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Statistical analysis of single-case experimental designs:

Conditional equivalence of the general-linear-model approach of GLASS, WILLSON & GOTTMAN with the intervention model of BOX & TIAO

C. Möbus, G. Göricke and P. A. Kröh

Summary

In single case diagnostics the researcher has often to evaluate the influence of his intervention. If the data are at the level of the interval-scales the impact assessment can be pursued by the GLASS, WILLSON & GOTTMAN (1975) approach, which can be regarded as a mixture of the general linear model and time series modelling of the error component, or by the intervention model of BOX & TIAO (1975), which is a descendant of BOX & JENKINS' transfer model (1970).

In clinical research only the former method is used for impact assessment. The latter model is up to now not widely known. Even in statistical literature there is no unified treatment[1] of both approaches. We want to show that they are equivalent under certain conditions. But in spite of this, researchers are advised to abandon the GLASS, WILLSON & GOTTMAN method in favor of the intervention model because of its greater elegance and practicability. This means that the danger of misspecifying the intervention effects is far more negligible in the intervention model than in the GLASS, WILLSON & GOTTMAN method.

Zusammenfassung

In der Einzelfalldiagnostik sollen oft Interventionseffekte evaluiert werden. Sind die Daten intervallskaliert, kann das nach dem Ansatz von GLASS, WILLSON & GOTTMANN (1975) erfolgen. Er beinhaltet die Modellierung des Effektes nach dem allgemeinen linearen Modell und die des Fehlers nach dem ARIMA-Modell von BOX & JENKINS. Eine andere Möglichkeit der Interventionsevaluation bietet das Interventionsmodell von BOX & TIAO (1975), das sich aus dem Transfermodell von BOX & JENKINS (1970) herleitet.

In der klinischen Forschung wird meist nur die erste Methode verwendet. Das zweite Modell ist bis jetzt nicht sehr bekannt. Sogar in der statistischen Literatur gibt es keine einheitliche Darstellung[2] beider Ansätze. Wir wollen zeigen, daß sie unter bestimmten Bedingungen äquivalent sind. Dennoch sollte in Untersuchungen das Interventionsmodell wegen seiner größeren Eleganz und Praktikabilität vorgezogen werden. Außerdem ist im Interventionsmodell die Gefahr von Fehlspezifikationen des Interventionseffektes kleiner als im Ansatz von GLASS, WILLSON & GOTTMANN.

[1] JENKINS (1979) does not mention the GLM approach. Only KEESER (1979, p. 268) gives a short informal hint concerning possible parallelism in both approaches. Even the new book of GOTTMAN (1981, p. 365ff) treats both approaches separately without any reference to their partial identity.

[2] JENKINS (1979) erwähnt nicht den Ansatz von GLASS, WILLSON & GOTTMAN. Nur KEESER (1979, S. 268) gibt einen kurzen informellen Hinweis auf mögliche Parallelen zwischen den beiden Modellen. Sogar das neue Buch von GOTTMAN (1981, S. 365 ff.) behandelt beide Ansätze separat, ohne einen Hinweis auf ihre partielle Identität zu geben.

1. The general linear model and many-case experimental designs

The analysis of cross-sectional data arising from N>1-experiments with the general linear model is well known (WOTTAWA, 1974; MOOSBRUGGER, 1978; TIMM, 1975; BOCK, 1975). The structure of the model is in the univariate case

(1.1) $${}_N\underline{y}_1 = {}_N\underline{X}_M\underline{\beta}_1 + {}_N\underline{\varepsilon}_1$$

where:
- N = number of subjects
- M = number of variables
- ${}_N\underline{y}_1$ = N x 1 vector of the dependent variable
- ${}_N\underline{X}_M$ = N x M matrix of M predictors (= design matrix)
- ${}_M\underline{\beta}_1$ = M x 1 vector of parameters
- ${}_N\underline{\varepsilon}_1$ = N x 1 vector of error variables

To obtain the best linear unbiased (blue) estimators of $\underline{\beta}$ it is usual to formulate a »weak« set of assumptions (1.2a–1.2c):

(1.2a) $$Y_i = \underline{x}_i'\underline{\beta} + \varepsilon_i$$

where the ε_i (i=1,...,N) are independent random variables with

(1.2b) $$E(\varepsilon_i) = 0 \quad \text{or} \quad E(\underline{\varepsilon}) = \underline{0}$$

and variance

(1.2c) $$\mathrm{var}(\varepsilon_i) = \sigma_\varepsilon^2 \quad \text{or} \quad E(\underline{\varepsilon}\,\underline{\varepsilon}') = \sigma_\varepsilon^2 \cdot \underline{I}$$

Two additional assumptions are often not stated explicitly, though they will be important in this context:

(1.3a) $\underline{X}$ must not contain measurement errors (GOLDBERGER, 1973). Especially $\underline{X}$ must not contain estimators of parameters.

and

(1.3b) $\underline{X}$ has to be known apriori and must not contain unknown parameters.

As an example we want to write down the familiar t-test with the two hypotheses H_0: $\mu_A = \mu_B$ and H_1: $\mu_A \neq \mu_B$ in terms of (1.1). The full model representing the alternative H_1 hypothesis can be written

$$
(1.4a)\quad
\begin{array}{l}
\\ \\ \text{group A} \\ \\ \\ \\ \\ \\ \\ \text{group B} \\ \\
\end{array}
\begin{array}{c}
1 \\ 2 \\ 3 \\ \vdots \\ N_A \\ -- \\ 1 \\ 2 \\ 3 \\ \vdots \\ N_B
\end{array}
\begin{bmatrix}
Y_{11} \\ Y_{12} \\ Y_{13} \\ \vdots \\ Y_{1N_A} \\ ---- \\ Y_{21} \\ Y_{22} \\ Y_{23} \\ \vdots \\ Y_{2N_B}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ -- & -- \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1
\end{bmatrix}
\begin{bmatrix}
L_F \\ \beta_F
\end{bmatrix}
+
\begin{bmatrix}
\varepsilon_{11F} \\ \varepsilon_{12F} \\ \varepsilon_{13F} \\ \vdots \\ \varepsilon_{1N_AF} \\ ---- \\ \varepsilon_{21F} \\ \varepsilon_{22F} \\ \varepsilon_{23F} \\ \vdots \\ \varepsilon_{2N_BF}
\end{bmatrix}
$$

or

$$(1.4b)\qquad \underline{y} = \underline{X}_F\underline{\beta}_F + \underline{\varepsilon}_F$$

with: L = levelparameter
β_F = stepparameter
$\underline{X}_F$ = designmatrix
$\underline{\beta}_F$ = vector of parameters in full model

The ordinary least squares (OLS) estimator is

$$(1.5)\qquad \hat{\underline{\beta}}_F = (\underline{X}_F'\,\underline{X}_F)^{-1}\underline{X}_F'\,\underline{y}$$

and the estimate of the residual vector

$$(1.6)\qquad \hat{\underline{\varepsilon}}_F = \underline{e}_F = \underline{y} - \underline{X}_F\hat{\underline{\beta}}_F$$

Under H_0 we expect the equality of population means: $\mu_A = \mu_B$. The equivalent hypothesis for our model (1.4) is: $\beta_F = 0$. To check the hypothesis, we insert $\beta_F = 0$ in (1.4) and formulate the reduced model

$$
(1.7a)\quad
\begin{array}{l}
\\ \\ \text{group A} \\ \\ \\ \\ \\ \\ \\ \text{group A} \\ \\
\end{array}
\begin{array}{c}
1 \\ 2 \\ 3 \\ \vdots \\ N_A \\ --- \\ 1 \\ 2 \\ 3 \\ \vdots \\ N_B
\end{array}
\begin{bmatrix}
Y_{11} \\ Y_{12} \\ Y_{13} \\ \vdots \\ Y_{1N_A} \\ ---- \\ Y_{21} \\ Y_{22} \\ Y_{23} \\ \vdots \\ Y_{2N_B}
\end{bmatrix}
=
\begin{bmatrix}
1 \\ 1 \\ 1 \\ \vdots \\ 1 \\ -- \\ 1 \\ 1 \\ 1 \\ \vdots \\ 1
\end{bmatrix}
\begin{bmatrix}
L_R
\end{bmatrix}
+
\begin{bmatrix}
\varepsilon_{11R} \\ \varepsilon_{12R} \\ \varepsilon_{13R} \\ \vdots \\ \varepsilon_{1N_AR} \\ ---- \\ \varepsilon_{21R} \\ \varepsilon_{22R} \\ \varepsilon_{23R} \\ \vdots \\ \varepsilon_{2N_BR}
\end{bmatrix}
$$

or

$$(1.7b)\qquad \underline{y} = \underline{X}_R\underline{\beta}_R + \underline{\varepsilon}_R$$

The OLS-estimator is

$$(1.8)\qquad \hat{\underline{\beta}}_R = (\underline{X}_R'\,\underline{X}_R)^{-1}\underline{X}_R'\,\underline{y}$$

and the estimate of the residual vector

$$(1.9)\qquad \hat{\underline{\varepsilon}}_R = \underline{e}_R = \underline{y} - \underline{X}_R\hat{\underline{\beta}}_R$$

Now we can test H_0 with the F-ratio

$$(1.10)\qquad F_{df_1,df_2} = \frac{(\underline{e}_R'\underline{e}_R - \underline{e}_F'\underline{e}_F)/(p_F - p_R)}{(\underline{e}_F'\underline{e}_F)/(N - p_F)}$$

where: N = number of subjects
p_F = number of linear independent columns in $\underline{X}_F$ (here: 2)
p_R = number of linear independent columns in $\underline{X}_R$ (here: 1)
$df_1 = p_F - p_R$
$df_2 = N - p_F$

If we take the square root of (1.10) we get the t-ratio with $df = N - p_F$.

2. The general linear model and single-case experimental designs

In the single-case experimental design, which is not a cross-sectional but a longitudinal design, one subject is observed under various experimental conditions (CHASSAN, 1972). The impact of the experimental conditions should be assessed by statistical methods (BARLOW & HERSEN, 1973; HERSON & BARLOW, 1976; KAZDIN, 1976; FICHTER, 1979; TYLER & BROWN, 1968).

One of the most familiar designs is the $A_1B_1A_2B_2$-design. The observation time is partitioned into four intervals. Two intervals (A_1,A_2) are without experimental intervention. A_1 is sometimes called »base line«. B_1 represents the first intervention phase. An example for an even more complicated $A_1B_1C_1B_2C_2$-experiment is demonstrated in Figure 1.

Without loss of generality we want to restrict our attention to the simple AB-design. The impact of the intervention can be analyzed according the general linear model along (1.1)–(1.9) if the statistical assumptions (1.2a)–(1.3b) are met

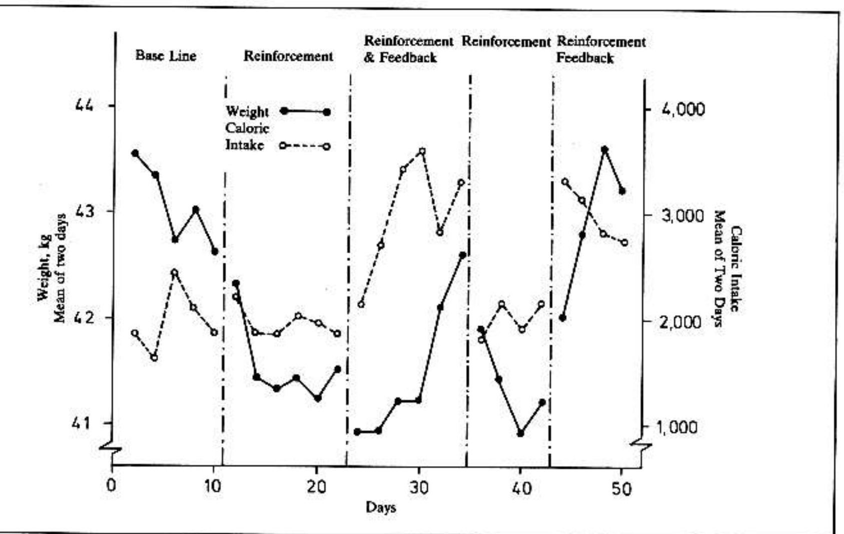


Fig. 1. Data from an experiment examining the effect of feedback on the eating behaviour of a patient with anorexia nervosa (Patient 4). (Fig. 3, p. 283, from: AGRAS, W. S., BARLOW, D. H., CHAPIN, H. N., ABEL, G. G., and LEITENBERG, H. Behaviour modification of anorexia nervosa. *Archives of General Psychiatry*, 1974, **30**, 279–286. Reproduced by permission.)

by the data. In that case the full model is (1.4) and the reduced model is (1.7). N_A and N_B are now the number of time points, where measurements are taken in phase A and B so that $N_A + N_B = T$. Whereas in a cross-sectional design it is assumed that the ε_i and Y_i are independent, in a time-series experiment it is possible to test this assumption. We have to prove that the time-series of estimated residuals $\hat{\varepsilon}_t = e_t$ is sampled from a time-discrete white-noise process a_t. For »diagnostic checks applied to residuals« we refer to BOX & JENKINS (1976, p. 287–299).

If the residuals ε_i are dependent and not a white-noise process, then

$$\text{(2.1)} \qquad E(\underline{\varepsilon}\ \underline{\varepsilon}') = \sigma^2 \cdot \underline{\Omega} \neq \sigma^2 \cdot \underline{I}$$

and the estimators are no longer blue. But more important for impact assessment is the fact, that the F-value (1.10) is »too large« or »too small«, depending on the correlation structure of the ε-process (HIBBS, 1974). Nonsignificant effects could appear to be significant and vice versa!

In the case of (2.1) it is at least in principle possible to estimate the parametervector $\underline{\beta}$ with the generalized least squares (GLS)-method (AITKEN, 1935). The linear model (1.1), (1.4) and (1.7) must be transformed, so that the new residuals ε_t^* will be independent and will follow a white-noise process. We have to look for a T × T transformation matrix $\underline{A}$, with

$$\text{(2.2a)} \qquad \underline{A} \cdot \underline{\Omega} \cdot \underline{A}' = \underline{I}$$

$$\text{(2.2b)} \qquad \underline{A}\ \underline{y} = \underline{AX\beta} + \underline{A}\ \underline{\varepsilon}$$
$$\underline{y}^* = \underline{X}^*\underline{\beta} + \underline{\varepsilon}^*$$
$$\text{with: } \underline{y}^* = \underline{A}\ \underline{y}, \quad \underline{X}^* = \underline{A} \cdot \underline{X}, \quad \underline{\varepsilon}^* = \underline{A}\ \underline{\varepsilon}$$

and

$$\text{(2.3)} \qquad E(\underline{\varepsilon}^*\underline{\varepsilon}^{*\prime}) = E(\underline{A}\ \underline{\varepsilon}\ \underline{\varepsilon}'\underline{A}') = \underline{A} \cdot E(\underline{\varepsilon}\ \underline{\varepsilon}') \cdot \underline{A}'$$
$$= \underline{A} \cdot \sigma_\varepsilon^2 \underline{\Omega} \cdot \underline{A}' = \sigma_\varepsilon^2 \cdot \underline{A} \cdot \underline{\Omega} \cdot \underline{A}' = \sigma_\varepsilon^2 \cdot \underline{I}$$

Because of the positive-definiteness of $\underline{\Omega}$, $\underline{A}$ is a triangular matrix (HIBBS, 1974; REVENSTORFF & KEESER, 1979). The GLS-estimator of $\underline{\beta}$ is

$$\text{(2.4)} \qquad \hat{\underline{\beta}} = (\underline{X}^{*\prime}\underline{X}^*)^{-1}\underline{X}^{*\prime}\underline{y}^*$$
$$= (\underline{X}'\underline{A}'\underline{A}\ \underline{X})^{-1}\underline{X}'\underline{A}'\underline{A}\ \underline{y} = (\underline{X}'\underline{\Omega}^{-1}\underline{X})^{-1}\underline{X}'\underline{\Omega}^{-1}\underline{y}$$

The GLS-model is equivalent to transformation (2.2) and successive OLS. But usually the autocovariance matrix $\underline{\Omega}$ of the residuals is unknown and has to be estimated. The estimation is impossible, if all parameters in $\underline{\Omega}$ are free and if the time-series is finite. So we have to assume some simplifying structure in $\underline{\Omega}$, so that the number of unknown parameters decreases sharply. It is the merit of GLASS, WILLSON & GOTTMAN (1975) who provided us with a wide variety of new design matrices $\underline{X}^*$ for various ε_t-processes.

3. The approach of GLASS, WILLSON & GOTTMAN

On the basis of an early article of BOX & TIAO (1965), GLASS, WILLSON & GOTTMAN (1975) expanded the GLS-approach covering a wide variety of intervention designs and autocorrelation structures of Y_t and ε_t.

3.1 If e.g. the raw data follow a moving-average-process of order 1 (= ARIMA (0,0,1) in the BOX & JENKINS terminology), we expect the data for the preintervention phase A to be construed in accordance with the model

$$\text{(3.1a)} \qquad Y_t = L - \theta_1 a_{t-1} + a_t = f(L, \theta_1, a_{t-1}, a_t)$$
$$\text{with} \quad \underline{a} \sim N(\underline{0}, \sigma^2 \cdot \underline{I})$$

and for the intervention-phase B along the model

$$\text{(3.1b)} \qquad Y_t = (L + \beta) - \theta_1 a_{t-1} + a_t = f(L, \beta, \theta_1, a_{t-1}, a_t)$$
$$\text{with} \quad -1 < \theta_1 < +1$$

which can be combined with the linear model (full model):

$$\text{(3.2a)} \quad
\begin{array}{l}
\\ \\ \text{phase A} \\ \text{(e.g.:} \\ \text{baseline)} \\ \\ \\ \\ \\ \text{phase B} \\ \text{(e.g.:} \\ \text{inter-} \\ \text{vention)}
\end{array}
\begin{array}{cc}
1 & 1 \\ 2 & 2 \\ 3 & 3 \\ \vdots & \vdots \\ T_A & T_A \\ --- & \\ . & 1 \\ . & 2 \\ . & 3 \\ \vdots & \vdots \\ T & T_B
\end{array}
\begin{bmatrix}
Y_{11} \\ Y_{12} \\ Y_{13} \\ \vdots \\ Y_{1T_A} \\ --- \\ Y_{21} \\ Y_{22} \\ Y_{23} \\ \vdots \\ Y_{2T_B}
\end{bmatrix}
=
\begin{bmatrix}
1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ --- & --- \\ 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1
\end{bmatrix}
\begin{bmatrix}
L_F \\ \beta_F
\end{bmatrix}
+
\underbrace{
\begin{bmatrix}
0 & a_1 \\ a_1 & a_2 \\ a_2 & a_3 \\ \vdots & \vdots \\ . & . \\ --- & --- \\ . & . \\ . & . \\ . & . \\ \vdots & \vdots \\ a_{T-1} & a_T
\end{bmatrix}
\begin{bmatrix}
-\theta_1 \\ 1
\end{bmatrix}}_{\underline{\varepsilon}}$$

or

$$\text{(3.2b)} \qquad \underline{y} = \underline{X}\ \underline{\beta} + \underline{\varepsilon}$$
$$\text{with: } \varepsilon_t = -\theta_1 a_{t-1} + a_t = (1 - \theta_1 B)a_t$$

where B is the backshift operator

$$Ba_t = a_{t-1} \quad \text{and in general} \quad B^k a_t = a_{t-k}$$

and:

$$\text{(3.3)} \qquad E(\underline{\varepsilon}\ \underline{\varepsilon}') = \sigma_\varepsilon^2 \cdot \underline{\Omega} \neq \sigma_\varepsilon^2 \cdot \underline{I}$$

The dependence of the residuals (3.3) can be seen very quickly in (3.2a). ε_{t-1} and ε_t share a common term, which is a_{t-1}.

Now we want to determine the matrices $\sigma_\varepsilon^2 \cdot \underline{\Omega}$, $\underline{A}$ and $\underline{X}^*$. The variance of the correlated residuals is

$$\text{(3.4)} \qquad \text{var}(\varepsilon_t) = \text{var}(-\theta_1 a_{t-1}) + \text{var}(a_t) = (1 + \theta_1^2)\sigma_a^2$$

and the covariance of lag k=1

$$\text{(3.5)} \qquad \text{cov}(\varepsilon_t, \varepsilon_{t-1}) = \text{cov}(-\theta_1 a_{t-1} + a_t, -\theta_1 a_{t-2} + a_{t-1})$$
$$= -\theta_1 \sigma_a^2$$

so the autocorrelation of both time series Y_k and ε_k with their lagged counterparts Y_{t-k} and ε_{t-k} is

$$\text{(3.6)} \qquad \varrho_k = \begin{cases} \dfrac{-\theta_1}{1+\theta_1^2} & k = 1 \\ 0 & k > 1 \end{cases}$$

The autocovariance matrix of the residuals can be written as the symmetric matrix

$$\text{(3.7)} \quad E(\underline{\varepsilon}\underline{\varepsilon}') =
\begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ T \end{array}
\begin{bmatrix}
1 & \cdot & \cdot & \cdot & \cdot \\
\dfrac{-\theta_1}{1+\theta_1^2} & 1 & \cdot & \cdot & \cdot \\
0 & \dfrac{-\theta_1}{1+\theta_1^2} & 1 & \cdot & \cdot \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \dfrac{-\theta_1}{1+\theta_1^2} & 1
\end{bmatrix}
\sigma_a (1+\theta_1^2)$$

(3.7) shows that only in the case of $\theta_1 = 0$, the residuals are independent and the familiar t-test is justified to test the intervention hypothesis H_0: $\beta_F = 0$.

The transformation matrix $\underline{A}$ is (REVENSTORFF & KEESER, 1979):

$$(3.8) \quad \underline{A} = \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ T \end{array} \begin{bmatrix} 1 & & & & \\ \theta_1 & 1 & & & \\ \theta_1^2 & \theta_1 & 1 & & \\ \vdots & \vdots & \vdots & \ddots & \\ \theta_1^{T-1} & \theta_1^{T-2} & \theta_1^{T-3} & & 1 \end{bmatrix} \atop \quad 1 \quad 2 \quad 3 \quad \ldots \quad T$$

For the transformed designmatrix $\underline{X}^* = \underline{A}\ \underline{X}$ we get:

$$(3.9) \quad \underline{X}^* =$$

$$\begin{array}{cc} 1 & 1 \\ 2 & 2 \\ 3 & 3 \\ \vdots & \vdots \\ T_A & T_A \\ \hline . & 1 \\ . & 2 \\ \vdots & \vdots \\ T & T_B \end{array} \begin{bmatrix} 1 & 0 \\ 1+\theta_1 & 0 \\ 1+\theta_1+\theta_1^2 & 0 \\ \vdots & \vdots \\ 1+\theta_1+\theta_1^2+\ldots+\theta_1^{(T_A-1)} & 0 \\ \hline 1+\theta_1+\theta_1^2+\ldots+\theta_1^{T_A} & 1 \\ 1+\theta_1+\theta_1^2+\ldots+\theta_1^{(T_A+1)} & 1+\theta_1 \\ \vdots & \vdots \\ 1+\theta_1+\theta_1^2+\ldots+\theta_1^{(T-1)} & 1+\theta_1+\theta_1^2+\ldots+\theta_1^{(T_B-1)} \end{bmatrix}$$

and as the transformed vector of the dependent variable (= timeseries Y_t):

(3.10a)

$$\begin{bmatrix} Y_{11}^* \\ Y_{12}^* \\ Y_{13}^* \\ \vdots \\ Y_T^* \end{bmatrix} = \begin{bmatrix} Y_{11} \\ Y_{12}+\theta_1 Y_{11} \\ Y_{13}+\theta_1 Y_{12}+\theta_1^2 Y_{11} \\ \vdots \quad \vdots \quad \vdots\vdots\vdots\vdots\vdots\vdots\vdots\vdots\ldots \\ Y_T+\theta_1 Y_{T-1}+\ldots\ldots\ldots+\theta_1^{T-1}Y_{11} \end{bmatrix}$$

$$= \begin{bmatrix} Y_{11}^* \\ Y_{12}+\theta_1 Y_{11}^* \\ Y_{13}+\theta_1 Y_{12}^* \\ \vdots \\ Y_T+\theta_1 Y_{T-1}^* \end{bmatrix}$$

$$(3.10b) \quad \underline{y}^* = \underline{A}\ \underline{y}$$

The transformed residual factor $\underline{\varepsilon}^* = \underline{A}\ \underline{\varepsilon}$ takes the simple form

$$(3.11) \quad \underline{\varepsilon}^* = \underline{A}\ \underline{\varepsilon} = \underline{a}$$

As a result we get the transformed linear model (2.2) with independent errors

$$(3.12) \quad \underline{y}^* = \underline{X}^*\underline{\beta} + \underline{\varepsilon}^* = \underline{X}^*\underline{\beta} + \underline{a}$$

$$\text{with } \underline{a} \sim N(\underline{0}, \sigma_a^2 \cdot \underline{I})$$

If we estimate the full and reduced models, we can test the intervention hypothesis with the familiar F-ratio (1.10). Higher order moving average processes (ARIMA(0,0,q)-models) can be transformed in a similar fashion.

3.2 Now the general transformation method will be demonstrated for the general nonstationary ARIMA(p,d,q)-model. In literature three alternative formulations of the same ARIMA-process (version I, II, III) are used.

3.2.1 Three formulations for the general ARIMA(p,d,q)-model: We postulate that the preintervention data are construed in accordance with the ARIMA(p,d,q)-model (HIBBS, 1977, p. 140; MÖBUS & NAGL 1983)

$$\begin{array}{l} (3.13a) \\ \text{Version I} \\ (3.13b) \end{array} \left\{ \begin{array}{l} \Phi_p(B)(1\text{-}B)^d Y_t = \theta_o + \theta_q(B)a_t \\ Y_t = \dfrac{\theta_o + \theta_q(B)a_t}{\Phi_p(B)(1-B)^d} = \dfrac{\theta_q(B)}{\Phi_p(B)(1-B)^d}a_t + \dfrac{\theta_o}{\Phi_p(B)(1-B)^d} \end{array} \right.$$

where:
a) $\Phi_p(B)$ is the autoregressive operator: $\Phi_p(B) = (1 - \Phi_1 B - \ldots - \Phi_p B^p)$
b) $\theta_q(B)$ is the moving average operator: $\theta_q(B) = (1 - \theta_1 B - \ldots - \theta_q B^q)$
c) $(1-B)^d$ is the difference operator: e.g.: $(1-B)^2 Y_t = (1 - 2B + B^2)Y_t = Y_t - 2Y_{t-1} + Y_{t-2}$ which is exactly taking differences of differences e.g.: $(1-B)^2 Y_t = \nabla^2 Y_t = (\nabla Y_t - \nabla Y_{t-1}) = (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$
d) θ_o is an unknown constant
e) and the inverse of the differenceoperator is e.g. for (1–B) defined as: $\frac{1}{(1-B)} = (1-B)^{-1} = (1+B+B^2+\ldots) = \sum_{k=o}^{\infty} B^k$ as can be shown by multiplication: $(1 + B + B^2 + \ldots)(1-B) = 1$
f) and the inverse of e.g. $\Phi_1(B)$ is $(1 - \Phi_1 B)^{-1} = (1 + \Phi_1 B + \Phi_1^2 B^2 + \ldots) = \sum_{k=0}^{\infty} \Phi_1^k B^k$ as can be shown too: $(1 + \Phi_1 B + \Phi_1^2 B^2 + \ldots)(1 - \Phi_1 B) = 1$

Coefficients of a general $\Phi_p^{-1}(B)$ an $(1-B)^{-d}$ can be obtained by equating coefficients.

Various authors use a second model formulation. If we put the differencing operator $(1-B)^d$ on the left side of the equality sign of (3.13b), we can express the nonstationary model in differences $w_t = (1-B)^d Y_t$ (JENKINS, 1977, p. 98):

$$\begin{array}{l} (3.14a) \\ \text{Version II} \\ (3.14b) \end{array} \left\{ \begin{array}{l} (1\text{-}B)^d Y_t = \dfrac{\theta_q(B)}{\Phi_p(B)} a_t + \dfrac{\theta_o}{\Phi_p(B)} \\ w_t = \dfrac{\Phi_q(B)}{\Phi_p(B)} a_t + \mu_w \end{array} \right.$$

where: $\mu_w = E(w_t) = \frac{\theta_o}{\Phi_p(B)} =$ mean of the d-th differences

A third representation is preferred by GLASS, WILLSON & GOTTMAN (1975). They choose

$$L = \frac{\mu_w}{(1-B)^d} = \frac{\theta_o}{\Phi_p(B)(1-B)^d}$$

and put L on the left side of (3.13b), so that we may write

$$\begin{array}{l} (3.15a) \\ \text{Version III} \\ (3.15b) \end{array} \left\{ \begin{array}{l} Y_t - L = \dfrac{\theta_q(B)}{\Phi_p(B)\,(1-B)^d} a_t \\ \Phi_p(B)(1\text{-}B)^d(Y_t - L) = \theta_q(B)a_t \end{array} \right.$$

(3.15) is an infinite polynominal in B for $d>0$ and/or $p>0$. This means, that the deviation $(Y_t - L)$ can be expressed as an infinite moving average process. This is called the »random shock« form (BOX & JENKINS, 1976, p. 95 ff).

(3.16a) $$Y_t - L = \frac{\theta_q(B)}{\Phi_p(B)(1-B)^d}\, a_t = \psi(B)a_t$$

where: $\psi(B) = \psi_o + \psi_1 B + \psi_2 B^2 + \ldots$
with $\psi_o = 1$

or

(3.16b) $$Y_t = L + \sum_{k=1}^{\infty} \psi_k a_{t-k} + a_t = L + \sum_{k=0}^{\infty} \psi_k a_{t-k}$$

To get the unknown ψ-weights we multiply (3.16a) on both sides with $(1-B)^d\Phi_p(B)$.

(3.17) $$\Phi_p(B)(1-B)^d(Y_t - L) = \Phi_p(B)(1-B)^d\psi(B)a_t$$

Following (3.15b), the left side of (3.17) is $\theta_q(B)a_t$. So we get

(3.18) $$\theta_q(B)a_t = \Phi_p(B)(1-B)^d\psi(B)a_t = \varphi(B)\psi(B)a_t$$

The equation of operators is

(3.19a) $$\theta_q(B) = \varphi(B)\psi(B)$$

where: $\varphi(B)$ is the general autoregressive operator (BOX & JENKINS, 1976, p. 95):

$$\varphi(B) = \Phi_p(B)(1-B)^d = \varphi_o - \varphi_1 B - \varphi_2 B^2 - \ldots - \varphi_{p+d}B^{p+d}$$

with $\varphi_o = 1$

and can be written explicitly as:

(3.19b) $$(1-\theta_1 B-\ldots-\theta_q B^q) = (1-\varphi_1 B-\ldots-\varphi_{p+d}B^{p+d})(1+\psi_1 B+\psi_2 B^2+\ldots)$$

Therefore, the ψ-weights can be obtained by equating coefficients in B on the left side of (3.19) to the coefficients in B of the right side.

Theoretically there are countably infinite coefficients ψ_k. GLASS et al. (1975, p. 152) argue,that the values of unobserved time series Y_k, a_k ($k \leqq 0$) should be set to their expected value, so that only $\psi_1, \ldots, \psi_{t-1}$ have to be derived (ψ_o is set to 1).

(3.20) $$Y_t = L' + \sum_{k=0}^{t-1} \psi_k a_{t-k} + a_t$$

The derivation of the ψ-weights is shown explicitly in appendix A, because the treatment of GLASS et al. (1975, p. 153–154) is confusing.

3.2.2. Transformation of the ψ-weight form of the intervention-model to the general linear model

The linear intervention-model (full model) in ψ-weight form is

(3.21a)
$$\begin{array}{r} \\ \text{phase A} \\ \\ \\ \\ \\ \text{phase B} \\ \\ \\ \end{array}
\begin{bmatrix} Y_{11} \\ Y_{12} \\ Y_{13} \\ \vdots \\ Y_{1T_A} \\ \hline Y_{21} \\ Y_{22} \\ Y_{23} \\ \vdots \\ Y_{2T_B} \end{bmatrix}
=
\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ \hline 1 & 1 \\ 1 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}
\begin{bmatrix} L' \\ \beta_1 \end{bmatrix}
+
\begin{bmatrix} 1 & 0 & 0 & \ldots\ldots & 0 \\ \psi_1 & 1 & 0 & \ldots\ldots & 0 \\ \psi_2 & \psi_1 & 1 & \ldots\ldots & 0 \\ \vdots & \vdots & & & \vdots \\ . & . & . & & . \\ . & . & & . & . \\ . & . & & . & . \\ . & . & & . & . \\ \vdots & \vdots & & & \vdots \\ \psi_{T-1} & \psi_{T-2} & \psi_{T-3} & \ldots\ \psi_1 & 1 \end{bmatrix}
\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ . \\ . \\ . \\ . \\ \vdots \\ a_T \end{bmatrix}$$

(3.21b) $$\underline{Y} = \underline{X}\,\underline{\beta} + \underline{\Psi}\,\underline{a}$$

where $\underline{\varepsilon} = \underline{\Psi}\,\underline{a}$

Some authors prefer instead of using matrix $\underline{\Psi}$, the formulation with the operator $\psi(B)$

(3.21c) $$\underline{Y} = \underline{X}\,\underline{\beta} + \psi(B)\underline{a}$$

or

(3.21d) $$Y_t = f_t(L', \beta_1, \ldots, \beta_{s+1}) + \psi(B)a_t$$

The residuals ε_t are not independent

(3.22) $$E(\underline{\varepsilon}\,\underline{\varepsilon}') \neq \sigma_\varepsilon^2 \cdot \underline{I}$$

which is due to the overlap of common terms in $\underline{\varepsilon}$.

(3.23) $$\underline{\varepsilon} = \begin{bmatrix} \psi_o a_1 \\ \psi_1 a_1 + \psi_o a_2 \\ \psi_2 a_1 + \psi_1 a_2 + \psi_o a_3 \\ \psi_3 a_1 + \psi_2 a_2 + \psi_1 a_3 + \psi_o a_4 \\ \text{-------------------------} \end{bmatrix}$$

The variance-covariance matrix of the correlated residuals ε_t is:

(3.24) $$\text{cov}(\underline{\varepsilon}\,\underline{\varepsilon}') = \sigma_a^2 \cdot \underline{\Omega} =$$

$$\sigma_a^2 \cdot
\begin{array}{r|ccccccc|}
 & 1 & 2 & 3 & & l & & \infty \\
1 & \psi_o^2 & \psi_o\psi_1 & \psi_o\psi_2 & \ldots & \psi_o\psi_{l-1} & \ldots & \psi_o\eta \\ \hline
2 & \psi_o\psi_1 & \psi_o^2 + \psi_1^2 & \sum_{i=1}^{2}\psi_{2-i}\psi_{3-i} & \ldots & \sum_{i=1}^{2}\psi_{2-i}\psi_{l-i} & \ldots & \eta\sum_{i=1}^{2}\psi_{2-i} \\ \hline
3 & \psi_o\psi_2 & \sum_{i=1}^{2}\psi_{2-i}\psi_{3-i} & \sum_{i=1}^{3}\psi_{i-1}^2 & \ldots & \sum_{i=1}^{3}\psi_{3-i}\psi_{l-i} & \ldots & \eta\sum_{i=1}^{3}\psi_{3-i} \\ \hline
4 & \psi_o\psi_3 & \sum_{i=1}^{2}\psi_{2-i}\psi_{4-i} & \sum_{i=1}^{3}\psi_{3-i}\psi_{4-i} & \ldots & \sum_{i=1}^{4}\psi_{4-i}\psi_{l-i} & \ldots & \eta\sum_{i=1}^{4}\psi_{4-i} \\ \hline
 & \vdots & \vdots & \vdots & & \vdots & & \vdots \\ \hline
k & \psi_o\psi_{k-1} & \sum_{i=1}^{2}\psi_{2-i}\psi_{k-i} & \sum_{i=1}^{3}\psi_{3-i}\psi_{k-i} & \ldots & \sum_{i=1}^{\min\{k,l\}}\psi_{k-i}\psi_{l-i} & \ldots & \eta\sum_{i=1}^{k}\psi_{k-i} \\ \hline
 & \vdots & \vdots & \vdots & & \vdots & & \vdots \\ \hline
\infty & \psi_o\eta & \eta\sum_{i=1}^{2}\psi_{2-i} & \eta\sum_{i=1}^{3}\psi_{3-i} & \ldots & \eta\sum_{i=1}^{l}\psi_{l-i} & \ldots & \sum_{i=1}^{\infty}\psi_{i-1}^2
\end{array}$$

where: $\eta = \lim_{j\to\infty} \psi_j$

The intervention-model with correlated residuals (3.21) can be transformed to one with uncorrelated residuals. This can be done either by multiplying (3.21) by $\underline{\Psi}^{-1}$

(3.25) $$\underline{\Psi}^{-1} = \sum_{i=0}^{\infty}(\underline{I} - \underline{\Psi})^i$$

(3.26a) $$\underline{\Psi}^{-1}\underline{Y} = \underline{\Psi}^{-1}\underline{X}\,\underline{\beta} + \underline{a}$$

(3.26b) $$\underline{Y}^* = \underline{X}^*\underline{\beta} + \underline{a}$$

or by using the recursive relationships

(3.27a) $$Y_t^* = Y_t - \sum_{j=1}^{t-1}\psi_j Y_{t-j}^*$$

(3.27b) $$x_{tk}^* = x_{tk} - \sum_{j=1}^{t-1}\psi_j x_{t-j,k}^*$$

(3.27c) $$a_t = \varepsilon_t - \sum_{j=1}^{t-1}\psi_j a_{t-j}$$

For T=5 time points we get the following matrix $\underline{\Psi}^{-1}$, where T_A=2 and T_B=3:

(3.28) $${}_5\underline{\Psi}_5^{-1} =$$

$$\begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
-\psi_1 & 1 & 0 & 0 & 0 \\
\psi_1^2-\psi_2 & -\psi_1 & 1 & 0 & 0 \\
-\psi_1^3+2\psi_1\psi_2-\psi_3 & \psi_1^2-\psi_2 & -\psi_1 & 1 & 0 \\
\psi_1^4-3\psi_1^2\psi_2+2\psi_1\psi_3+\psi_2^2-\psi_4 & -\psi_1^3+2\psi_1\psi_2-\psi_3 & \psi_1^2-\psi_2 & -\psi_1 & 1
\end{bmatrix}$$

The burden of computations is taken by computer programs, distributed by BOWER, PADIA & GLASS (1974) and GOTTMAN (1981) and will thus not be demonstrated here.

4. The intervention-model-approach of BOX & TIAO

BOX & TIAO (1975) developed their intervention model on the basis of BOX & JENKINS' transfer function model (BOX & JENKINS, 1970). Their intervention model is given by:

(4.1) $\delta(B)Y_t = \Omega(B)I_t + \delta(B)\varepsilon_t$

where ε_t is assumed to be an ordinary ARIMA(p,d,q)-model according (3.13b)

$$(4.2a) \qquad \varepsilon_t = \frac{\theta_o}{(1-B)^d\Phi_p(B)} + \frac{\theta_q(B)}{(1-B)^d\Phi_p(B)}\, a_t$$

$$\text{with} \quad L = \frac{\theta_o}{(1-B)^d\Phi_p(B)}$$

According (3.16a) we obtain

$$(4.2b) \qquad \varepsilon_t = L + \psi(B)a_t$$

Substituting equation (4.2b) into (4.1), we get the form:

$$(4.3a) \quad \delta(B)Y_t = \Omega(B)I_t + \delta(B)\cdot\{L + \psi(B)a_t\}$$

or equivalently

$$(4.3b) \qquad Y_t = \underbrace{\frac{\Omega(B)}{\delta(B)}\cdot I_t}_{\mathcal{Y}_t} + L + \underbrace{\psi(B)a_t}_{N_t}$$

or equivalently

$$(4.3c) \quad Y_t - L = \left\{\begin{matrix}\text{effects}\\ \text{of intervention}\\ \mathcal{Y}_t\end{matrix}\right\} + \left\{\begin{matrix}\text{noncontrollable}\\ \text{effects}\\ N_t\end{matrix}\right\}$$

$\mathcal{Y}_t$ is that part of time-dependent data, which is controlled by the experimenter. The intervention variable I_t is at apriori determined time points set to '1' (=»on«) or '0' (=»off«). The aim of intervention analysis is the splitting of the time-series $\mathcal{Y}_t$ and N_t and the estimation of the parameters of the $\mathcal{Y}_t$-process. These parameters are effect-parameters which can be tested.

»The function $\mathcal{Y}_t$ represents the additional effects of the intervention over the noise. In particular, when N_t is nonstationary, large changes could occur in the output even with no intervention. Fitting the model can make it possible to distinguish between what can and what cannot be explained by the noise.« (BOX & TIAO, 1975, p. 72), HIBBS (1975) calls the transfer function model

$$(4.4a) \qquad \mathcal{Y}_t = \frac{\Omega(B)}{\delta(B)}\cdot I_t$$

or equivalently

$$(4.4b) \quad \delta(B)\mathcal{Y}_t = \Omega(B)I_t$$

or equivalently

$$(4.4c) \quad \delta(B)\mathcal{Y}_t = \omega(B)B^bI_t$$

or equivalently

$$(4.4d) \quad (1 - \delta_1B - \ldots - \delta_rB^r)\,\mathcal{Y}_t = (\omega_o - \omega_1B - \ldots - \omega_sB^s)B^bI_t$$

or equivalently

$$(4.4e) \quad \mathcal{Y}_t - \delta_1\mathcal{Y}_{t-1} - \ldots - \delta_r\mathcal{Y}_{t-r} = \omega_oI_{t-b} - \omega_1I_{t-b-1} - \ldots - \omega_sI_{t-b-s}$$

the »general intervention effects model«. The indices reflect the »memory« of the intervention component. If we have a nonintervention phase A and an intervention phase B, the intervention variable I_t takes the values

t	1	2	...	T_a	1	2	...	T_b
I_t	0	0	...	0	1	1	...	1

If there is an abrupt change in level of Y_t without time-delay, we use the intervention model of order zero

(4.5) $\mathcal{Y}_t = \omega_oI_t$ step change without time-delay (Fig. 2a)

If there is a step change with time delay, we use

(4.6) $\mathcal{Y}_t = \omega_oI_{t-b}$ step change with time delay b (Fig. 2b)

In the case of a nonstationary N_t, the level of Y_t shows a different reaction.

Is the effect $\mathcal{Y}_t$ changing slowly, we use the transferfunction of order one

(4.7a) $\mathcal{Y}_t = \delta_1\,\mathcal{Y}_{t-1} + \omega_oI_{t-b}$

or

(4.7b) $(1 - \delta_1B)\mathcal{Y}_t = \omega_oI_{t-b}$

or

(4.7c) $\mathcal{Y}_t = \dfrac{\omega_o}{(1 - \delta_1B)}\cdot I_{t-b}$

ramp change with time delay b (Fig. 2c)

Is $\delta_1 = 0$ and $b = 0$, we get again the step change (Fig. 2d) and in the case of $\delta_1 = 1$, we get the model

(4.8) $\mathcal{Y}_t = \dfrac{1}{(1-B)}\cdot I_{t-b}$ nondamped increase (Fig. 2e)

which is not stable.

The modelling of various other intervention effects is shown in MÖBUS & NAGL (1983).

The stability of the intervention model (4.4) is guaranteed, if the roots $B_1,\ldots,B_j,\ldots,B_r$ of the characteristic equation

$$\delta(B) = (1 - \delta_1B - \delta_2B^2 - \ldots - \delta_rB^r) = 0$$

satisfy the conditions $|B_j| > 1$ for $j = 1,\ldots,r$.

For the intervention model of order two

$$(1 - \delta_1B - \delta_2B^2)\mathcal{Y}_t = \omega_oI_t$$

the roots B_j of the polynomial

$$(1 - \delta_1B - \delta_2B^2) = 0$$

have to be of absolute value $|B_j| > 1$

$$\text{with} \quad B_j = \frac{\delta_1 \pm \sqrt{\delta_1^2 + 4\delta_2}}{2\delta_2}$$

The conditions $|B_j| > 1$ can only be met, if the parameters lie in these regions:

$$-1 < \delta_2 < +1$$
$$\delta_1 + \delta_2 < +1$$
$$\delta_2 - \delta_1 < +1$$

If the Intervention variable I_t is held constant $I_t = 1$ for $t > T_A$, $\mathcal{Y}_t$ will reach the steady state:

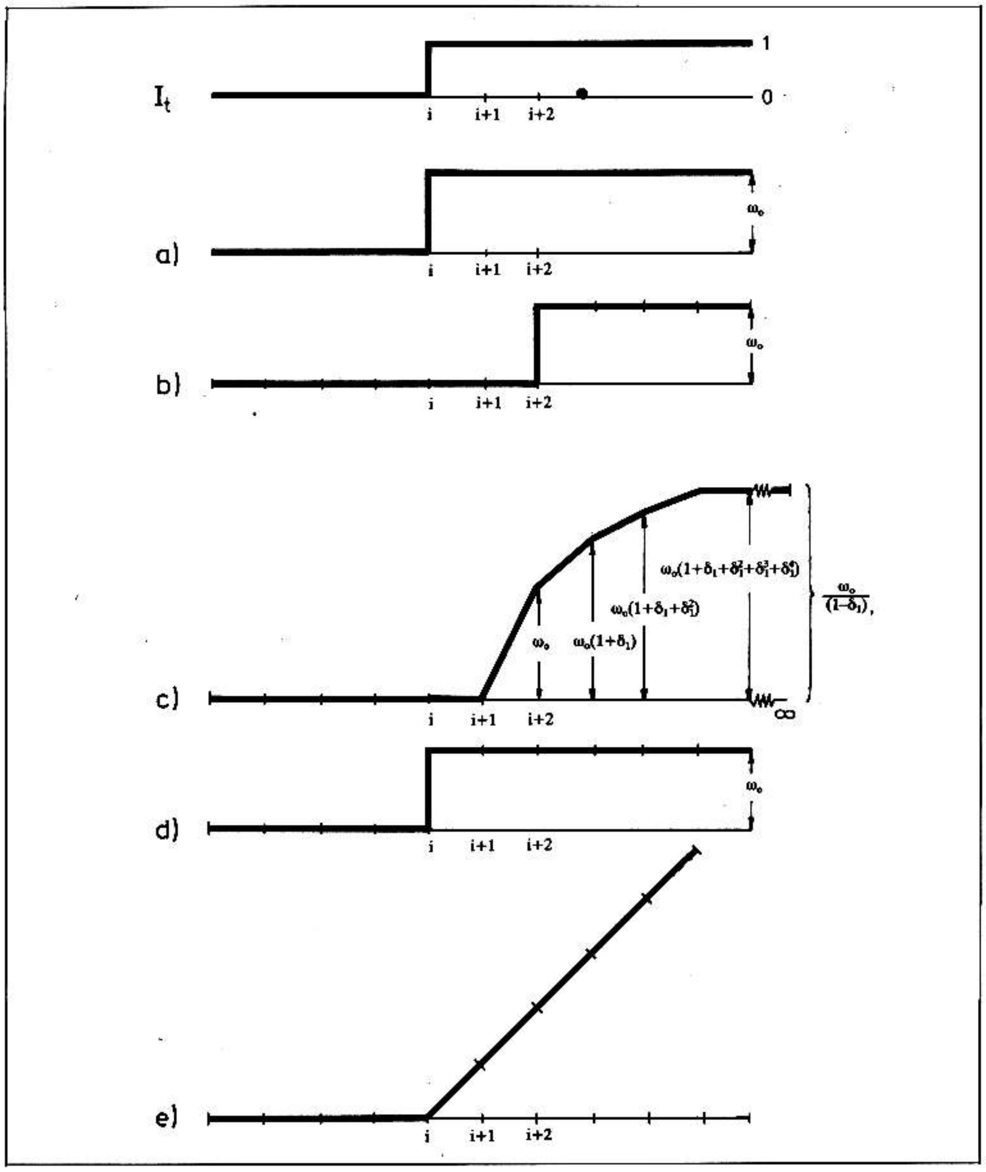


Fig. 2. Various deterministic responses $\mathcal{Y}_t$ to step input I_t.

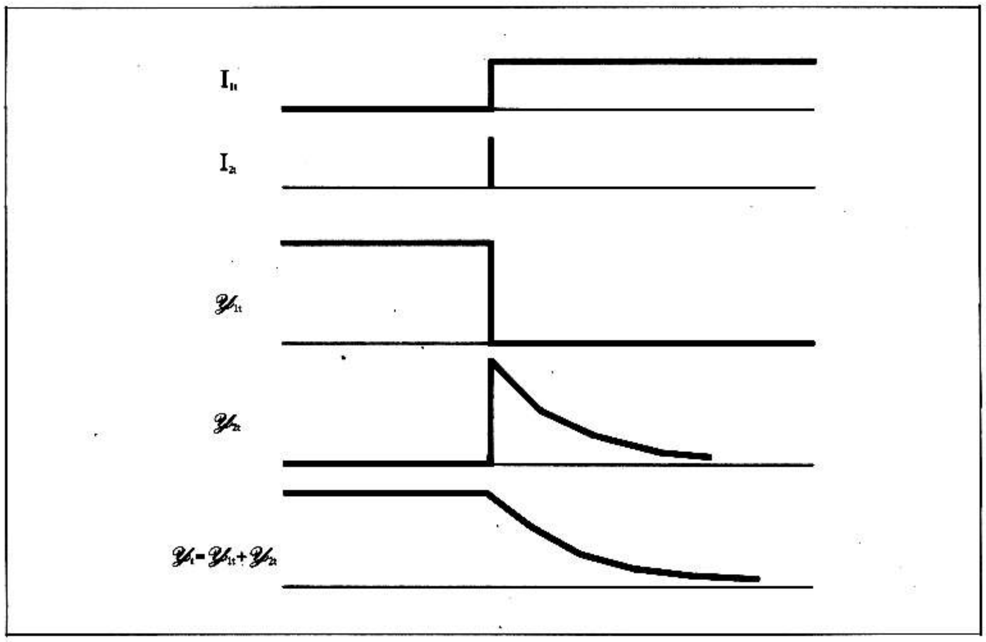


Fig. 3. Addition of deterministic responses to one response $\mathcal{Y}_t$.

$$(4.9)\qquad E(\mathcal{Y}_t) = \mathcal{Y}_\infty = \frac{(\omega_o - \omega_1 - \ldots - \omega_s)}{(1 - \delta_1 - \ldots - \delta_r)}$$

BOX & TIAO (1975) and other authors distinguish between two forms of the indicator variable I_t:

the step input

$$(4.10)\qquad I_t = S_t = \begin{cases} 0 & \text{for } t \leqq T_A \\ 1 & \text{for } t > T_A \end{cases}$$

and the pulse input

$$(4.11)\qquad I_t = P_t = \begin{cases} 0 & \text{for } t \neq T_A+1 \\ 1 & \text{for } t = T_A+1 \end{cases}$$

where the pulse input P_t can be derived by differencing the step input S_t

$$(4.12)\qquad P_t = (1-B)S_t$$

If there are k intervention variables acting simultaneously, the general intervention-effects-model can be extended similar to the statistical model of a multifactorial anova-model

$$(4.13)\qquad \mathcal{Y}_t = \sum_{j=1}^{k} \mathcal{Y}_{jt} = \sum_{j=1}^{k} \left\{ \frac{\Omega_j(B)}{\delta_j(B)} \right\} \cdot I_{jt}$$

For example it is reasonable to hypothesize two effects on the deterministic component of perceptual speed of a person in a drug experiment (Figure 3)

$$(4.14)\qquad \mathcal{Y}_t = \mathcal{Y}_{1t} + \mathcal{Y}_{2t} = f_1(I_{1t}) + f_2(I_{2t}) \quad \text{see (5.1c)}$$

where: I_{1t} is a step input: »no drug« vs. »drug«
I_{2t} is a pulse input: $(1-B)I_{1t}$, which can be interpreted as the change of experimental conditions

The observable time series »perceptual speed« is contaminated by noncontrollable effects N_t, so that we get the time series model according to (4.3c)

$$Y_t - L = \mathcal{Y}_t + N_t$$

In practical data analysis it is advised to test a fixed sequence of intervention models. Hints can be found into CLEARY & HAY (1980) and MÖBUS & NAGL (1983).

5. The comparison of the GLS-approach of GLASS, WILLSON & GOTTMAN with the Intervention-model of BOX & TIAO

As shown in (3.21) the approach of GLASS et al. (1975) is given as a matrix equation by

$$(5.1a)\qquad \underline{Y} = \underline{X}\,\underline{\beta} + \underline{\Psi}\,\underline{a}$$

with: $\underline{\Psi}$ = lower triangular matrix with ψ-weights

or in operator form either as

$$(5.1b)\qquad \underline{Y} = \underline{X}\,\underline{\beta} + \psi(B)\underline{a}$$

or for a single observation

$$(5.1c)\qquad Y_t = f_t(L', \beta_1, \ldots, \beta_{s+1}) + \psi(B)a_t$$

with: L' = L only in the case of nontruncated white noise process a_t. If a_t is set to $E(a_t)$ for $t < 0$, we have a truncated ψ-weight form and $L' \neq L$. In the further derivation it is not important wether we assume $L = L'$ or $L \neq L'$.

Equation (5.1) represents the general form for testing s+1 effects of one or more interventions. The approach of BOX & TIAO (1975) for testing s+1 effects of one intervention is (4.3)

$$(5.2)\qquad Y_t - L = \mathcal{Y}_t + \psi(B)a_t = \frac{\Omega(B)}{\delta(B)} \cdot I_t + \psi(B)a_t$$

If there are more than one interventions, with more than s+1 effects, we are allowed to extend the model as was done in (4.13). We want to show the equality of the functions f_t and $L + \mathcal{Y}_t$ for fixed parameters δ_i $(i = 1,\ldots,r)$ of $\delta(B)$:

$$(5.3a)\qquad L + \mathcal{Y}_t = f_t(L, \beta_1, \ldots, \beta_{s+1})$$

or in matrix form

$$(5.3b)\qquad {}_T\underline{L + \mathcal{Y}}_1 = {}_T\underline{X}_{s+2}\underline{\beta}_1$$

Without loss of generality we suppose that we have to test s+1 effects of one intervention-variable I_t. The general intervention-effect model $\mathcal{Y}_t$ can be written in the form (4.4d) or

$$(5.4)\qquad \mathcal{Y}_t - \delta_1\mathcal{Y}_{t-1} - \ldots - \delta_r\mathcal{Y}_{t-r} = [I_{t-b}\ I_{t-b-1} \ldots I_{t-b-s}]\ \underline{\omega}$$

where $\underline{\omega}' = [\omega_o\ -\omega_1 \ldots -\omega_s]$

or as

$$(5.5a)\ \mathcal{Y}_t = [I_{t-b}\ I_{t-b-1} \ldots I_{t-b-s}]\ \underline{\omega} + \delta_1\mathcal{Y}_{t-1} + \ldots + \delta_r\mathcal{Y}_{t-r}$$

In analogy to (5.4) we can write

$$(5.5b)\ \mathcal{Y}_{t-1} = [I_{t-b-1}\ I_{t-b-2} \ldots I_{t-b-s-1}]\underline{\omega} + \delta_1\mathcal{Y}_{t-2} + \ldots + \delta_r\mathcal{Y}_{t-r-1}$$

$$(5.5c)\ \mathcal{Y}_{t-2} = [I_{t-b-2}\ I_{t-b-3} \ldots I_{t-b-s-2}]\ \underline{\omega} + \delta_1\mathcal{Y}_{t-3} + \ldots + \delta_r\mathcal{Y}_{t-r-2}$$

. . . .

$$(5.5d)\ \mathcal{Y}_{t-r} = [I_{t-b-r}\ I_{t-b-r-1} \ldots I_{t-b-s-r}]\ \underline{\omega} + \delta_1\mathcal{Y}_{t-r-1} + \ldots + \delta_r\mathcal{Y}_{t-2r}$$

In principle we need for the computation of $\mathcal{Y}_t$ an infinite number of previous $\mathcal{Y}$'s. But in practice $\mathcal{Y}_t = 0$ for $t \leqq T_A+b$, because $I_t = 0$ for $t \leqq T_A$.
As an example we'll compute $\mathcal{Y}_t = h(I_t, I_{t-1}, \ldots)$ for $t = T_A+b+3$ and $r = 2$.

$$(5.6a)\ \mathcal{Y}_{T_A+b+3} = [I_{T_A+3}\, I_{T_A+2}\, I_{T_A+1}\, 0 \ldots 0]\ \underline{\omega} + \delta_1\mathcal{Y}_{T_A+b+2} + \delta_2\mathcal{Y}_{T_A+b+1}$$

$$(5.6b)\ \mathcal{Y}_{T_A+b+2} = [I_{T_A+2}\, I_{T_A+1}\, 0 \quad 0 \ldots 0]\ \underline{\omega} + \delta_1\mathcal{Y}_{T_A+b+1}$$

$$(5.6c)\ \mathcal{Y}_{T_A+b+1} = [I_{T_A+1}\, 0 \quad 0 \quad 0 \ldots 0]\ \underline{\omega}$$

$$\begin{aligned}(5.7a)\ \mathcal{Y}_{T_A+b+3} = {} & [I_{T_A+3}\, I_{T_A+2}\, I_{T_A+1}\, 0 \ldots 0]\ \underline{\omega} \\ & + \delta_1 [I_{T_A+2}\, I_{T_A+1}\, 0 \quad 0 \ldots 0]\ \underline{\omega} \\ & + \delta_2 [I_{T_A+1}\, 0 \quad 0 \quad 0 \ldots 0]\ \underline{\omega} \\ & + \delta_1^2 [I_{T_A+1}\, 0 \quad 0 \quad 0 \ldots 0]\ \underline{\omega}\end{aligned}$$

Collecting terms of $\underline{\omega}$:

$$(5.7b)\ \mathcal{Y}_{T_A+b+3} = \left[\begin{array}{c|c|c|c} I_{T_A+3} & I_{T_A+2} & & \\ + \delta_1 I_{T_A+2} & + \delta_1 I_{T_A+1} & I_{T_A+1} & 0 \ldots 0 \\ + (\delta_1^2 + \delta_2) I_{T_A+1} & & & \end{array}\right] \underline{\omega}$$

Equation (5.7b) can be written shortly

$$(5.8)\quad \mathcal{Y}_{T_A+b+3} = [g_{T_A+b+3,1}\ g_{T_A+b+3,2}\ g_{T_A+b+3,3} \ldots g_{T_A+b+3,s+1}]\ \underline{\omega}$$

In general we can express $\mathcal{Y}_t$ similar to the example (5.8)

$$(5.9)\qquad \mathcal{Y}_t = [g_{t,1}\ g_{t,2} \ldots g_{t,s+1}]\ \underline{\omega} \quad \text{for all } t > T_A + b$$

$$= \underline{g}_t'\ \underline{\omega}$$

The k–th component $g_{t,k}$ $(k = 1,\ldots,s+1)$ of the vector g_t' can be computed as

$$(5.10)\qquad g_{t,k} = I_{t-b-k+1} + \sum_{l=1}^{r} \delta_l g_{t-l,k}$$

where $g_{t,k} = 0$ for $t \leqq T_A + b + k - 1$

Now we define $\beta_o = L$, $\beta_1 = \omega_o$ and $\beta_j = -\omega_{j-1}$ $(j = 2,\ldots,s+1)$ and are able to write down the matrix equation (5.11)

$$(5.11a)\qquad {}_T\underline{L} + \underline{\mathcal{Y}}_1 = {}_T\underline{G}_{s+1}\underline{\beta}_1$$

or explicitly

$$(5.11b)\quad \begin{bmatrix} L + \mathcal{Y}_1 \\ \vdots \\ L + \mathcal{Y}_{T_A+b} \\ \hline L + \mathcal{Y}_{T_A+b+1} \\ L + \mathcal{Y}_{T_A+b+2} \\ \vdots \\ L + \mathcal{Y}_T \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & 0 & 0 & \ldots & 0 \\ \hline 1 & g_{T_A+b+1,1} & 0 & \ldots & 0 \\ 1 & g_{T_A+b+2,1} & g_{T_A+b+2,2} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & g_{T,1} & g_{T,2} & \cdots & g_{T,s+1} \end{bmatrix} \begin{bmatrix} L \\ \beta_1 \\ \vdots \\ \beta_{s+1} \end{bmatrix}$$

The elements $g_{t,k}$ of the matrix $\underline{G}$ are fixed, because we assumed that the δ's are known a priori. In this case the design-matrix $\underline{X}$ can be set to $\underline{G}$ and (5.3) holds. Therefore the approaches of GLASS, WILLSON & GOTTMAN (1975) and BOX & TIAO (1975) are conditionally equivalent. This means that equivalence can be met only in the case of fixed δ parameters.

When testing intervention hypothesis with the approach of BOX & TIAO (1975) we don't need any a priori information about the δ's. We estimate both the δ's and the ω's. If we try to evaluate the intervention-effect along GLASS et al., we have to know the estimates $\hat{\delta}$, to specify the design-matrix $\underline{X}$. This is shown in the following example.

The transfer function model

$$(5.12)\qquad \mathcal{Y}_t = \frac{\omega_o}{1 - \delta_1 B} \cdot P_t$$

with $0 < \delta_1 < 1$ and $P_t = \begin{cases} 0 \text{ for } t \neq T_A + 1 \\ 1 \text{ for } t = T_A + 1 \end{cases}$

describes an abrupt but temporary intervention effect. It follows from equation (5.12)

$$(5.13)\qquad \mathcal{Y}_t = \delta_1 \mathcal{Y}_{t-1} + \omega_o P_t$$

and we get

$$(5.14)\qquad \mathcal{Y}_t = \begin{cases} 0 & \text{for } t \leqq T_A \\ \delta_1^{t-T_A-1} \cdot \omega_o & \text{for } t > T_A \end{cases}$$

The pattern of effect may be called a »decaying spike«. The parameter δ_1 is to be interpreted as the momentary rate of decay. When δ_1 is large, say $\delta_1 = 0.9$, the effect persists for a long period of time (1.0, 0.9, 0.81, ...). If $\delta_1 = 0.5$ the decay is faster (1.0, 0.5, 0.25, ...). Testing the same hypothesis with the approach of GLASS et al. (1975) we have to know a priori the exact value of δ_1 in the design-matrix $\underline{X}$:

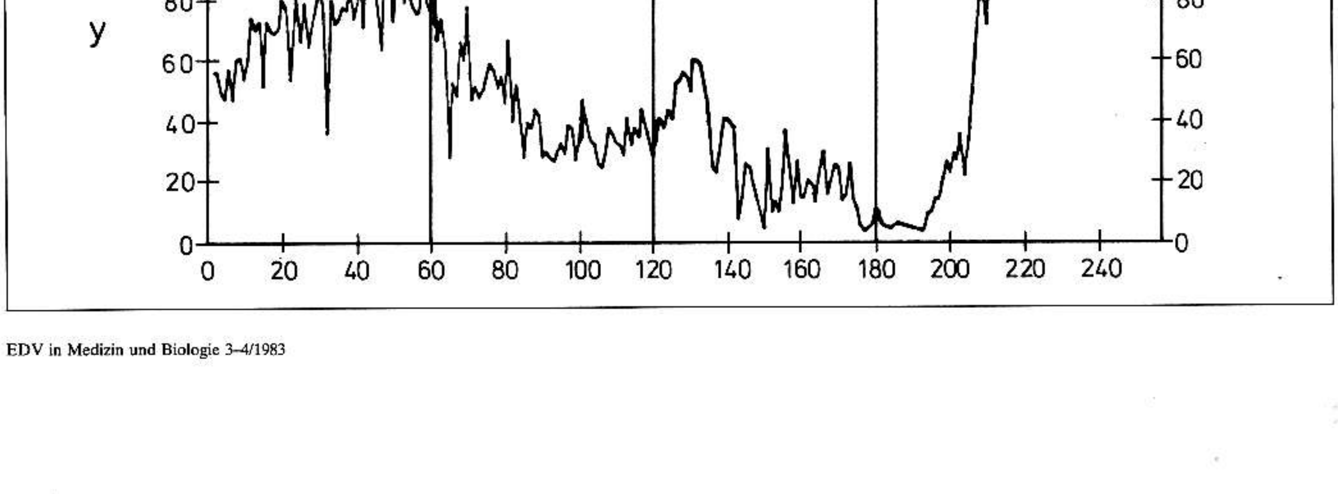
$$(5.15)\qquad \underline{X} = \begin{matrix} 1 \\ \vdots \\ T_A \\ \\ \cdot \\ \cdot \\ \cdot \\ \vdots \\ T \end{matrix} \begin{bmatrix} 1 & 0 \\ \vdots & \vdots \\ 1 & 0 \\ \hline 1 & 1 \\ 1 & \delta_1 \\ 1 & \delta_1^2 \\ \vdots & \vdots \\ 1 & \delta_1^{T-T_A-1} \end{bmatrix}$$

This example has shown that the statistical hypothesis in the approach of GLASS, WILLSON & GOTTMAN (1975) has to be formulated more precisely than in the approach of BOX & TIAO (1975).

6. Illustrative analysis for an ARIMA(0,1,1)-model

The data set to illustrate the testing of intervention effects stems from a study by Meffert (see HOLTZMANN, 1963). One single schizophrenic patient's performance on a perceptual speed task was observed for 245 days (Fig. 4).

Fig. 4. Perceptual speed of a schizophrenic before, during and after drug administration.



During the first 60 days (phase I), the patient daily received a placebo drug, and during the second 60 days (phase II) chlorpromazine. From day 120 to day 180 (phase III) chlorpromazine was continued but with electroshock therapy superimposed. The final 65-day period (phase IV) was similar to the first. The analysis of the autocorrelations and partial correlations of every single phase shows that we can suggest an ARIMA(0,1,1)-model:

(6.1) $(1-B)(Y_t - L) = (1 - \theta_1 B)a_t$

The maximum likelihood (ML) estimation of θ_1 in phase I to III is approximately 0.76. Only for the last 65-day period θ_1 was estimated with 0.22.

The last estimation of θ_1 is significantly different from $\hat{\theta}_1 = 0.76$. This is an indication for a change in the model. Therefore we restrict the analysis to the effects of the first and second intervention.

The graph of the data shows some instability of level, which is typical for a nonstationary process. It seems also obvious that the treatments had markedly different effects upon the client. The introduction of the tranquilizer led to a downward shift in level of the series. The second intervention shows two effects, first a transient upward shift in level over a time period of 11 days and second a dynamic decreasing effect. Now we are testing the described intervention hypothesis with the model

(6.2) $$(1-B)(Y_t - L) = \omega_o S_t^{(60)} + \omega_1(S_t^{(120)} - S_t^{(131)}) + \frac{\omega_2}{1 - \delta_1 B} \cdot B^{12} \cdot S_t^{(120)} + (1 - \theta_1 B)a_t$$

Because of $(1-B)(Y_t - L) = (1-B)Y_t$ we need not estimate the levelparameter L or the constant θ_o. The ML estimation of the parameters are given in:

(6.3)

	MLE	t-statistic
$\hat{\theta}_1$	0.764	15.570
$\hat{\omega}_o$	–21.986	–3.656
$\hat{\omega}_1$	9.137	2.040
$\hat{\omega}_2$	–3.071	–3.120
$\hat{\delta}_1$	0.922	14.931

With the a priori information about δ we can estimate the parameters L, ω_o, ω_1, ω_2 of the model (6.2) with the general linear model approach of GLASS et al. (1975). The designmatrix $\underline{X}$ is given by:

(6.4) $$\underline{X} = \begin{array}{r} \\ 1 \\ 2 \\ \vdots \\ 60 \\ 61 \\ \vdots \\ 120 \\ 121 \\ \vdots \\ 131 \\ 132 \\ 133 \\ 134 \\ \vdots \\ 180 \end{array} \begin{bmatrix} L & \omega_o & \omega_1 & \omega_2 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 0 & 0 \\ \hline 1 & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 0 & 0 \\ \hline 1 & 1 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1+\delta_1 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 1 & 0 & \sum_{i=o}^{47} \delta_1^i \end{bmatrix}$$

Table (6.5) shows the results of the estimation with a priori known $\delta_1 = 0.92$:

(6.5)

	MLE	t-statistic
$\hat{L}$	54.06	8.98
$\hat{\omega}_o$	–21.83	–3.63
$\hat{\omega}_1$	9.09	2.00
$\hat{\omega}_2$	–3.09	–3.03

All the parameters are significantly different from zero on the $\alpha = 0.05$ level.

Appendix A

The derivation of the ψ-weights:

The general autoregressive operator $\varphi(B)$ is a polynominal in B of degree p+d:

(A1) $$\varphi(B) = \varphi_o - \varphi_1 B - \varphi_2 B^2 - \ldots - \varphi_{p+d}B^{p+d}$$
with $\varphi_o = 1$

We have to derive the coefficients of this polynominal. Using the Binominal Expansion Theorem, we are able to express the difference operator as:

(A2) $$(1-B)^d = \binom{d}{o} - \binom{d}{1}B + \binom{d}{2}B^2 - \binom{d}{3}B^3 + \ldots + (-1)^d\binom{d}{d}B^d = \sum_{r=o}^{d}\binom{d}{r}(-1)^r B^r$$

The autoregressive operator can be expanded as:

(A3) $$\Phi_p(B) = (1 - \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p) = \sum_{j=o}^{p}(-\Phi_j)B^j$$
with $-\Phi_o = 1$

We may combine (A2) with (A3), to get the general autoregressive operator

(A4) $$\varphi(B) = \left\{\sum_{r=o}^{d}\binom{d}{r}(-1)^r B^r\right\} \cdot \left\{\sum_{j=o}^{p}(-\Phi_j)B^j\right\}$$

If we collect the coefficients of B^k in (A4), we get

$$\varphi_o = -\Phi_o = 1$$

and for k = 1,…,p+d

(A5) $$\begin{aligned} \varphi_1 &= \Phi_1 + \binom{d}{1} \\ \varphi_2 &= \Phi_2 - \binom{d}{1}\Phi_1 - \binom{d}{2} \\ \varphi_3 &= \Phi_3 - \binom{d}{1}\Phi_2 + \binom{d}{2}\Phi_1 + \binom{d}{3} \\ \varphi_4 &= \Phi_4 - \binom{d}{1}\Phi_3 + \binom{d}{2}\Phi_2 - \binom{d}{3}\Phi_1 - \binom{d}{4} \\ &\vdots \\ \varphi_k &= \sum_{r=o}^{k}(-1)^{r+1}\binom{d}{r}(-\Phi_{k-r}) \qquad k = 0,1,\ldots,p+d \end{aligned}$$

where: $\binom{d}{r} = 0$ for r>d and $-\Phi_{k-r} = 0$ for k–r > p

Now we know the coefficients in $\theta_q(B)$ and $\varphi(B)$. Using (3.19) we can collect the coefficients of B^k on the left and right side of (3.19) in a similar fashion as we did in (A1) – (A5). The coefficients of B^k can be arranged to a recursive equation-system:

(A6) $$\begin{bmatrix} -\theta_o \\ -\theta_1 \\ -\theta_2 \\ -\theta_3 \\ -\theta_4 \\ \vdots \\ -\theta_h \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & \ldots & 0 \\ -\varphi_1 & 1 & 0 & 0 & 0 & \ldots & 0 \\ -\varphi_2 & -\varphi_1 & 1 & 0 & 0 & \ldots & 0 \\ -\varphi_3 & -\varphi_2 & -\varphi_1 & 1 & 0 & \ldots & 0 \\ -\varphi_4 & -\varphi_3 & -\varphi_2 & -\varphi_1 & 1 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ -\varphi_h & -\varphi_{h-1} & -\varphi_{h-2} & -\varphi_{h-3} & -\varphi_{h-4} & \ldots & 1 \end{bmatrix} \begin{bmatrix} \psi_o \\ \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \\ \vdots \\ \psi_h \end{bmatrix}$$

where: $-\theta_h = 0$ for h>q, $\psi_o = 1$ and $-\theta_o = 1$

Now the ψ-weights can be obtained recursively

$$
(A7)\quad
\begin{aligned}
\psi_o &= -\theta_o = 1\\
\psi_1 &= -\theta_1 + \psi_o\varphi_1\\
\psi_2 &= -\theta_2 + \psi_1\varphi_1 + \psi_o\varphi_2\\
\psi_3 &= -\theta_3 + \psi_2\varphi_1 + \psi_1\varphi_2 + \psi_o\varphi_3\\
&\text{- -}\\
\psi_h &= -\theta_h + \sum_{i=1}^{h}\psi_{h-i}\varphi_i \qquad h = 1,\ldots,t-1\\
&\text{with } \varphi_i = 0 \text{ for } i > p+d
\end{aligned}
$$

As an example we want to derive the ψ-weights for the nonstationary ARIMA(1,1,1)-process

$$
(A8)\quad
\begin{aligned}
(1 - \Phi_1B)(1-B)(Y_t - L) &= (1 - \theta_1B)a_t\\
(1 - (1+\Phi_1)B + \Phi_1B^2)(Y_t - L) &= (1 - \theta_1B)a_t\\
\updownarrow \qquad \updownarrow \qquad \updownarrow \qquad &\\
(\varphi_o \; - \; \varphi_1B \; - \; \varphi_2B^2)(Y_t - L) &= (1 - \theta_1B)a_t
\end{aligned}
$$

The coefficients of the general autoregressive operator are

$$
(A9a)\quad
\begin{aligned}
\varphi_o &= 1\\
\varphi_1 &= (1 + \Phi_1)\\
\varphi_2 &= -\Phi_1
\end{aligned}
$$

The same results could be obtained by (A5)

$$
(A9b)\quad
\begin{aligned}
\varphi_o &= 1\\
\varphi_1 &= (-1)^1\tbinom{d}{o}(-\Phi_1) + (-1)^2\tbinom{d}{1}(-\Phi_o) = \Phi_1 + 1\\
\varphi_2 &= (-1)^1\tbinom{d}{o}\underbrace{(-\Phi_2)}_{=0} + (-1)^2\tbinom{d}{1}(-\Phi_1) + (-1)^3\underbrace{\tbinom{d}{2}}_{=0}(-\Phi_o) = -\Phi_1
\end{aligned}
$$

Inserting the φ_o, φ_1, φ_2 into (A7) we get the ψ-weights

$$
\begin{aligned}
\psi_o &= 1 && = \frac{1-\theta_1}{1-\Phi_1} + \frac{\theta_1-\Phi_1}{1-\Phi_1}\cdot\Phi_1^o\\
\psi_1 &= \Phi_1 + (1-\theta_1) && = \frac{1-\theta_1}{1-\Phi_1} + \frac{\theta_1-\Phi_1}{1-\Phi_1}\cdot\Phi_1^1\\
\psi_2 &= \Phi_1(\Phi_1 + (1-\theta_1)) + (1-\theta_1) && = \frac{1-\theta_1}{1-\Phi_1} + \frac{\theta_1-\Phi_1}{1-\Phi_1}\cdot\Phi_1^2\\
\psi_3 &= \Phi_1(\Phi_1(\Phi_1 + (1-\theta_1))) + (1-\theta_1) && = \frac{1-\theta_1}{1-\Phi_1} + \frac{\theta_1-\Phi_1}{1-\Phi_1}\cdot\Phi_1^3\\
\vdots\quad & \quad\vdots && \quad\vdots\\
\psi_j &= \Phi_1^j + (1-\theta_1)\sum_{i=0}^{j-1}\Phi_1^i && = \frac{1-\theta_1}{1-\Phi_1} + \frac{\theta_1-\Phi_1}{1-\Phi_1}\cdot\Phi_1^j
\end{aligned}
$$

(A10)

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The authors' address: Prof. Dr. Claus Möbus, Fachbereich Mathematik/Informatik, Universität Oldenburg, Postfach, D-2900 Oldenburg.