On Set-Membership Estimation of Hybrid Systems via SAT Mod ODE

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Martin FRANZLE (Carl von Ossietzky Univ., Oldenburg, Germany).

SWIM 2012, June 4-6, Oldenburg, Germany

main reference

A. Eggers, N. Ramdani, N. S. Nedialkov, and M.Fränzle.
***Set-membership estimation of hybrid systems via SAT Mod ODE**.*
*In 16th IFAC Symp. on System Identification, **SYSID 2012**, July 11-13, Bruxelles, 2012.*

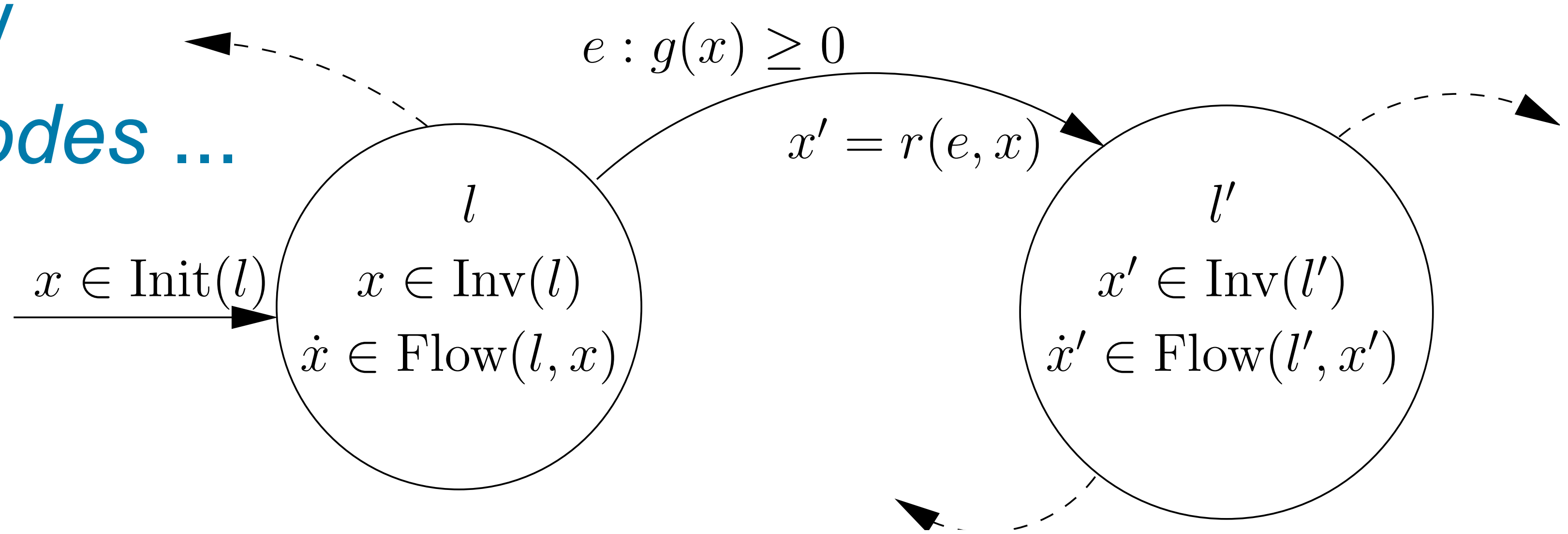
Hybrid Cyber-Physical Systems

- Interaction discrete
  + continuous dynamics
- Safety-critical
  embedded systems
- Networked
  autonomous systems

Estimation of Hybrid State

Modelling → hybrid automaton

- Non-linear continuous dynamics
- Bounded uncertainty
- *may include fault modes ...*



State Estimation → reconstruct system variables

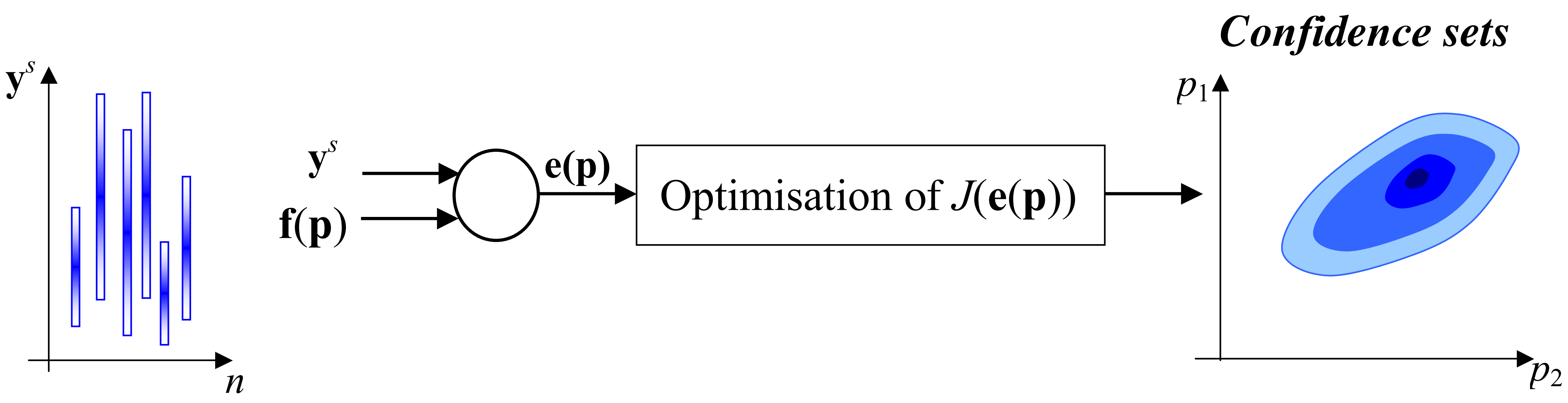
- continuous variables
- switching sequence

Important issue

- Control & Diagnosis ...

Bounded-error estimation

- Classical estimation is probabilistic

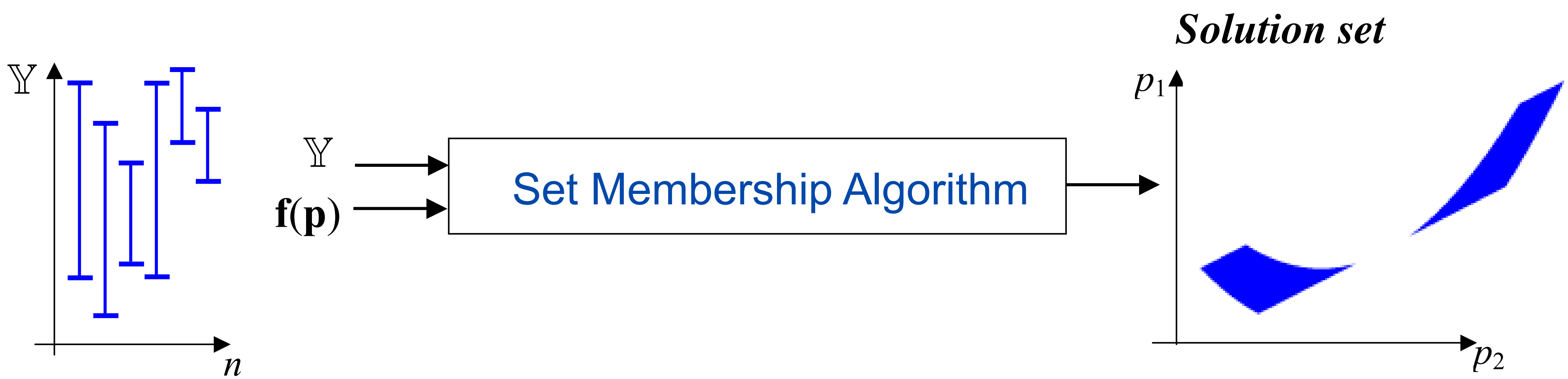


Yield valid results only if

- Perturbations, errors and model uncertainties with statistical properties known *a priori*
- Model structure is correct, no modeling errors

Bounded-error estimation

- Unknown but bounded-error framework



Hypothesis

Uncertainties and errors are bounded with known prior bounds

A set of feasible solutions

$$\mathbb{S} = \{\mathbf{p} \in \mathbb{P} | \mathbf{f}(\mathbf{p}) \in \mathbb{Y}\} = \mathbf{f}^{-1}(\mathbb{Y}) \cap \mathbb{P}$$

Bounded-error estimation

- **Continuous systems**
  - (Milanese & Novara, 2011), (Kieffer & Walter, 2011), (Jaulin, 2011), (Le Bars, et al., 2012)
  - (Moisan, et al. 2009), (Meslem & Ramdani, 2011)
- **Hybrid systems**
  - Piecewise affine systems (Bemporad, et al. 2005)
  - ODE + CSP (Goldsztejn, et al., 2010)
  - Nonlinear systems (Benazera & Travé-Massuyès, 2009)

Bounded-error estimation

- Interval Solving : Branch-&-Bound, Branch-&-Prune algorithms

$$\mathbb{S} = \{\mathbf{z} \in \mathcal{Z}, \mid f(\mathbf{z}) \in \mathcal{Y}\} \quad \rightarrow \underline{\mathbb{S}} \subseteq \mathbb{S} \subseteq \overline{\mathbb{S}}$$

$f([\mathbf{z}]) \subseteq \mathcal{Y}$ $\Rightarrow [\mathbf{z}] \subseteq \underline{\mathbb{S}}$: inner approximation

$f([\mathbf{z}])) \cap \mathcal{Y} = \emptyset$ $\Rightarrow [\mathbf{z}] \not\subseteq \overline{\mathbb{S}}$: outer approximation $\Rightarrow [\mathbf{z}] \subseteq \mathcal{Z} \backslash \overline{\mathbb{S}}$

otherwise partition ...

Bounded-error estimation

- Interval Solving : Branch-&-Bound, Branch-&-Prune algorithms

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otherwise partition ...

outer approximation ⟷ **unsatisfiability**

- **How to use iSAT-ODE to solve SME for HDS ?**

A SAT mod ODE approach

- ***Predicative encoding***

$$
\begin{aligned}
\Phi = {} & \mathrm{decl}[0] \\
& \wedge \ldots \\
& \wedge \mathrm{decl}[k] \\
& \wedge \mathrm{init}[0] \\
& \wedge \mathrm{trans}[0,1] \\
& \wedge \ldots \\
& \wedge \mathrm{trans}[(k-1),k] \\
& \wedge \mathrm{target}[k]
\end{aligned}
$$

instantiation of variables' bounds.
a priori knowledge ...

discrete or continuous
transitions

A SAT mod ODE approach

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\end{aligned}
$$

instantiation of variables' bounds.

a priori knowledge ...

feasible domains for measurements

discrete or continuous transitions

A SAT mod ODE approach

Outer approximation in Set Membership Estimation

↕

iSAT-ODE

1. *if needed*, **infere number of transition steps**
2. **Deduction & no splitting** : prune off inconsistant ranges
3. **Deduction & splitting** : search candidate boxes
4. **Evaluate range bounds** for solution boxes.

Experimental Evaluation

- Example 1.
  - Parameter estimation in a chemical reaction

$$2HI \longrightarrow H_2 + I_2$$

$$\dot{c}_{I_2} = k \cdot c_{HI}^2,\ \dot{c}_{H_2} = k \cdot c_{HI}^2,\ \dot{c}_{HI} = -2 \cdot k \cdot c_{HI}^2$$

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Experimental Evaluation

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$$c_{H_2}(0), c_{I_2}(0) \in [0, 0.02],\ c_{HI}(0) \in [0.95, 1]$$ ← decl[0]

$$t \in [5.6, 5.8],\ c_{HI}(t) \in [0.08, 0.12]$$ ← target[1]

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step	k	c_{HI}	c_{H_2}	c_{I_2}
0	$(0.622, 1.038)$	$[0.949, 1.000]$	$[0.000, 0.021]$	$[0.000, 0.021]$
1	$(0.612, 1.041)$	$[0.079, 0.120]$	$[0.411, 0.489]$	$[0.409, 0.487]$

$$k \in [0.622, 1.038] \cap [0.612, 1.041]$$

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Experimental Evaluation

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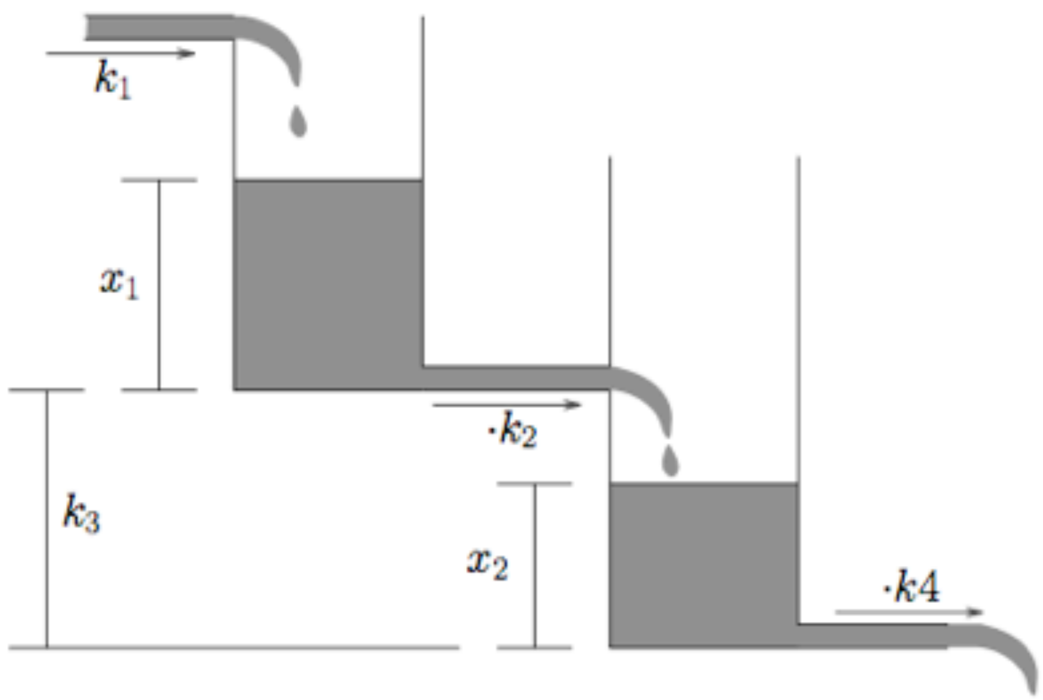
$k \in [0.622, 1.038] \cap [0.612, 1.041]$

CPU time 8h to 56h according tuning. (Intel Core i7 2.6 GHz)

Experimental Evaluation

- Example 2.
  - Estimation of Hybrid System
    - (Stursberg, et al. 1997)

$$\begin{aligned}\Phi = \ & \mathrm{decl}[0] \wedge \cdots \wedge \mathrm{decl}[k] \\ & \wedge \mathrm{init}[0] \\ & \wedge \mathrm{trans}[0,1] \wedge \cdots \wedge \mathrm{trans}[(k-1),k] \\ & \wedge \mathrm{target}[k]\end{aligned}$$



For $x_2 > k_3$:

$$\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2\end{pmatrix} = \begin{pmatrix} k_1 - k_2\sqrt{x_1 - x_2 + k_3} \\ k_2\sqrt{x_1 - x_2 + k_3} - k_4\sqrt{x_2}\end{pmatrix}$$

For $x_2 \leq k_3$:

$$\begin{pmatrix}\dot{x}_1 \\ \dot{x}_2\end{pmatrix} = \begin{pmatrix} k_1 - k_2\sqrt{x_1} \\ k_2\sqrt{x_1} - k_4\sqrt{x_2}\end{pmatrix}$$

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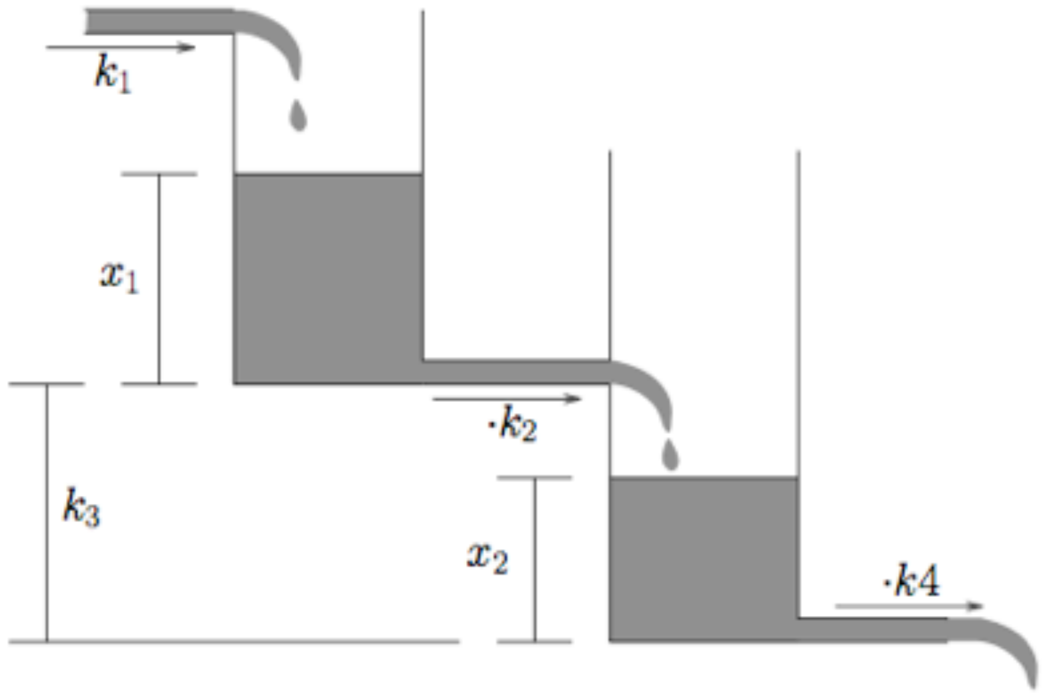

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- Measuring at discrete time instants.
- Measuring height threshold (Koutsoukos, 2003).

Experimental Evaluation

- Example 2.
  - Estimation of Hybrid System
    - (Stursberg, et al. 1997)

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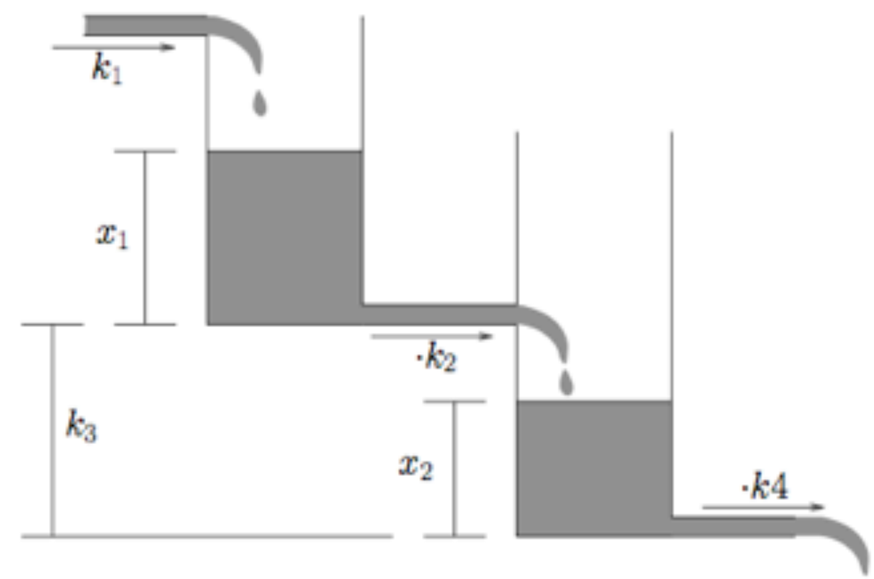
- Measuring at discrete time instants,
  .... added as new transition steps.

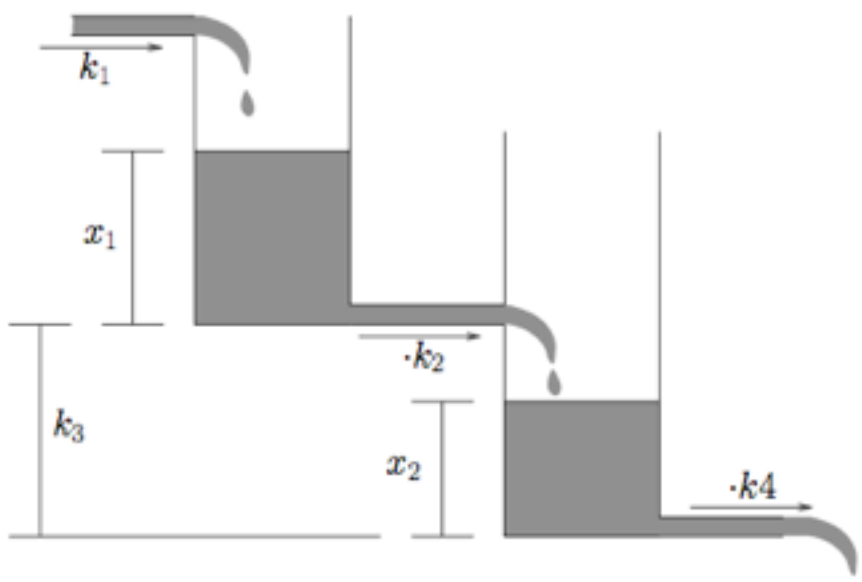


- *Stage 1*
- Infere minimal number of unwinding steps
  target = one measure. missed
  or final time not reached.
- UNSAT → depth 7

Experimental Evaluation

- Example 2.
  - Estimation of Hybrid System
    - (Stursberg, et al. 1997)

$$\Phi = \mathrm{decl}[0] \wedge \cdots \wedge \mathrm{decl}[k] \\ \wedge \mathrm{init}[0] \\ \wedge \mathrm{trans}[0,1] \wedge \cdots \wedge \mathrm{trans}[(k-1),k] \\ \wedge \mathrm{target}[k]$$



- Measuring at discrete time instants,
  .... added as new transition steps.

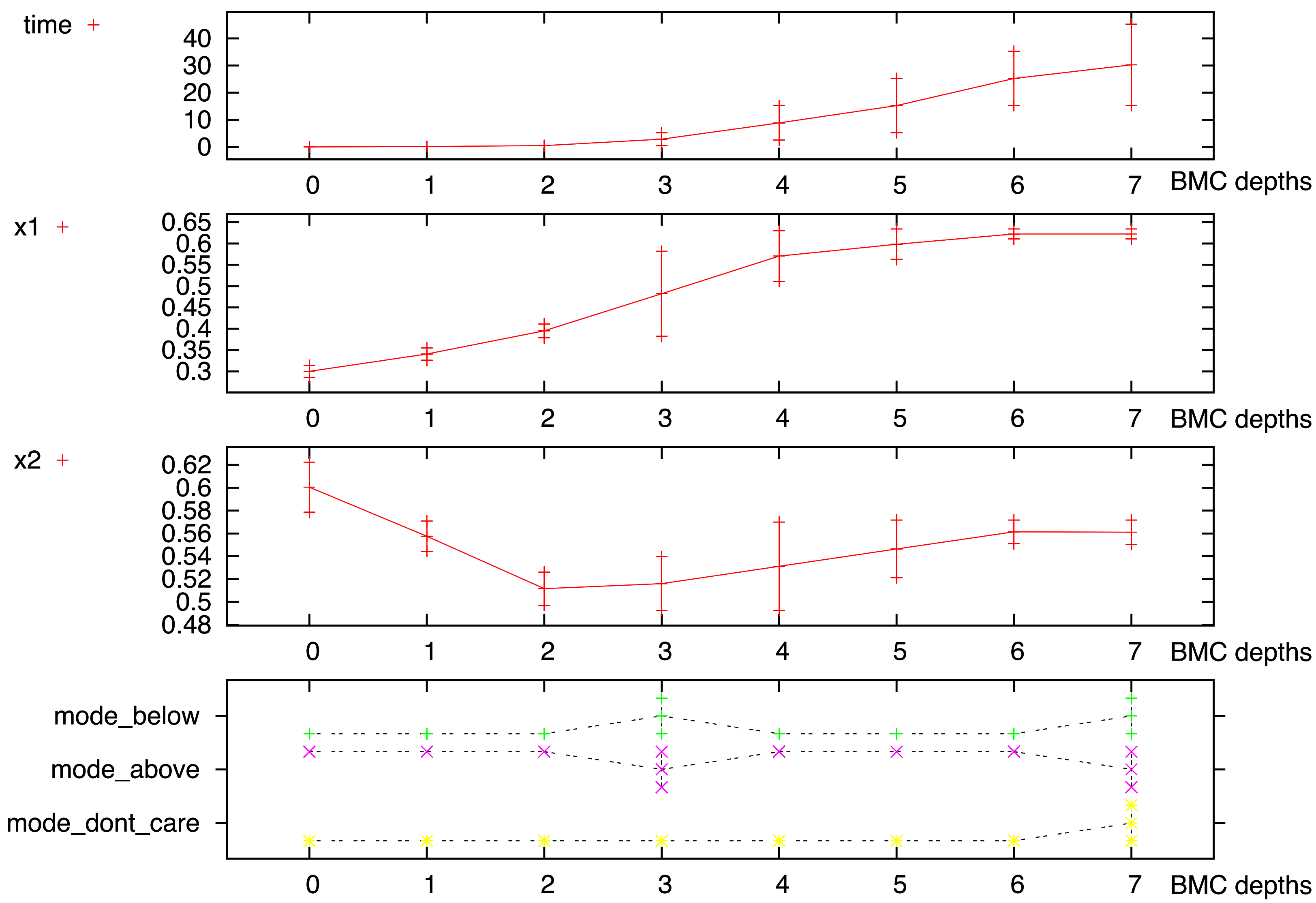
- *Stage 2*
- target = all measurements used.
- → reduce range of initial conditions,
- → bound range of hybrid state variables.

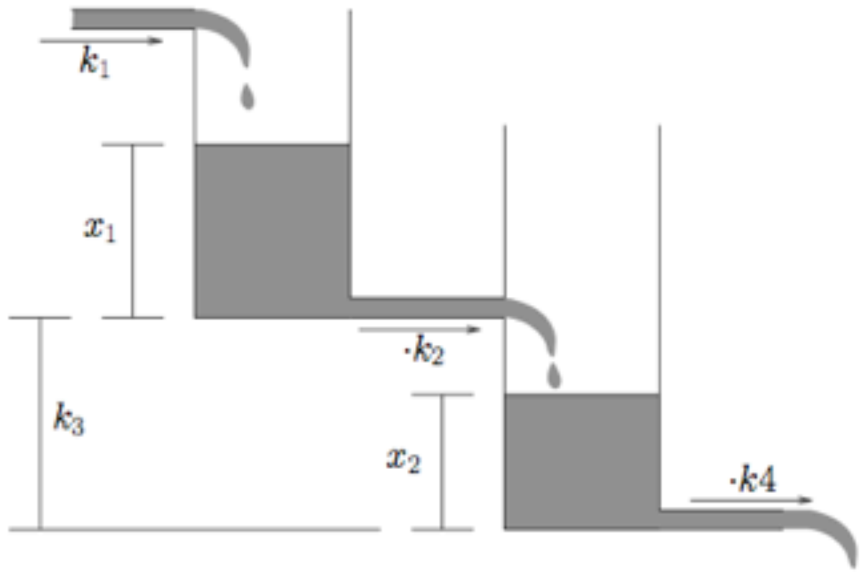
Experimental Evaluation

Example 2.

- Estimation of Hybrid System
  - Measuring at discrete time instants,

$$\begin{aligned}\Phi = \ & \mathrm{decl}[0] \land \cdots \land \mathrm{decl}[k] \\ & \land \mathrm{init}[0] \\ & \land \mathrm{trans}[0,1] \land \cdots \land \mathrm{trans}[(k-1),k] \\ & \land \mathrm{target}[k]\end{aligned}$$





Experimental Evaluation

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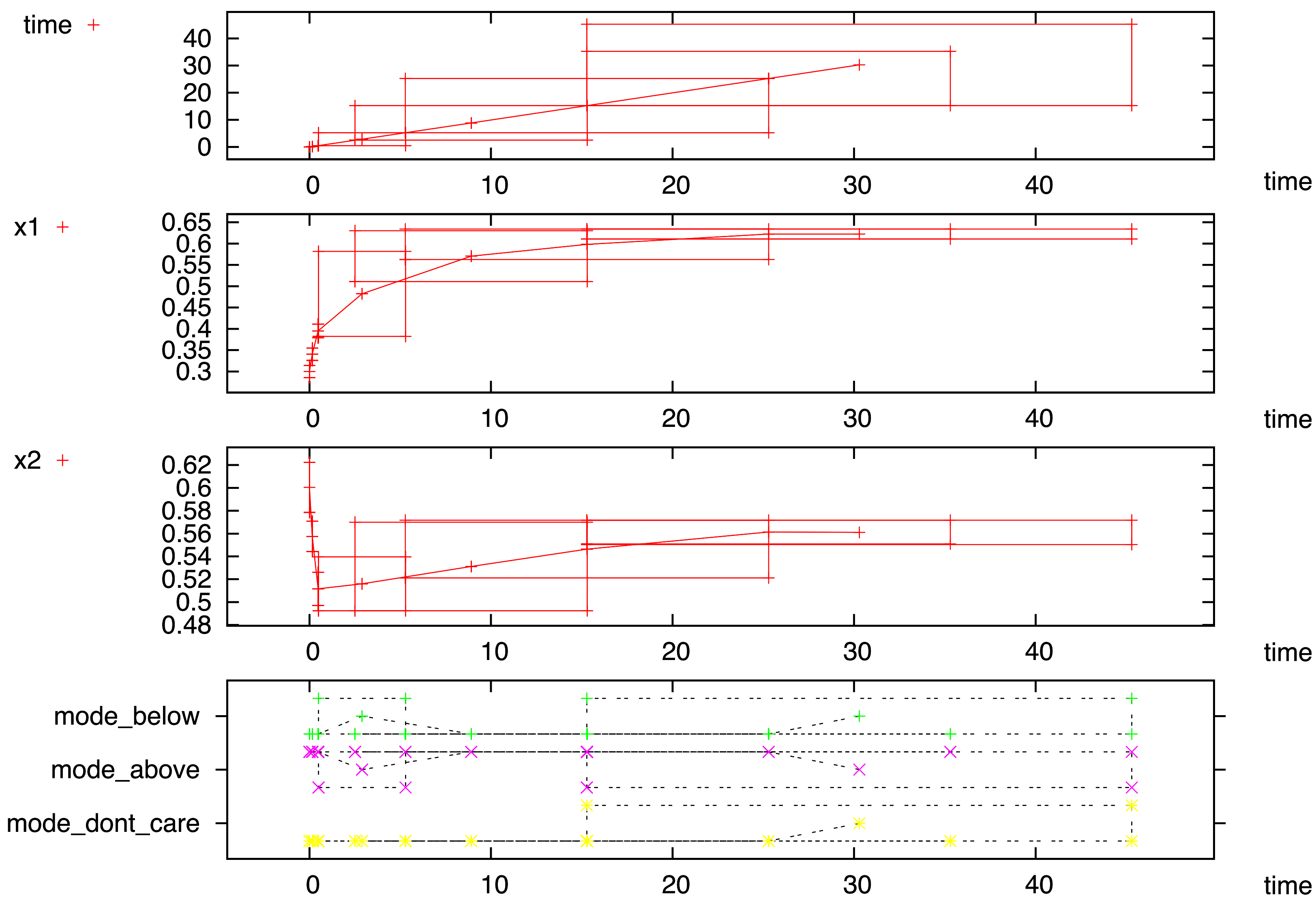

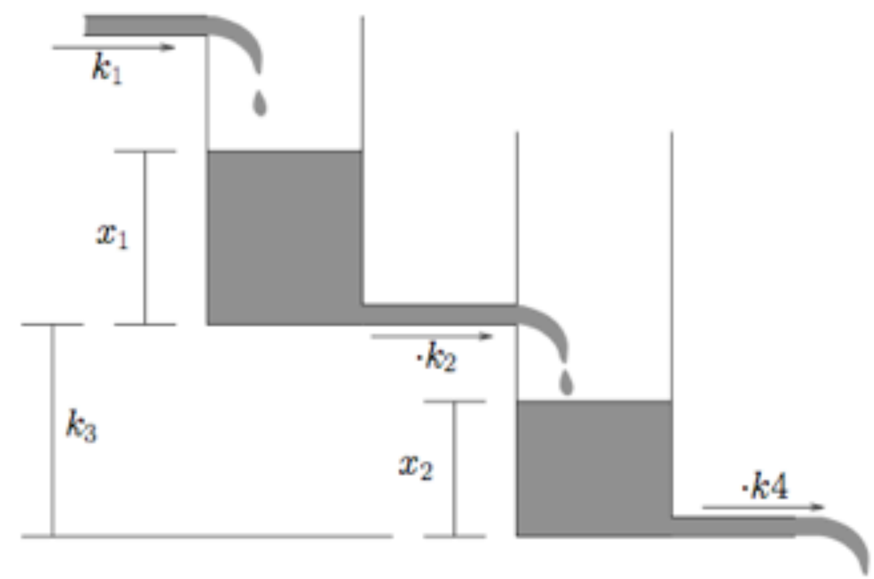

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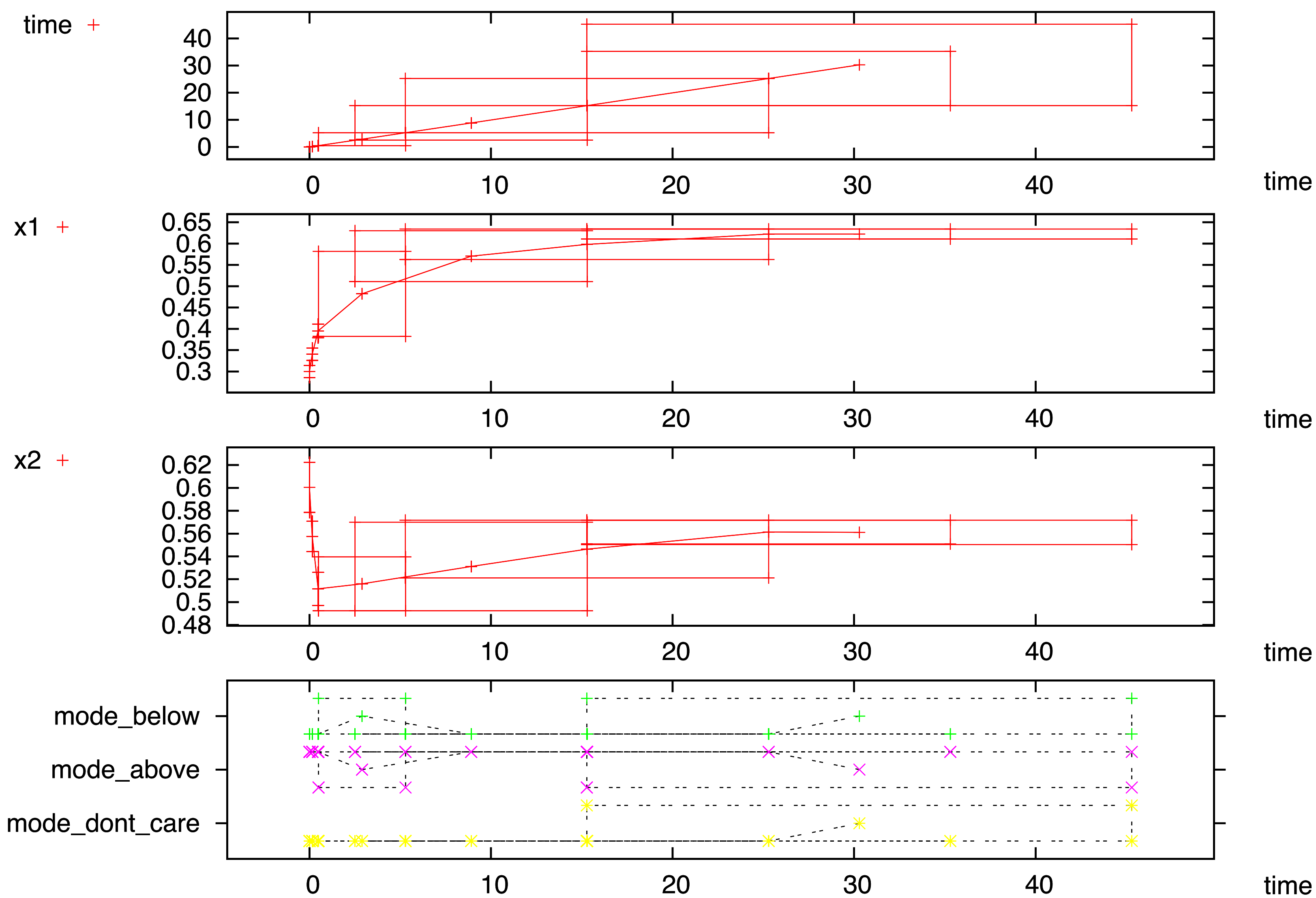
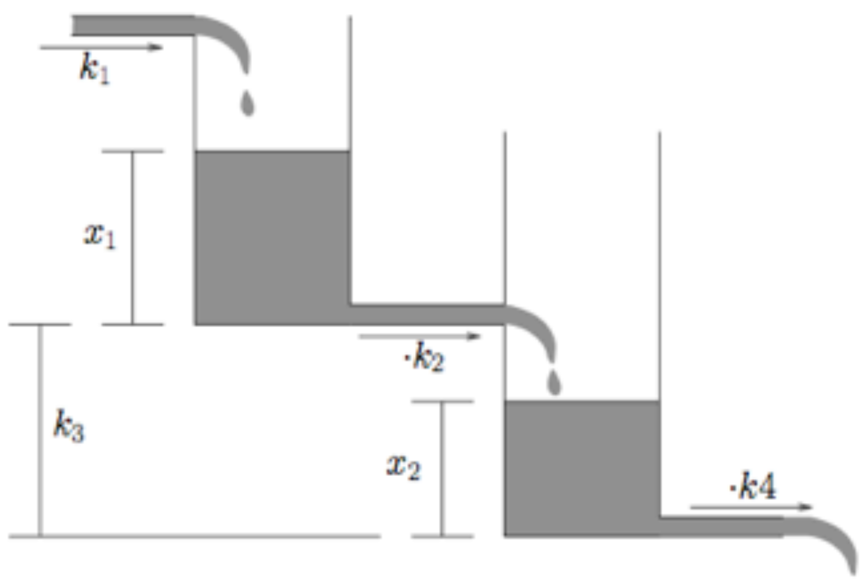




CPU time on AMD Opteron 8378 2.4 GHz 64bits Linux

depth 7 ⟶ 87mn

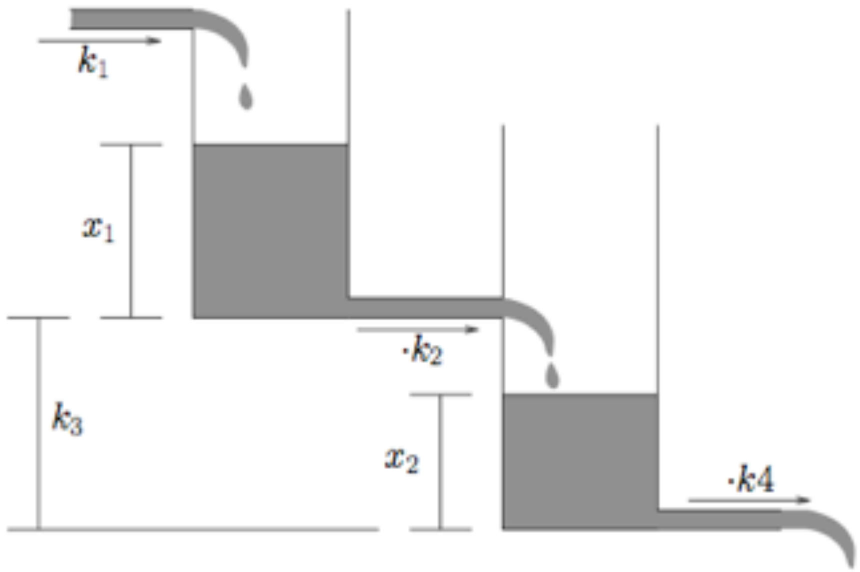
bounds ⟶ 5.7h

Experimental Evaluation

- Example 2b
  - Estimation of Hybrid System
    - (Stursberg, et al. 1997)

$$\begin{aligned}\Phi = \; & \mathrm{decl}[0] \wedge \cdots \wedge \mathrm{decl}[k] \\ & \wedge \mathrm{init}[0] \\ & \wedge \mathrm{trans}[0,1] \wedge \cdots \wedge \mathrm{trans}[(k-1),k] \\ & \wedge \mathrm{target}[k]\end{aligned}$$

- Measuring height threshold
    - (Koutsoukos, 2003)
  - Assume uncertain measurement time.
  - ... add additional interruptions at points of time.

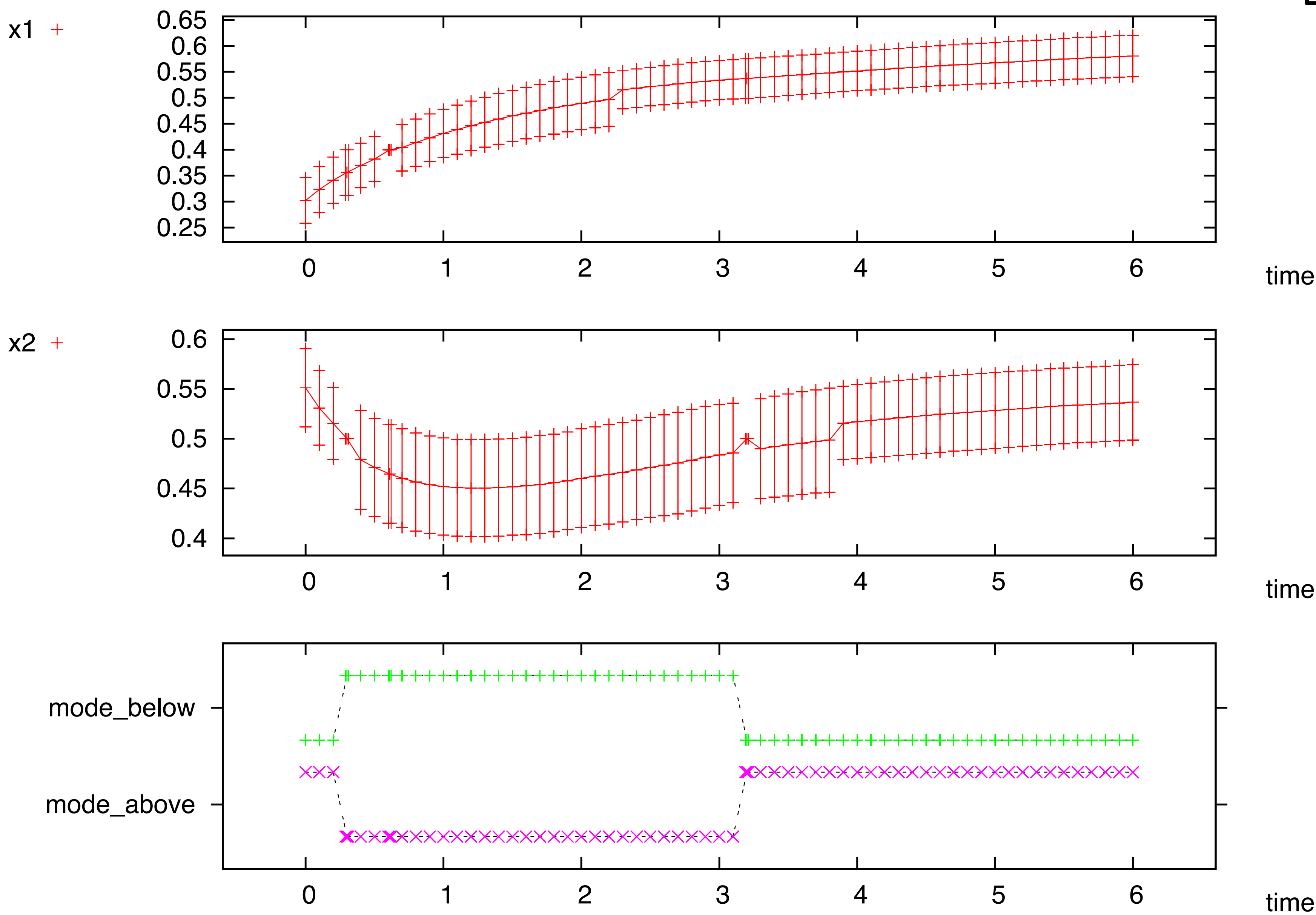


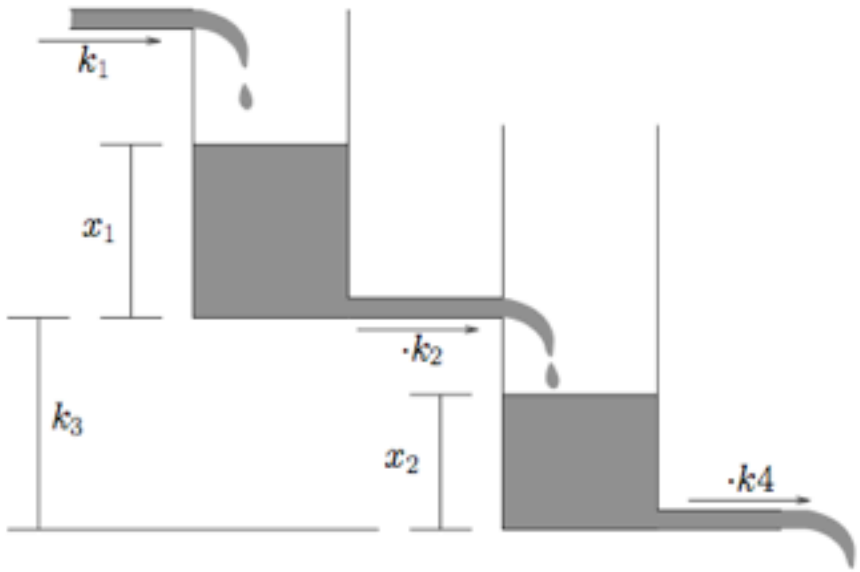
Experimental Evaluation

Example 2b

- Estimation of Hybrid System
  - (Stursberg, et al. 1997)

$$\Phi = \text{decl}[0] \wedge \cdots \wedge \text{decl}[k] \wedge \text{init}[0] \wedge \text{trans}[0,1] \wedge \cdots \wedge \text{trans}[(k-1),k] \wedge \text{target}[k]$$





Experimental Evaluation

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- Estimation of Hybrid System
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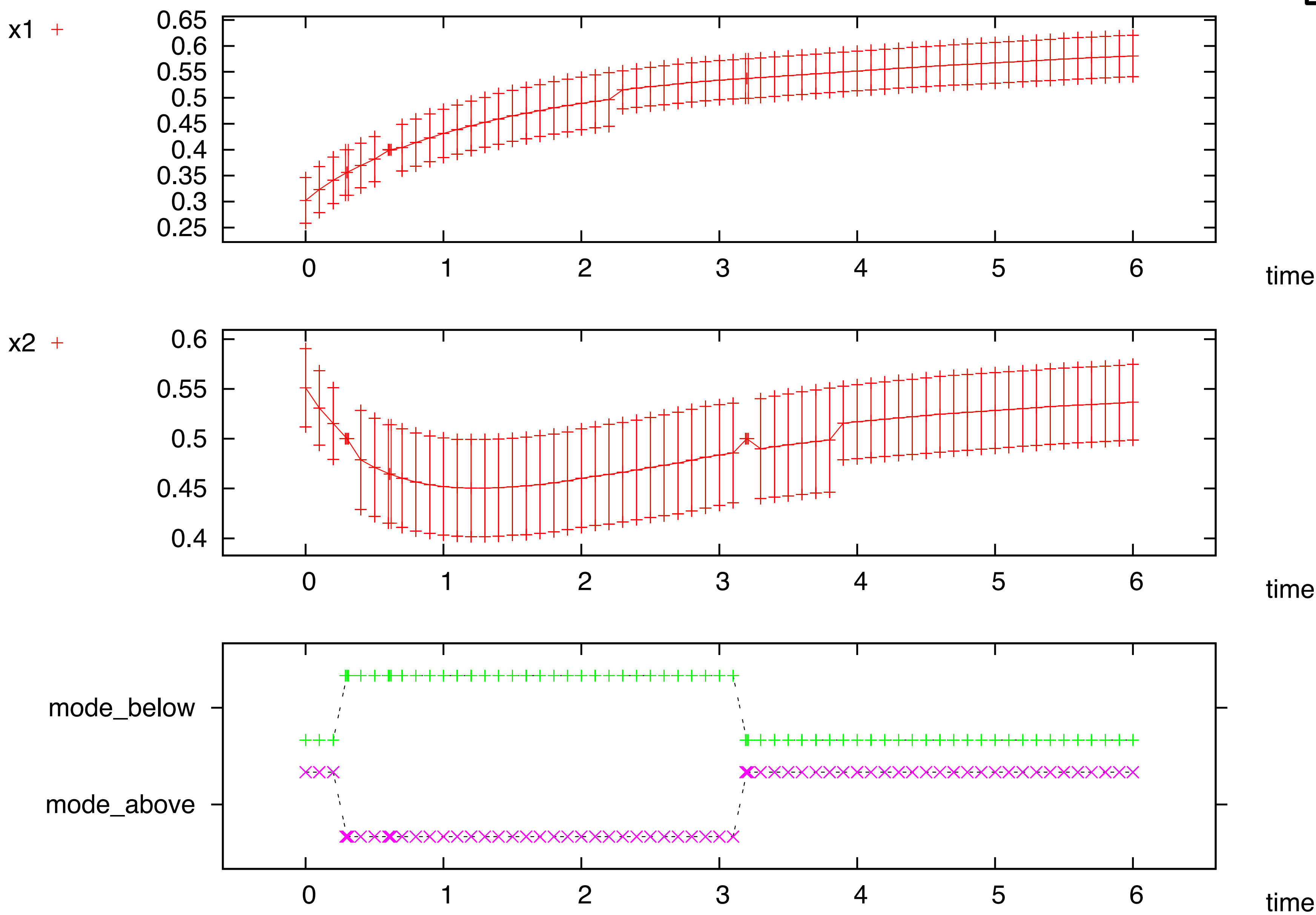

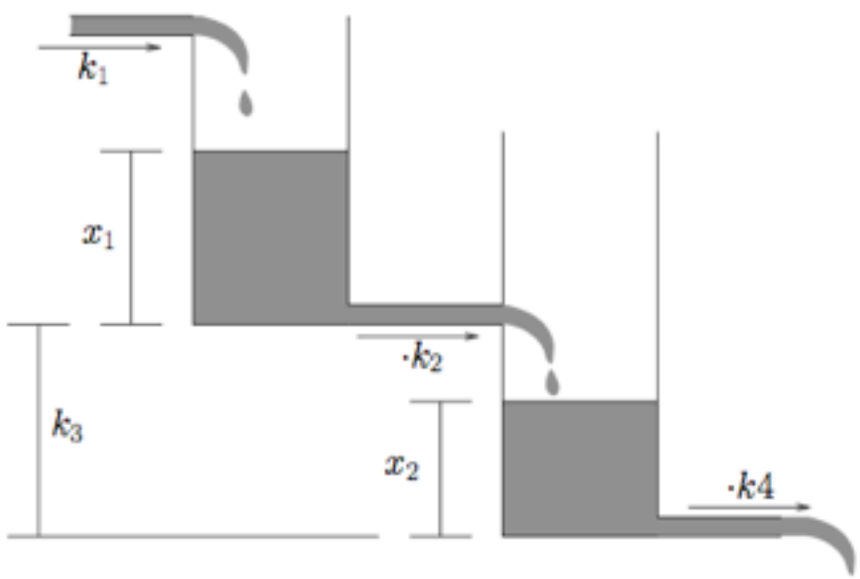

CPU time on AMD Opteron 8378 2.4 GHz 64bits Linux

bounds $\longrightarrow$ 7.3h.

Concluding Remarks

- SME of HDS via SAT Mod ODE
- Further evaluation with
  enhanced version of iSAT-ODE
- Inner approximation ...

Danke!   Thanks!   谢谢!