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The decomposition of inequality reconsidered: Weakly decomposable measures

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Abstract

The paper characterizes the class of weakly decomposable (aggregable) inequality measures which satisfy a new (weak) decomposition (and aggregation) property. These measures can be decomposed into the sum of the usual within-group and a between-group term which is based on the inequality between all pairs of individuals belonging to the groups involved. The measures therefore depend on the inequality index for two-person distributions and are proportional to the total sum of the inequality values between all pairs of individuals. Extending Gini's mean difference, the Gini coefficient, and the variance of logarithms we characterize three families of measures. By choosing other basic measures further (families of) weakly decomposable measures can be defined.

Keywords: Inequality measures, decomposition, aggregation, Gini's mean difference, Gini coefficient, variance of logarithms

JEL-codes: D63, D31, C43

1. Introduction[1]

Measuring income inequality one is often interested in the contribution of different (sub)groups of the population to overall inequality. Then one has to employ appropriate inequality measures which allow to identify the impact of subgroups on total inequality. The measures have to be decomposable for a given partition of the population. This paper suggests a new method of decomposing inequality and provides a complete characterization of the class of measures satisfying the corresponding decomposition property.

Thirty years ago a number of papers were published dealing with the decomposition of inequality measures.[2] These measures were supposed to be additively decomposable: If a given population is split into any two mutually exclusive and exhausting subgroups, overall income inequality can be decomposed into a within-group and a between-group inequality term. The first one is a weighted sum of the subgroup income inequality values. The between-group term measures the inequality between both subgroups by considering a smoothed income distribution for each subgroup – which is generated by replacing the actual incomes by the respective average income or by using some representative income.

A simple example demonstrates a shortcoming of this (conventional) approach. Consider a population which consists of two subgroups each containing two individuals and let the income vector of group 1 be given by $X^1 = (10,20)$ and that of group 2 by $X^2 = (15,15)$. Then the average incomes are identical (and equal to 15). If the average income is chosen as representative income, between-group inequality is measured by the inequality of the income vector $(15,15,15,15)$, i.e.; it is equal to zero since all (representative) incomes are the same. We obtain the same result if subgroup 2 had the income vector $\overline{X}^2 = (5,25)$ as the average income in group 2 is still 15. On the other hand, things have changed drastically: The individual in group 1 having income 10 (20) is now no longer the poorest (richest) one in the population, i.e., there is someone in group 2 with less (more) income. Thus the inequality between both subgroups is now different from the inequality in the original situation. But this change is not reflected by the between-group term in the decomposition of inequality.

This paper presents an alternative to the conventional decomposition method and investigates its implications for inequality measures. They can be decomposed into the usual within-group

[1] I thank Rolf Aaberge, Martin Duensing, Peter Lambert, Shlomo Yitzhaki, and two anonymous referees for helpful comments.

[2] Bourguignon (1979), Shorrocks (1980, 1984), Cowell and Kuga (1981), Blackorby, Donaldson, and Auersperg (1981), and Foster (1983) dealt with the additive decomposition of inequality measures. Cf. also Ebert (1999), Foster and Shneyerov (1999, 2000) and Chakravarty (1999) on this topic.

and a new, simple between-group term. The representation of the inequality between two subgroups is based on an intuitive idea. We measure this inequality directly by a comparison of *all* pairs of income; i.e., we compare the income of each individual in the first group with the income of each individual in the second group. The between-group term is then formed by the sum of all these inequality values. As usual overall inequality is equal to the sum of the within-group term and the between-group term. In this case, the change of income in our example from X^2 to $\bar{X}^2$ is properly reflected by the between-group term.

For both methods the decomposition process is a top-down approach: The level of overall inequality is known and is then decomposed into several meaningful components. The new decomposition property can, however, also be interpreted the other way around: Starting from two given groups we can combine them to a new overall population. In this case overall inequality is constructed from the inequality values within the two groups and the additional inequality generated by combining both (sub)groups which is represented by the corresponding between-group term. Then we obtain an aggregation property and a bottom-up approach (cf. also the interpretation provided in Shorrocks (1984)). This idea can be made more precise if we consider two particular subgroups. We can enlarge a given subgroup by exactly one individual. Since there is no within-group inequality if the group consists only of one individual, the within-group inequality term is determined by the inequality value for the original (sub)group. The between-group term boils down to the inequality values comparing each income belonging to an individual in the original group with the new individual's income. Overall inequality is then again a sum of both terms and the aggregation process is additive.

In the following, Section 2 introduces the framework, the notation used, and the basic set of properties for inequality measures: Normalization requires that there is no inequality if all incomes are identical. Symmetry postulates that the identity of individuals is irrelevant. The principle of population makes the inequality measures for different population sizes consistent. Two principles for the redistribution of income are considered. The Pigou-Dalton principle of transfers requires that a rank-preserving transfer of income from a richer individual to a poorer one decreases inequality, and the concentration principle postulates that a concentration[3] of income diminishes the level of inequality. In section 3 the new decomposition and aggregation properties are proposed and motivated. Some examples are given. Then the class of weakly decomposable measures is characterized which satisfy the new decomposition axiom. Under weak assumptions the decomposition property and the population principle

[3] By definition a concentration preserves the mean. It redistributes income by reducing the distance between each income and the average income in the same proportion.

jointly determine the general structure of an inequality measure. These properties imply that for a given population size an inequality measure has to be proportional to the sum of the levels of income inequality between *all* pairs of individuals. Therefore the measure is based on pairwise comparisons of income and depends on the choice of an arbitrary inequality measure defined for two individuals. Conversely, it is shown that the type of measure derived satisfies the decomposition property and the principle of population; i.e., we obtain a simple characterization of the class of weakly decomposable inequality measures. In this characterization the decomposition property can be replaced equivalently by the aggregation property.

In section 4 we apply this result by choosing three different families of measures for two individuals. The first one is given by the absolute income differential (or a power of it). We obtain a characterization of a family of measures which are equal to the average of the (power of the) income differentials between all pairs of individuals. These measures form a one parameter (the exponent) extension of Gini's mean difference. Then the investigation is extended to the corresponding family of relative measures by exploiting the fact that the family derived consists of compromise measures which are absolute or relative measures depending on the way they are represented. We introduce a variant of the decomposition property and then get a corresponding one parameter extension of the Gini coefficient. This family of relative measures takes into account mean incomes. The result in particular demonstrates that the Gini coefficient satisfies an aggregation property. Finally, we consider a family of measures extending the variance of logarithms. Though these indices satisfy the original form of the decomposition property they are relative measures. This outcome demonstrates that this property is flexible and not limited to absolute measures. The families considered have – to the best knowledge of the author – not yet been characterized in the literature. Section 5 concludes.

In summary, the contribution of the paper is fourfold: First, it suggests a new decomposition and an aggregation property for the measurement of inequality which have intuitive appeal. Second, it characterizes the corresponding class of weakly decomposable and weakly aggregable inequality measures. Third, by applying this result the properties of three families of inequality measures are described. They extend Gini's mean difference, the Gini coefficient, and, respectively, the variance of logarithms. Fourth, the results presented can also be used to measure the dispersion inherent in any distribution and are therefore also relevant for other applications.

2. Framework and basic properties

To begin with we introduce the framework and the notation. For every population size $n \geq 1$ there are n individuals numbered by i, for $i = 1,\ldots,n$. Individual i's income is denoted by $X_i \in \Omega$ where $\Omega = \mathbb{R}$ or $\mathbb{R}_{++}$. An income distribution is described by a vector $\boldsymbol{X} = (X_1,\ldots,X_n) \in \Omega(n)$. Here $\Omega(n)$ is equal to $\mathbb{R}^n$ or to $\mathbb{R}^n_{++}$. Income distributions can be replicated. Let $\boldsymbol{X}^{(m)} \in \Omega(mn)$ be the vector containing m times the components of $\boldsymbol{X}$ for $\boldsymbol{X} \in \Omega(n)$ and $m \geq 2$, i.e. $\boldsymbol{X}^{(m)} := (\boldsymbol{X},\ldots,\boldsymbol{X}) := ((X_1,X_2,\ldots,X_n),\ldots,(X_1,X_2,\ldots,X_n))$ (in this order). If the population of size n is partitioned into two disjoint and exhaustive subgroups of size n^1 and n^2 where $n^1 + n^2 = n$, an income distribution $\boldsymbol{X} \in \Omega(n)$ can also be partitioned into $\boldsymbol{X} = (\boldsymbol{X}^1, \boldsymbol{X}^2)$ where $\boldsymbol{X}^1 \in \Omega(n^1)$ and $\boldsymbol{X}^2 \in \Omega(n^2)$. Such a partition is characterized by $\boldsymbol{n} = (n^1, n^2)$. Finally, $\mu(\boldsymbol{X}) = \mu(X_1,\ldots,X_n) = (1/n)\,\Sigma_i X_i$ denotes the arithmetic mean, $g(\boldsymbol{X}) = \Pi_i\, X_i^{1/n}$ the geometric mean, and $\mathbf{1}_n$ a vector consisting of n ones.

For $n \geq 1$ an inequality *index* is defined by a function $I(\cdot,n)\colon\ \Omega(n) \to \mathbb{R}_+$. An inequality *measure* I consists of a countable sequence of inequality indices $\{I(\cdot,n)\}_{n\geq 1}$ which contains exactly one inequality index for every population size $n \geq 1$. These indices are not necessarily related for different population sizes, but we suppose that either $\Omega(n) = \mathbb{R}^n$ or $\Omega(n) = \mathbb{R}^n_{++}$ for all $n \geq 1$. We always set $I(X,1) \equiv 0$ for $X \in \Omega$ since for $n = 1$ there cannot be any inequality.

Given this setting we introduce some standard[4] properties for inequality indices and inequality measures:

NORM(alization): *For all* $\boldsymbol{X} \in \Omega(n)$: $I(\boldsymbol{X},n) = 0 \Leftrightarrow$ *there is* $\lambda \in \Omega$ *s.t.* $\boldsymbol{X} = \lambda \mathbf{1}_n$.

SYM(metry): $I(\cdot,n)$ *is symmetric if* $I(\boldsymbol{X},n) = I(\boldsymbol{X}^{\pi},n)$ *for all* $\boldsymbol{X} \in \Omega(n)$ *where* $\boldsymbol{X}^{\pi} = (X_{\pi(1)},\ldots,X_{\pi(n)})$ *for a permutation* π *of* $\{1,\ldots,n\}$.

PP (Principle of Population): $I(\boldsymbol{X}^{(m)},mn) = I(\boldsymbol{X},n)$ *for all* $\boldsymbol{X} \in \Omega(n)$ *and* $m,n \geq 2$.

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[4] See Kolm (1976a, 1976b, 1999) for a thorough discussion of inequality measurement.

NORM postulates that an inequality index equals zero if and only if all incomes are equal. This property normalizes the index $I(\cdot,n)$. We furthermore obtain $I(\mathbf{X},n)>0$ whenever the incomes considered are not identical. Thus $I(\cdot,n)$ cannot be constant. As in our framework individuals can differ only with respect to income there is no reason to treat them differently: SYM guarantees anonymity ($I(\cdot,n)$ is symmetric). The principle of population requires that inequality should depend only on the statistical *distribution* of income. A view at the literature demonstrates that this property has in general two consequences: It makes indices for different population sizes consistent and comparable, and it gives some additional structure to the indices involved.

For the measurement of inequality we have to consider the redistribution of income and introduce two principles. First, the Pigou-Dalton principle of transfers going back to Pigou (1912) and Dalton (1920). It requires that a rank-preserving transfer of income from a richer individual to a poorer one *de*creases inequality. It is based on the definition of a progressive transfer

Definition: *$\mathbf{Y}\in\Omega(n)$ is obtained from $\mathbf{X}\in\Omega(n)$ by a progressive transfer if there are $\eta>0,\;\; i,j\in\{1,\dots,n\}$ such that $Y_k=X_k$ for $k\neq i, k\neq j$ and $X_i<Y_i=X_i+\eta\le Y_j=X_j-\eta<X_j$*

and given by

PT (Pigou-Dalton Principle of Transfers): *For all $\mathbf{X}\in\Omega(n)$: If $\mathbf{Y}\in\Omega(n)$ is obtained from* **X** *by a progressive transfer, then $I(\mathbf{Y},n)<I(\mathbf{X},n)$.*

PT is the classical principle of redistribution. Second, the principle of concentration postulates that any equiproportional reduction of the distance between each income and the (fixed) average income decreases inequality. It has been mentioned by Kolm (1996, 1999), is discussed in detail by Ebert (2009), and has also been used by Aaberge (1997, 2001).[5] A concentration is defined by the transformation

$$T_\kappa(\mathbf{X}):=\mathbf{X}+\kappa\left(\mu(\mathbf{X})\mathbf{1}_n-\mathbf{X}\right) \text{ for } \mathbf{X}\in\Omega(n) \text{ and } 0<\kappa\le 1$$

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[5] A concentration T_κ is equivalent to a lump sum transfer $\kappa\mu(\mathbf{X})$ in combination with a proportional tax (the tax rate is equal to κ). Aaberge (1997) uses this tool in order to interpret changes in rank-dependent inequality measures. In Aaberge (2001) this transformation is employed in the characterization of preference relations defined on the set of Lorenz curves and of rank-dependent measures of inequality.

and the corresponding principle is given by

PC (Principle of Concentration): *For all* $0<\kappa\leq 1$ *and* $X\in\Omega(n)$ *with* $X\neq\mu(X)\mathbf{1}_n$*:* $I(T_\kappa(X),n)<I(X,n)$.

Both principles coincide for $n=2$. For $n>3$ the Pigou-Dalton principle of transfers implies the principle concentration, i.e., PC is weaker than PT. Consider e.g. the income distribution $X:=(4,12,14)$. Then $\mu(X)=10$. A concentration T_κ leads to $T_\kappa(X)=X+\kappa(6,-2,-4)$. For $\kappa=1/2$ we obtain $Y:=(7,11,12)$. All incomes move towards the average income which is not changed by this kind of redistribution. Since the components of the vector $(\mu(X)\mathbf{1}_n-X)$ sum up to zero it is obvious that the sum of the negative entries is equal to the sum of the positive entries. Therefore a concentration can always be decomposed into a series of progressive transfers (in our case: $X\rightarrow(5,11,14)=:Z\rightarrow(7,11,12)=Y$). A concentration is a particular combination of progressive transfers. Then both principles imply that $I(Y)<I(X)$, but PC is silent when only a progressive transfer is applied yielding Z. On the other hand $I(Z)<I(X)$ is an implication of PT. Therefore it is worthwhile to derive the implications of both principles.[6]

PP is an axiom for an inequality measure whereas the other properties are concerned with inequality indices. Accordingly we define for any property $Q\in\{SYM, NORM, PT, PC\}$ that an inequality measure I satisfies Q if every inequality index $I(\cdot,n)$ satisfies Q for $n\geq 2$.

3. Weakly decomposable and weakly aggregable inequality measures

In this section the decomposition property and aggregation property are introduced and discussed. Weakly decomposable and weakly aggregable inequality measures are defined. Then their relationship is clarified. Finally the class of weakly decomposable (aggregable) measures is characterized.

3.1 Definitions

At first we consider the (new) decomposition method in more detail and suggest

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6 Amiel and Cowell (1999) report that in their empirical studies the principle PT is rejected by a majority of respondents. Ebert (2009) demonstrates that PC represents the attitude towards the redistribution of income revealed in this empirical work.

DEC(omposition): *For every* $\boldsymbol{n}=(n^1,n^2)$, *where* $n^1 \geq 1$ *and* $n^2 \geq 1$ *there exist strictly positive weighting functions* $\alpha^1(\boldsymbol{n})$, $\alpha^2(\boldsymbol{n})$, *and* $\beta(\boldsymbol{n})$, *such that*

$$I(\boldsymbol{X}^1,\boldsymbol{X}^2,n^1+n^2)=\alpha^1(\boldsymbol{n})I(\boldsymbol{X}^1,n^1)+\alpha^2(\boldsymbol{n})I(\boldsymbol{X}^2,n^2)+\beta(\boldsymbol{n})\sum_{i=1}^{n^1}\sum_{j=1}^{n^2}I(X_i^1,X_j^2,2) \qquad (1)$$

for all $\boldsymbol{X}^1 \in \Omega(n^1)$ *and* $\boldsymbol{X}^2 \in \Omega(n^2)$.

Property DEC allows us to decompose overall inequality in a population of size n into a within-group term I_W and between-group term I_B for any partition $\boldsymbol{n}=(n^1,n^2)$ and $n=n^1+n^2$. The first one is a weighted sum of the inequality values of the two subgroups, i.e., $I_W=\alpha^1(\boldsymbol{n})I(\boldsymbol{X}^1,n^1)+\alpha^2(\boldsymbol{n})I(\boldsymbol{X}^2,n^2)$. It corresponds to the usual within-group term used in the literature (cf. e.g., Bourguignon (1979), Shorrocks (1980)). The weighting functions $\alpha^1(\boldsymbol{n})$ and $\alpha^2(\boldsymbol{n})$ depend only on the population and subgroup size(s) and have to be strictly positive. They do not necessarily sum to unity and they are assumed to be independent of the average incomes. The second one, $I_B=\beta(\boldsymbol{n})\sum_{i=1}^{n^1}\sum_{j=1}^{n^2}I(X_i^1,X_j^2,2)$, is based on a pairwise comparison of incomes. Usually the income distributions $\boldsymbol{X}^1$ and $\boldsymbol{X}^2$ are smoothed and between-group inequality is calculated by means of the respective average incomes as $I(\mu(\boldsymbol{X}^1)\mathbf{1}_{n^1},\mu(\boldsymbol{X}^2)\mathbf{1}_{n^2},n^1+n^2)$. In our case, no smoothing is required. We compare the inequality directly, i.e., we consider the inequality between all pairs of individuals – one belonging to subgroup 1, the other one belonging to subgroup 2. Then, of course, we have to take into account the total between-group inequality by adding up these inequality values. $\beta(\boldsymbol{n})$ represents a scaling factor.

An inequality measure I is called weakly decomposable if it satisfies DEC. It should be emphasized that the decomposition method is considered for two subgroups. It can, however, be extended to more than two subgroups by repeated (recursive) application of (1). Furthermore, it is interesting to classify the property DEC more precisely. The present paper is obviously normative since it characterizes various families of inequality measures by a small number of relevant properties. On the other hand, a decomposition property is mainly descriptive since it allows us to find a particular representation of the underlying inequality ordering. This representation is essentially cardinal and can be employed to calculate the contribution of each subgroup to overall inequality.

The definition of the between-group term is based on an elementary approach and seems to be more natural than using the smoothed income distributions for a comparison. It assigns a particular role to the inequality index for two individuals. But it is easy to see that the between-group term I_B reduces to the conventional (smoothed) between-group term when there is no inequality within groups. This observation provides a link between DEC and the conventional subgroup decompositions. Closer inspection of the term demonstrates that for weakly decomposable measures between-group inequality in general depends on the distribution of income within the subgroups considered. The entire term can also be interpreted as an index measuring the distance between the income distributions of both subgroups. In principle there are infinitely many ways of calculating the distance between two income distributions (cf. the statistical literature on this topic, e.g. Bickel and Doksum (2000), and also Ebert (1984) and Deutsch and Silber (1999)). One extreme possibility of determining this distance is to use only the average incomes (the conventional method). The other extreme is to utilize the full information available, i.e., to consider the inequality between all pairs of income (the new method). Choosing an extreme form of the term (as is done also in this paper) represents one possibility. Of course, it is also possible to consider further intermediate cases and to distinguish between them by imposing appropriate conditions. This will be done in another paper.

On the other hand the definition provided in (1) is therefore quite intuitive. The between-group inequality term necessarily depends on the distribution of incomes whenever its definition is not based on the arithmetic means. Ethical inequality measures use the corresponding equally distributed incomes for measuring between-group inequality and depend on the distribution of income within the subgroups (cf. e.g., Blackorby, Donaldson and Auersperg (1981) and Ebert (1999)). The same is true for the measures investigated by Foster and Shneyerov (1999, 2000) which employ a generalized mean as representative income. The issues addressed here have therefore always to be dealt with in these cases (cf. also the discussion of Foster and Shneyerov (1999) in their concluding section).

Using the conventional definition of the between-group term has the advantage that any transfer within a subgroup does not change the value of the term. Thus at first sight the new concept of between-group inequality seems to be not as clear-cut as it is in the classical case of decomposition if between-group inequality is measured by the inequality between the average incomes of the subgroups (cf. Shorrocks (1980, 1984)). On the other hand the example provided in the introduction clearly demonstrates that employing only the average incomes is restrictive and actually is a disadvantage as distributional considerations are entirely ignored. Therefore the new form of the between-group term is appealing and justified since the distributions of income in both subgroups are taken into account.

Furthermore, the between-group term also reacts in particular situations in a way one would expect intuitively. Suppose that there are two subgroups, each containing two individuals. Let their incomes be given by $X^1 = (b,c)$, $X^2 = (a,d)$ such that $a < b < c < d$ and $a,b,c,d \in \Omega$. In this case there is some overlapping and a specific pattern of incomes. The incomes of subgroup 2 are more extreme than those of subgroup 1. Then any regressive transfer within subgroup 2 will increase the between-group inequality term (if the measure satisfies the Pigou-Dalton principle of transfers).

Thus property DEC has a lot of appeal since the between-group term takes into account all incomes and does not require any smoothing. The property is satisfied by several well-known measures: Let $G(\boldsymbol{X},n) := \frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|X_i - X_j\right|$ denote Gini's mean difference for $\boldsymbol{X} \in \Omega(n)$.

Then we have[7,8,9]

$$G(\boldsymbol{X}^1,\boldsymbol{X}^2,n^1+n^2) = \frac{(n^1)^2}{(n^1+n^2)^2}G(\boldsymbol{X}^1,n^1) + \frac{(n^2)^2}{(n^1+n^2)^2}G(\boldsymbol{X}^2,n^2) + \frac{4}{(n^1+n^2)^2}\sum_{i=1}^{n^1}\sum_{j=1}^{n^2}G(X_i^1,X_j^2,2)$$

for $\boldsymbol{X}^1 \in \Omega(n^1)$, $\boldsymbol{X}^2 \in \Omega(n^2)$, $\boldsymbol{n} = (n^1,n^2)$, and $n^1+n^2 \geq 2$.

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7 A referee points at the fact that this decomposition of Gini's mean difference can be obtained by a decomposition of the cumulative income distribution function F (in the continuous framework). Suppose that $F(X) = \sum_{i=1}^{2} p_i F_i(X)$ where p_i is the population share of subgroup i. Since $G = \int F(X)(1-F(X))\,dX$ (cf. e.g. Yitzhaki (1998)) we get by insertion $G = \sum_{i=1}^{2} p_i^2 G_i + 2p_1p_2\int F_1(X)(1-F_2(X))\,dX$ where G_i is Gini's mean difference associated with the cumulative distribution function F_i.

8 Dagum (1997) presents a particular decomposition of the Gini-coefficient into the inequality within the subgroups, the net inequality between subgroups, and a third term related to the intensity of overlapping between the subgroups. The sum of his second and third terms is identical to the between-group inequality term defined here.

9 The Gini-coefficient can also be decomposed by income components – if there are different sources of income (cf. Rao (1969)). This topic is not dealt with here.

We get the same result for the variance $V(\boldsymbol{X},n)$ since

$$V(\boldsymbol{X},n):=\frac{1}{n}\sum_{i=1}^{n}\left(X_i-\mu(\boldsymbol{X})\right)^2=\frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|X_i-X_j\right|^2 \text{ for } \boldsymbol{X}\in\mathbb{R}^n.$$

Similarly, the variance of logarithms $VL(\boldsymbol{X},n)$ also satisfies DEC.[10] It can also be defined with respect to the geometric mean:

$$VL(\boldsymbol{X},n):=\frac{1}{n}\sum_{i=1}^{n}\left(\ln X_i-\mu(\ln(\boldsymbol{X}))\right)^2=\frac{1}{n}\sum_{i=1}^{n}\left(\ln X_i-\ln g(\boldsymbol{X})\right)^2 \text{ for } \boldsymbol{X}\in\mathbb{R}^n_{++}$$

and it can be rewritten as

$$VL(\boldsymbol{X},n)=\frac{1}{2n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\ln X_i-\ln X_j\right|^2.$$

Now we turn to a different interpretation of the decomposition property DEC. Given two groups of size n^1 and n^2 the property DEC describes the inequality in an *overall* population of size $n=n^1+n^2$. In other words, DEC can also be interpreted as an aggregation property. If we know the indices $I(\cdot,n^1)$, $I(\cdot,n^2)$ and $I(\cdot,2)$ we are able to define an inequality index for a population of size $n=n^1+n^2$ by using (1). Following this idea we introduce a particular case of (1) by considering a partition described by $\boldsymbol{n}=(n,1)$:

AGG(regation): *For all* $n\geq 1$ *there exist strictly positive weighting functions* $\gamma(n+1)$ *and* $\delta(n+1)\in\mathbb{R}_{++}$ *such that*

$$I(\boldsymbol{X},X_{n+1})=\gamma(n+1)I(\boldsymbol{X},n)+\delta(n+1)\sum_{i=1}^{n}I(X_i,X_{n+1},2) \tag{2}$$

for all $\boldsymbol{X}\in\Omega(n)$ *and* $X_{n+1}\in\Omega$.

Here we start from a population of size n and enlarge it by one individual. Then the income inequality for a population of size $n+1$ is determined in two steps: At first, the inequality for the subgroup of the individuals 1 to n is computed and taken into account by $\gamma(n+1)I(\boldsymbol{X},n)$. It corresponds to the (total) within-group term since individual $n+1$ forms a subgroup of size 1, in which by assumption no within-group inequality exists. In a second step the income inequality between each individual belonging to the subgroup of size n and

[10] It is well known that VL violates the Pigou-Dalton principle of transfers.

individual $n+1$ is calculated and taken into consideration by $\delta(n+1)\Sigma_i I(X_i, X_{n+1}, 2)$. This sum represents the between-group inequality term. The aggregation property requires that the overall inequality value is a sum of both components (within-group and between-group term).

This basic idea can be used for all $n \geq 1$. Therefore we obtain a recursive definition of inequality indices. It is obvious that an inequality measure, i.e., a sequence of indices, satisfying AGG is already uniquely determined by the inequality index $I(\cdot,2)$ and the sequence of weights $\{\gamma(n), \delta(n)\}_{n\geq 2}$. An inequality measure I is called weakly aggregable if it satisfies AGG.

3.2 Relationship

Next we have to clarify the relationship between the properties DEC and AGG. Formally property AGG describes a particular variant of DEC: We have $\gamma(n+1) = \alpha^1(n,1)$ and $\delta(n+1) = \beta(n,1)$. Thus we immediately obtain

Proposition 1: *Every weakly decomposable inequality measure I is weakly aggregable.*

Conversely, we can establish

Proposition 2: *Assume that I is a weakly aggregable inequality measure and that $I(\cdot,2)$ satisfies NORM and SYM. Then I is weakly decomposable.*

Thus both properties are equivalent given weak assumptions on the inequality index for two individuals. Indeed, below it will turn out that the classes of weakly decomposable and weakly aggregable inequality measures coincide if the principle of population is additionally imposed.

Proof of Proposition 2

The assertion is proved in four steps: (a) At first a simple (technical) implication of AGG is derived. (b) Then it is shown that – given AGG – symmetry of $I(\cdot,n)$ is inherited from symmetry of $I(\cdot,2)$. (c) This fact allows us to prove that every inequality index is proportional to the sum of the inequality values between all pairs of income. (d) This representation implies weak decomposability.

(a) **Claim:** *Assume that I satisfies AGG and that $I(\cdot,2)$ satisfies NORM. Then*

$$I(\boldsymbol{X},n)=\sum_{j=2}^{n}\sigma_j(n)\sum_{i=1}^{j-1}I(X_i,X_j,2) \tag{3}$$

for $\boldsymbol{X}\in\Omega(n)$ *and* $n\geq 2$ *where*[11] $\delta(2)=1$ *and* $\sigma_j(n)=\delta(j)\prod_{k=j+1}^{n}\gamma(k)$ *for* $j=2,\ldots,n$.

Proof: The statement is correct for $n=2$. Now suppose that it has been proved for $n\geq 2$. Then AGG yields that

$$I(\boldsymbol{X}_i,X_{n+1})=\gamma(n+1)I(\boldsymbol{X},n)+\delta(n+1)\sum_{i=1}^{n}I(X_i,X_{n+1},2)$$

$$=\gamma(n+1)\sum_{j=2}^{n}\delta(j)\left(\prod_{k=j+1}^{n}\gamma(k)\right)\sum_{i=1}^{j-1}I(X_i,X_j,2)+\delta(n+1)\sum_{i=1}^{n}I(X_i,X_{n+1},2)$$

$$=\sum_{j=2}^{n}\delta(j)\left(\prod_{k=j+1}^{n+1}\gamma(k)\right)\sum_{i=1}^{j-1}I(X_i,X_j,2)+\delta(n+1)\sum_{i=1}^{n}I(X_i,X_{n+1},2)$$

$$=\sum_{j=2}^{n+1}\delta(j)\left(\prod_{k=j+1}^{n+1}\gamma(k)\right)\sum_{i=1}^{j-1}I(X_i,X_j,2).$$

Furthermore, $\delta(2)=1$ for $\boldsymbol{X}\in\Omega(n)$ and $X_{n+1}\in\Omega$ is implied by AGG and NORM since $I(\boldsymbol{X},2)=\gamma(2)I(X_1,1)+\delta(2)I(\boldsymbol{X},2)=\delta(2)I(\boldsymbol{X},2)$.

(b) **Claim:** *Assume that I satisfies (3) and that* $I(\cdot,2)$ *satisfies SYM. Then I is symmetric.*

Proof: Symmetry of $I(\cdot,2)$ yields $I(X_i,X_j,2)=\frac{1}{2}\left(I(X_i,X_j,2)+I(X_j,X_i,2)\right)$. Using (3) we obtain

$$I(\boldsymbol{X},n)=\tfrac{1}{2}\sum_{j=2}^{n}\sum_{i=1}^{j-1}\tau_{ij}(n)\,I(X_i,X_j,2)+\tfrac{1}{2}\sum_{j=2}^{n}\sum_{i=1}^{j-1}\tau_{ij}(n)\,I(X_j,X_i,2) \tag{4}$$

where $\tau_{ij}(n):=\sigma_j(n)$ for $i<j$. We define $\tau_{ij}(n):=\sigma_i(n)$ for $i>j$ and get $\tau_{ij}(n)=\tau_{ji}(n)$ for $i,j=1,\ldots,n,\quad i\neq j$.

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[11] Here: $\prod_{k=j+1}^{n}\gamma(k):=1$ for $j+1>n$.

The first term of the right hand side of (4) can be rearranged to $\frac{1}{2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\tau_{ij}(n)\,I(X_i,X_j,2)$.

Interchanging the indices i and j and observing the symmetry of $\tau_{ij}(n)$ we obtain $\frac{1}{2}\sum_{i=2}^{n}\sum_{j=1}^{i-1}\tau_{ij}(n)\,I(X_i,X_j,2)$ for the second term of the right hand side. Therefore

$$I(\boldsymbol{X},n)=\tfrac{1}{2}\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\tau_{ij}(n)\,I(X_i,X_j,2) \tag{5}$$

which proves symmetry of $I(\cdot,n)$ for $n\geq 2$.

(c) **Claim:** *Assume that I satisfies AGG and that $I(\cdot,2)$ satisfies NORM and SYM. Then*

$$I(\boldsymbol{X},n)=\frac{\delta(n)}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}I(X_i,X_j,2) \tag{6}$$

for all $\boldsymbol{X}\in\Omega(n)$ and $n\geq 2$ where $\delta(2)=1$ and $\delta(n)=\delta(2)\left(\prod_{k=3}^{n}\gamma(k)\right)$.

Proof: The statement is true for $n=2$ (use claim (a), and NORM and SYM of $I(\cdot,2)$). The proof is by induction.

We define $\boldsymbol{X},\boldsymbol{Y}\in\Omega(n+1)$ by setting $X_i:=x$ for $i=1,\ldots,n$, $X_{n+1}:=y$ and $Y_1:=y$, $Y_i:=x$ for $i=2,\ldots,n+1$ for any $x,y\in\mathbb{R}_{++}\subset\Omega$ and $x\neq y$.

Then $I(\boldsymbol{X},n+1)=I(\boldsymbol{Y},n+1)$ since *I* is symmetric (employ claim (a) and (b)).

Using AGG, the result already proved for *n*, and the specific structure of ***X*** and ***Y***, we get

$$I(\boldsymbol{X},n+1)=\gamma(n+1)\frac{\delta(n)}{2}n^2I(x,x,2)+\delta(n+1)nI(x,y,2)$$

and

$$I(\boldsymbol{Y},n+1)=\gamma(n+1)\frac{\delta(n)}{2}\left[(n-1)^2I(x,x,2)+2(n-1)I(x,y,2)+I(y,y,2)\right]$$
$$+\delta(n+1)\left[I(y,x,2)+(n-1)I(x,x,2)\right].$$

NORM implies that $I(x,x,2)=I(y,y,2)=0$. Equating both equations and rearranging terms we obtain

$$\left[\gamma(n+1)\delta(n)(n-1)+\delta(n+1)-n\delta(n+1)\right]I(x,y,2)=0.$$

Since $I(\cdot,2)$ satisfies NORM and $x \neq y$ we get $I(x,y,2)>0$. Then the bracket on the LHS has to be zero and we obtain $\delta(n+1)=\gamma(n+1)\delta(n)=\delta(2)\left(\prod_{k=3}^{n+1}\gamma(k)\right)$.

Therefore

$$\sigma_i(n)=\delta(j)\prod_{k=j+1}^{n}\gamma(k)=\delta(2)\prod_{k=3}^{j}\gamma(k)\prod_{k=j+1}^{n}\gamma(k)=\delta(n).$$

Then (5), NORM of $I(\cdot,2)$, and the definition of $\tau_{ij}(n)$ imply (6).

(d) **Claim:** *Assume that I satisfies (6). Then I is weakly decomposable.*

Proof: Given (5) one defines

$$\alpha^i(n^1,n^2):=\delta(n^1+n^2)/\delta(n^i) \text{ for } i=1,2$$

and

$$\beta(n^1,n^2)=\delta(n^1+n^2) \text{ for } n^1 \geq 1 \text{ and } n^2 \geq 1.$$

Direct computation demonstrates that DEC is satisfied (even if $n^1=1$ or $n^2=1$). □

3.3 Characterization

Now we are in a position to investigate the consequences of the principle of population.

Theorem 1: *Assume that I is an inequality measure and that $I(\cdot,2)$ satisfies NORM.*

Then I satisfies DEC (or AGG) and PP if and only if

$$I(X,n)=\frac{2}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}I(X_i,X_j,2) \tag{7}$$

for all $X \in \Omega(n)$ and $n \geq 2$.

It turns out that – given that $I(\cdot,2)$ satisfies NORM – property DEC (or AGG) and the principle of population already determine the structure of an inequality measure uniquely: The only degree of freedom left is the choice of the index $I(\cdot,2)$. It is easy to see that the measure *I* is then symmetric and also satisfies NORM. The weighting functions used in DEC and AGG are determined implicitly. Employing (7) we obtain the decomposition

$$I\left(X^1, X^2, n^1+n^2\right) = \frac{\left(n^1\right)^2}{\left(n^1+n^2\right)^2} I\left(X^1, n^1\right) + \frac{\left(n^2\right)^2}{\left(n^1+n^2\right)^2} I\left(X^2, n^2\right)$$

$$+ \frac{4}{\left(n^1+n^2\right)^2} \sum_{i=1}^{n^1} \sum_{j=1}^{n^2} I\left(X_i^1, X_j^2, 2\right)$$

for all $X^1 \in \Omega\left(n^1\right)$, $X^2 \in \Omega\left(n^2\right)$, $\boldsymbol{n} = \left(n^1, n^2\right)$, and $n^1 + n^2 \geq 2$

and the aggregation rule

$$I\left(X, X_{n+1}, n+1\right) = \frac{n^2}{\left(n+1\right)^2} I\left(X, n\right) + \frac{4}{\left(n+1\right)^2} \sum_{i=1}^{n} I\left(X_i, X_{n+1}, 2\right)$$

for all $X \in \Omega(n)$, $X_{n+1} \in \Omega$ and $n \geq 1$.

Obviously the weights are used to 'renormalize' the indices $I\left(\cdot, n^1\right)$, $I\left(\cdot, n^2\right)$, $I(\cdot, n)$ and $I(\cdot, 2)$. The size of the (sub)groups has to be taken into account properly. The weights of the within-group term do not sum up to unity – a phenomenon which is well known from the literature (cf. e.g. the generalized entropy class in Shorrocks (1980)). This fact is an implication of the principle of population imposed in Theorem 1. Consider the measure $I^*(X, n) := nI(X, n)$ – where I is defined in (7). It also satisfies DEC (and AGG), but violates PP. Its weighting functions are given by $\alpha^i\left(n^1, n^2\right) = n^i / \left(n^1 + n^2\right)$ for $i = 1, 2$, add up to unity, and are equal to the population share of the subgroup.

To sum up, any inequality index $I(\cdot, 2)$ which satisfies NORM and only two axioms – DEC (or AGG) and PP – yields a unique inequality measure *I*. This result demonstrates that the decomposition and, respectively, the aggregation property is powerful. Few axioms suffice to obtain the measure described in (7). In particular, no regularity condition like continuity has to be imposed: the characterization is parsimonious.

Proof of Theorem 1

The assertion is proved in four steps: (a)-(c) demonstrate necessity. Conversely, sufficiency is proved in (d).

(a) **Claim:** *DEC implies AGG* (Proposition 1).

(b) **Claim:** *Assume that I satisfies AGG and PP and that* $I(\cdot,2)$ *satisfies NORM. Then I is symmetric.*

Proof: The principle of population and claim (a) of the proof of Proposition 2 imply

$$I(X_1,X_2,2)=I(X_1,X_2,X_1,X_2,4)$$
$$=\sigma_2(4)I(X_1,X_2,2)+\sigma_3(4)\big(I(X_1,X_1,2)+I(X_2,X_1,2)\big)$$
$$+\sigma_4(4)\big(I(X_1,X_2,2)+I(X_2,X_2,2)+I(X_1,X_2,2)\big).$$

Thus

$$\begin{aligned}I(X_1,X_2,2)&=(\sigma_2(4)+2\sigma_4(4))I(X_1,X_2,2)+\sigma_3(4)I(X_2,X_1,2)\\&\quad+\sigma_3(4)I(X_1,X_1,2)+\sigma_4(4)I(X_2,X_2,2).\end{aligned}\tag{8}$$

Analogously we can consider $I(X_2,X_1,2)$ and get

$$I(X_2,X_1,2)=(\sigma_2(4)+2\sigma_4(4))I(X_2,X_1,2)+\sigma_3(4)I(X_1,X_2,2)$$
$$+\sigma_3(4)I(X_2,X_2,2)+\sigma_4(4)I(X_1,X_1,2).$$

Subtraction and some rearrangement yield

$$(1-\sigma_2(4)+\sigma_3(4)-2\sigma_4(4))[I(X_1,X_2,2)-I(X_2,X_1,2)]$$
$$=(\sigma_3(4)-\sigma_4(4))[I(X_1,X_1,2)-I(X_2,X_2,2)]$$

NORM implies that $I(X_1,X_1,2)=I(X_2,X_2,2)=0$ for all X_1,X_2. Thus the right hand side equals zero.

We know from (8) that $1-\sigma_2(4)-2\sigma_4(4)>0$ since NORM satisfied by $I(\cdot,2)$ implies that $\sigma_3(4)I(X_2,X_1,2)>0$ for $X_1\neq X_2$.

Therefore also $1-\sigma_2(4)+\sigma_3(4)-2\sigma_4(4)>0$ since $\sigma_3(4)>0$ and we obtain $I(X_1,X_2,2)=I(X_2,X_1,2)$.

The rest follows from claim (b) of the proof of Proposition 2.

(c) **Claim:** *Assume that I satisfies AGG and PP and that* $I(\cdot,2)$ *satisfies* NORM. *Then* $\delta(n)=4/n^2$ *for* $n\geq 1$.

Proof: Since (b) implies that I is symmetric, we can use claim (c) of the proof of Proposition 2 in order to get $I(\boldsymbol{X},n)=\frac{\delta(n)}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}I(X_i,X_j,2)$ for $n\geq 2$.

Now choose any $\boldsymbol{X}\in\Omega(n)$ and $m\geq 2$. Then by the definition of the replicated income distribution we obtain

$$\sum_{i=1}^{mn}\sum_{j=1}^{mn}I\left(X_i^{(m)},X_j^{(m)},2\right)=m^2\sum_{i=1}^{n}\sum_{j=1}^{n}I(X_i,X_j,2)$$

and therefore

$$I(\boldsymbol{X},n)=I\left(\boldsymbol{X}^{(m)},mn\right)=\frac{\delta(mn)}{2}\sum_{i=1}^{mn}\sum_{j=1}^{mn}I\left(X_i^{(m)},X_j^{(m)},2\right)$$

$$=\frac{\delta(mn)}{2}m^2\sum_{i=1}^{n}\sum_{j=1}^{n}I(X_i,X_j,2)=\frac{\delta(mn)}{2}m^2\frac{2}{\delta(n)}I(\boldsymbol{X},n)$$

which implies $\delta(mn)=\delta(n)/m^2$. Analogously $\delta(mn)=\delta(m)/n^2$.

Then we get $\delta(n)\,n^2=$ constant for $n\geq 2$. For $n=2$ we have $\delta(2)\cdot 2^2=4$ since $\delta(2)=1$. Thus $\delta(n)=4/n^2$.

(d) ***Claim***: *If I satisfies (7) and $I(\cdot,2)$ satisfies NORM, then I fulfills NORM, SYM, AGG (DEC) and PP.*

It is obvious that (7) implies PP (and NORM and SYM). Furthermore, we define

$$\alpha^i\left(n^1,n^2\right):=\frac{\left(n^i\right)^2}{\left(n^1+n^2\right)^2}\ \text{for } i=1,2,\qquad \beta\left(n^1,n^2\right):=\frac{4}{\left(n^1+n^2\right)^2}.$$

and, respectively,

$$\gamma(n+1):=\frac{n^2}{(n+1)^2},\qquad \delta(n+1):=\frac{4}{(n+1)^2}$$

Then DEC and, respectively, AGG are also fulfilled. □

4. Some families of measures

The above analysis demonstrates that the inequality index for two individuals is crucial for any inequality measure satisfying the properties considered in Theorem 1. This index can in principle be an arbitrary indicator defined for two individuals which evaluates the inequality inherent in a distribution and which satisfies the necessary properties. In the following we consider three families of measures by using an appropriate index for two individuals: First, we derive (an extension of) Gini's mean difference. Second, we suggest a variant of the decomposition property which allows us to characterize (an extension) of the Gini coefficient. Third, we introduce a simple family of indices which extend the variance of logarithms and have not been considered in the literature up to now.

4.1 Extension of Gini's mean difference

We introduce the family of indices $K^{\varepsilon}(\boldsymbol{X},2):=\frac{1}{4}\sum_{i=1}^{2}\sum_{j=1}^{2}\left|X_i-X_j\right|^{\varepsilon}=\left|X_1-X_2\right|^{\varepsilon}/2$ for all $\boldsymbol{X}\in\Omega(2):=\mathbb{R}^2$ and $\varepsilon>0$. These indices represent the same inequality *ordering* (cf. also Ebert (1988b)) and are therefore ordinally equivalent for all $\varepsilon>0$. $K^{\varepsilon}(\cdot,2)$ essentially measures the absolute difference between both incomes.

Using Theorem 1 and this family we obtain a characterization of a single parameter family[12] of inequality measures K^{ε} where

$$K^{\varepsilon}(\boldsymbol{X},n):=\frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|X_i-X_j\right|^{\varepsilon}\ \textit{for all}\ \boldsymbol{X}\in\Omega(n)=\mathbb{R}^n\ \textit{and}\ n\geq 2.$$

These measures are absolute since the addition of the same amount to each income leaves inequality unchanged. Since $K^{\varepsilon}(\cdot,2)$ satisfies NORM automatically, this property required in Theorem 1 has no longer to be postulated explicitly. The family includes Gini's mean difference[13] for $\varepsilon=1$. The variance is characterized for $\varepsilon=2$, i.e., $K^2(\boldsymbol{X},n)=V(\boldsymbol{X},n)$. Given the structure of the measure the parameter ε can be chosen to reflect the attitude to inequality.

To the best knowledge of the author the family K^{ε} has not been characterized before. In the literature several (one parameter) generalizations of Gini's mean difference [the Gini coefficient] have been dealt with: Donaldson and Weymark (1980) and Weymark (1981) investigate ethical equality measures (S-Ginis) which are defined on a rank-ordered income vector. S-Ginis are also discussed by Yitzhaki (1983). Chakravarty (1988) suggests the family of E-Ginis by extending one of the many representations of the Gini-coefficient (cf. Yitzhaki

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12 Ramasubban (1958, 1959, 1960) considers a subfamily in more detail. He is concerned with measures of variability and not of inequality and confines himself to particular distributions.

13 See Yitzhaki (2003) for a survey of the properties of Gini's mean difference.

(1998)). Ebert (1988a) characterizes two more families of ethical inequality measures which are generalizations of Gini's mean difference and which are implied by particular aggregation properties of the underlying social welfare function.

It is interesting to investigate the reaction of the measures K^{ε} to a redistribution of income. We get

Proposition 3: *K^{ε} satisfies PC [PT] if and only if $\varepsilon > 0$ [$\varepsilon \geq 1$].*

Thus they satisfy the principle of concentration for every $\varepsilon > 0$ whereas the parameter ε must not be smaller than unity if PT is to be fulfilled.

Proof of Proposition 3

We only have to prove:

Claim: *Assume that $n > 2$. Then $K^{\varepsilon}(\cdot, n)$ satisfies the Pigou-Dalton principle of transfers if and only if $\varepsilon \geq 1$.*

Proof: For $0 < x < y$ we define two income distributions $\boldsymbol{X}, \boldsymbol{Y} \in \Omega(n)$:

$$\boldsymbol{X} = (x, y, \ldots, y) \text{ and } \boldsymbol{Y} = (x+\eta, y-\eta, y, \ldots, y) \text{ for } 0 < \eta \leq \frac{y-x}{2}.$$

Using (2) and leaving a factor of proportionality aside we obtain

$$I(\boldsymbol{X}, n) = (n-1)(y-x)^{\varepsilon} \text{ and } I(\boldsymbol{Y}, n) = (y-x-2\eta)^{\varepsilon} + (n-2)(y-x-\eta)^{\varepsilon} + (n-2)\eta^{\varepsilon}.$$

Now set $y = x + 2\eta$. Then

$$I(\boldsymbol{X}, n) = (n-1)(2\eta)^{\varepsilon} \text{ and } I(\boldsymbol{Y}, n) = 0^{\varepsilon} + (n-2)\eta^{\varepsilon} + (n-2)\eta^{\varepsilon}.$$

PT requires that $I(\boldsymbol{X}, n) > I(\boldsymbol{Y}, n)$ which is equivalent to

$$(n-1)(2\eta)^{\varepsilon} > 2(n-2)\eta^{\varepsilon} \Leftrightarrow \frac{(n-1)}{(n-2)} > 2^{1-\varepsilon}.$$

The left hand side tends to unity for $n \to \infty$. Thus we obtain $\varepsilon \geq 1$.

Conversely, if $\varepsilon \geq 1$ the indices $K^{\varepsilon}(\cdot, n)$ satisfy PT which can be shown by induction:

It is obvious that $K^{\varepsilon}(\boldsymbol{X}, 2)$ fulfills PT for $\varepsilon \geq 1$.

Now for $n \geq 2$ let $\boldsymbol{Y}$ be obtained from $\boldsymbol{X} \in \Omega(n+1)$ by a progressive transfer from individual j to individual i. By AGG we have

$$K^{\varepsilon}(\boldsymbol{X}, n+1) = \gamma(n+1) K^{\varepsilon}(X_1, \ldots, X_n, n) + \delta(n+1) \sum_{k=1}^{n} K^{\varepsilon}(X_k, X_{n+1}, 2).$$

Since K^{ε} is symmetric and $n \geq 2$ we can assume that $X_{n+1} = Y_{n+1}$. Then

$$K^{\varepsilon}(Y_1, \ldots, Y_n, n) < K^{\varepsilon}(X_1, \ldots, X_n, n)$$

by assumption. Furthermore we have

$$|Y_k - Y_{n+1}|^{\varepsilon} = |X_k - X_{n+1}|^{\varepsilon} \text{ for } k \neq i, j \text{ and}$$

$$|X_i + \eta - X_{n+1}|^{\varepsilon} + |X_j - \eta - X_{n+1}|^{\varepsilon} \leq |X_i - X_{n+1}|^{\varepsilon} + |X_j - X_{n+1}|^{\varepsilon}$$

since $f(x) := |x - X_{n+1}|^{\varepsilon}$ is a convex function for $\varepsilon \geq 1$. □

4.2 Extension of Gini's coefficient

Looking at the functional form of the family of measures K^{ε} we recognize that an index $K^{\varepsilon}(\cdot, n)$ can be transformed to a relative inequality index $\hat{K}^{\varepsilon}(\cdot, n)$ by a simple transformation. We define $\hat{K}^{\varepsilon}(\boldsymbol{X}, n) := K^{\varepsilon}(\boldsymbol{X}, n)/\mu(\boldsymbol{X})^{\varepsilon}$ for $\mu(\boldsymbol{X}) > 0$. Then $\hat{K}(\cdot, n)$ is invariant with respect to equal proportional changes of all incomes, i.e., it is homogeneous of degree zero. In Proposition 3 we have got a family of compromise measures which are absolute or relative depending on the way they are represented (cf. Blackorby and Donaldson (1980) and Ebert (1988b) for the concept of compromise measures). In particular, for $\varepsilon = 1$ we obtain (a multiple of) the Gini coefficient[14] $\hat{K}^{1}(\cdot, n)$.

In order to characterize the corresponding one parameter family of *relative* measures directly and independently of the characterization of the absolute measures, we now confine ourselves to $\Omega(n) = \mathbb{R}^{n}_{++}$ and suggest an alternative form of the decomposition property for every $\varepsilon > 0$:

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[14] If we renormalize K^{1} and divide the index $K^{1}(\cdot, 2)$ by $1/2$, we obtain the usual definition, i.e. $G(\boldsymbol{X}, n) = \hat{K}^{1}(\boldsymbol{X}, n)/2$. The Gini coefficient measures the area between the Lorenz curve and the 45° line as a fraction of the total area under the 45° line.

$\widehat{\text{DEC}}(\varepsilon)$ **(Decomposition):** *For every* $\boldsymbol{n}=(n^1,n^2)$, *where* $n^1 \geq 1$ *and* $n^2 \geq 1$, *there exist strictly positive weighting functions* $\alpha^1(\boldsymbol{n})$, $\alpha^2(\boldsymbol{n})$, *and* $\beta(\boldsymbol{n})$ *such that*

$$I(\boldsymbol{X}^1,\boldsymbol{X}^2,n^1+n^2)=\alpha^1(\boldsymbol{n})\frac{\mu(\boldsymbol{X}^1)^\varepsilon}{\mu(\boldsymbol{X})^\varepsilon}I(\boldsymbol{X}^1,n^1)+\alpha^2(\boldsymbol{n})\frac{\mu(\boldsymbol{X}^2)^\varepsilon}{\mu(\boldsymbol{X})^\varepsilon}I(\boldsymbol{X}^2,n^2)$$

$$+\beta(\boldsymbol{n})\sum_{i=1}^{n^1}\sum_{j=1}^{n^2}\frac{\mu(X_i^1,X_j^2)^\varepsilon}{\mu(\boldsymbol{X})^\varepsilon}I(X_i^1,X_j^2,2)$$

for all $\boldsymbol{X}^1\in\Omega(n^1)$ *and* $\boldsymbol{X}^2\in\Omega(n^2)$.

In this case the weights employed depend also on the respective arithmetic means. A similar kind of weighting (for $\varepsilon=1$) has been proposed by Foster (1983) in his characterization of the Theil measure of inequality. Also the weights used in a decomposition of the generalized entropy class possess this form (cf. equation (32) in Shorrocks (1980)). (Relative) inequality measures $\hat{I}$ satisfying $\widehat{\text{DEC}}(\varepsilon)$ will also be called weakly decomposable. Using this property we can establish an analogue for relative measures to the result discussed in subsection 4.1:

Proposition 4: *Consider the inequality measure* $\hat{I}$ *and assume that* $\hat{I}(\boldsymbol{X},2)=\hat{K}^\varepsilon(\boldsymbol{X},2)$ *for* $\boldsymbol{X}\in\Omega(2)=\mathbb{R}^2_{++}$ *and* $\varepsilon>0$.

The inequality measure $\hat{I}$ *satisfies* $\widehat{DEC}(\varepsilon)$, *PP and PC [PT] if and only if*

$$\hat{I}(\boldsymbol{X},n)=\hat{K}^\varepsilon(\boldsymbol{X},n):=\frac{1}{n^2\mu(\boldsymbol{X})^\varepsilon}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|X_i-X_j\right|^\varepsilon \text{ for all } \boldsymbol{X}\in\Omega(n)=\mathbb{R}^n_{++} \text{ and } n\geq 2$$

[$\varepsilon\geq 1$].

Thus we also obtain a characterization of the one parameter family of inequality measures $\hat{K}^\varepsilon=\left\{\hat{K}^\varepsilon(\cdot,n)\right\}_{n\geq 2}$. In particular for $\varepsilon=1$, i.e., if $\widehat{\text{DEC}}(1)$ is used, the Gini coefficient is characterized.[15] For $\varepsilon=2$ we get an axiomatization of the normalized variance (or the square of the coefficient of variation). An important result is that the Gini coefficient satisfies the decomposition property $\widehat{\text{DEC}}(\varepsilon)$ for $\varepsilon=1$ (or an analogous aggregation property). Thus it is *weakly decomposable.* In the literature there are numerous attempts to find some kind of

[15] Direct characterizations of the Gini coefficient can also be found in Sen (1974), Thon (1982), Ben-Porath and Gilboa (1994), and Aaberge (2001). See also Barrett and Salles (1995) for the class of I-Ginis which form a one-parameter generalization of the Gini coefficient.

decomposability of the Gini coefficient. It is well known that it is decomposable for rank-ordered income vectors and *non-overlapping* subgroups (see e.g. Ebert (1988c)), but it is not decomposable in the conventional sense if arbitrary subgroups can be chosen (see e.g. Zagier (1983) and also Lambert and Decoster (2005)).

Proof of Proposition 4

The proof is obvious: If $\left\{\hat{I}(\cdot,n)\right\}_{n\geq 1}$ satisfies $\widehat{\text{DEC}}(\varepsilon)$ the absolute measure $\left\{\mu(\cdot)^{\varepsilon}\,\hat{I}(\cdot,n)\right\}_{n\geq 1}$ satisfies DEC. Then use Theorem 1. □

4.3 Extension of the variance of logarithms

Finally we define another family of indices $L^{\varepsilon}(X,2) := \frac{1}{4}\sum_{i=1}^{2}\sum_{j=1}^{2}\left|\ln X_i - \ln X_j\right|^{\varepsilon} = \left|\ln X_1 - \ln X_2\right|^{\varepsilon}/2$ for all $X \in \Omega(2) := \mathbb{R}^2_{++}$ and $\varepsilon > 0$.

These indices are also ordinally equivalent for all $\varepsilon > 0$ and are essentially based on the absolute differences of ln-incomes. Since $\left(\ln X_i - \ln X_j\right) = \ln\left(X_i/X_j\right)$ the indicators $L^{\varepsilon}(\cdot,2)$ are relative indices (equal proportional changes do alter their value). Using Theorem 1 we obtain a characterization of the corresponding family of relative measures L^{ε}, where a typical index is given by

$$L^{\varepsilon}(X,n) := \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\ln X_i - \ln X_j\right|^{\varepsilon}$$

for $X \in \Omega(n) = \mathbb{R}^n_{++}$ and $n \geq 2$. We get an axiomatization of the variance of logarithms for $\varepsilon = 2$, i.e. $L^2(X,n) = VL(X,n)$ for $n \geq 2$, which has been considered e.g. by Anand (1983), Sen (1997), Foster and Ok (1999), and Foster and Shneyerov (1999). The measures for $\varepsilon \neq 2$ have been ignored in the literature. All these measures do not fulfill the Pigou-Dalton principle of transfers. We obtain

Proposition 5

(a) *L^{ε} does not satisfy PT for all $\varepsilon > 0$.*

(b) *L^{ε} satisfies PC for $\varepsilon > 0$.*

Thus the principle of concentration is fulfilled. These measures properly react to a redistribution of income according to PC. Therefore they can be used as inequality measures. It is worth to emphasize that the measures can be axiomatized by means of the decomposition (aggregation) property DEC (AGG). Though it seems at first sight that this property has been formulated for absolute measures, it can also be used to characterize the relative measures L^ε.

Proof of Proposition 5

An index $L^\varepsilon(X,n)$ is proportional to

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\ln X_i-\ln X_j\right|^\varepsilon=\sum_{i=1}^{n}\sum_{j=1}^{n}\left|\ln\left(X_j/X_i\right)\right|^\varepsilon. \tag{9}$$

(a) **Claim:** *PT is violated by* L^ε.

Proof: Define a vector $\boldsymbol{X}$ for any $k=n-2$ and $n\geq 3$:

$$X_i:=1 \text{ for } i=1,\dots,k,\ X_{k+1}:=y \text{ and } X_{k+2}:=z \text{ for } y,z\in\Omega \text{ and } 1<y<z.$$

Then we obtain

$$L^\varepsilon(\boldsymbol{X},n)=2\left(k\left[(\ln y)^\varepsilon+(\ln z)^\varepsilon\right]+\left(\ln(z/y)\right)^\varepsilon\right)$$

by the definition of $\boldsymbol{X}$.

Let $\boldsymbol{X}^\eta$ be generated by a particular progressive transfer; i.e., we assume that there is $\eta>0$ such that $X_i^\eta=X_i$ for $i=1,\dots,k$, $X_{k+1}^\eta:=X_{k+1}+\eta=y+\eta$, $X_{k+2}^\eta:=X_{k+2}-\eta=z-\eta$, and $y+\eta\leq z-\eta$. Then

$$\begin{aligned}&L^\varepsilon(\boldsymbol{X},n)-L^\varepsilon(\boldsymbol{X}^\eta,n)\\&\quad=2\Big(k\left[(\ln y)^\varepsilon-\left(\ln(y+\eta)\right)^\varepsilon+(\ln z)^\varepsilon-\left(\ln(z-\eta)\right)^\varepsilon\right]\\&\qquad+\left(\ln(z/y)\right)^\varepsilon-\left(\ln((z-\eta)/(y+\eta))\right)^\varepsilon\Big).\end{aligned} \tag{10}$$

Now we demonstrate that the expression in square brackets on the RHS of (10) is strictly negative for $y:=e^\varepsilon>1$, $z:=y+1$, $\eta:=1/2$: The bracket can be rewritten as

$$[\ ]=g(y)-g(z-\eta)$$

where $g(t)=f(t)-f(t+\eta)$ and $f(t)=(\ln t)^\varepsilon$.

The derivatives of f are given by $f'(t)=\varepsilon(\ln t)^{\varepsilon-1}/t$ and $f''(t)=\varepsilon(\ln t)^{\varepsilon-2}(\varepsilon-1-\ln t)/t^2$. Thus we have $f''(t)<0$ for $t>\max\{e^{\varepsilon-1},1\}$.

Then $g'(t) = f'(t) - f'(t+\eta) > 0$ and therefore

$$g(y) - g(z-\eta) < 0$$

since $z-\eta \geq y+\eta > y = e^{\varepsilon} > \max\{e^{\varepsilon-1}, 1\}$.

Then the RHS of (10) can be made negative by increasing *k* sufficiently. In this case we get

$$L^{\varepsilon}(X^{\eta}, n) > L^{\varepsilon}(X, n),$$

i.e., the Pigou-Dalton principle of transfers is violated.

(b) **Claim:** *PC is satisfied for any* $\varepsilon > 0$.

Proof: Choose any $X \in \Omega(n)$. Because of symmetry we can assume that the incomes in X are increasingly arranged. Then we get

$$1 \leq \frac{X_j + \kappa(\mu(X) - X_j)}{X_i + \kappa(\mu(X) - X_i)} < \frac{X_j}{X_i}$$

for all $\kappa > 0$. Therefore a concentration of income decreases each term $\left|\ln(X_j/X_i)\right|^{\varepsilon}$ □

5. Conclusion

The paper has presented a new decomposition and aggregation property for inequality measures and derived their implications for the structure of measures when the principle of population is additionally imposed. The characterization of weakly decomposable and, respectively, weakly aggregable measures does not require any regularity condition (like continuity or differentiability) and is based on a small number of properties. Applying this result we obtain a one parameter family of compromise measures which contains and extends Gini's mean difference and, respectively, the Gini coefficient. The paper therefore contributes to the debate about the decomposability of the Gini measures. These particular measures are very popular in empirical and theoretical work. Furthermore, a family extending the variance of logarithms is also axiomatized. The reaction of these measures with respect to progressive transfers and concentration is investigated, as well. The results derived are always helpful when a decision on the choice of an inequality measure has to be made since they reveal the value judgments underlying these measures.

Furthermore, it should be emphasized that the analysis performed in this paper is not only important for the measurement of inequality. The measures discussed can in principle be based on an arbitrary 'distance' measure defined for two individuals and be used for an evaluation of the dispersion inherent in any distribution. For example, any index for two individuals having the form $I(X_1, X_2, 2) = f(g(X_1) - g(X_2))$ where $g(t)$ is monotonic and where $f(t)$ is nonnegative, monotonic for $t > 0$ and satisfies $f(0) = 0$ and $f(t) = f(-t)$ could be used. By choosing appropriate functions $f(t)$ and $g(t)$ other families of measures can be generated. Thus the characterization presented lends itself to various applications.

References

Aaberge, R. (1997), Interpretation of changes in rank-dependent measures of inequality, Economics Letters 55, 215-219.

Aaberge, R. (2001), Axiomatic characterization of the Gini coefficient and Lorenz curve orderings, Journal of Economic Theory 101, 115-132.

Amiel, Y. and F.A. Cowell (1999), Thinking about inequality, Cambridge University Press, Cambridge.

Anand, S. (1983), Inequality and Poverty in Malaysia, Oxford University Press, London.

Barrett, C.R. and M. Salles (1995), On a generalization of the Gini coefficient, Mathematical Social Sciences 30, 235-244.

Ben-Porath, E., and I. Gilboa (1994), Linear measures, the Gini index, and the income-equality trade-off, Journal of Economic Theory 64, 443-467.

Bickel, P. and K. Doksum (2000), Mathematical Statistics: Basic Ideas and Selected Topics I, Prentice Hall, Eaglewood Cliffs.

Blackorby, C. and D. Donaldson (1980), A theoretical treatment of indices of absolute inequality, International Economic Review 21, 107-136.

Blackorby, C., D. Donaldson, M. Auersperg (1981), A new procedure for the measurement of inequality, within and among population subgroups: ethically consistent subindices, Canadian Journal of Economics 14, 665-685.

Bourguignon, F. (1979), Decomposable income inequality measures, Econometrica 47, 901-920.

Chakravarty, S.R. (1988), Extended Gini indices of inequality, International Economic Review 29, 147-156.

Chakravarty, S.R. (1999), Measuring inequality in the axiomatic approach, in: J. Silber (Ed.), Handbook of income inequality measurement, Kluwer Academic Publishers, Boston, 163-184.

Cowell, F.A., and K.Kuga (1981), Additivity and the entropy concept: an axiomatic approach to inequality measurement, Journal of Economic Theory 25, 131-143.

Dagum, C. (1997), A new approach to the decomposition of the Gini income inequality ratio, Empirical Economics 23, 515-531.

Dalton, H. (1920), The measurement of inequality of income, Economic Journal 30, 348-361.

Deutsch, J. and J. Silber (1999), Inequality decomposition by population subgroups and the analysis of interdistributional inequality, in: J. Silber (Ed.), Handbook of income inequality measurement, Kluwer Academic Publishers, Boston, 363-397.

Donaldson, D. and J.A. Weymark (1980), A single-parameter generalization of the Gini indices of inequality, Journal of Economic Theory 22, 67-86.

Ebert, U. (1984), Measures of distance between income distributions, Journal of Economic Theory 32, 266-274.

Ebert, U. (1988a), Measurement of inequality: an attempt at unification and generalization, Social Choice and Welfare 5, 147-169.

Ebert, U. (1988b), A family of aggregative compromise inequality measures, International Economic Review 29, 363-376.

Ebert, U. (1988c), On the decomposition of inequality: partitions into non-overlapping subgroups, in: W. Eichhorn (ed.), Measurement in Economics, Physica-Verlag, 1988, 399-412.

Ebert, U. (1999), Dual decomposable inequality measures, Canadian Journal of Economics 32, 234-246.

Ebert, U. (2009), Taking empirical evidence seriously: The principle of concentration and the measurement of welfare and inequality, Social Choice and Welfare 32, 555-574.

Foster, J.E. (1983), An axiomatic characterization of the Theil measure of income inequality, Journal of Economic Theory 31, 105-121.

Foster, J.E., and A.A. Shneyerov (1999), A general class of additively decomposable inequality measures, Economic Theory 14, 89-111.

Foster, J.E. and E.A. Ok (1999), Lorenz dominance and the variance of logarithms, Econometrica 67, 901-907.

Foster, J.E., and A.A. Shneyerov (2000), Path independent inequality measures, Journal of Economic Theory 91, 199-222.

Kolm, S.-C. (1976a), Unequal inequalities I, Journal of Economic Theory 12, 416-442.

Kolm, S.-C. (1976b), Unequal inequalities II, Journal of Economic Theory 13, 82-111.

Kolm, S.-C. (1996), Modern Theories of Justice, MIT Press, Cambridge (Mass.).

Kolm, S.-C. (1999), The rational foundations of income inequality measurement, in: J. Silber (Ed.), Handbook of income inequality measurement, Kluwer Academic Publishers, Boston, 19-94.

Lambert, P.J. and A. Decoster (2005), The Gini coefficient reveals more, Metron 63, 373-400.

Pigou, A.C. (1912), Wealth and Welfare, London: Macmillan.

Ramasubban, T.A. (1958), The mean difference and the mean deviation of some discontinuous distributions, Biometrika 45, 549-556.

Ramasubban, T.A. (1959), The generalized mean difference of the binomial and Poisson distributions, Biometrika 46, 223-229.

Ramasubban, T.A. (1960), Some distributions arising in the study of generalized mean differences, Biometrika 47, 469-473.

Rao, V.M. (1969), Two decompositions of concentration ratio, Journal of the Royal Statistical Society, Series A, 132, 418-425.

Sen, A. (1974), Informational bases of alternative welfare approaches, Journal of Public Economics 3, 387-403.

Sen, A. (1997): On economic inequality, enlarged edition, Oxford: Clarendon Press.

Shorrocks, A.F. (1980), The class of additively decomposable inequality measures, Econometrica 48, 613-625.

Shorrocks, A.F. (1984), Inequality decomposition by population subgroups, Econometrica 52, 1369-1386.

Thon, D. (1982), An axiomatization of the Gini coefficient, Mathematical Social Sciences 2, 131-143.

Weymark, J.A. (1981), Generalized Gini inequality indices, Mathematical Social Sciences 1, 409-430.

Yitzhaki, S. (1983), On an extension of the Gini inequality index, International Economic Review 24, 617-628.

Yitzhaki, S. (1998), More than a dozen alternative ways of spelling Gini, Research on Economic Inequality 8, 13-30.

Yitzhaki, S. (2003), Gini's mean difference: A superior measure of variability, Metron 61, 285-316.

Zagier, D. (1983), Inequalities for the Gini coefficient of composite populations, Journal of Mathematical Economics 12, 103-118.

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