Endpoint and Midpoint Interval Representations - Theoretical and Computational Comparison ⋆

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In classical interval analysis [2] a real value x is represented by an interval $x \in [x_{lo}, x_{hi}]$ where x_{lo} and x_{hi} are two floating point numbers. There are further possible representations of the value of x using two or three floating point numbers:

- $x \in [x_{mid} - e, x_{mid} + e]$ using two floating point numbers x_{mid} and e
- $x \in [x_{mid} - e_{lo}, x_{mid} + e_{hi}]$ using three floating point values x_{mid}, e_{lo} and e_{hi}

Intervals of the form $[x_{mid} - e, x_{mid} + e]$ and operations with those were used in [3], but their main purpose was multi-precision arithmetic. In our work, we introduce intervals of the form $[x_{mid} - e_{lo}, x_{mid} + e_{hi}]$ and we show, that intervals in both alternative forms provide tighter enclosures compared to the classical interval form. We also compare all interval representations on computational examples.

To motivate our work, let us consider an example where $x = 1/15$. Using the classical interval format, the tightest possible interval that contains x using standard double precision floating point format [1] is

$$[6.66666666666666657415 \times 10^{-2}, 6.66666666666666796193 \times 10^{-2}]$$

The width of this interval is approximately 1.387779×10^{-17}. Using the alternative representation with $x_{mid} \doteq 6.66666666666666657415 \times 10^{-2}$, we can obtain

$$e = e_{hi} \doteq 9.252 \times 10^{-19}; e_{lo} = 0$$

With either one of the alternative representations, we can obtain an order of magnitude tighter enclosure of the actual value of x. In general, the obtained precision is not that high. Still, both alternative representations provide tighter interval enclosures of x on average compared to classical interval analysis.

References

1. IEEE Standards Board. IEEE standard for binary floating-point arithmetic. Technical report, The Institute of Electrical and Electronics Engineers, 1985. Technical Report IEEE Std 754-1985.
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3. A. Wittig and M. Berz. Rigorous high precision interval arithmetic in COSY INFINITY. *Proceedings of the Fields Institute*, 2009.

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