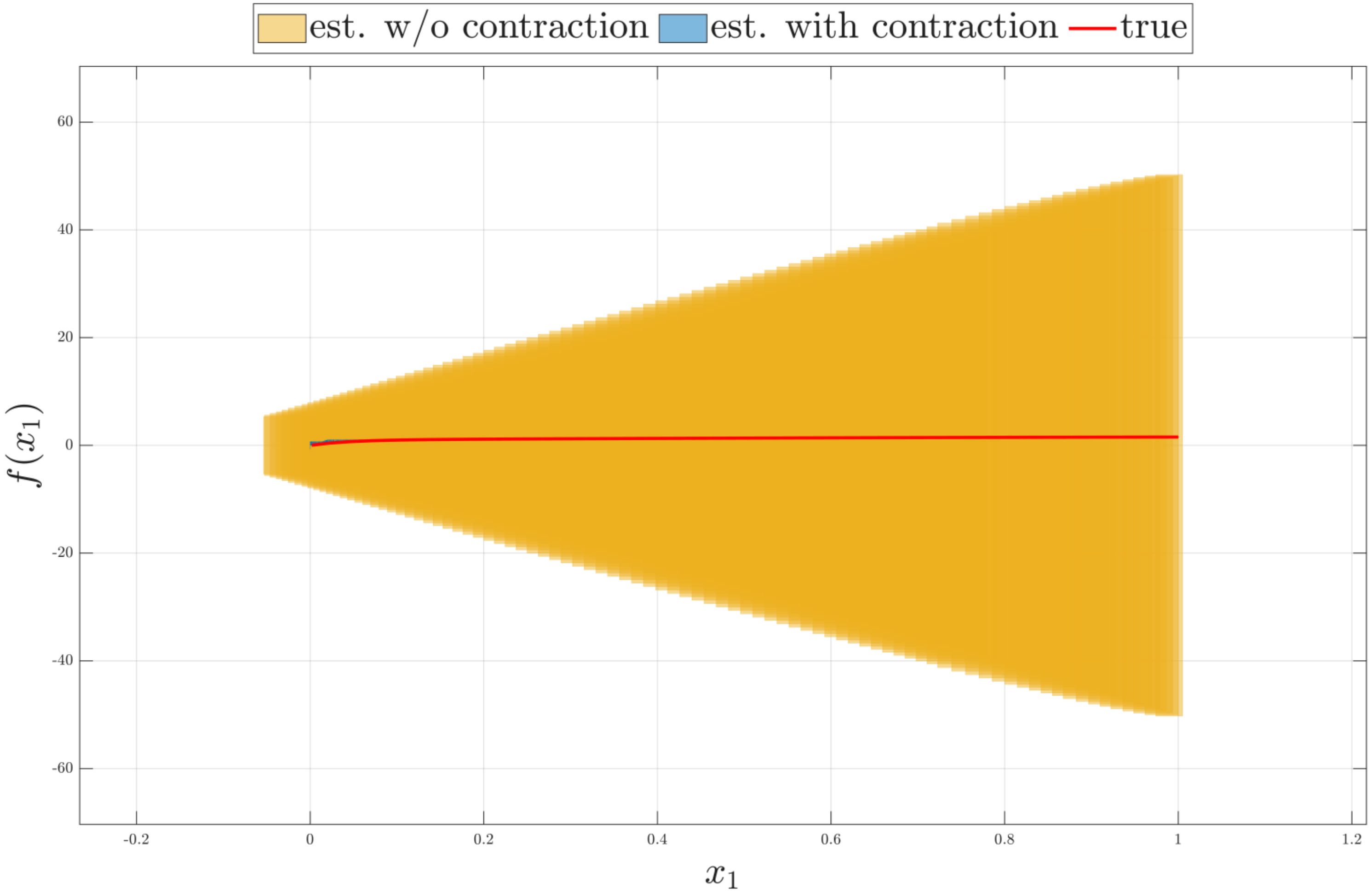

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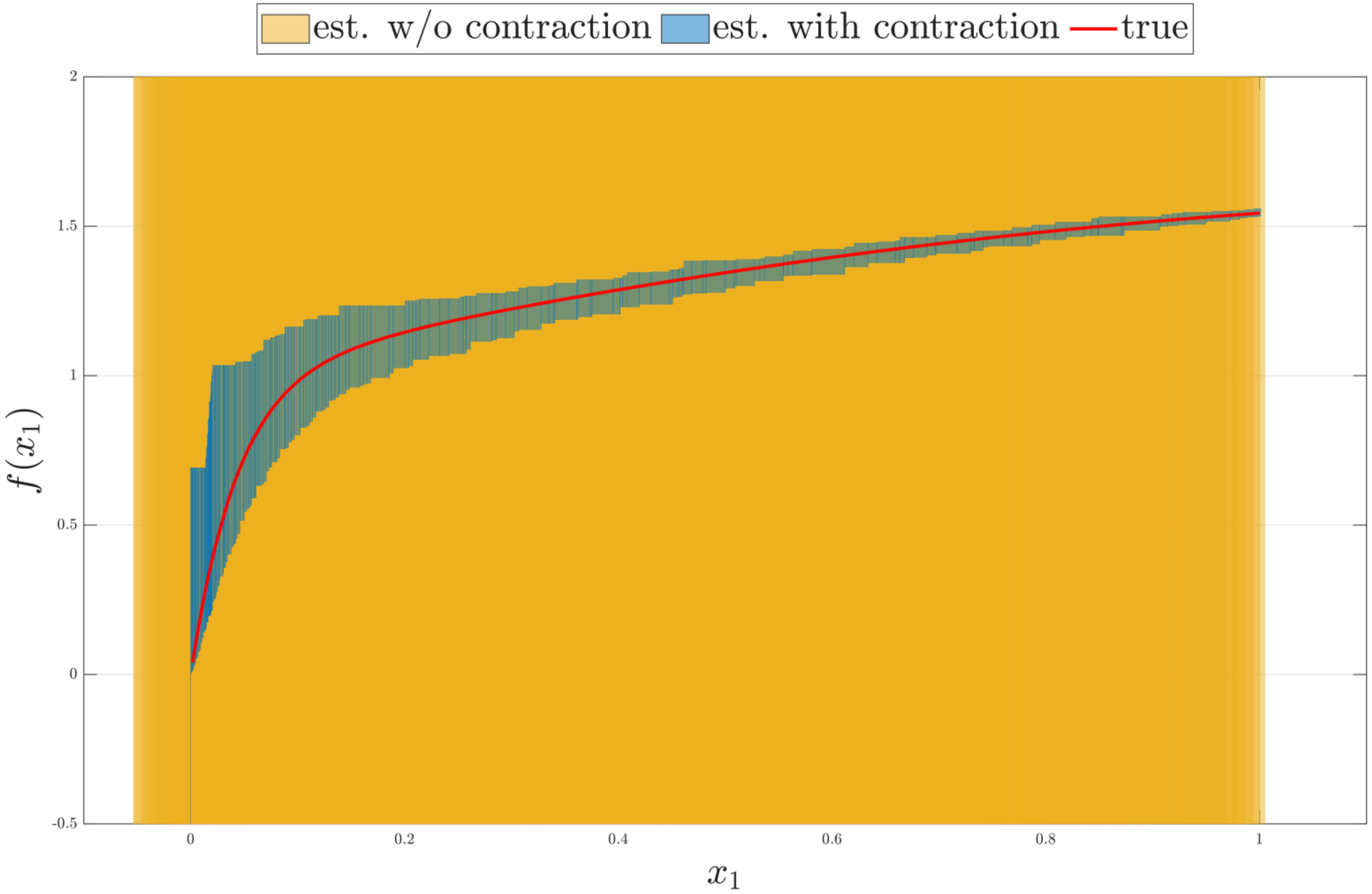
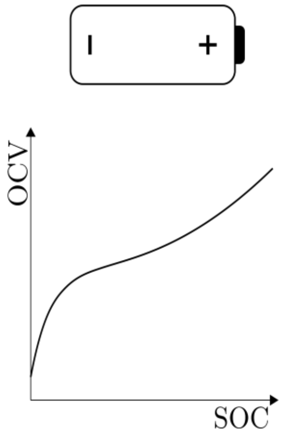
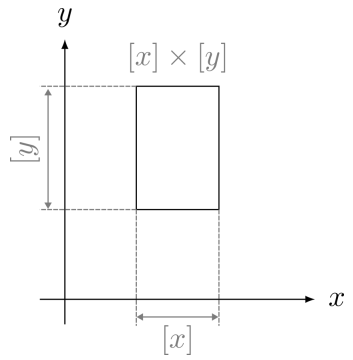


Conclusions

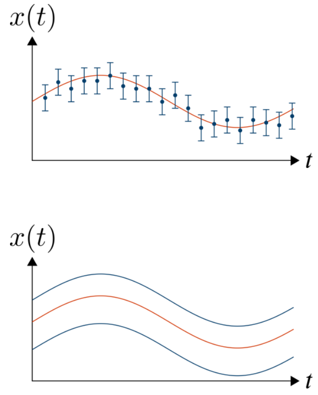
Objective



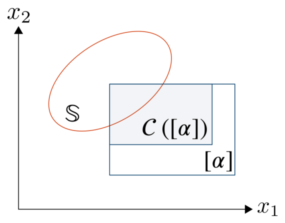
Uncertainty Quantification



Obtaining Interval Boxes



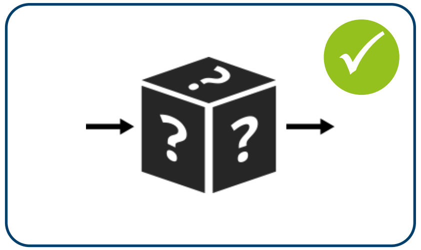
Improving the Approximation



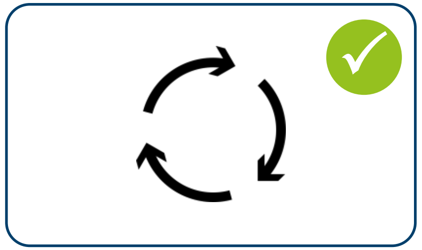
$\mathcal{C}_{Int}$ $\mathcal{C}_{FB}$

Conclusions

Guaranteed uncertainty bounds



Without knowledge about structure



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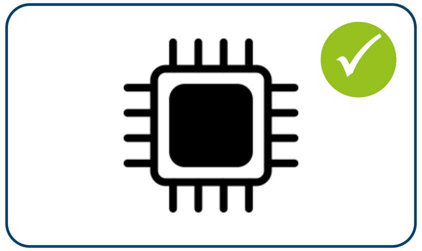
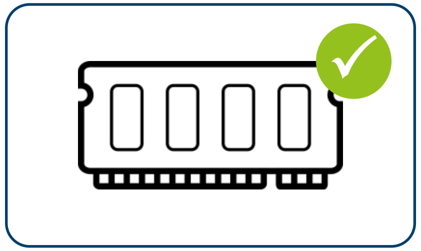


Reasonable computational effort



Reasonable memory requirements

Challenges and Future Work

- Precise state estimation (Wrapping effect)
- State observer design
- Identifying functions with $f(0) \neq 0$

- Extension to nonlinearities that depend on multiple state variables
- Extension to functions with $f(0) \neq 0$

Questions?

Marit Lahme
marit.lahme@uni-oldenburg.de

www.uol.de/en/computingscience/dcis

Related Publications

[1] L. Jaulin, M. Kieffer, O. Didrit, and É. Walter, *Applied Interval Analysis: With Examples in Parameter and State Estimation, Robust Control and Robotics*. Springer London Ltd, 2001. doi: 10.1007/978-1-4471-0249-6.

[2] M. Lahme and A. Rauh, “Approaches Towards A Systematic Tuning of Interval Observers via Norm-Bounded Error Dynamics and LMIs,” *Acta Cybernetica*, vol. 27, no. 3, pp. 359–382, 2026, doi: 10.14232/actacyb.318090.

[3] M. Lahme, A. Rauh, and G. Defresne, “Interval Observer Design for an Uncertain Time-Varying Quasi-Linear System Model of Lithium-Ion Batteries,” in *Proc. of the 2024 European Control Conference (ECC)*, Stockholm, Sweden, 2024, pp. 3696–3702. doi: 10.23919/ECC64448.2024.10591102.

[4] M. Lahme and A. Rauh, “Set-Valued Approach for the Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries,” *Acta Cybernetica,* vol. 26, no. 4, pp. 855–869, 2024, doi:10.14232/actacyb.301185.

[5] M. Lahme and A. Rauh, “Online Identification of the Open-Circuit Voltage Characteristic of Lithium-Ion Batteries with a Contractor-Based Procedure,” in *Proc. of the 27th International Conference on Methods and Models in Automation and Robotics (MMAR)*, Międzyzdroje, Poland: IEEE, 2023, pp. 39–44. doi: 10.1109/MMAR58394.2023.10242524.

[6] M. Lahme and A. Rauh, “Combination of Stochastic State Estimation with Online Identification of the Open-Circuit Voltage of Lithium-Ion Batteries,” in *Proc. of the 1st IFAC Workshop on Control of Complex Systems (COSY)*, Bologna, Italy: IFAC-PapersOnLine 55(40), 2022, pp. 97–102. doi: 10.1016/j.ifacol.2023.01.055.

[7] Z. Wang, C.-C. Lim, and Y. Shen, “Interval Observer Design For Uncertain Discrete-Time Linear Systems,” *Systems & Control Letters*, vol. 116, pp. 41–46, 2018, doi: 10.1016/j.sysconle.2018.04.003.