Oldenburg Discussion
Papers in Economics

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Carsten Helm and Franz Wirl

V – 336 – 11

June 2011

Department of Economics
Universität Oldenburg, D-26111 Oldenburg

International Environmental Agreements: Incentive Contracts with Multilateral Externalities*

Carsten Helm[†] and Franz Wirl[‡]

June 1, 2011

Abstract

We consider how one party can induce another party to join an international emission compact given private information. Due to multilateral externalities the principal uses her own emissions besides subsidies to incentivize the agent. This leads to a number of non-standard features: Optimal contracts can include a boundary part, which is not a copy of the no contract outcome. Compared to this, a contract can increase emissions of the principal for inefficient types, and reduce his payoff for efficient types. Subsidies can be constant or even decreasing and turn negative, i.e., the agent reduces emissions and pays the principal.

Keywords: private information, multilateral externalities, mechanism design, restricted contracts, environmental agreements.

JEL: D82, Q54, H87

1 Introduction

In this paper we analyse optimal incentive contracts characterized by asymmetric information and multilateral externalities. The presentation focuses on the example of international climate change negotiations. However, several further applications exist such as fishing, transboundary pollution, common water resources as well as joint ventures and other team problems for which the output depends on the overall effort or investment level.

One of the most important issues of ongoing climate negotiations is how industrialized countries can induce developing countries to accept a contract in

*The authors acknowledge helpful comments from Fuhai Hong, Jon Strand and others after presenting a first draft at the Fourth World Congress of Environmental and Resource Economists, June 28 to July 2, 2010, Montreal, Canada as well as from Andreas Novak, Jun Honda and Klaus Eisenack.

†University of Oldenburg, Department of Economics and Law, 26111 Oldenburg, Germany, phone +49 441 798-4113, fax +49 441 798-4116, carsten.helm@uni.oldenburg.de.

‡University of Vienna, Center of Business Studies, Brünnerstr. 72, 1210 Wien, Austria, franz.wirl@univie.ac.at.

which both sides commit to binding emission targets. These negotiations are aggravated by the fact that countries have private information, especially about their costs of emissions — respectively their willingness-to-pay (WTP) for abatement —, which they present strategically in order to negotiate more favorable terms for themselves. Contract theory appears as an obvious tool to investigate this issue. In particular, we use the principal-agent model which assumes complete bargaining power of the principal, unilateral asymmetric information and binding agreements.

Obviously, these are strong simplifications of the rather complex climate change negotiations, which we now motivate in turn. First, most observers would agree that industrialized countries are in a first-mover position and have more bargaining power than developing countries. Second, there is probably less information about the WTP in developing countries for several reasons: there exists less reliable scientific studies about the damages from climate change and the adaptation potential, the public press is less well developed, political processes are less transparent, and there is less domestic action which may reveal the underlying preferences (Mäler 1989).

Third, although the Kyoto Protocol lacks a stringent enforcement mechanism (Finus 2008), this has become an important issue for the negotiations of a Post-Kyoto Protocol; and there are other examples of binding international agreements such as the WTO/GATT or the Montreal Protocol on Substances that Deplete the Ozone Layer. Moreover, especially economists often argue that an international system of tradable permits should become a cornerstone of a successful climate change policy. Obviously, to prevent participants from overselling permits such a system needs a credible enforcement mechanism; hence it presumes the possibility of binding agreements.

Given this assumption, the non-cooperative coalition theory approach (e.g., Carraro and Siniscalco 1993; Barrett 1994), which is perhaps the most widely used approach to analyse international environmental agreements (IEAs), would predict the first-best solution. This would also be the outcome of a standard coalition model with only two-players. The reason is that the inefficient solution that usually results in these models is driven by the interaction of the countries' strategic participation decisions without commitment. By contrast, we assume binding participation decisions so that this mechanism is absent in our model. Instead, inefficiencies are caused by private information. To analyse this, a two-player setup is sufficient and strengthens the focus. Given the complexity of the analysis and the novelty of this approach, it seems a reasonable first pass to better understand the strategic effects of private information in the negotiation of climate treaties.

The prevalence of uncertainty in the climate change context has been widely acknowledged and several papers examine its implications for the negotiation of IEAs (e.g., Kolstad 2007; Morath 2010). However, applications of contract theory with its focus on asymmetric information are rare, although this has been pointed out as a valuable extension long ago (e.g., Carraro and Siniscalco 1993, 327). An exception are Caparrós, Péreau, and Tazdaït (2004), who consider a bargaining model in which Northern countries negotiate about transfers in exchange for a given level of emission reductions from Southern countries, which have private information about the minimum amount of transfers that they would accept. There is also a literature that analyses the role of private information about emission reduction projects for joint implementation and the clean development mechanism (e.g., Hagem (1996), Montero (2000), Fischer (2005)). For the case of sulphur emissions in Europe, Huber and Wirl (1996) consider the optimal incentive contract between a West European country (the principal) that faces high damages and an East European country (the agent) that has high emissions.

Apart from the analysis of climate change negotiations, we contribute to the contract theory literature by extending the principal-agent model to an environment of multilateral externalities between the principal and the agent. This allows the principal to use not only the standard instrument of subsidies to incentivize the agent, but also her own emissions. An additional consequence of multilateral externalities is that the agent's outside option depends on his type (e.g., his WTP for emission abatement) because it affects the players' strategic interaction in the case of contract failure. This leads to countervailing incentives (Lewis and Sappington 1989). On the one, the agent has an incentive to overstate his costs of climate damages in order to get a higher compensation. On the other hand, he wants to understate his costs in order to pretend a better outside option. In combination, these effects of multilateral externalities lead to a number of results that differ from the standard screening model – such as the non-linear pricing problem.

Type-dependent outside options have also been analysed in other papers. For a general analysis see Maggi and Rodriguez-Clare (1995) and Jullien (2000). Related applications from environmental economics are Wirl and Huber (2005a, 2005b) as well as Huber and Wirl (1998), where a pollutee (the principal) offers the polluter a subsidy in exchange for a reduction in pollution. McKelvey and Page (2002) consider a similar setting but focus on bargaining outcomes. Although these studies also consider externalities, they are only unilateral from the agent to the principal. Segal (1999) analyses multilateral externalities, but these are restricted to a set of $n \geq 2$ agents, which are offered a contract by the principal (see also Gomes (2005), Genicot and Ray (2006)). Accordingly, in both cases there is no externality from the principal to the agent, which is crucial for our paper.

A particular feature of this combination of multilateral externalities and type dependent reservation prices is that an 'interior boundary' solution can be part of the optimal contract. More precisely, types receiving a (boundary) contract whose payoff equals their reservation price are prescribed (interior) emissions that differ from the no contract solution. Intuitively, by reallocating emissions so as to equalize marginal abatement costs the principal can raise the joint surplus and use this to pay subsidies. Further surprising properties can characterize the optimal contract: First, although contracts raise the principal's expected payoff, she may loose from contracts with efficient types. Second, subsidies can be negative, i.e. the agent is asked to reduce emissions yet pays the principal for issuing this order. Third, subsidies can decline in the efficiency of the agent or, respectively, increase with emissions so that countries which reduce emissions less are rewarded by higher monetary payments. Finally, emissions and subsidies in the optimal contract allocation can be implemented alternatively by a system of competitive permit trading.

The outline of the remaining paper is as follows. Section 2 introduces the model. Section 3 considers the reference situations of no contracts and Pareto-efficient cooperation. Section 4 analyses the optimal contract, and section 5 its alternative implementation via tradable permits. Section 6 analyses some properties of the optimal contract in more detail and discusses a specific example. Finally, section 7 concludes by interpreting the main results in the climate change context. An appendix contains all proofs.

2 The model

We analyse contractual mitigation of a global public bad with heterogenous damages that are private information. Specifically, consider two groups of countries – say industrialized and developing – signing an emission compact. The two players (countries) are indexed alternatively by $i, j = 1, 2$, $i \neq j$, where we always use subscript 1 for the principal ('she') and subscript 2 for the agent ('he'). Countries have benefits $B_i(x_i)$, which are increasing and strictly concave in their own emissions, $x_i \in \mathbb{R}_+$, and satisfy the Inada conditions. In addition, they face costs $\theta_i D(X)$ that are strictly increasing and convex in aggregate emissions, $X := x_1 + x_2$, and also depend on a country-specific damage parameter $\theta_i \in (0, 1]$. When we consider emission reductions, we often refer to $B_i'(x_i)$ as the marginal cost of emission abatement and to $\theta_i D'(X)$ as the marginal benefit of abatement. A country's payoff from emissions is

$$V_i := B_i(x_i) - \theta_i D(X), \quad i = 1, 2. \tag{1}$$

We assume that the benefit functions, $B_i(x_i)$, and the damage function, $D(X)$, are common knowledge. In the climate change context, this could be interpreted as information that is publicly available from the IPCC or other sources. However, governments may have additional private information about the (anticipated) damages in their own country. Moreover, one can interpret θ_i more broadly as a valuation parameter that determines a country's WTP for abatement, which depends not only on physical damages, but also on the preferences of voters and politicians or lobbying activities. In the context of international environmental agreements the resulting WTP of industrialized countries is often much better known than that of developing countries for the reasons mentioned in the introduction.

We capture this in a stylized way by assuming that the principal's valuation parameter is common knowledge, and normalize it to $\theta_1 = 1$.[1] By contrast, the

[1] Otherwise, the reservation price of the agent depends also on the principal's private information parameter. In this case, it is either optimal for the principal to conceal or to reveal the type (Maskin and Tirole 1990). The following results extend to bilateral private information only if the latter applies.

agent's valuation $\theta := \theta_2$ is private information with a known distribution: f denotes the density with support $[\underline{\theta}, 1]$, F the cumulative distribution function, and $h := f/(1-F)$ the hazard rate that is assumed to be increasing in θ, i.e., $\dot{h} > 0$ (throughout the text we use dots to refer to the (total) derivative w.r.t. θ). Before we turn to the optimal contract, we determine the first-best solution and the outcome without contracts as reference points.

3 Emissions: first-best and out-of-contract

3.1 First-best emissions

First-best emissions, indicated by superscripts ^{1}, follow from maximizing the aggregate payoff $V_1 + V_2$. They satisfy Samuelson's rule for public goods that the sum of countries' marginal benefits from emission abatement (the public good) is set equal to its marginal costs,

$$B_i'\left(x_i^1\right) = (1+\theta)\, D'\left(X^1\right), \quad i = 1, 2. \tag{2}$$

Implicit differentiation of these first-order conditions yields

$$\dot{x}_i^1 = \frac{D' B_j''}{B_i'' B_j'' - \left(B_i'' + B_j''\right)(1+\theta)\, D''} < 0, \quad i, j = 1, 2,\ j \neq i. \tag{3}$$

so that the principal's and the agent's emissions are both decreasing in θ. Intuitively, higher damages are associated with less emissions.

3.2 Out-of-contract emissions

In the non-cooperative solution without contracts, each country chooses its emissions as a best response to those of the other country. Two reasonable specification of the timing exist: (*i*) countries choose their emissions simultaneously (Cournot scenario), or (*ii*) country 1 (the principal) acts as leader and chooses its emissions first (Stackelberg scenario). The Cournot scenario is more widely used in the climate change literature. However, the Stackelberg scenario is more in line with the principal-agent framework, in which the principal can commit herself to a certain emission level.

In both cases, the agent's emissions contingent on the principal's choice of emissions and his type are (superscripts ^{0} indicate the non-cooperative solution)

$$x_2^0(x_1, \theta) = \arg\max_{x_2} B_2(x_2) - \theta D\left(x_1^0 + x_2\right) \tag{4}$$

so that by implicit differentiation of the first-order condition

$$\dot{x}_2^0 = \frac{D'}{B_2'' - \theta D''} < 0 \quad \text{and} \quad \frac{\partial x_2^0}{\partial x_1} = \frac{\theta D''}{B_2'' - \theta D''} \in (-1, 0]. \tag{5}$$

Accordingly, an increase of the principal's emissions (weakly) lowers the agent's emissions but below the principal's expansion.

The principal does not know the agent's valuation θ so that $\dot{x}_1^0 = 0$. In particular, she takes a Bayesian perspective and maximizes her (expected) payoff over all possible values of θ,

$$\max_{x_1} B_1(x_1) - \int_{\underline{\theta}}^{1} D\left(x_1 + x_2^0(x_1, \theta)\right) dF(\theta). \tag{6}$$

The resulting first-order conditions are for Cournot interaction:

$$B_1'\left(x_1^0\right) = \int_{\underline{\theta}}^{1} D'\left(x_1^0 + x_2^0(x_1, \theta)\right) dF(\theta), \tag{7}$$

and for Stackelberg interaction:

$$B_1'\left(x_1^0\right) = \int_{\underline{\theta}}^{1} D'\left(x_1^0 + x_2^0(x_1, \theta)\right)\left(1 + \frac{\partial x_2^0}{\partial x_1}\right) dF(\theta). \tag{8}$$

In the latter case, the agent (weakly) reduces his emissions in response to emissions of the principal (by 5) who, therefore, chooses (weakly) higher emissions than in the Cournot scenario. However, for the case of linear damage costs that we consider in section 6, $D'' = 0$ so that $\partial x_2^0 / \partial x_1 = 0$, and the Cournot and Stackelberg outcome coincide. This reflects that with constant marginal damages countries' non-cooperative emission choices are independent of each other.

Aggregate emissions will always exceed the first-best. The reason is that each player and consequently also player 1 accounts just for own harm, and any strategically motivated increase will only be inadequately compensated by player 2 due to (5). However, one can think of scenarios where player 2's emission as a consequence of a strategic Stackelberg move of high emissions falls below his first-best allocation.

4 Optimal contract

We now turn to the analysis of the optimal contract, using the standard assumption that the principal makes a take-it-or-leave offer to the agent. In particular, the principal writes a contract $\{x_1(\theta), x_2(\theta), s(\theta), \theta \in [\underline{\theta}, 1]\}$, specifying emissions and transfers $s(\theta) \in \mathbb{R}$ from the principal to the agent that maximize her expected payoff:

$$\max_{x_1(\theta), x_2(\theta), s(\theta)} \int_{\underline{\theta}}^{1} \left[B_1(x_1(\theta)) - D(x_1(\theta) + x_2(\theta)) - s(\theta)\right] dF(\theta). \tag{9}$$

This optimization faces the usual incentive compatibility and participation constraints. The revelation principle ensures that one can restrict attention to contracts inducing agents to tell the truth. Let θ denote the true and $\hat{\theta}$ the reported type, then incentive compatibility requires

$$U(\theta) := U(\theta, \theta) \geq U\left(\hat{\theta}, \theta\right) \ \forall \theta, \hat{\theta} \in [\underline{\theta}, 1], \tag{10}$$

where

$$U\left(\hat{\theta}, \theta\right) := B_2\left(x_2\left(\hat{\theta}\right)\right) - \theta D\left(x_1\left(\hat{\theta}\right) + x_2\left(\hat{\theta}\right)\right) + s\left(\hat{\theta}\right) \tag{11}$$

is the payoff of a type θ who pretends to be of type $\hat{\theta}$. This implies the following first-order conditions for the agent's optimization problem when reporting his type:

$$B_2'\left(x_2\left(\hat{\theta}\right)\right)\frac{dx_2\left(\hat{\theta}\right)}{d\hat{\theta}} - \theta D'\left(X\left(\hat{\theta}\right)\right)\left[\frac{dx_1\left(\hat{\theta}\right)}{d\hat{\theta}} + \frac{dx_2\left(\hat{\theta}\right)}{d\hat{\theta}}\right] + \frac{ds\left(\hat{\theta}\right)}{d\hat{\theta}} = 0. \tag{12}$$

Evaluated at $\hat{\theta} = \theta$ this is called the "local incentive compatibility constraint", which can also be written in terms of the agent's payoff $U(\theta)$ as

$$\dot{U}(\theta) = -D\left(x_1(\theta) + x_2(\theta)\right) < 0. \tag{IC}$$

If the agent declines the offered contract, then the out-of-contract solution as described in section 3.2 obtains. Accordingly, the agent's participation (or individual rationality) constraint is

$$U(\theta) \geq R(\theta) := \max_{x_2} B_2(x_2) - \theta D\left(x_1^0 + x_2\right) \ \forall \theta \in [\underline{\theta}, 1], \tag{IR}$$

where the principal's choice x_1^0 follows from (7) or (8), depending on whether one assumes Cournot or Stackelberg interaction. The envelope theorem implies

$$\dot{R}(\theta) = -D\left(x_1^0 + x_2^0\left(x_1^0, \theta\right)\right) < 0, \tag{13}$$

and thus a reservation price that declines in the agent's type.

It is well known (see, e.g., Bolton and Dewatripont (2005)) that the principal's problem of finding the optimal contract can be stated equivalently as maximizing the principal's expected payoff (9) subject to the local incentive compatibility constraint (IC), the participation constraint (IR), and the monotonicity constraint, $\dot{x}_2(\theta) \leq 0$. In contrast to the standard solution procedure, however, we can not simply replace the general participation constraint by that of the lowest (or highest) type because the agent's outside option depends on his type.

No general results exist for this class of problems (Maggi and Rodriguez-Clare 1995). Therefore, several authors have used simplifying assumptions for the reservation utility, e.g. that it changes linearly in the agent's type (Lewis and Sappington 1989; Feenstra and Lewis 1991). Such a simplification is not feasible in our model because the out-of-contract solution and the associated reservation utility are determined endogenously. Hence we have to explicitly account for the participation constraint (IR) over the whole range of types.

In conclusion, solving (11) at $\hat{\theta} = \theta$ for $s(\theta)$ and substitution into (9), the optimal contract is the solution of the following optimal control problem (x_i are the controls and U is the state variable):

$$\max_{x_1(\theta), x_2(\theta)} \int_{\underline{\theta}}^{1} \left[B_1\left(x_1\left(\theta\right)\right) + B_2\left(x_2\left(\theta\right)\right) - \left(1+\theta\right) D\left(X\left(\theta\right)\right) - U\left(\theta\right) \right] f\left(\theta\right) d\theta, \tag{14}$$

subject to the 'dynamic' constraint (IC), the 'state' constraint (IR) and the monotonicity constraint, $\dot{x}_2(\theta) \leq 0$. The proposition below summarizes the major properties of the optimal contract.

Proposition 1

(i) *Abatement is undertaken cost-effectively, i.e.,*

$$B_1'\left(x_1\left(\theta\right)\right) = B_2'\left(x_2\left(\theta\right)\right) \text{ for all } \theta \in \left[\underline{\theta}, 1\right]. \tag{15}$$

(ii) *Emissions of the principal and of the agent are above the first-best – except at the top – and declining in* θ*; i.e.,* $\dot{x}_i < 0$ *for all* $\theta \in \left[\underline{\theta}, 1\right], i = 1, 2$*.*

(iii) *An interior solution of the contract (i.e., the participation constraint (IR) is not binding) is uniquely determined by the relaxed program (identified by superscript* ^{r}*),*

$$B_i'\left(x_i^r\left(\theta\right)\right) - \left(1 + g\left(\theta\right)\right) D'\left(X^r\left(\theta\right)\right) = 0, \qquad i = 1, 2, \tag{16}$$

where

$$g\left(\theta\right) := \theta - \frac{1}{h\left(\theta\right)} = \theta - \frac{1 - F(\theta)}{f\left(\theta\right)} \leq \theta, \quad \text{and } \dot{g} > 1. \tag{17}$$

No interior solution exists for $g(\theta) \leq -1$*.*

(iv) *A boundary solution of the contract (i.e., along which the participation constraint (IR) is binding; identified by superscript* ^{b}*) is uniquely determined by (15) and*

$$X^b\left(\theta\right) = X^0\left(\theta\right); \tag{18}$$

i.e., aggregate emissions are the same as in the out-of-contract solution. Yet individual emissions differ, $x_i^b\left(\theta\right) \neq x_i^0\left(\theta\right)$*, and are below relaxed program emissions,* $x_i^b\left(\theta\right) < x_i^r\left(\theta\right)$*.*

A proof is given in the appendix so that the following discussion can focus on economic intuition. There are two ways to raise the overall surplus as compared to the out-of-contract solution: first, increasing cost effectiveness by reducing differences in marginal abatement costs; second, internalizing the externality by reducing overall emissions. The incentive compatibility constraint (IC) depends only on aggregate emissions. Therefore, the principal uses the first instrument to the full extent and allocates emissions so as to equalize marginal abatement costs. By contrast, the optimal internalization of the externality depends on the agent's type. In a private information context, revealing this type requires payment of an information rent, $U(\theta) - R(\theta)$, that accrues to the agent. Hence internalization will be incomplete, which is reflected in the difference $\theta - g(\theta) \geq 0$ when comparing the relaxed program solution (16) with first-best (2). Only for $\theta = 1$, which implies $g(\theta) = \theta$, emissions are first-best.

The result that the contract binds for low types and $\theta = 1$ is the efficient type crucially depends on the type-dependence of the agent's outside option $R(\theta)$. To see this, use (13) to write the incentive compatibility constraint (IC) in terms of the agent's information rent as

$$\dot{U}(\theta) - \dot{R}_2(\theta) = -D(X(\theta)) + D\left(X^0(\theta)\right). \tag{19}$$

For the moment, suppose that the reservation value were constant for all θ (i.e., the second term would be cancelled on both sides). Then the information rent would fall in θ, which reflects the agent's incentive to overstate its damages to obtain a higher compensation. However, once accounting for type-dependence, the agent also has a countervailing incentive to understate θ so as to pretend a better outside option. This effect suggests that the information rent should increase in θ, as it is reflected in the positive second term on the right-hand side of (19). Whenever the optimal contract (partly) internalizes the externality so that $X(\theta) < X^0(\theta)$, the second effect dominates.

As argued above, the principal gains from internalization so that the contract binds from below. Moreover, in the standard relaxed program solution it would bind only for the lowest type, while all others receive an information rent. If this leads to overall emissions below those out-of-contract for all types, it will be the optimal contract. However, the incentive compatibility constraint (IC) requires that lower types are associated with higher overall emissions. Therefore, it may happen that relaxed program emissions exceed those out-of-contract. From (16) this is the case if $1 + g(\theta)$ becomes sufficiently small, which depends on the distribution of θ.[2] Obviously, the principal then prefers a boundary contract in which overall emissions and the agent's payoff equal their respective values out-of-contract.

From (19), this boundary contract satisfies the agent's participation and monotonicity constraint. Hence types are revealed, and the principal can condition individual emissions on θ so as to equalize marginal abatement costs. Thereby she extracts at least the surplus that obtains from achieving cost-effectiveness. Observe that this result crucially depends on the presence of multilateral externalities, which allows the principal to reallocate individual emissions without affecting their overall level and, therefore, the type-specific damages that determine incentive compatibility.

[2] $1 + g(\theta)$ can even be negative for certain specifications, e.g., for linear increasing densities (mode at $\theta = 1$) that are sufficiently steep. In this case, no interior solution as given by (16) exists because that would require negative marginal benefits, which are ruled out by the Inada assumptions.

The preceding discussion already suggests that the interior and boundary parts of the contract are joined at the type for which overall relaxed program emissions are equal to those in the out-of-contract solution. To see that this is actually the case, define with

$$\theta_{IR} := max\{\theta_0 : X^r(\theta_0) = X^0(\theta_0)\} \tag{20}$$

the highest type at which aggregate relaxed program emissions, X^r, cross aggregate out-of-contract emissions, X^0.[3] Observe that this crossing must be from above because $X^r(1) < X^0(1)$ due to the first-best at $\theta = 1$. Moreover, denote by θ_m the 'marginal' type at which the interior or relaxed program solution (16) is joined with the boundary solution (15) and (18). In the appendix we show that the optimization problem leads to a concave Hamiltonian which implies that emissions of principal and agent are continuous functions of θ. This includes the marginal type so that

$$x_i^b(\theta_m) = x_i^r(\theta_m), \quad i = 1, 2. \tag{21}$$

Remembering that $X^b(\theta) = X^0(\theta)$, it follows that the marginal type θ_m, if existing, is determined by the intersection of aggregate emissions under no contract with the counterpart implied by the relaxed program, i.e., $\theta_m = \theta_{IR}$.

Proposition 2

(i) *The boundary and interior parts of the optimal contract are joined at*

$$\theta_m = \theta_{IR}, \tag{22}$$

and the relaxed program solution satisfies the participation constraint for all $\theta \geq \theta_m$. At this junction not only aggregate but also the principal's and agent's emissions are continuous. As a consequence, $\theta_{IR} > \underline{\theta}$ is necessary and sufficient that a boundary solution applies to a subset of types.

(ii) *Along the interior part of the optimal contract, overall emissions are below their out-of-contract level.*

(iii) *However, emissions of either the principal or the agent can exceed their out-of-contract level, along the boundary and interior parts of the optimal contract.*

[3] Although none or a unique intersection seems very intuitive (we could not construct a contradicting example), the multilateral externalities make it hard to prove this property in general and would require assumptions about third order derivatives and about the hazard rate (more precisely, about $\dot{g}$). If multiple crossings were existing (this requires at least three since the last one must be from above and so is the first by the pole of x_i^r) then segments of an interior solution bordering on two boundary solutions cannot characterize an optimal contract as this would lead to a discontinuity in U, violating incentive compatibility. Instead, the optimal interior contract must be restricted to types above the last intersection, as expressed in Proposition 2 below.

Statement (*ii*) follows straightforwardly from the internalization of the externality, but result (*iii*) is surprising. The reason is that high types care more about emissions so that it becomes less attractive for them to underreport their willingness-to-pay, θ, if this triggers an increase of emissions. Therefore, the principal can reduce the information rent by raising emissions for low types. Moreover, from proposition 1 the allocation of overall emissions between the two countries is governed solely by the equalization of marginal abatement costs. Hence if the abatement cost functions are very asymmetric, one country may undertake most of the abatement while the other has substantially higher emissions. For the principal we have the additional effect that her out-of-contract emissions are based on the expected type, while in the contract solution low types are associated with higher emissions.

The agent's payoffs associated with the optimal contract are illustrated in Fig. 1 for a simple example of logarithmic benefits, linear damages[4] and a uniform distribution,

$$B_1(x_1) = \ln x_1,\ B_2(x_2) = a \ln x_2,\ D(X) = X,\ f(\theta) = \frac{1}{1-\underline{\theta}},\ a > 0. \tag{23}$$

The bold curve represents the agent's contract payoff, the dashed curve the hypothetical relaxed program payoff for boundary types (U^r) and the out-of-contract payoff for interior types (R). At the junction of boundary and interior solution, the payoff associated with the relaxed program, denoted by U^r (and thus only hypothetical to the left of θ_m), is more convex than R since

$$\ddot{U}^r(\theta_m) = -D'\dot{X}^r(\theta_m) > -D'\dot{X}^0(\theta_m) = \ddot{R}(\theta_m) \tag{24}$$

due to $\dot{X}^r(\theta_m) < \dot{X}^0(\theta_m) < 0$. Therefore, using $U(\theta_m) = R(\theta_m)$ and $\dot{U}(\theta_m) = \dot{R}(\theta_m)$, R envelops U^r at θ_m.

5 Alternative implementation via permits

The optimal contract that we have described above (now indicated by superscript ^{c}) specifies emissions and subsidies, $\{x_1^c(\theta), x_2^c(\theta), s^c(\theta), \theta \in [\underline{\theta}, 1]\}$. In the following we show that the same allocation can be implemented by a different contract that specifies permit endowments, $\{\omega_1(\theta), \omega_2(\theta), \theta \in [\underline{\theta}, 1]\}$. From a policy point of view this is interesting because a system of international permit trading often features prominently in scenarios for a future climate policy.

It is well known that competitive permit trading equalizes marginal abatement costs.[5] Therefore, if the overall permit number equals overall emissions in the optimal contract, i.e.,

$$\omega_1(\theta) + \omega_2(\theta) = X^c(\theta), \tag{25}$$

[4] Finus, Ierland, and Dellink (2006) show that discounted climate change damages that are linear in emissions are a good approximation of the figures in the DICE model (Nordhaus and Yang 1996), which models damages as a non-linear function of the change in termperature.

[5] With only two countries competitive trading would result if permit endowments are forwarded to firms, an assumption that is widely used in the literature (e.g., Helm 2003).

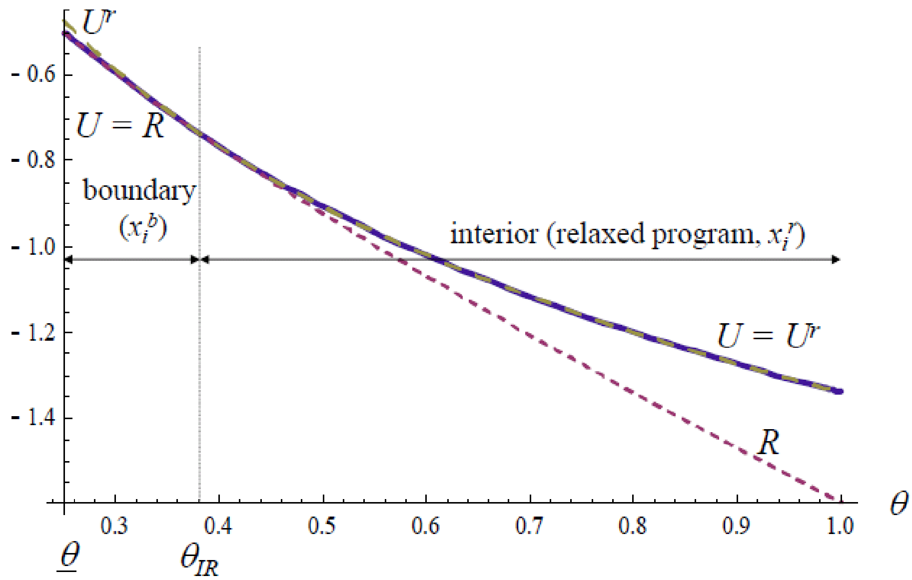

Figure 1: Agent‘s payoff with contract (U) and out-of-contract (R) for specification (23) with $\underline{\theta} = \frac{1}{4}$, and $a = \frac{1}{4}$.

then after-trade emissions are the same by proposition 1(i), i.e.,

$$x_1^p(\theta) = x_1^c(\theta), x_2^p(\theta) = x_2^c(\theta) \quad \text{for all } \theta \in [\underline{\theta}, 1], \tag{26}$$

where superscript ^{p} indicates the emissions after trading under the permit contract. Using this, profit maximization on the permit market implies an equilibrium permit price $\pi^p(\theta) = B_1'(x_1^c(\theta))$. Accordingly, if the permit endowment of an agent of type θ solves

$$[\omega_2(\theta) - x_2^c(\theta)] B_1'(x_1^c(\theta)) = s^c(\theta), \tag{27}$$

then also the same transfers as in the original contract result. Given this equivalence, the agent will truthfully reveal his type θ.

Proposition 3 *Emissions and transfers of the optimal contract can be implemented alternatively by a competitive permit market with an endowment allocation that satisfies (25) and (27).*

It is interesting to note that the principal sets $\omega_2(\theta)$ – i.e. the agent's decision variable that it wants to regulate – so as to induce a subsidy target $s^c(\theta)$; and that she can do so without affecting the incentive compatibility constraint. This is again a consequence of multilateral externalities due to which damages – and, therefore, incentive compatibility – depend only on overall emissions. Hence the principal can compensate changes in $\omega_2(\theta)$ by a corresponding change in her own endowment $\omega_1(\theta)$.

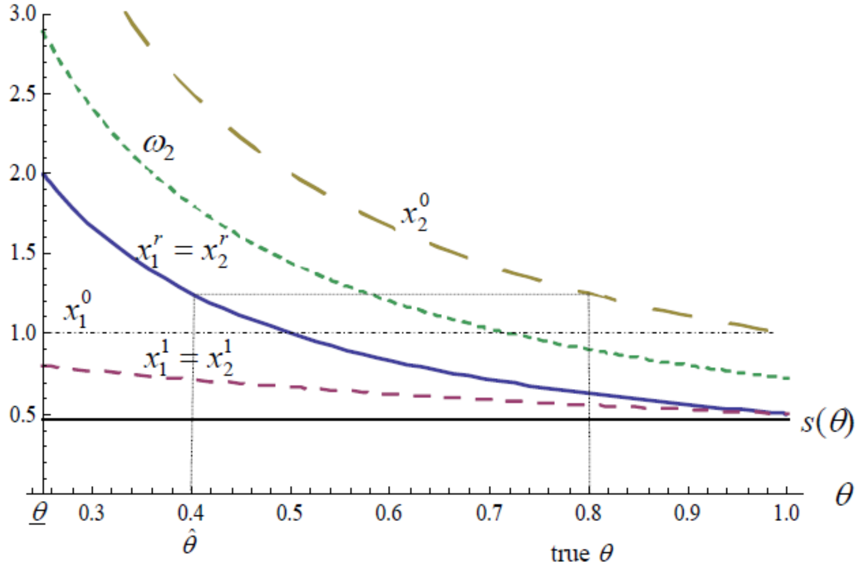

Figure 2: Emissions and subsidies for specification (23) with $\underline{\theta} = \frac{1}{4}$, and $a = 1$.

6 Example and discussion of optimal contract

We now return to the contract in emissions and subsidies because it is more similar to the standard screening contract. This facilitates a comparison and, thereby, enables us to highlight some further non-standard features that arise in contracting with multilateral externalities. These enable the principal to use not only subsidies but also her emissions as an incentive instrument, which plays a crucial role in the following elaborations.

In the discussion we will sometimes use the simple example (23). The payoffs for a specification of this example ($\underline{\theta} = 0.25, a = 0.25$) that leads to a contract with an interior and boundary part have already been depicted in Figure 1. Figure 2 shows emissions and subsidies for equal abatement costs ($a = 1$) because this simplifies the plots due to symmetric allocations of emissions. In this case, the interior solution is globally optimal. The figure also includes the allocation of permits to the agent, $\omega_2(\theta)$, in a permit contract (see proposition 3). Although the subsidy, $s(\theta)$, is constant across types, the number of permits that the agents sells, $\omega_2(\theta) - x_2^r(\theta)$ is decreasing. This reflects the higher permit price as emissions are reduced.[6]

The constant subsidies imply that the principal relies exclusively on her emissions to incentivize the agent. In the depicted specification this leads to emissions of the principal which exceed their out-of-contract level for all $\theta < 0.5$,

[6] An algebraic closed form solution for specification (23) including the permit allocation is available upon request from the authors.

an outcome that we have already discussed after proposition 2. We now analyse more systematically the role of subsidies in the optimal contract. These follow from (11) and (12) for $\hat{\theta} = \theta$, which implies that their level and slope depend in a non-trivial way on the type distribution and benefit functions. To disentangle these effects, we sometimes impose a particularly simple assumption for one of these determinants in order to focus on the other.

Proposition 4

(*i*) *If θ has a uniform distribution, then along an interior solution,*

$$-B_1''(x_1^r(\theta)) \gtreqless -B_2''(x_2^r(\theta)) \iff \dot{s} \lesseqgtr 0. \tag{28}$$

(*ii*) *Subsidies $s(\theta)$ can be positive, zero or negative, both along boundary and interior solutions.*

From the optimal contract we know that marginal benefits of emissions are equalized for all θ. Hence, for any two types θ, θ' we have

$$\int_{x_1^c(\theta)}^{x_1^c(\theta')} -B_1''(x_1)\,dx_1 = \int_{x_2^c(\theta)}^{x_2^c(\theta')} -B_2''(x_2)\,dx_2. \tag{29}$$

It follows that for a marginal increase of θ the reduction in emissions is lower for the actor that has the higher $|B_i''(x_i)|$. Intuitively, if the agent has a steeper marginal benefit function – i.e., $B_2''(x_2^r(\theta)) < B_1''(x_1^r(\theta))$ – then his marginal benefits are adjusted over a smaller interval to assure that they remain equalized for the higher θ. Accordingly, the principal's emissions are reduced by more than those of the agent. The agent benefits from this due to the externality from the principal so that subsidies – the standard incentive instrument – can be reduced.

The example in figure 2 depicts a situation with equal benefit functions. In this case $\dot{s} = 0$; hence the principal uses subsidies only to satisfy the lowest type's participation constraint, but not to incentivize the agent to reveal his type. In a model without externalities this would contradict the revelation principle and the implied incentive compatibility constraint. For example, in figure 2 an agent of type $\theta = 0.8$ could pretend $\hat{\theta} \approx 0.42$, exercise his emission level outside the contract, and still collect the constant subsidy $s(\hat{\theta})$. This cheating, however, is deterred by the implicit commitment to the principal's emission, because then she would also choose the higher emission level that corresponds to $\hat{\theta}$. This harms the agent, particularly those with significant environmental concerns.

Moreover, the principal's own emission reductions do not only serve as an incentive instrument, but they also reduce her damage costs. As a consequence, the principal may lower her own emissions so much that subsidies can even turn negative (statement (*ii*)); especially for high types which benefit most from the principal's emission reductions. In the specification on which figure 1 is based ($\underline{\theta} = 0.25, a = 0.25$), this is the case for types $\theta \gtrapprox 0.595$.

However, positive subsidies are necessary to get inefficient types to sign because they benefit only little from the principal's emission reductions. Therefore, subsidies are positive if there is a sufficient probability mass of inefficient types – for specification (23) and $a = 1$, if $\underline{\theta} < \ln 2$. Conversely, consider the limiting case where all types are close to 1. Then proposing an (almost) efficient allocation increases the total surplus substantially above the non-cooperative outcome, and the principal can use a negative subsidy to accrue the bulk of this surplus, including a part of the agent's gain from reduced emissions.

Not only the level but also the dynamics of the subsidy for interior solutions depend on the type distribution. Moreover, although the distribution has no impact on the out-of-contract allocation, it affects the critical type θ_m and, therefore, the range of types for which the boundary and interior solution applies.

Proposition 5

(i) *Consider two hazard rates $h_1(\theta)$ and $h_2(\theta)$. The hazard rate h_1 induces higher emissions along the interior solution for type θ, if and only if $h_1(\theta) < h_2(\theta)$.*

(ii) *Assuming equal benefit functions, the slope of subsidies along an interior solution is determined by the hazard rate of the prior relative to the hazard rate of the uniform distribution:*

$$h(\theta) \gtreqless \frac{1}{1-\theta} \iff \dot{s} \gtreqless 0. \tag{30}$$

Both statements are related to the notion of *hazard rate dominance*: given two densities, f_1 dominates f_2 iff

$$h(f_1(\theta)) \leq h(f_2(\theta)) \text{ for all } \theta. \tag{31}$$

This means that the probability of observing an outcome within a neighborhood of θ, conditional on the outcome being no less than θ, is smaller under f_1 than under f_2 *for all θs* (by contrast, statement (i) considers individual θs). Hazard rate dominance implies first order stochastic dominance of f_1 over f_2 and thus a higher expected value of θ. Comparing a monotonic density function f with the uniform distribution, then hazard rate dominance of f is equivalent to $\dot{f} > 0$, i.e., *an optimistic prior.* From the proposition it follows that the "more optimistic" prior induces higher emissions along the interior (relaxed) program for each type.

Intuitively, higher emissions along the relaxed program have two effects. On the one hand, more low types will receive a boundary solution, for which overall emissions are equal to those out-of-contract.[7] Hence for these low types the

[7] Remember that the critical type θ_m is defined as the highest type at which relaxed program emissions intersect out-of-contract emissions from above (from proposition 2 and (20)). If relaxed program emissions are higher for each type, then this intersection must be further to the right.

principal forgoes the benefits that would result from internalizing the externality. On the other hand, the principal has to pay a lower information rent to the interior types, which follows straightforwardly from the dynamics of the information rent (see 19). For a distribution that has less probability mass on low types and more on high types, the second effect dominates so that emissions increase.

Turning to statement (*ii*), remember that the highest type implements efficient effort. Furthermore, we have just shown that for a distribution which stochastically dominates the uniform one according to the hazard rate order (i.e., $h(\theta) < (1-\theta)^{-1}$), the critical type θ_m lies further to the right. Accordingly, emissions fall more rapidly in the interval $[\theta_m, 1]$ of interior types. Hence underreporting of types is deterred by the associated stronger increase of emissions. If this effect is strong enough, then the principal can even pay subsidies that are lower for higher types.

The final non-standard result that obtains from the combination of multilateral externalities and type dependent outside option concerns the principal's payoff.

Proposition 6 *Although the optimal contract raises the principal's expected payoff as compared to the out-of-contract solution, she may loose for efficient types.*

Assuming specification (23) and $a = 1$, this happens for θ's close to 1 if $\underline{\theta} < \ln 4 - 1$. In particular, the contract shown in figure 2 assumes $\underline{\theta} = 0.25$; and the principal's gains over and above the out-of-contract payoff turn negative for $\theta > 0.87$, even if only moderately. The reason is that efficient types have a high valuation for abatement and would choose low emissions even without any bribe. This makes the laisser-faire state more attractive for the principal. Moreover, in the contract state the agent's incentive compatibility constraint prevents the principal from stopping to subsidize efficient types because they would then pretend less efficiency.

7 Concluding remarks

We have analysed a principal-agent model in an environment with multilateral externalities. As a consequence, the principal can use her own emissions – beside subsidies – to incentivize the agent. Moreover, both players' fall-back positions depend on the agent's type. We have shown that the optimal contract consists of an interior part and, possibly, also of a boundary part. The former is described by the relaxed program with the usual property of no distortion at the top. The boundary contract is an 'interior' solution in the sense that it differs from the out-of-contract allocation, although that allocation would be incentive compatible and also satisfies the agent's participation constraint. However, the principal can gain by allocating aggregate emissions between herself and the agent in a cost-efficient way.

Our motivating example has been international environmental agreements and, in particular, current efforts to convince developing countries to accept binding targets for greenhouse gas emissions. We will now discuss some of the paper's results in the context of this issue. Obviously, this should be done with care because our model is a rather strong simplification of the extremely complex climate negotiations (see the discussion in the introduction).

The first result is that the principal should condition her own emission reductions on those of the agent such that she reduces emissions more substantially if the agent also does so (or, equivalently, if the agent states a high WTP). As high WTP countries benefit most from the principal's emission reductions, this is a more effective instrument to separate agents along their type than subsidies. In addition, the principal benefits herself from her emission reductions, which is not the case for subsidies. Interestingly, this nexus between own and foreign emission reductions is often used in the context of international environmental agreements. For example, the European Union has offered to increase its emission reductions from 20% to 30% (of 1990 levels by 2020), on condition that other major emitting countries in the developed and developing worlds commit to do their fair share under a global climate agreement.[8]

Second, the above argument suggests that subsidies play a much smaller role to incentivize the agent than in the standard model without multilateral externalities. Emission reductions are often a better means of incentive payment. This may rationalize why monetary compensations are rarely used in international environmental agreements.

Third, the optimal contract allocates abatement cost-effectively; in the interior, as well as along the boundary solution. Therefore, the lower relative abatement costs in the group of developing countries (the agent), the lower the share of emission reductions that should be overtaken by the group of industrialized countries. This limits the extent to which the principal can use her own emission reductions as an incentive instrument. Accordingly, the larger her abatement cost relative to those of the agent, the more "standard" the contract should become, involving, in particular, subsidies that are positive and rising in the agent's type. Given the substantial differences in abatement costs for greenhouse gases, this suggests that substantial payments may be necessary to induce the meaningful participation of developing countries, be it by direct monetary payments or other forms of compensation such as technology transfers.

Fourth, there exists an alternative implementation of the optimal contract allocation by competitive permit markets. This equivalence follows from the cost efficient allocation of emissions under competitive permit trading, and the fact that the initial allocation of permit endowments allows to replicate the subsidies of the contract. This alternative seems highly policy relevant because international permit trading features prominently in political and theoretical debates about climate policies.

The results that have been derived with specification (23) are robust for

[8] Several other countries have made similar conditional pledges, e.g. Australia, New Zealand, Norway and Russia (see www.iccgov.org/policy-2_mitigation.htm).

simple variations of the model such as quadratic damage cost functions[9], or accounting for a different size of the agent (the mentioned examples are available upon request). However, there certainly exists substantial scope for more fundamental modifications, such as a more equal distribution of bargaining power, multilateral asymmetric information and more than two players. Obviously, the price of such modifications would be that the model becomes less tractable. An increased reliance on numerical simulations, possibly with models that are calibrated to a specific example such as climate change, may constitute a way out of this dilemma; albeit at the price of making results less transparent and general.

Appendix

A1: Proof of Proposition 1

The (IR) constraint is a pure state constraint of the first order, because the controls appear after differentiating,

$$\dot{U}(\theta) - \dot{R}(\theta) = -D(x_1(\theta) + x_2(\theta)) - \dot{R}(\theta). \tag{32}$$

This fact can be used to apply the indirect method (see, e.g., Chiang (1992)) by replacing the state constraint (IR) by

$$\dot{U}(\theta) \geq \dot{R}(\theta) \text{ whenever } U(\theta) = R(\theta). \tag{33}$$

Using this, the optimal control problem as stated before proposition 1 leads to the Hamiltonian ($\lambda(\theta)$ is the costate of $U(\theta)$ and the arguments are dropped from now on),

$$\mathcal{H} = [B_1(x_1) + B_2(x_2) - (1+\theta) D(X) - U] f - \lambda D(X), \tag{34}$$

and the Lagrangean ($\mu(\theta)$ is the Lagrangean multiplier)

$$\mathcal{L} = H + \mu \left(\dot{U} - \dot{R} \right). \tag{35}$$

[9] Indeed convex and in particular quadratic damages lead to even more pronounced effects due to the resulting strategic choice of x_1^0.

The conditions for the optimal contract are,

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_i} &= \left[B_i' - (1+\theta) D'\right] f - (\lambda + \mu) D' = 0, \quad i = 1, 2, && (36)\\
\dot{\lambda} &= -\frac{\partial \mathcal{L}}{\partial U} = f, && (37)\\
\dot{U} &= -D(x_1 + x_2), && (38)\\
\frac{\partial \mathcal{L}}{\partial \mu} &= \dot{U} - \dot{R} \geq 0, \quad \mu \geq 0, \quad \mu\left(\dot{U} - \dot{R}\right) = 0, && (39)\\
U(\theta) - R(\theta) &\geq 0, \qquad \mu\left[U(\theta) - R(\theta)\right] = 0, && (40)\\
\dot{\mu} &\leq 0 \qquad [= 0 \text{ when } U(\theta) > R(\theta)], && (41)\\
\lambda(\underline{\theta}) &\leq 0, \ \lambda(\underline{\theta})\left(U(\underline{\theta}) - R(\underline{\theta})\right) = 0, && (42)\\
\lambda(1) &\geq 0, \ \lambda(1)\left(U(1) - R(1)\right) = 0. && (43)
\end{aligned}
$$

Here, (37) and (38) are the standard differential equations for the co-state and state variable. The complementary slackness condition (40) assures that the constraint on the state variable (39) only applies when $U(\theta) - R(\theta) = 0$. (41) restricts the dynamics of the Lagrangean multiplier if the state constraint binds. Finally, (42) and (43) are the transversality conditions which reflect that we have a truncated vertical initial and terminal line.

Moreover, using (IC) and (13)

$$\dot{U} \geq \dot{R} \iff D(x_1(\theta) + x_2(\theta)) \leq D\left(x_1^0 + x_2^0\left(x_1^0, \theta\right)\right) \tag{44}$$

so that (39) can be stated alternatively as

$$X^0(\theta) - X(\theta) \geq 0, \quad \mu \geq 0, \quad \mu\left(X^0(\theta) - X(\theta)\right) = 0. \tag{45}$$

Statement (i) follows straightforwardly from (36). Turning to statement (iv), at a boundary solution $\mu(\theta) > 0$ so that $X^b(\theta) = X^0(\theta)$ from (45). Individual emissions will usually differ because only in the contract solution they are chosen to equalize marginal benefits.

Next, we analyse the transversality conditions, for which there are different combinations of binding and non-binding state constraints at the lowest and highest type. First, suppose $\lambda(1) > 0$. From (43), $U(1) = R(1)$ so that $X^b(1) = X^0(1)$ by Proposition 1(iv). However, at $\theta = 1$ from (36), $\left[B_i' - 2D'\right] f = [\lambda(1) + \mu(1)] D' > 0$ so that $X^b(1) < X^1(1) < X^0(1)$, a contradiction. Hence $\lambda(1) = 0$ so that the contract does not bind at the highest type.

Using $\lambda(1) = 0$, integration of the costate differential equation (37) over the interval $[\theta, 1]$ leads to $\lambda(\theta) = F(\theta) - 1$. Thus, assuming in addition $\lambda(\underline{\theta}) = 0$ leads to a contradiction since then $\lambda(\theta) = F(\theta)$ after integration of (37) over the interval $[\underline{\theta}, \theta]$. Accordingly, $\lambda(\underline{\theta}) < 0$ and the contract binds at the lowest type.

Using the above, statement (*iii*) follows straightforwardly. In particular, substitution of $\lambda(\theta) = F(\theta) - 1$ into (36) yields

$$B_i' - \left(1 + g(\theta) + \frac{\mu(\theta)}{f(\theta)}\right) D' = 0, \quad i = 1, 2, \tag{46}$$

where $\mu(\theta) = 0$ for interior solutions as given in (16). By the curvature assumptions, no such interior solution can exist for $g(\theta) \leq -1$. Hence in this case $\mu(\theta)$ must become positive to satisfy (46) and a boundary solution obtains. This leads to the following equivalences,

$$\mu(\theta) > 0 \Longleftrightarrow B_i' > (1 + g(\theta)) D' \Longleftrightarrow x_i^b(\theta) < x_i^r(\theta), \tag{47}$$

and thus to emissions below the relaxed program along a boundary solution.

Turning to (*ii*), observe that $g(\theta) \leq \theta$ with equality iff $\theta = 1$ (see 17). Moreover, $\mu(\theta) \geq 0$ with equality at interior solutions that include $\theta = 1$ (from the preceding analysis). Hence, for interior solutions comparing the conditions that determine emissions with a contract (16) and in the first-best solution (2) shows that emissions are above the first-best except at the top. For boundary solutions, overall emissions equal those in the out-of-contract solution and, therefore, are above the first-best. This must also be the case for individual emissions of the principal and the agent because emissions are chosen to equalize marginal benefits with contracts and in the first-best solution.

For the second statement in (*ii*) we use again the fact that contract emissions are determined by equation system (46). Implicit differentiation yields

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \frac{1}{B_1'' B_2'' - (B_1'' + B_2'')\left(1 + g(\theta) + \frac{\mu(\theta)}{f(\theta)}\right) D''} \begin{pmatrix} B_2''\left(\dot{g} + \frac{\dot{\mu} f - \mu \dot{f}}{f^2}\right) D' \\ B_1''\left(\dot{g} + \frac{\dot{\mu} f - \mu \dot{f}}{f^2}\right) D' \end{pmatrix}. \tag{48}$$

For interior solutions ($\mu(\theta) = 0$), $\dot{x}_i < 0$ follows immediately from $\dot{g} > 1$ and the curvature assumptions. For boundary solutions, observe that $sign(\dot{x}_1) = sign(\dot{x}_2)$. Moreover, by proposition 1(*iv*) overall emissions are the same as in the out-of-contract solution, which are decreasing in θ (by (5) and $\dot{x}_1 = 0$). Hence also at the boundary $\dot{x}_i < 0, i = 1, 2$. ■

A2: Proof of Proposition 2

The Hamiltonian is concave in the controls (i.e., in the emissions x_i) over the relevant domain,

$$B_i'' f - [(1 + \theta) f + \lambda + \mu] D'' < 0, \quad i = 1, 2, \tag{49}$$

because $[(1 + \theta) f + \lambda + \mu] D' = B_i' f > 0$ from (36) for positive emissions, and is thus also jointly concave given the linearity in the state. Moreover, the control problem with objective (14) and 'dynamic' (IC) and (pure) 'state' constraint

(IR) satisfies the regularity condition (Feichtinger and Hartl (1986, 165), condition (6.17)), i.e., the 'matrix'

$$\begin{pmatrix} -D' & -D' & U-R \end{pmatrix} \tag{50}$$

has the maximal rank of 1 in the interior and along the boundary since $D' > 0$. Therefore, the controls must be continuous for all types (Feichtinger and Hartl 1986, 167), including the marginal type θ_m, which yields (21).

For statement (*i*) it remains to prove that θ_m separates the contract in the sense that a boundary solution obtains for all $\theta \leq \theta_m$ and an interior one for all $\theta > \theta_m$. If there is a unique intersection of aggregate relaxed program emissions, X^r, with aggregate out-of-contract emissions, X^0, then this follows straightforwardly from the discussion in the main text. Hence, consider the (unlikely) case of multiple intersections between X^r and X^0, and suppose that an interior solution (for which $\mu = 0$) is followed by a boundary solution (for which $\mu > 0$) . This requires that $\dot{\mu} > 0$ at least for some θ, in contradiction to (41).

Summarizing, the solution as described in propositions 1 and 2 satisfies all first-order conditions (36) to (43), which are also sufficient given that the regularity condition holds and that the (maximized) Hamiltonian is concave with respect to the state U.

Statement (*ii*) follows straightforwardly from the fact that $X^b = X^0$ for all $\theta \leq \theta_m$ (by proposition 1), and that θ_m is the highest type at which aggregate relaxed program emissions, X^r, which prevail in the interior, cross aggregate out-of-contract emissions, X^0, from above.

Statement (*iii*) follows immediately for boundary solutions because they are characterized by $X^b = X^0$ and equalization of marginal abatement costs. Turning to interior solutions, an example which proves that the principal's contract emissions may exceed their out-of-contract level is given in section 6. Modifying this specification by assuming quadratic damages, a Stackelberg setup, $a = 2, \underline{\theta} = 0.5, f(\theta)$ linearly increasing, and $f(\underline{\theta}) = 0.5$ leads to the equivalent result for the agent's emissions (a detailed solution of this example is available upon request from the authors). ■

A3: Proof of Proposition 4

Rearranging (12), the dynamics of the subsidy are

$$\begin{aligned} \dot{s} &= -B_2'(x_2(\theta))\,\dot{x}_2 + \theta D'(X(\theta))\,(\dot{x}_1 + \dot{x}_2) && (51)\\ &= -\dot{x}_2\left[B_2'(x_2(\theta)) - 2\theta D'(X(\theta))\right] + \theta D'(X(\theta))\,(\dot{x}_1 - \dot{x}_2)\,. && (52) \end{aligned}$$

Moreover, for a uniform distribution $g(\theta) = 2\theta - 1$. Accordingly, along the relaxed program the first term in (52) is zero (by 16). Statement (*i*) then follows straightforwardly from (48), according to which the sign of $\dot{x}_1 - \dot{x}_2$ is equal to the sign of $B_2''(x_1^r(\theta)) - B_1''(x_2^r(\theta))$.

Turning to statement (*ii*), along a boundary solution overall emissions are the same as in the out-of-contract solution, but marginal abatement costs are equalized. Therefore, if the agent's contract emissions *fall below* those out-of-contract, he must be compensated by a positive subsidy - and vice versa. More formally, solving (11) for $s(\theta)$, using $U^b(\theta) = R(\theta) = B_2\left(x_2^0\right) - \theta D\left(X^0(\theta)\right)$ and $X^0(\theta) = X^b(\theta)$ yields

$$s^b(\theta) = U^b(\theta) - B_2\left(x_2^b(\theta)\right) + \theta D\left(X^b(\theta)\right) \quad (53)$$

$$= B_2\left(x_2^0(\theta)\right) - B_2\left(x_2^b(\theta)\right). \quad (54)$$

From proposition 2(*iii*) this expression can be positive or negative. Given that emissions are continuous, negative subsidies in the left neighborhood of θ_m (boundary contract) imply that these are also negative in the right neighborhood of θ_m (interior contract). The same argument applies to positive subsidies. Alternatively, the result can be proved by solving specification (23), which yields that $s(\theta) = -\underline{\theta} + \ln 2$. ■

A4: Proof of Proposition 5

Rearranging the relaxed program condition (16) yields

$$\frac{B_1'\left(x_1^r(\theta)\right)}{D'\left(X^r(\theta)\right)} - (1+\theta) = -\frac{1}{h(\theta)}.$$

Lowering the hazard rate lowers the right-hand side in the relaxed program. Thus, the positive first term on the left-hand side must becomes smaller. By the curvature assumptions, this requires higher emissions; possibly exceeding the out-of-contract emissions that are independent of assumptions about the distribution.

Turning to statement (*ii*), the assumption of equal benefit functions and the equalization of marginal benefits in the optimal contract imply $x_1 = x_2$ and $\dot{X} = 2\dot{x}_2$. Upon substitution into (51),

$$\dot{s} = -\dot{x}_2\left[B_2'\left(x_2(\theta)\right) - 2\theta D'(X)\right]$$

Substitution for $B_2'\left(x_2(\theta)\right)$ from the relaxed program condition (16) yields that along the relaxed program

$$\dot{s} = -\dot{x}_2\left[(1+g(\theta)) - 2\theta\right] D'(X^r).$$

From (17), $1 + g(\theta) - 2\theta = 1 + \theta - \frac{1}{h(\theta)}$. Hence $\dot{s} > 0$ if this term is positive. ■

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Appendix B: Algebraic solution of example (23)

In the out-of-contract solution, the agent's best response,

$$x_2^0 = \frac{a}{\theta}, \tag{55}$$

is independent of the principal's emission. Therefore, the Cournot and Stackelberg solutions coincide, and the principal's emissions are simply[10]

$$x_1^0 = 1. \tag{56}$$

For a uniform distribution, $g(\theta) = 2\theta - 1$. Hence the relaxed program (16) exists for all θ (by Proposition 1), and its emissions are

$$x_1^r(\theta) = \frac{1}{2\theta} \quad \text{and} \quad x_2^r(\theta) = \frac{a}{2\theta}. \tag{57}$$

The aggregate of the relaxed program emissions exceeds the out-of-contract counterpart iff

$$\frac{1+a}{2\theta} > \frac{\theta + a}{\theta} \quad \Longleftrightarrow \quad \theta < \frac{1-a}{2}, \tag{58}$$

which requires $a < 1$ and a sufficiently small lower bound $\underline{\theta}$. Furthermore, if there are types $\theta > \underline{\theta}$ that satisfy (58), then there exists a unique type

$$\theta_m = \theta_{IR} = \frac{1-a}{2}, \tag{59}$$

at which aggregate relaxed program emissions, X^r, cross aggregate out-of-contract emissions, X^0. Moreover, differentiating emissions yields

$$\dot{X}^r = -\frac{1+a}{2\theta^2} \quad \text{and} \quad \dot{X}^0 = \dot{x}_2^0 = -\frac{a}{\theta^2} \tag{60}$$

so that $\dot{X}^r < \dot{X}^0$ for any $a < 1$, which is necessary for a unique intersection from above. Accordingly, for types $\theta \le \theta_m$ a boundary solution obtains, for which emissions follow from (15) and (18) as

$$x_1^b(\theta) = \frac{a+\theta}{(1+a)\theta}, \qquad x_2^b(\theta) = a\frac{a+\theta}{(1+a)\theta}, \qquad \theta \in \left[\underline{\theta}, \frac{1-a}{2}\right]. \tag{61}$$

The subsidies that are required to implement the emission targets for boundary and interior solutions are (assuming that $\theta_{IR} = \frac{1-a}{2} > \underline{\theta}$)[11]

$$s^b(\theta) \;=\; a \ln \frac{1+a}{a+\theta} > 0 \quad \text{for } \theta \le \theta_{IR}, \tag{62}$$

$$s^r(\theta) \;=\; a \ln 2 + \frac{1-a}{2} \ln \frac{1-a}{2\theta} \quad \text{for } \theta > \theta_{IR}. \tag{63}$$

[10] This depends, of course, crucially on the linear damage function. With convex damages, the principal has a strong incentive to choose much higher emissions in the Stackelberg set up because that reduces the agent's emissions free of charge for the principal.

[11] Subsidies along the boundary follow from substituting the above emission levels into the identity $U = R$. Subsidies along the interior follow from $U = V_2 + s$ and integrating (IC) to determine $U(\theta) = R(\theta_m) - \int_{\theta_m}^{\theta} D\left(x_1^r(t) + x_2^r(t)\right) dt$.

Differentiation yields that subsidies are always declining with respect to the agent's type along a boundary solution. By contrast, for the interior solution[12]

$$\dot{s}^r(\theta) = \frac{a-1}{2\theta} \tag{64}$$

so that subsidies are declining if the agent has lower abatement costs ($a < 1$).

Considering a concrete case, say $\underline{\theta} = 1/4$ and $a = 1/4$, the boundary contract (61) is applied for all $\theta \leq 0.375 = \theta_m$ and the relaxed program (57) for the remaining types.

For an alternative permit contract one can also calculate the endowment allocated to the agent as (using 27)

$$\omega_2(\theta) = x_2^c(\theta) + \frac{s^c(\theta)}{p} = \frac{1}{\theta} + \frac{\ln 2 - \underline{\theta}}{2\theta} \tag{65}$$

The endowment must exceed after-trade emissions ($\omega_2 > x_2^r$) in order to deliver the necessary transfers.

In the main text we have claimed that for specification (23) and $a = 1$ subsidies are positive if $\underline{\theta} < \ln 2$. This can be shown as follows. For $a = 1$, the subsidy is constant and the relaxed program defines the solution for all θ; hence we cannot simply use (63). However, it suffices to calculate the subsidy for $\underline{\theta}$, for which the participation constraint binds so that

$$B_2(x_2^r(\underline{\theta})) - \underline{\theta} D(X^r(\underline{\theta})) + s(\underline{\theta}) = B_2\left(x_2^0(\underline{\theta})\right) - \theta D\left(X^0(\underline{\theta})\right). \tag{66}$$

Solving for $s(\underline{\theta})$ and substituting for specification (23) yields

$$\begin{aligned} s(\underline{\theta}) &= \ln\frac{1}{\underline{\theta}} - \underline{\theta}\left(1 + \frac{1}{\underline{\theta}}\right) - \ln\frac{1}{2\underline{\theta}} + 1 & (67)\\ &= -\underline{\theta} + \ln 2. & (68) \end{aligned}$$

In order to show that the principal can loose from contracting with certain types, we calculate for specification (23) the principal's payoff from a type θ in the out-of-contract solution,

$$R_1(\theta) = B_1\left(x_1^0(\theta)\right) - D\left(X^0(\theta)\right) = -\frac{1+\theta}{\theta}, \tag{69}$$

where the subscript 1 refers to the principal in order to differentiate from the agent, and with contracts (using the constant subsidy (68))

$$U_1(\theta) = B_1(x_1^r(\theta)) - D(X^r(\theta)) - s(\theta) = -\ln\theta - \frac{1}{\theta} + \underline{\theta} - 2\ln 2. \tag{70}$$

Hence the principal's gain from contracts is

$$U_1(\theta) - R_1(\theta) = -\ln\theta - \frac{1}{\theta} + \underline{\theta} - 2\ln 2 + \frac{1+\theta}{\theta}, \tag{71}$$

[12] This follows from solving the local incentive constraint (i.e., (12) evaluated at $\hat{\theta} = \theta$) for $\dot{s}(\theta)$. Obviously, if a boundary part exists (i.e., if $\theta_{IR} = \frac{1-a}{2} > \underline{\theta}$) it also follows directly from differentiation of (63).

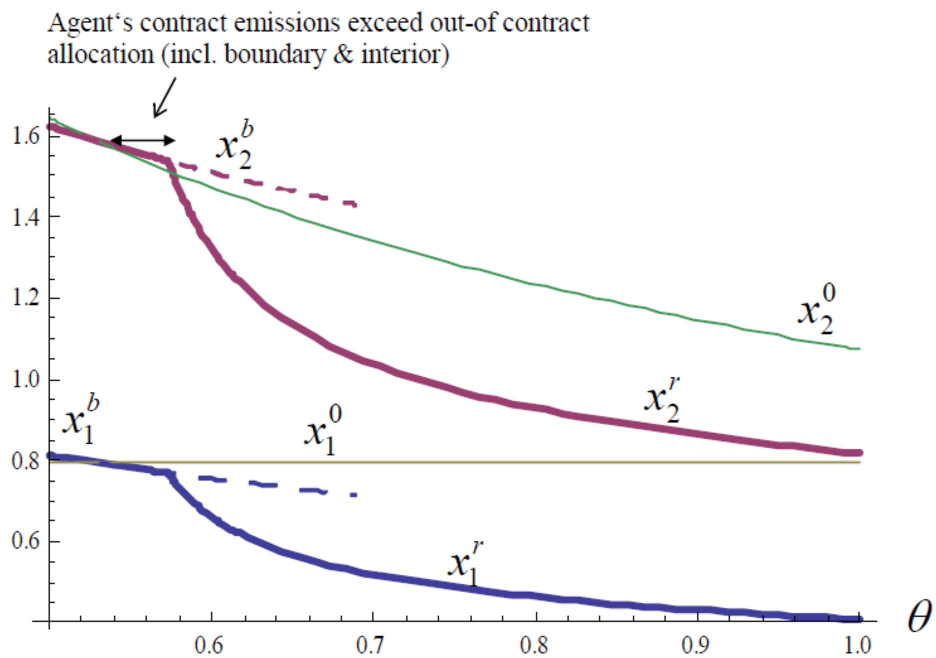

Figure 3: Agent‘s contracted emissions (x_2^b, x_2^r) can exceed their out-of-contract level (x_2^0): quadratic damages, Stackelberg setup, $a = 2, \underline{\theta} = 0.5, f(\theta)$ linearly increasing, $f(\underline{\theta}) = 0.5$.

which decreases in θ but increases in $\underline{\theta}$. Accordingly, the highest $\underline{\theta}$ such that the principal looses at least from a contract with the highest type, $\theta = 1$, is

$$\underline{\theta} = 2\ln 2 - 1 \approx 0.386. \quad \blacksquare \tag{72}$$

Appendix C: Example which illustrates statement that also the agent's emissions may exceed their out-of-contract level

From the discussion in text, the following properties favour such a case: (i) high abatement costs of the agent (e.g., $a > 1$ in the simple specification (23)), (ii) low θs, and (iii) large out-of-contract emissions of the principal, e.g., due to the Stackelberg outcome and convex damages. In such cases a reallocation of emissions to the agent can increase total welfare. Fig. 3 shows an example along these lines, where the agent's contract emissions exceed his no contract allocation (albeit mostly in a small domain), both along the boundary and interior part of the optimal contract.

Appendix D: Sensitivity analysis

The purpose of this Appendix is to document that the results are by no means sensitive to the chosen simple specification. Moreover, the first charts complement graphically the example already discussed in the paper.

D1: Illustration of examples from main text

Figs. 4 and 5 show emissions (interior and boundary) payoffs and subsidies corresponding to the example in Fig. 1. Aggregate emissions in the relaxed program exceed those outside a contract for types $\theta < 0.375 = \theta_m$ (see Fig. 4). Fig. 5 shows the corresponding subsidies, which have the unusual properties of being declining (see propositions 4 and 5) and turning negative for high types θ at the claimed level (see p. 14). This means that high types which choose low emissions are 'penalized' by negative subsidies, while higher emissions fetch higher and positive subsidies (unless emissions are so high that they violated the participation constraint). Nevertheless, high types are deterred from cheating by underreporting their willingness to pay because this would trigger higher emissions by the principal, hurting the agent.

D2: Linear and increasing densities

Figs 6-8 are based on an alternative example with identical benefits ($a = 1$), which simplifies the plots compared with the reference example above due to symmetric allocation of emissions within a contract. However, the inclusion of an 'interior boundary' contract requires a distribution different from the uniform one. Thus a linearly increasing density is used with the mode at $\theta = 1$, i.e., inefficient types are relatively rare. Interestingly, the opposite of a pessimistic prior with the mode at $\underline{\theta}$ implies a global interior contract. Although, one can characterize the outcome still in closed form, figures provide a much more informative picture because of cumbersome formulas (e.g., a cubic equation determines θ_m).

Fig. 6 plots the crucial elements g and h highlighting that the relaxed program cannot exist at small levels of θ since $g < -1$. Fig. 7 shows the emissions, without a contract, with a contract that consists of a boundary $\left(x_1^b = x_2^b\right)$ and interior segment $(x_1^r = x_2^r)$, and for the first best $\left(x_1^1 = x_2^1\right)$. Finally, Fig. 8 shows the agent's payoffs and subsidies on the left-hand side, and subsidies contingent on the agent's emission on the right-hand side. The latter is only a projection of the three-dimensional optimal incentive scheme because that includes in addition the crucial commitment of the principal to match the agent's emissions.

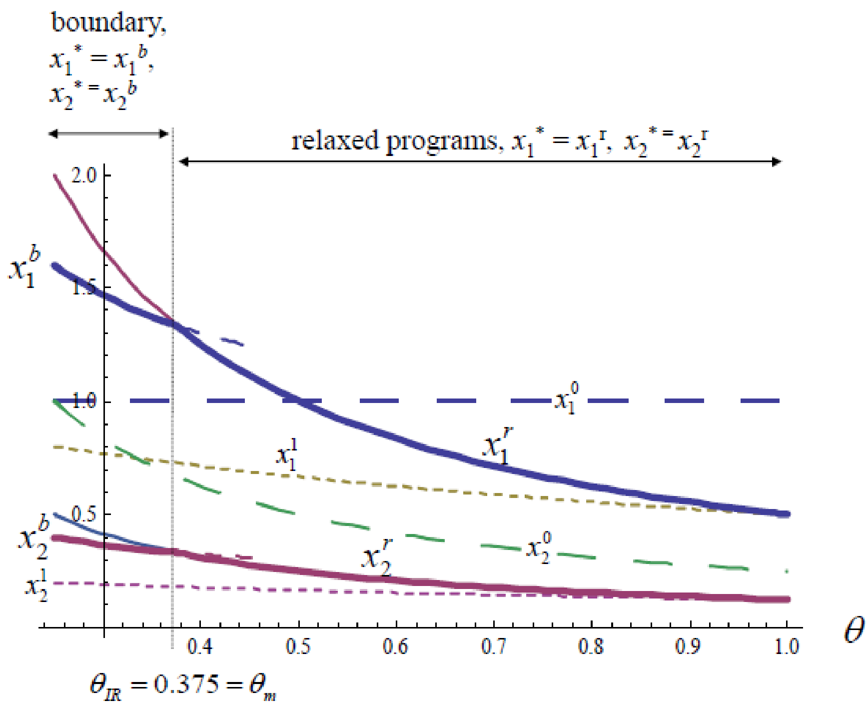

Figure 4: Emissions for (23) with $\underline{\theta} = \frac{1}{4}$, and $a = \frac{1}{4}$. Optimal contract emissions (bold and asterisks), first best and out-of-contract.

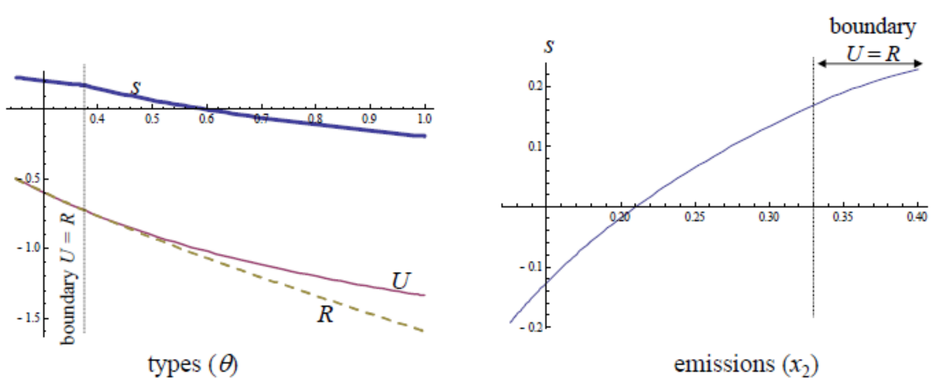

Figure 5: Agent's payoffs for (23) & incentive scheme, $\underline{\theta} = \frac{1}{4}$, and $a = \frac{1}{4}$

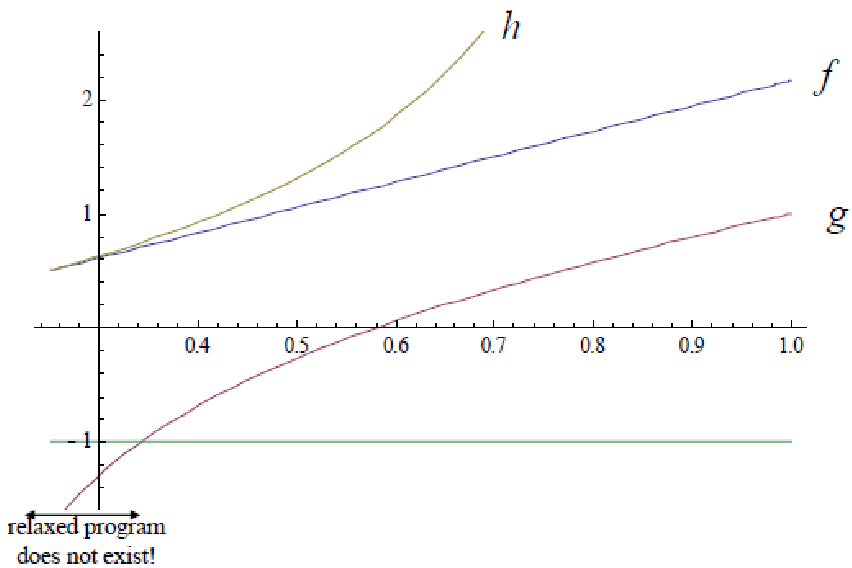

Figure 6: Linearly increasing densities: $\underline{\theta} = \frac{1}{4}$ and $f(\underline{\theta}) = \frac{1}{2} \implies f(1) = 13/6$. No relaxed program exists for $g \leq -1$ since that would require $B_i' \leq 0$.

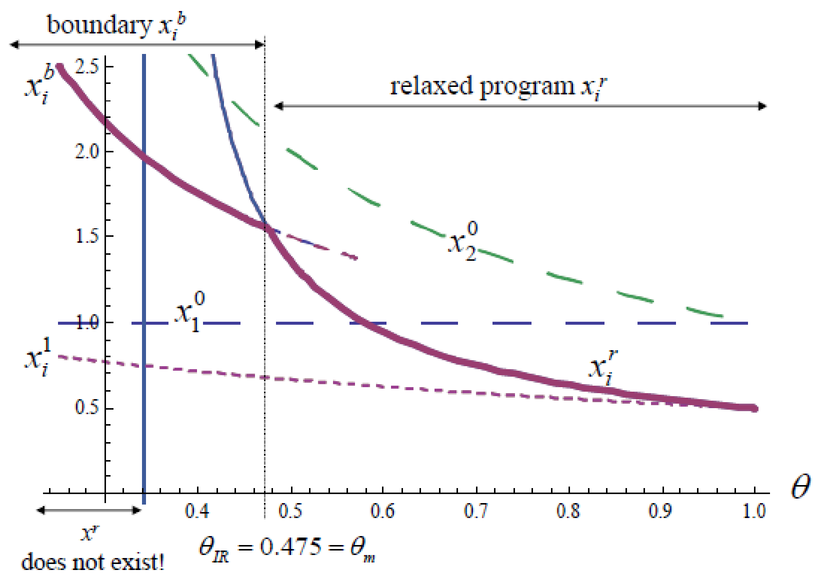

Figure 7: Example (23) except for probability distribution function from Fig. 6, and $a = 1$.

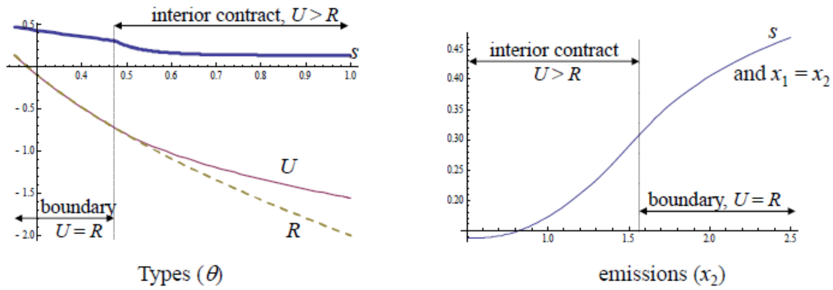

Figure 8: Payoff and subsidies (s) for example (23), $a = 1$, but distribution from Fig. 6, $\underline{\theta}= \frac{1}{4}$ and $f(\underline{\theta}) = \frac{1}{2}$.

D3: Scaling the Size of the Agent

Both players have identical benefit (B increasing and concave satisfying the Inada conditions) from individual emissions ($x_i, i = 1, 2$) but face the damage (D increasing and convex) depending on aggregate emissions. Hence

$$V_i = A_i \left[B\left(x_i\right) - \theta_i D\left(A_i x_i + A_j x_j\right)\right], i = 1, 2, j \neq i, \tag{73}$$

where A_i is the size and the principal's size is normalized, $A_1 = 1$. As a consequence, the first-best as well as the relaxed program emissions are identical, $x_1^r = x_2^r$. Fig. 9 shows a corresponding example for a larger agent $A > 1$ (but saving here on the computation of the boundary strategy) and the interpretation is analogue to the one in the paper (for $a < 1$).

D4: Quadratic damages

Alternatively, consider quadratic damages, identical size and uniform distribution,

$$B\left(x_i\right) = \ln\left(x_i\right), \; D\left(X\right) = \frac{1}{2}X^2, \; f\left(\theta\right) = \frac{1}{1-\underline{\theta}}. \tag{74}$$

The first-best emissions are

$$x_1^1\left(\theta\right) = \frac{1}{\sqrt{2\left(1+\theta\right)}} = x_2^1\left(\theta\right). \tag{75}$$

The agent's first-order condition of his Cournot reaction is

$$x_2^0\left(\theta\right) = \frac{\sqrt{\left(x_1^0\right)^2 + 4/\theta} - x_1^0}{2}, \tag{76}$$

$$\frac{\partial x_2^0}{\partial x_1^0} = \frac{x_1^0 - \sqrt{\left(x_1^0\right)^2 + 4/\theta}}{2\sqrt{\left(x_1^0\right)^2 + 4/\theta}} = \frac{1}{2}\left(\frac{x_1^0}{\sqrt{\left(x_1^0\right)^2 + 4/\theta}} - 1\right). \tag{77}$$

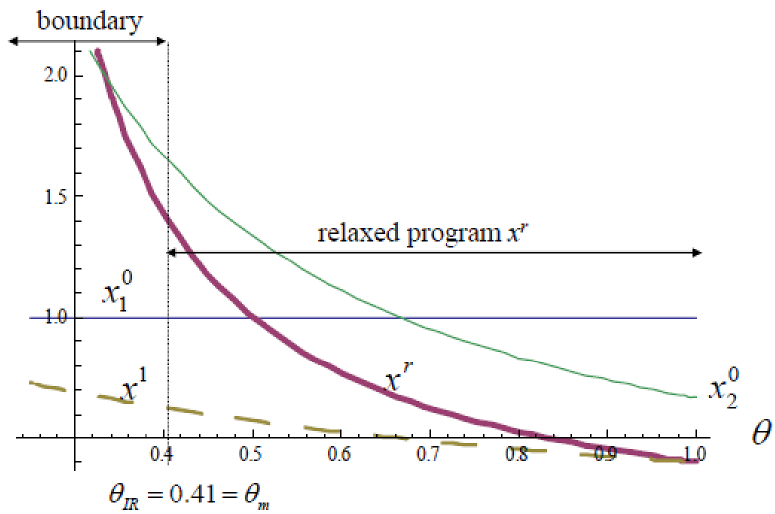

Figure 9: Emissions for example (23) but payoff (74), $\underline{\theta}= \frac{1}{4}$ and larger agent $(A = \frac{3}{2})$.

Therefore, the first-order condition for the Stackelberg Bayesian decision of the principal becomes

$$\frac{1}{x_1} = \int_{\underline{\theta}}^{1} D'\left(x_1 + x_2^0(x_1, \theta)\right)\left(1 + \frac{\partial x_2^0}{\partial x_1^0}\right) f(\theta)\, d\theta, \tag{78}$$

$$= \frac{1}{4(1-\underline{\theta})}\int_{\underline{\theta}}^{1} \frac{\left(x_1 + \sqrt{x_1^2 + 4/\theta}\right)^2}{\sqrt{\theta x_1^2 + 4/\theta}} d\theta, \tag{79}$$

$$= \frac{x_1}{2} + \frac{\sqrt{x_1^2+4} - \underline{\theta}\sqrt{x_1^2 + 4/\underline{\theta}}}{2(1-\underline{\theta})}, \tag{80}$$

and the integral can be even analytically computed. However, trying to solve this first-order condition, implies a high order polynomial that yields a clumsy solution which is not worth reporting. Therefore, consider the example $\underline{\theta} = 1/4$ for which $x_1^0 = 0.8426$. Similar calculations yield for the Cournot game,

$$\frac{1}{x_1} = \int_{\underline{\theta}}^{1} D'(x_1 + x_2(\theta)) f(\theta)\, d\theta = \frac{1}{2(1-\underline{\theta})}\int_{\underline{\theta}}^{1} x_1 + \sqrt{x_1^2 + 4/\theta} d\theta, \tag{81}$$

so that the first-order condition becomes

$$\frac{1}{x_1} = \frac{x_1}{2} + \frac{\sqrt{x_1^2+4} + \frac{\ln\left(4+2x_1\left(x_1+\sqrt{x_1^2+4}\right)\right)}{x_1}}{2\left(1-\underline{\theta}\right)} \tag{82}$$

$$-\frac{\underline{\theta}\sqrt{x_1^2+4/\underline{\theta}} + \frac{\ln\left(4+2\underline{\theta}x_1\left(x_1+\sqrt{x_1^2+4/\underline{\theta}}\right)\right)}{x_1}}{2\left(1-\underline{\theta}\right)}, \tag{83}$$

for which no closed form solution (not even a clumsy one) could be obtained. Therefore, considering the above example, $\underline{\theta} = 1/4$, the corresponding Cournot emission is $x_1^0 = 0.5997$ and thus substantially below the Stackelberg outcome for the reason given in the main text.

Given the principal's emission x_1^0 outside a contract, the agents reservation price is

$$R\left(\theta\right) = \ln\left(\frac{\sqrt{\left(x_1^0\right)^2+4/\theta} - x_1^0}{2}\right) - \frac{\theta}{2}\left(\frac{\sqrt{\left(x_1^0\right)^2+4/\theta} + x_1^0}{2}\right)^2, \tag{84}$$

and the principal's reservation price is,

$$R_1\left(\theta\right) = \ln\left(x_1^0\right) - \frac{1}{2}\left(\frac{\sqrt{\left(x_1^0\right)^2+4/\theta} + x_1^0}{2}\right)^2. \tag{85}$$

The relaxed program is

$$x^r\left(\theta\right) = \frac{1}{2\sqrt{\theta}} < x_2^0\left(\theta\right) = \frac{\sqrt{\left(x_1^0\right)^2+4/\theta} - x_1^0}{2} \leq \frac{\sqrt{\frac{1}{2}+4/\theta} - \frac{1}{\sqrt{2}}}{2}. \tag{86}$$

Fig. 10 shows an example.

The agent's payoff follows from integrating the incentive compatibility constraint using the reservation price from (84) at $\underline{\theta}$ as the initial condition,

$$U\left(\theta\right) = U\left(\underline{\theta}\right) - \int_{\underline{\theta}}^{\theta} D\left(2x^*\left(v\right)\right)dv = U\left(\underline{\theta}\right) - \frac{1}{2}\int_{\underline{\theta}}^{\theta}\frac{1}{v}dv \tag{87}$$

$$= U\left(\underline{\theta}\right) + \frac{\ln\underline{\theta} - \ln\theta}{2}. \tag{88}$$

Since,

$$\theta D = \frac{\theta}{2}\left(\frac{1}{\sqrt{\theta}}\right)^2 = \frac{1}{2},\ B = \ln\left(x_i\right) = \ln\left(\frac{1}{2\sqrt{\theta}}\right) = -\ln 2 - \frac{1}{2}\ln\theta \tag{89}$$

the subsidies,

$$s\left(\theta\right) = U\left(\theta\right) + \theta D - B = U\left(\underline{\theta}\right) + \frac{\ln\underline{\theta}}{2} + \ln 2 + \frac{1}{2} \tag{90}$$

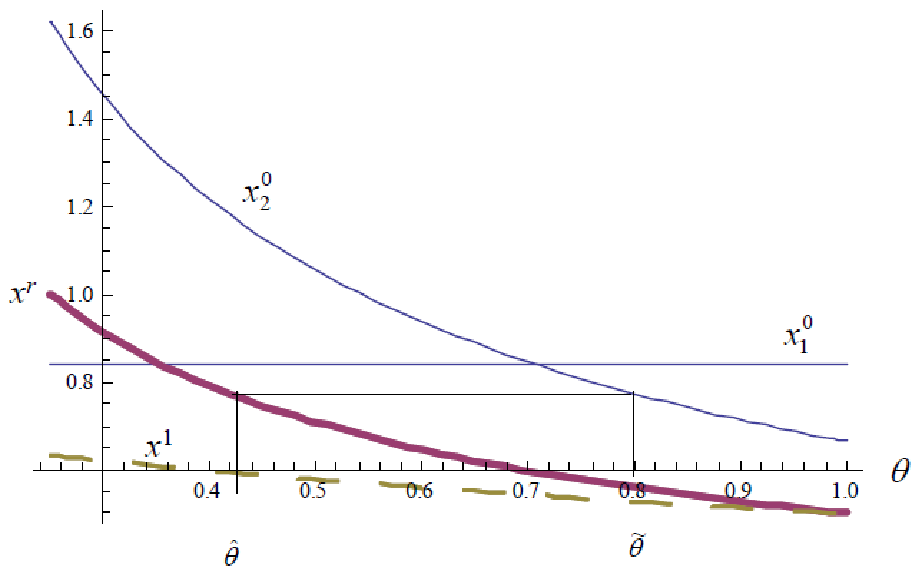

Figure 10: Quadratic damages: Emissions efficient (dashing), without a contract and optimal mechanism (bold) for $\theta = \frac{1}{4}$.

are constant.

The quadratic damage case is particularly suited to demonstrate that the principal can loose from contracts with efficient types. This follows from comparing the principal's gain under the optimal contract over and above no contract outcome (see Fig. 11).

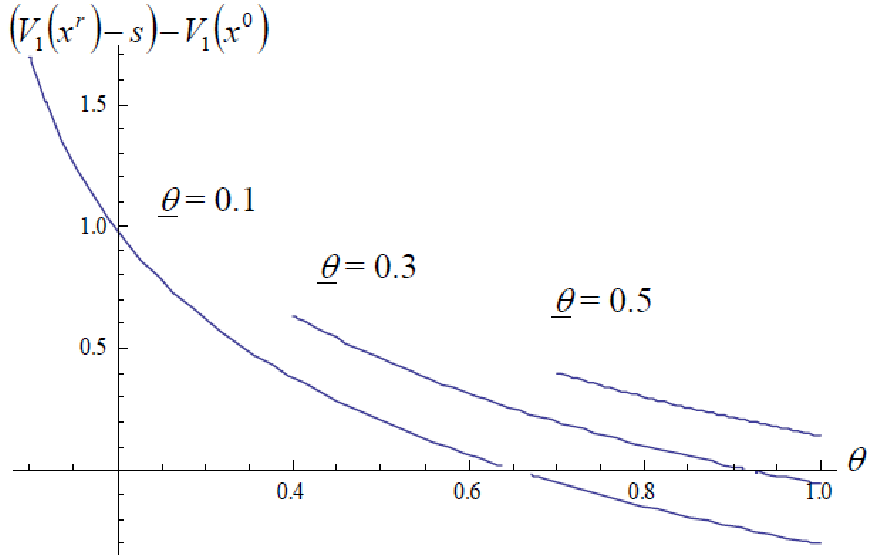

Figure 11: Quadratic damages: Principal's gain from contracts as a function of the agent's types for different values of $\underline{\theta}$.