Computation of the Secure Zone

Antoine MORVAN, Luc JAULIN, Natacha CAOUREN

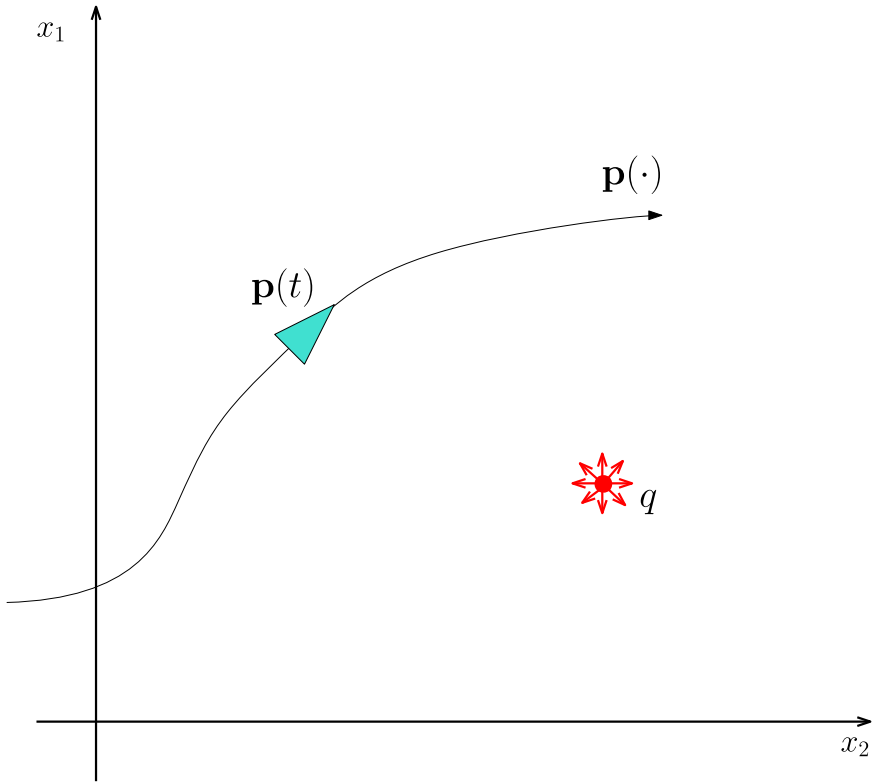
July 7, 2026

Introduction

Conclusion and perspectives

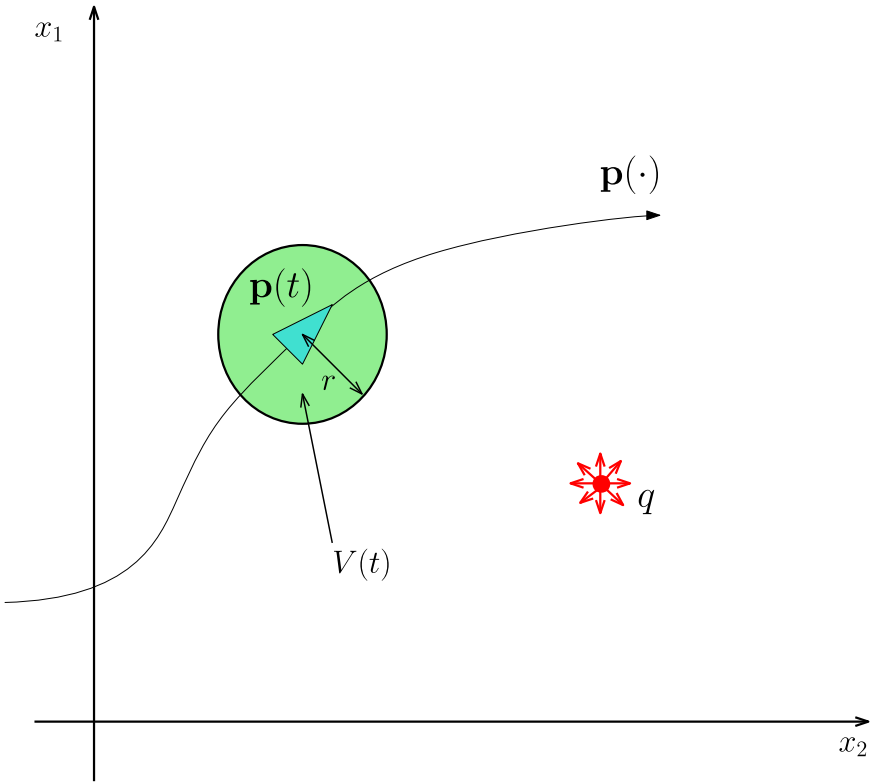
Introduction

Introduction



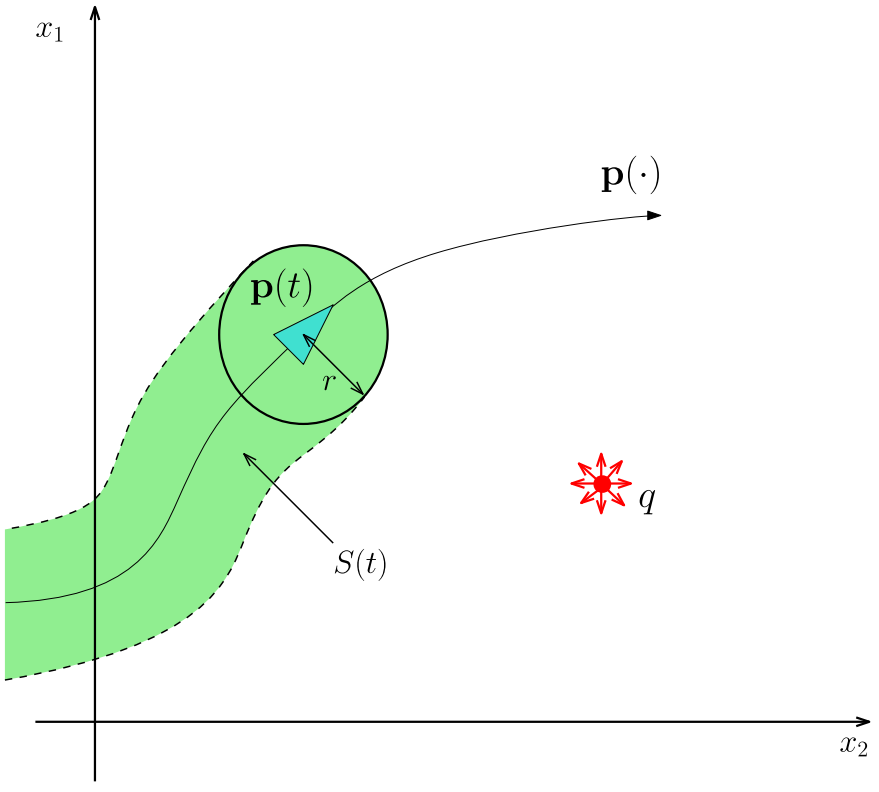
$$\|\dot{q}\| \leqslant v_{max}$$

Definition of the Visibility Zone



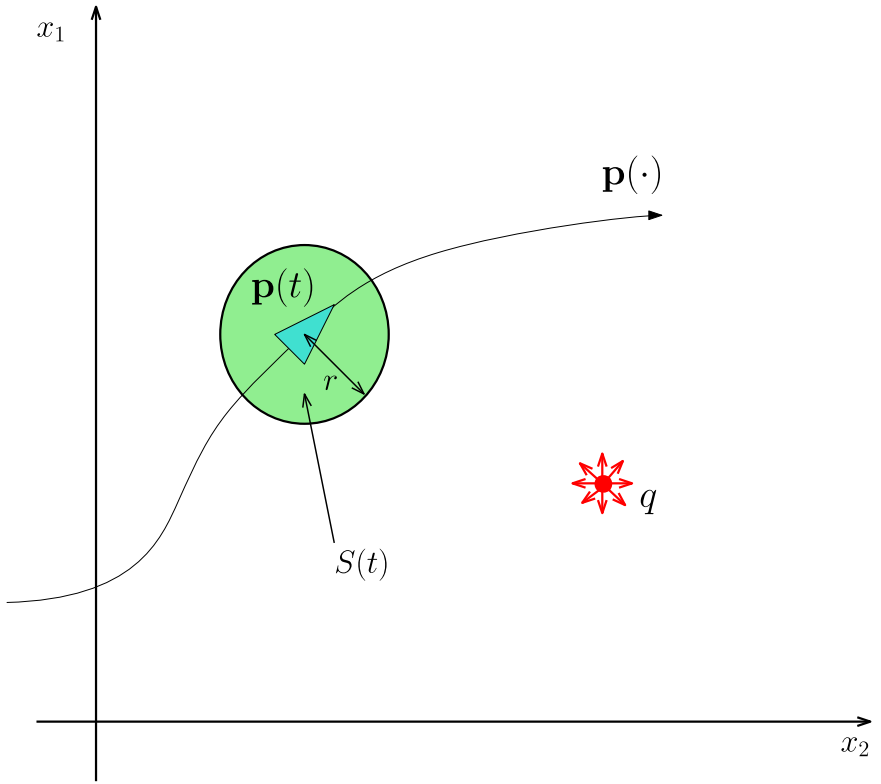
$$V(t) = \left\{ \mathbf{x} \in \mathbb{R}^2 | v(\mathbf{x}, \mathbf{p}(t)) \leqslant 0 \right\}, \; v(\mathbf{x}, \mathbf{p}) = \|\mathbf{x} - \mathbf{p}\| - r$$

Definition of the Secure Zone



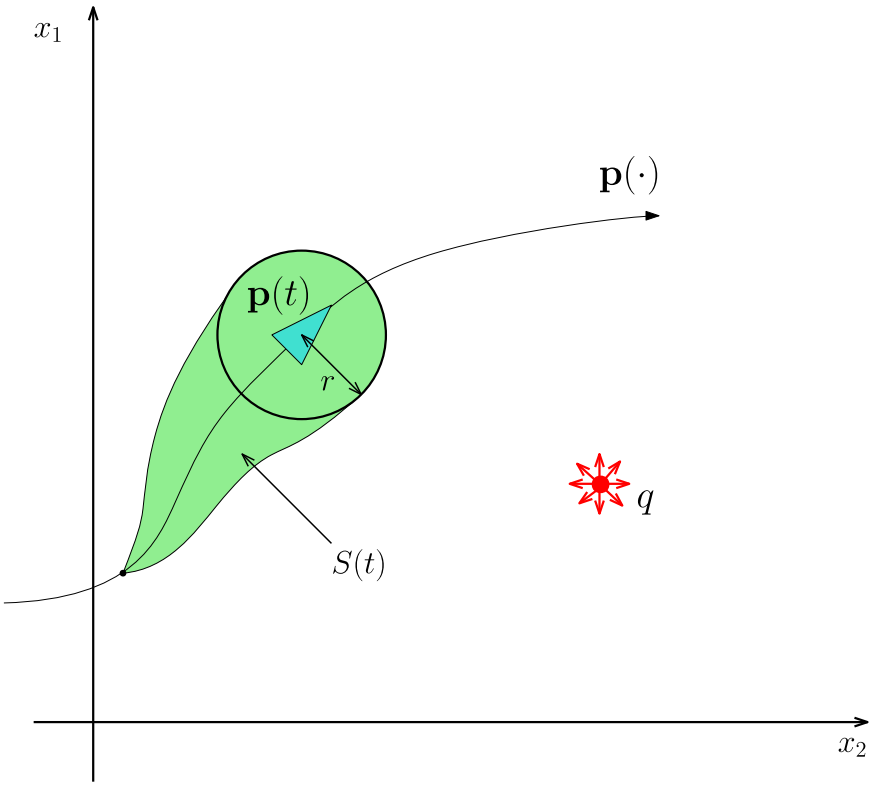
$$v_{max} = 0$$

Definition of the Secure Zone



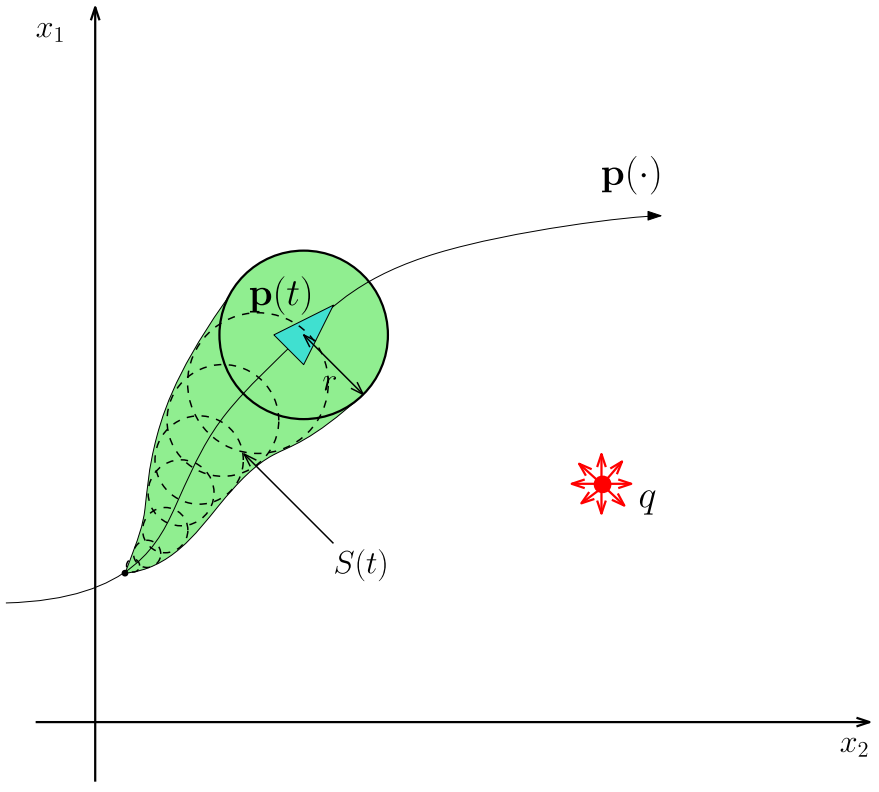
$$\forall t \in \mathbb{R},\ v_{max} > \|\dot{p}(t)\|$$

Definition of the Secure Zone



$$S(t) = \left\{ \mathbf{x} \in \mathbb{R}^2 | \exists \tau \in [-\infty, t], \phi_t(\mathbf{x}, \tau) \leqslant 0 \right\}$$

Definition of the secure zone

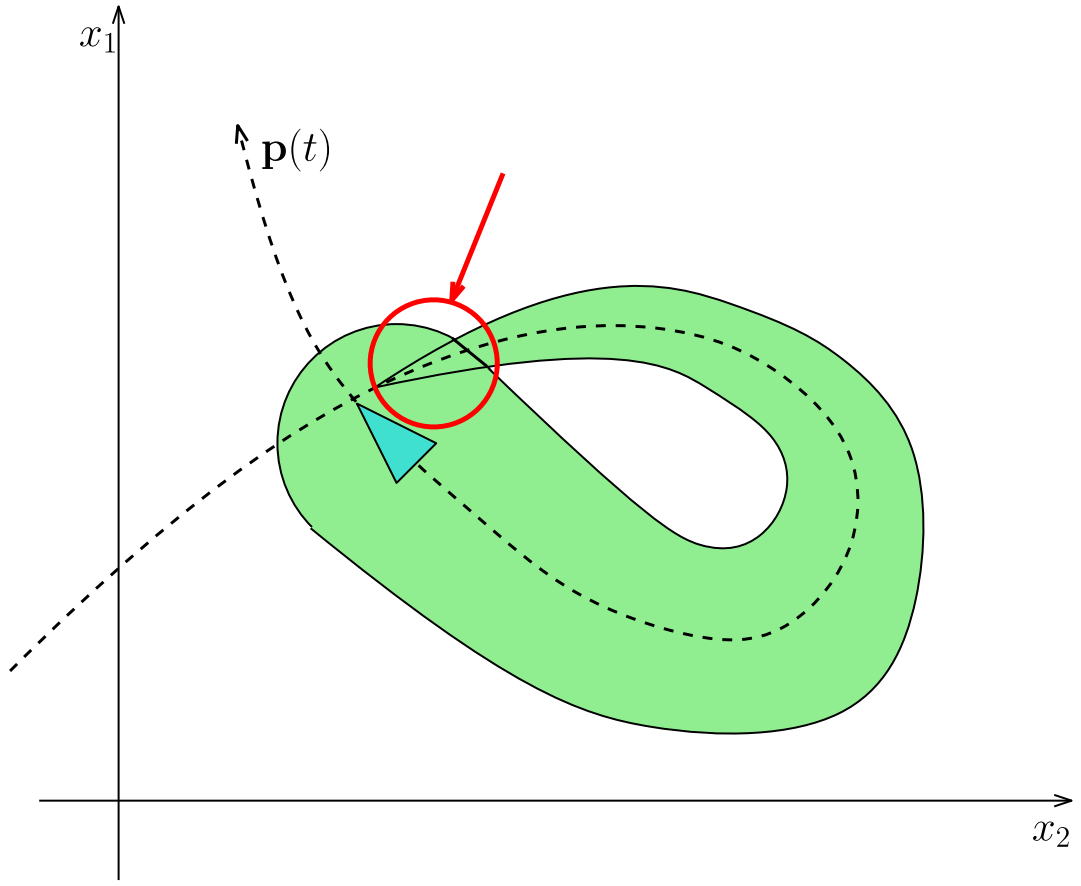


$$\phi_t(\mathbf{x}, \tau) = \|p(t-\tau) - \mathbf{x}\| - r + v_{max}\tau$$

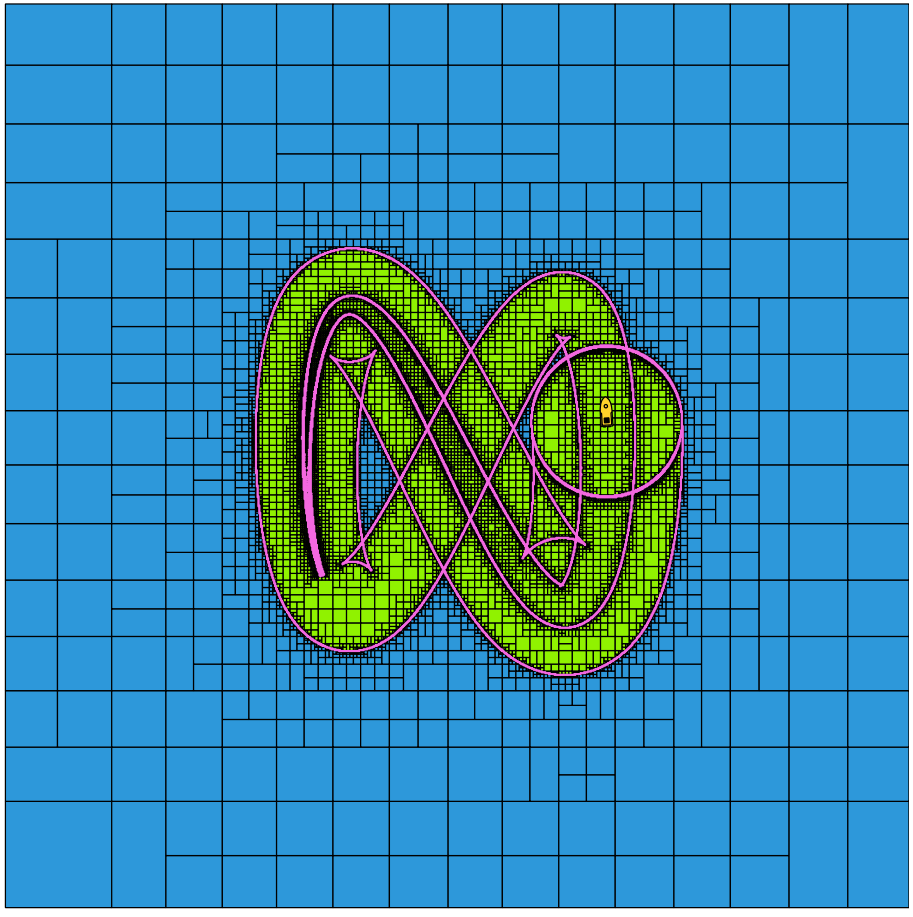
Boundary Approach

$$\partial S \subseteq S^{\partial}(t) = \left\{ \mathbf{x} \in \mathbb{R}^2 | \exists \tau \in [0, t] \, , \left\{ \begin{array}{l} \phi_t(\mathbf{x}, \tau) = 0 \\ \nabla \phi_t(\mathbf{x}, \tau)^T |_3 = 0 \end{array} \right. \right\}$$

Fake boundaries



Illustration

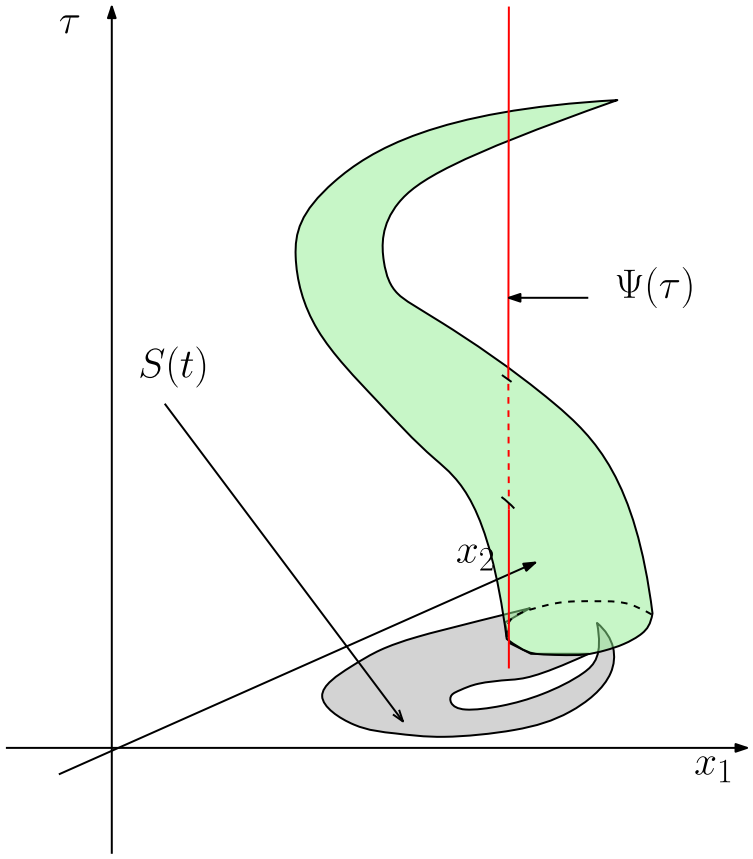


$$\mathbf{p}(t) = \begin{pmatrix} cos(t) \\ sin(2t) \end{pmatrix}$$

$$v_{max} = 0.05$$

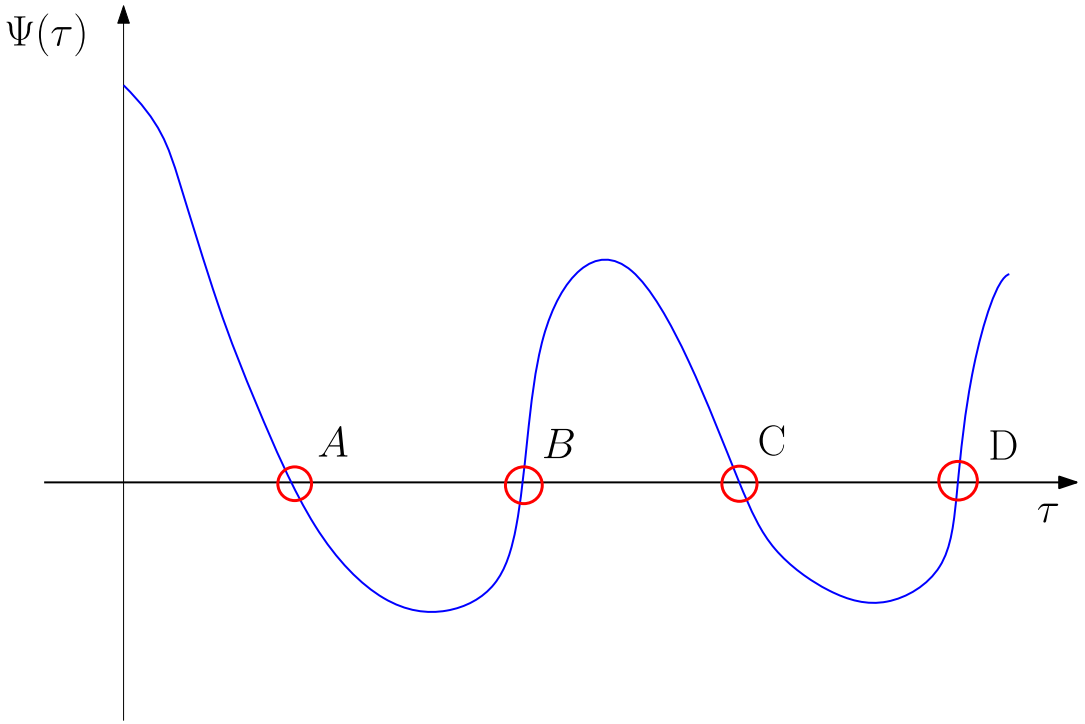
Projected volume

Fake Boundary identification concept

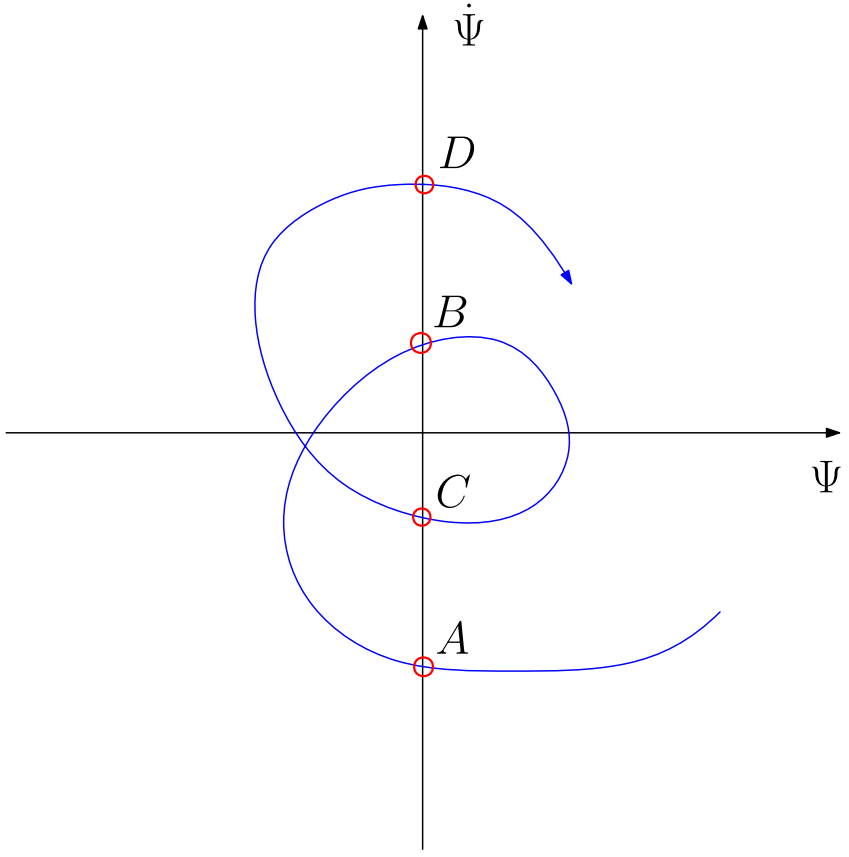


$$\Psi(\tau) = \phi_t(\mathbf{x}, \tau)$$

Roots counting

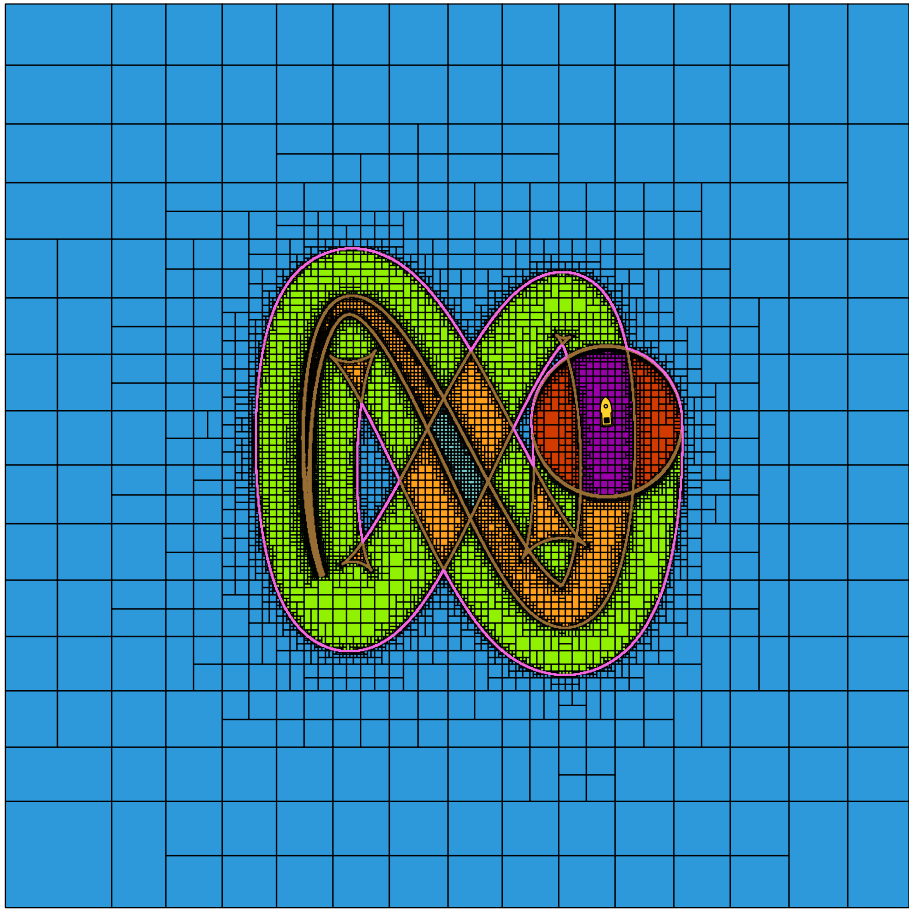


Roots counting



$$\Psi(\tau) = \phi_t(\mathbf{x}, \tau)$$

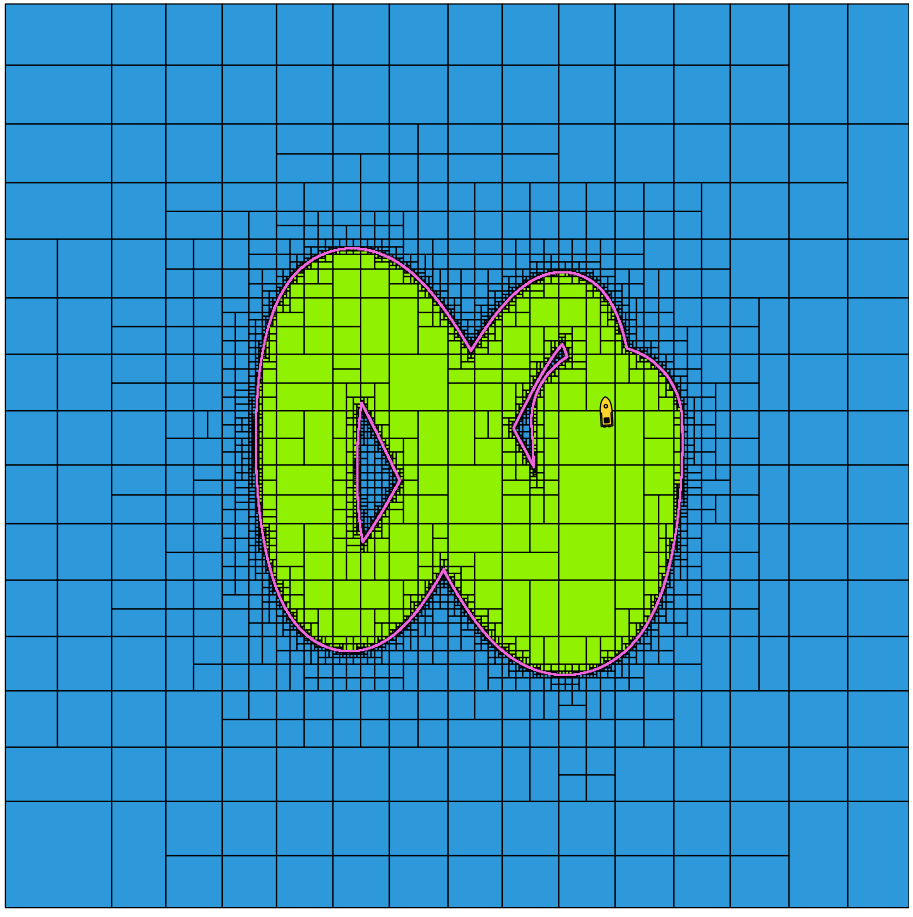
Fake boundaries identification



$$\mathbf{p}(t) = \begin{pmatrix} cos(t) \\ sin(2t) \end{pmatrix}$$

$$v_{max} = 0.05$$

Fake boundaries elimination



$$\mathbf{p}(t) = \begin{pmatrix} cos(t) \\ sin(2t) \end{pmatrix}$$

$$v_{max} = 0.05$$

Conclusion and perspectives

Conclusion and perspectives

In conclusion:

- Faster more accurate computation (~6s vs ~15min)
- Guaranteed computation

In perspectives:

- Capture zones (duality), cone of Mach, cone of light
- Optimization for real-time use (<1s)
- Adding pursuers
- Different shapes of V

Limitations

- Does not take account for proximity effects with other pursuers
- Fake erosion (too pessimistic)

Thank you for your attention