Sliding Mode Control for Uncertain Thermal SOFC Models with Physical Actuator Constraints

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Thomas Dötschel¹, Ekaterina Auer², Andreas Rauh¹, Harald Aschemann¹

¹ Chair of Mechatronics
University of Rostock, Germany

² Faculty of Engineering, INKO
University of Duisburg-Essen, Germany
Contents

- Working principles of Solid Oxide Fuel Cell (SOFC) systems
- Thermal modeling of SOFC stack modules
- Physical actuator constraints in SOFC systems
- Design of robust sliding mode control strategies
- Numerical validation and verification
- Conclusions and outlook
Working principles

- Fluid supply (fuel gas, air)
- Independent preheaters for fuel gas and air
- Stack module containing fuel cells in electric series connection
- Variable electric load as a disturbance
Modeling Approach

Energy balance of the SOFC stack module

- Control-oriented modeling of a SOFC stack module for the derivation of control and observer strategies
- Integral balancing of an instationary energy conversion process in the whole stack module as well as in individual finite volume elements
- Impact of the variation of the internal energy on the local temperature distribution in the stack module
Modeling Approach

- Relation between the variation of the internal energy and the stack temperature for constant material parameters $c_{FC}$ and $m_{FC}$

$$\frac{dE_{FC}(t)}{dt} = c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt}$$

- Modeling of the effects on the internal energy

$$\frac{dE_{FC}(t)}{dt} = C_{AG}(\vartheta_{FC}, t) \left( \vartheta_{AG,in}(t) - \vartheta_{FC}(t) \right)$$
$$+ C_{CG}(\vartheta_{FC}, t) \left( \vartheta_{CG,in}(t) - \vartheta_{FC}(t) \right)$$
$$+ \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)$$

- Reaction heat flow of the hydrogen oxidation reaction

$$\dot{Q}_R = \frac{\Delta RH(\vartheta_{FC}) \cdot \dot{m}_{H_2}(t)}{M_{H_2}}$$
Modeling Approach

- Relation between the variation of the internal energy and the stack temperature for constant material parameters $c_{FC}$ and $m_{FC}$

$$c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt} = C_{AG}(\vartheta_{FC}, t) (\vartheta_{AG,in}(t) - \vartheta_{FC}(t))$$

$$+ C_{CG}(\vartheta_{FC}, t) (\vartheta_{CG,in}(t) - \vartheta_{FC}(t))$$

$$+ \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)$$

- Heat transfer including linearized heat radiation to ambient media

$$\dot{Q}_A = \frac{1}{R_A} (\vartheta_A - \vartheta_{FC})$$

- Ohmic loss effects in the stack material

$$P_{El}(t) = R_{El}I^2(t)$$
Modeling Approach

- Relation between the variation of the internal energy and the stack temperature for constant material parameters $c_{FC}$ and $m_{FC}$

\[
c_{FC} \cdot m_{FC} \cdot \frac{d\vartheta_{FC}(t)}{dt} = C_{AG}(\vartheta_{FC}, t) (\vartheta_{AG,in}(t) - \vartheta_{FC}(t)) \\
+ C_{CG}(\vartheta_{FC}, t) (\vartheta_{CG,in}(t) - \vartheta_{FC}(t)) \\
+ \dot{Q}_R(t) + P_{El}(t) + \dot{Q}_A(t)
\]

- Anode gas: Heat capacity approximated by 2nd-order polynomials for $c_\chi$ with $\chi \in \{H_2, N_2, H_2O\}$

\[
C_{AG}(\vartheta_{FC}, t) = c_{H_2}(\vartheta_{FC}) \dot{m}_{H_2}(t) \\
+ c_{N_2}(\vartheta_{FC}) \dot{m}_{N_2}(t) + c_{H_2O}(\vartheta_{FC}) \dot{m}_{H_2O}(t)
\]

- Cathode gas: Heat capacity approximated with 2nd-order polynomials for $c_{CG}$

\[
C_{CG}(\vartheta_{FC}, t) = c_{CG}(\vartheta_{FC}) \cdot \dot{m}_{CG}(t)
\]
Semi-Discretization: The Finite Volume Method

- Semi-discretization into \( n_x = L \cdot M \cdot N \) finite volume elements to describe the internal temperature distributions

- Local energy balances lead to a set of \( n_x \) coupled ODEs represented by a state vector \( x^T = [\vartheta_{1,1,1}, \ldots, \vartheta_{L,M,N}] \in \mathbb{R}^{n_x} \)

- System boundary includes the thermal stack insulation
Semi-Discretization: The Finite Volume Method

- ODE for the local temperature distribution in a SOFC stack module

\[
c_{i,j,k} \cdot m_{i,j,k} \cdot \dot{\vartheta}_{i,j,k}(t) = C_{AG,i,j,k}(\vartheta_{i,j,k}, t) \left( \vartheta_{i,j-1,k}(t) - \vartheta_{i,j,k}(t) \right) \\
+ C_{CG,i,j,k}(\vartheta_{i,j,k}, t) \left( \vartheta_{i,j-1,k}(t) - \vartheta_{i,j,k}(t) \right) \\
+ \dot{Q}_{\eta,i,j,k}(t) + \dot{Q}_{R,i,j,k}(t) + P_{El,i,j,k}(t)
\]

- Modeling of local temperature-dependent and time-varying influence factors

Heat flow:
\[
\dot{Q}_{\eta,i,j,k}(t) = \sum_{\eta \in \mathcal{N}} \frac{1}{R_{th,\eta}} \left( \vartheta_{\eta}(t) - \vartheta_{i,j,k}(t) \right)
\]

Reaction heat flow:
\[
\dot{Q}_{R,i,j,k}(t) = \frac{\Delta R_H(i,j,k) \cdot \dot{m}_{H_2,i,j,k}(t)}{M_{H_2}}
\]

Ohmic losses:
\[
P_{El,i,j,k}(t) = R_{El,i,j,k} I^2_{i,j,k}(t)
\]
Semi-Discretization: The Finite Volume Method

**Case 1:** Semi-discretization into a single finite volume element leads to the global energy balance described before

- State variable $x$ and output variable $y$
  \[ x(t) = \vartheta_{FC}(t) \]
  \[ y(t) = h(x) = \vartheta_{FC}(t) \]
- Nonlinear ordinary differential equation
  \[ \dot{\vartheta}_{FC} = \Phi (\vartheta_{FC}(t), u(t)) \]

**Case 2:** Semi-discretization into three finite volume elements oriented in the direction of mass flow

- State vector $x$ and output variable $y$
  \[ x(t) = [\vartheta_{111}(t), \vartheta_{121}(t), \vartheta_{131}(t)]^T \]
  \[ y(t) = h(x) = \vartheta_{131}(t) \]
- Set of coupled nonlinear ordinary differential equations
  \[ \dot{x}(t) = \Phi (x(t), u(t)) \]
State Equations — Simplification for Control Synthesis

• Input-affine description of the nonlinear thermal subsystem

\[ \dot{x}(t) = f(x(t)) + g(x(t)) \cdot u(t), \quad x \in \mathbb{R}^{n_x} \]

\[ y(t) = h(x(t)), \quad y \in \mathbb{R}^{n_y} \]

\[ u(t) = m_{CG} \cdot \Delta \vartheta(t) \]

• Realization of the temperature difference \( \Delta \vartheta \) in the control input by an underlying controller for the preheating devices

\[ \Delta \vartheta(t) := \begin{cases} 
\vartheta_{CG}(t) - \vartheta_{FC}(t) & \text{for } x(t) = \vartheta_{FC}(t) \\
\vartheta_{CG}(t) - \vartheta_{111}(t) & \text{for } x(t) = [\vartheta_{111}(t), \vartheta_{121}(t), \vartheta_{131}(t)]^T 
\end{cases} \]

• Exact input-output linearization with relative degree \( \delta \) (Computation of the Lie-Derivatives of \( y \))

\[ \frac{d^i y}{dt^i} = L^i_f h(x) = L_f \left( L^{i-1}_f h(x) \right), \quad i = 0, \ldots, \delta - 1 \]

• Relative degree \( \delta \) determines the smallest order that explicitly depends on the input \( u \)
Modeling Approach — Transformation of the State-Space

- Nonlinear transformation of the state equations with the relative degree $\delta = n_x$ according to

$$z^T = [z_1 \ z_2 \ z_3] = [y \ \dot{y} \ \ddot{y}] = [h(x) \ L_fh(x) \ L_f^2h(x)]$$

- Nonlinear controller normal form (NCNF)

$$\dot{z} = \begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} L_fh(x) \\ L_f^2h(x) \\ L_f^3h(x) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ L_gL_f^2h(x) \end{bmatrix} u$$

- Feedback linearizing control law for sufficiently small variations of the mass flow used for the heat-up phase of the SOFC

$$u := \frac{-L_f^3h(x) - \alpha_0 h(x) - \alpha_1 L_f h(x) - \alpha_2 L_f^2 h(x) + \mu(t)}{L_gL_f h(x)}$$
Robust Sliding Mode Control

- Rejection of disturbances in the neighborhood of a desired operating point by means of sliding mode control regarding physical actuator constraints

- Online application of interval analysis to account for uncertainties in measurements and for state reconstruction errors

- Quality criterion for choosing adequate values for $\dot{m}_{CG}$ and $\Delta \vartheta$ to manipulate the enthalpy flow of the cathode gas

- Online subdivision strategy allows for converting the interval-based controller output $[v(t)]$ into a point-valued system input $u(t)$

- Guarantee of asymptotical stability inspite of system uncertainties
Robust Sliding Mode Control

- Mathematical model of the SOFC system in an input-affine description is extended by a bounded disturbance $d \in [d]$

$$
\begin{bmatrix}
\dot{z}_1 \\
\dot{z}_2 \\
\dot{z}_3
\end{bmatrix} =
\begin{bmatrix}
z_2 \\
z_3 \\
\tilde{a}(z, p, d)
\end{bmatrix} +
\begin{bmatrix}
0 \\
0 \\
\tilde{b}(z, p)
\end{bmatrix} v
$$

- Disturbance influences the system according to $\tilde{a} = L^3 f_h + d$

- Identification of interval parameters $p \in [p]$
Robust Sliding Mode Control

- Definition of an asymptotically stable sliding surface \( s(\tilde{z}) = 0 \) with the tracking error \( \tilde{z}_1^{(j)} = z_1^{(j)} - z_{1,d}^{(j)} \):
  \[
  s(\tilde{z}) = \tilde{z}_1^{(2)} + \alpha_1 \tilde{z}_1^{(1)} + \alpha_0 \tilde{z}_1^{(0)} = 0
  \]
  and the output \( z_1 \) and its time derivatives \( z_1^{(j)}, j = 1, \ldots, \delta - 1 = n_x - 1 \)

- Stabilization of the motion towards the sliding surface by a suitable Lyapunov function \( V \):
  \[
  V = \frac{1}{2} s^2 > 0 \quad \text{for} \quad s \neq 0, \quad \text{and its time derivative} \quad \dot{V} = s \dot{s}
  \]

- The condition \( \dot{V} = s \dot{s} \leq 0 \) for the time derivative of the Lyapunov function is fulfilled with
  \[
  s \dot{s} \leq -\eta s \text{ sign}\{s\} \quad \text{which is guaranteed for}
  \]
  \[
  \dot{s} + \eta \cdot \text{sign}\{s\} = -\beta \cdot \text{sign}\{s\}, \quad \eta, \beta > 0
  \]

- Control input \( v \) is obtained from
  \[
  \tilde{a}(z, p, d) + \tilde{b}(z, p)v - z_{1,d}^{(3)} + \alpha_1 \tilde{z}_1^{(2)} + \alpha_0 \tilde{z}_1^{(1)} = - (\beta + \eta) \cdot \text{sign}\{s\}
  \]
Robust Sliding Mode Control

- Control law for the disturbance rejection in the thermal subsystem

\[
\begin{align*}
[v] := & \left[ -\tilde{a}(z, [p], [d]) + z_1^{(3)} - \alpha_1 \tilde{z}_1^{(2)} - \alpha_0 \tilde{z}_1^{(1)} \right. \\
& \left. \frac{1}{\tilde{b}(z, [p])} \cdot (\eta + \beta) \cdot \text{sign}(s) \right] \mid \begin{array}{l}
 p \in [p] \\
 d \in [d]
\end{array}
\end{align*}
\]

- Appropriate choice of the switching amplitude \( \tilde{\eta} \) in the case of control design for interval parameters \( p \in [p] \) and interval disturbance \( d \in [d] \)

- Controller output for a guaranteed stabilization of the thermal SOFC system

\[
v := \begin{cases} 
\sup\{[v]\} & \text{for } s \geq 0 \\
\inf\{[v]\} & \text{for } s < 0
\end{cases}
\]
Robust Sliding Mode Control

- Instationary heating phase of the SOFC stack module using an exact linearizing control law to reach a desired operating point
Robust Sliding Mode Control

- Switching to the interval-based sliding mode control law in the point of time $t = 2.5 \cdot 10^4$ s

- **Objective:** Rejection of disturbances and stabilization of desired operating points accounting for parameter uncertainties and bounded state enclosures
Robust Sliding Mode Control

- Output of exact linearizing feedback control law $u(t)$ with switching to the output of the interval-based sliding mode controller $v(t)$ at the point of time $t = 2.5 \cdot 10^4 s$

**Problem:** Adequate setting of the SOFC system input $u = \dot{m}_{CG} \Delta \vartheta$ with an available sliding mode controller output $v(t)$
Robust Sliding Mode Control

- Subdivision strategy to determine appropriate control inputs $\dot{m}_{CG}$ and $\Delta \vartheta$ corresponding to $[v(t)]$

- The product of the mass flow $\dot{m}_{CG}$ and of the temperature difference $\Delta \vartheta$ determines the system input

$$u := (\dot{m}_{CG} \cdot \Delta \vartheta)$$

- Operating ranges of the actuators are defined by bounded intervals
Implementation of the Interval-Based Control Law in Simulations

- A **splitting procedure** is employed in each time step $k$ starting with the initial interval box described by $[\dot{m}_{CG}^{<0>}]$ and $[\Delta \vartheta^{<0>}]$ which is identical to the physical actuator constraints.

- Multi-sectioning of the input interval vector $[[\dot{m}_{CG}^{<1>}] ; [\Delta \vartheta^{<1>}] ]^T$ into the four interval boxes the mass flow and temperature difference in the time step $k$

$$
\begin{align*}
[[\dot{m}_{CG}^{<1>}] ; [\Delta \vartheta^{<1>}] ]^T & := 
\begin{bmatrix}
\inf ([\dot{m}_{CG}^{<1>}]) ; \text{mid} ([\dot{m}_{CG}^{<1>}]) \\
\inf ([\Delta \vartheta^{<1>}]) ; \text{mid} ([\Delta \vartheta^{<1>}])
\end{bmatrix} \\
[[\dot{m}_{CG}^{<L+1>}] ; [\Delta \vartheta^{<L+1>}] ]^T & := 
\begin{bmatrix}
\text{mid} ([\dot{m}_{CG}^{<L+1>}) ; \sup ([\dot{m}_{CG}^{<L+1>})) \\
\inf ([\Delta \vartheta^{<L+1>}) ; \text{mid} ([\Delta \vartheta^{<L+1>})]
\end{bmatrix} \\
[[\dot{m}_{CG}^{<L+2>}] ; [\Delta \vartheta^{<L+2>}] ]^T & := 
\begin{bmatrix}
\inf ([\dot{m}_{CG}^{<L+2>}) ; \text{mid} ([\dot{m}_{CG}^{<L+2>})) \\
\text{mid} ([\Delta \vartheta^{<L+2>}) ; \sup ([\Delta \vartheta^{<L+2>})]
\end{bmatrix} \\
[[\dot{m}_{CG}^{<L+3>}] ; [\Delta \vartheta^{<L+3>}] ]^T & := 
\begin{bmatrix}
\text{mid} ([\dot{m}_{CG}^{<L+3>}) ; \sup ([\dot{m}_{CG}^{<L+3>})) \\
\text{mid} ([\Delta \vartheta^{<L+3>}) ; \sup ([\Delta \vartheta^{<L+3>})]
\end{bmatrix}
\end{align*}
$$
Implementation of the Interval-Based Control Law in Simulations

- A validity test of \( [u^{l}] = [\dot{m}_{CG}^{l}] [\Delta \vartheta^{l}] \) is performed according to the controller output \([v]\) for classifying guaranteed consistent, undecided and guaranteed inconsistent input intervals.

- Consistency of \([u^{l}]\) in \([v]\) is proven if

\[
\sup\{[v]\} < \inf\{[u^{l}]\} \quad \text{for } s \geq 0
\]
\[
\inf\{[v]\} > \sup\{[u^{l}]\} \quad \text{for } s < 0
\]

- Illustration of the consistency test for \(s > 0\) bounded by actuator constraints (dashed lines)
Implementation of the Interval-Based Control Law in Simulations

- Compositions of $u(t)$ are assessed for $l$ subdivided intervals in each time step $k$

- Optimal interval box of $[\dot{m}_{CG}]$ and $[\Delta \vartheta]$ is detected with the quality criterion

$$[J_k^{<l>}] = \kappa_1 ([\Delta \vartheta_k^{<l>}] - [\Delta \vartheta_{nom}])^2 + \kappa_2 ([\Delta \vartheta_k^{<l>}])^2 + \kappa_3 ([\dot{m}_{CG,k}^{<l>}] - [\dot{m}_{nom}])^2$$

- The minimization of $J_{opt} = \min (\inf ([J_k^{<l>}]))$ yields

$$[\dot{m}_{CG}^{<opt>}] \text{ and } [\Delta \vartheta^{<opt>}]$$

- The guaranteed stabilizing control output for the SOFC system with $v \geq \sup ([v])$ is determined by

$$u(t) = \text{mid} ([\dot{m}_{CG}^{<opt>}] \cdot [\Delta \vartheta^{<opt>}])$$
Implementation of the Interval-Based Control Law in Simulations

- Depiction of the optimal system input with reference to the nominal values for $[\dot{m}_{nom}]$ and $[\Delta \vartheta_{nom}]$

- Cooling process with a value $s > 0$ in the sliding mode control design
Conclusions and Outlook

Conclusion

• Nonlinear modeling of the thermal subsystem of SOFCs including uncertainties in the parameterization and the system states

• Design of an interval-based sliding mode controller that is capable to cope with bounded uncertainties in a desired operating point

• Optimal adjustment of the enthalpy flow as a control input of the system employing a subdivision strategy regarding actuator constraints

Conclusions and Outlook

Outlook

• Proof of the robustness in case of a switching output $y$ where the remaining system dynamics have to be enclosed in state intervals

• Implementation of the presented approaches in the SOFC system available at the Chair of Mechatronics at the University of Rostock

• Translation of the software routines in INTLAB into the real-time capable C-XSC implementation