***I**nsurance **R**isk **M**anagement for catastrophic events*

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FAKULTÄT V
MATHEMATIK- UND NATURWISSENSCHAFTEN

AON

Content

- ***sources***
- ***geophysical and engineering aspects***
- ***mathematical aspects***
- ***case studies***

[1] K.M. CLARK (1997): *Current and Potential Impact of Hurricane Variability on the Insurance Industry*. In: H.F. DIAZ, R.S. PULWARTHY (Eds.): *Hurricanes.* Climate and Socioeconomic Impacts. Springer, N.Y., 273 – 283.

[2] K.M. CLARK (2002): *The Use of Computer Modeling in Estimating and Managing Future Catastrophe Losses.* The Geneva Papers on Risk and Insurance Vol. 27 No. 2, 181 – 195.

[3] W. DONG (2001): *Building a More Profitable Portfolio.* Modern Portfolio Theory with Aplication to Catastrophe Insurance. Reactions Publishing Group, London.

[4] P. GROSSI, H. KUNREUTHER (Eds.) (2005): *Catastrophe Modeling: A New Approach to Managing Risk.* Springer, N.Y.

[5] M. KHATER, D.E. KUZAK (2002): *Natural Catastrophe Loss Modelling.* In: M. LANE (Ed.): Alternative Risk Strategies. RISK Books, London, 271 – 299.

[6] M. STEEL (2002): *Integrated Simulation Techniques.* In: M. LANE (Ed.): Alternative Risk Strategies. RISK Books, London, 533 – 543.

[7] SWISS RE (2003): *Natural Catastrophes and Reinsurance.* Swiss Reinsurance Company, Zürich.

[8] D. WHITAKER (2002): *Catastrophe Modelling.* In: N. GOLDEN (Ed.): Rational Reinsurance Buying. RISK Books, London, 103 – 122.

[9] D. PFEIFER (2004): *Solvency II: neue Herausforderungen an Schadenmodellierung und Risikomanagement?* In: Risikoforschung und Versicherung. Festschrift für Elmar Helten, VVW Karlsruhe, 467 – 481.

[10] D. PFEIFER, J. NESLEHOVA (2004): *Modeling and generating dependent risk processes for IRM and DFA*. ASTIN Bulletin 34, 333 – 360.

[11] D. PFEIFER (2001): *Study 4: Extreme value theory in actuarial consulting: windstorm losses in central Europe*. In: R.-D. Reiss, M. Thomas: Statistical Analysis of Extreme Values. With applications to insurance, finance, hydrology and other fields. 2nd ed., Birkhäuser, Basel, 373 – 378.

Main Modelling Companies:

EQECAT, Inc. (ABS Consulting); founded 1981

EQECAT

AIR (Applied Insurance Research)
(Insurance Services Office, Inc. (ISO)); founded 1987

AIR

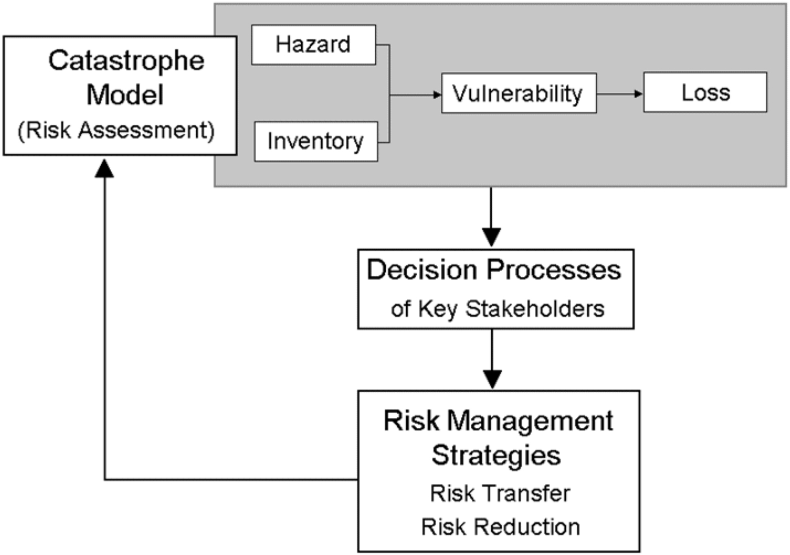
RMS (Risk Management Solutions, DMG Information);
founded 1988 [Stanford University]

Risk Management Solutions

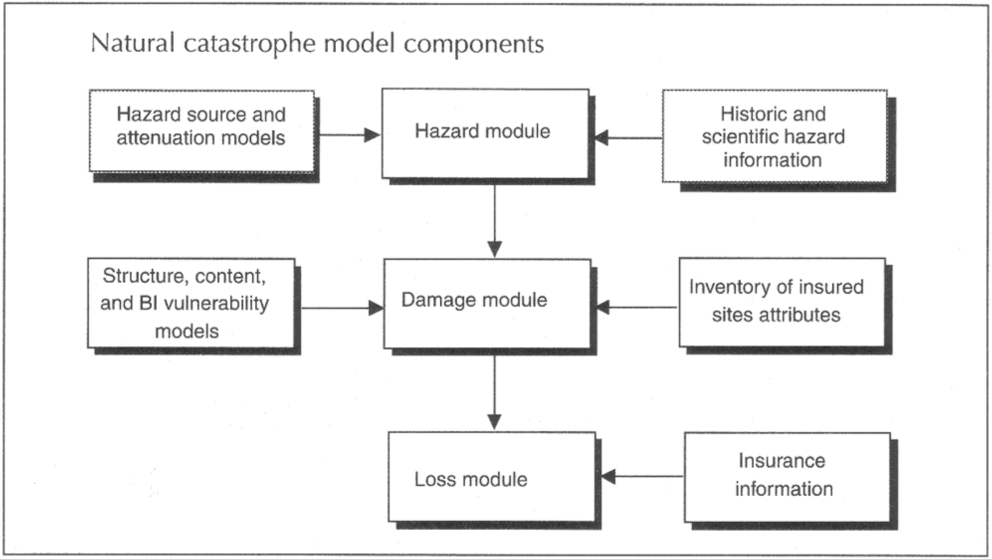
RMS™

aims and scopes:

- ***quantifying risk exposure under "natural" conditions***
- ***quantifying unobserved risk exposure (⇨ earthquakes)***
- ***optimization of re-insurance concepts***
- ***implementation into "internal models" (⇨ DFA, Solvency II)***


Catastrophe Model
(Risk Assessment)
Hazard
Inventory
Vulnerability
Loss
Decision Processes
of Key Stakeholders
Risk Management Strategies
Risk Transfer
Risk Reduction

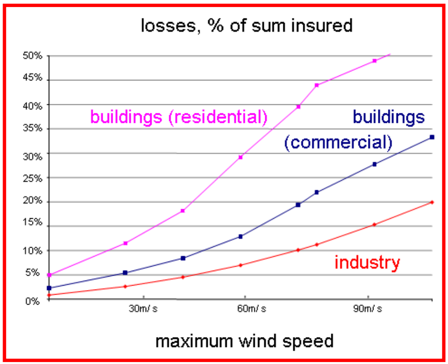
Source: [4], p. 40

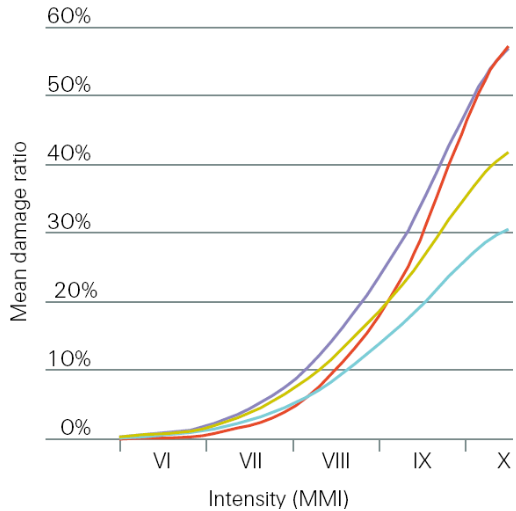


Source: [5], p. 276

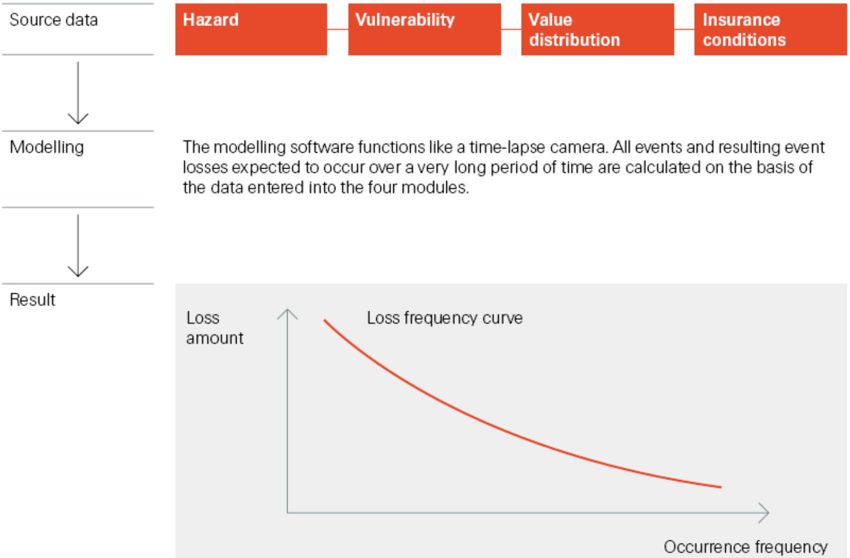
Typical earthquake vulnerability curves for:

- Residental buildings (single family)
- Residental contents (single family)
- Commercial buildings mix
- Industrial equipment and machinery





Source: [7], p. 21


Source data
Hazard
Vulnerability
Value distribution
Insurance conditions
Modelling
The modelling software functions like a time-lapse camera. All events and resulting event losses expected to occur over a very long period of time are calculated on the basis of the data entered into the four modules.
Result
Loss amount
Loss frequency curve
Occurrence frequency

Source: [7], p. 17

The Collective Model of Risk Theory

Basic mathematical assumptions for this model:

- The number N of claims (losses) within a certain period is a non-negative, integer valued random variable, called *frequency*.
- The *individual claims* (losses) occurring during this period, $X_1, X_2, \cdots$, are stochastically independent, identically (as X) distributed, *positive* random variables, independent also from the frequency *N*.

The *aggregate claim* or *aggregate loss* (for the period under consideration) is given by

$$S := \sum_{k=1}^{N} X_k .$$

Assumptions: probability distributions for the claims (losses) are continuous with a density function (df) f and a cumulative distribution function (cdf) F, given by

$$F(x) = \int_0^x f(u)\,du, \;\; x \geq 0.$$

The corresponding survival function (sf) is given by

$$\bar{F}(x) := 1 - F(x) = \int_x^\infty f(u)\,du, \;\; x \geq 0.$$

Lemma 1. The cdf of the aggregate claim (loss) F_S is given by:

$$P(S \leq z) = F_S(z) = p_0 + \sum_{n=1}^{\infty} p_n F^{n*}(z), \;\; z \geq 0.$$

Here $p_n := P(N = n)$ for $n = 0, 1, \cdots$, and F^{n*} denotes the n-fold convolution of F.

Definition (generating functions)**.** Let X be a real-valued random variable such that, for some subset $I \subseteq \mathbb{R}$, the expression

$$\psi_X(t) := E\left(e^{tX}\right),\ t \in I$$

remains finite for all $t \in I$. The mapping ψ_X, defined on I, is then called the *moment generating function* of X or of the distribution P^X, resp.

The mapping defined by

$$\varphi_X(s) := \psi_X(\ln s) = E\left(s^X\right),\ s \in e^I := \left\{e^t \mid t \in I\right\}$$

is called the *probability generating function* of X or of the distribution P^X, resp.

Theorem 1. Let X be a real-valued random variable such that, for some subset $I \subseteq \mathbb{R}$, the moment generating function ψ_X exists. Then the following holds true, under suitable conditions:

a) $\psi_X^{(k)}(0) = E\left(X^k\right),\ k \in \mathbb{N}$ and $\psi_X(t) = \sum_{k=0}^{\infty} \frac{E\left(X^k\right)}{k!} t^k,\ |t| \le \delta$

$\frac{\varphi_X^{(k)}(0)}{k!} = P(X = k),\ k \in \mathbb{N}$ and $\varphi_X(s) = \sum_{k=0}^{\infty} P(X = k)\, s^k,\ |s| \le 1.$

b) Let X and Y be stochastically independent, real-valued random variables with moment generating functions ψ_X and ψ_Y, then the random variable $Z = X + Y$ also possesses a moment generating function, which is given by

$$\psi_{X+Y}(t) = \psi_X(t) \cdot \psi_Y(t),\ t \in I.$$

Examples for discrete distributions:

P^X	distribution	$P(X=k)$	$\varphi_X(s)$	$E(X)$	$Var(X)$
$\mathcal{L}_n$	discrete uniform (Laplace)	$\frac{1}{n}$	$\frac{s}{n}\cdot\frac{s^n-1}{s-1}$	$\frac{n+1}{2}$	$\frac{n^2-1}{12}$
$\mathcal{B}(n,p)$	binomial	$\binom{n}{k}p^k(1-p)^{n-k}$	$(1-p+ps)^n$	np	$np(1-p)$
$\mathcal{NB}(\beta,p)$	negative binomial	$\binom{\beta+k-1}{k}p^\beta(1-p)^k$	$\left(\frac{p}{1-(1-p)s}\right)^\beta$	$\beta\frac{1-p}{p}$	$\beta\frac{1-p}{p^2}$
$\mathcal{P}(\lambda)$	Poisson	$e^{-\lambda}\frac{\lambda^k}{k!}$	$e^{\lambda(s-1)}$	λ	λ

Examples for continuous distributions:

P^X	distribution	density $f(x)$	$\psi_X(t)$	$E(X)$	$Var(X)$
$\mathcal{U}[a,b]$	continuous uniform	$\frac{1}{b-a},\quad a \leq x \leq b$	$\frac{e^{bt}-e^{at}}{t(b-a)}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
$\mathcal{E}(\lambda)$	exponential	$\lambda e^{-\lambda x},\quad x \geq 0$	$\frac{\lambda}{\lambda - t}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
$\Gamma(\alpha,\lambda)$	gamma	$\lambda^\alpha \frac{x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x},\quad x > 0$	$\left(\frac{\lambda}{\lambda - t}\right)^\alpha$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$
$\mathcal{N}(\mu,\sigma^2)$	normal	$\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$	$\exp\left(\frac{\sigma^2 t^2}{2} + \mu t\right)$	μ	σ^2

Theorem 2. If the probability generating function $\varphi_N(s)$ of the frequency exists for $0 \leq s < \eta$ with $\eta > 1$ and the moment generating function $\psi_X(t)$ of individual claim sizes X exists for $0 \leq t < \delta$ with some $\delta > 0$, then

$$\psi_S(t) = \varphi_N\left(\psi_X(t)\right),\ t \in I,$$

where I is a suitable interval, containing zero, with the property that $\psi_X(I) \subseteq [0,\eta)$. For a discrete claim size X with values in $\mathbb{N}$, there also holds

$$\varphi_S(t) = \varphi_N\left(\varphi_X(t)\right),\ t \in e^I \cup [0,1].$$

In particular, all (absolute) moments of the aggregate claim (loss) S exist, and there holds

$$E(S) = E(N)\cdot E(X),\ Var(S) = E(N)\cdot Var(X) + Var(N)\cdot \left\{E(X)\right\}^2.$$

Discretization:

$$X_\Delta := \left\lceil \frac{X}{\Delta} \right\rceil = \min\{k \in \mathbb{N} \mid k\Delta \geq X\}$$

with $\Delta > 0$, and with probabilities

$$P(X_\Delta = k) = P\left(\left\lceil \frac{X}{\Delta} \right\rceil = k\right) = P\left(k-1 < \frac{X}{\Delta} \leq k\right) = F(k\Delta) - F((k-1)\Delta), \quad k \in \mathbb{N}.$$

→ "aggregate claim (loss)" S_Δ has the probability generating function

$$\varphi_{S_\Delta}(s) = \varphi_N\left(\varphi_{X_\Delta}(s)\right), \ |s| \leq 1.$$

(⇨ Panjer-recursion, FFT, series expansion, …)

Model Output

The general data basis for the geophysical modelling software are the so-called *Event Sets*, consisting, among others, of historical data like wind speed, wind direction, flooding levels, earthquake magnitudes etc. By random permutation of the physical parameters, these sets can be artificially enlarged, resulting in the so-called *Stochastic Event Sets*. Such sets can easily have up to 50000 entries and more.

When applied to a particular portfolio analysis, only those entries of these (stochastic) event sets are selected which refer directly to the portfolio under consideration, e.g. by looking at zip codes of the locations. A typical output is then given through a table like this one, called *Event Loss Table*:

Analysis Name	Scenario	Modelled Loss	Standard Deviation	Exposed SI	Rate
Example Wind Analysis	3656	1,940,550,920	36,794,128	68,947,100,000	0.0000062953
Example Wind Analysis	3968	1,563,781,833	49,352,347	95,221,396,000	0.0000129744
Example Wind Analysis	7264	1,482,396,982	41,468,066	69,668,353,333	0.0000113048
Example Wind Analysis	7219	1,461,229,040	43,029,488	72,023,880,000	0.0000113048
Example Wind Analysis	3665	1,431,950,171	47,062,942	73,402,510,667	0.0000047371
Example Wind Analysis	7222	1,332,616,058	40,221,122	78,780,377,333	0.0000113048
Example Wind Analysis	6283	1,169,279,403	35,134,601	74,784,286,000	0.0000468744

Mathematically speaking, the Event Loss Table contains a Collective Risk Model of its own in each row (i.e., for each scenario), where each frequency is of Poisson type and the claims (losses) are deterministic in the basic case, and are endowed with standard deviations in the extended case.

Notation:

n: number of scenarios in the Event Loss Table (= number of rows)

$N_1, N_2, \cdots, N_n$: the row-wise frequencies

$X_{ij},\ 1 \le i \le n,\ \ j \in \mathbb{N}:$ the individual claim sizes, same distribution Q_i.

Note that in the basic Event Loss Table, these distributions are Dirac distributions. Then:

$$S_i := \sum_{j=1}^{N_i} X_{ij},\ \ i = 1, \cdots, n \quad (\textit{Scenario Loss})$$

$$S := \sum_{i=1}^{n} S_i = \sum_{i=1}^{n} \sum_{j=1}^{N_i} X_{ij} \quad (\textit{Aggregate Loss}).$$

Theorem 3. Let $N_1, N_2, \cdots, N_n$ be stochastically independent, Poisson distributed random variables (frequencies) with parameters $\lambda_1, \lambda_2, \cdots, \lambda_n > 0$, and X_{ij}, $1 \le i \le n$, $j \in \mathbb{N}$ be independent, positive random variables (claims, losses), independent also of the frequencies, such that all $X_{i\bullet}$ follow the same distribution Q_i. Then the distribution of $S := \sum_{i=1}^{n} \sum_{j=1}^{N_i} X_{ij}$ is identical with the aggregate claims distribution for the loss $\tilde{S}$ given by $\tilde{S} := \sum_{k=1}^{\tilde{N}} \tilde{X}_k$ from a single Collective Risk Model where $\tilde{N}$ is a Poisson distributed frequency with parameter $\tilde{\lambda} = \sum_{i=1}^{n} \lambda_i$ and the $\tilde{X}_i$ are independent (also of $\tilde{N}$), with mixture distribution $\tilde{Q} = \sum_{i=1}^{n} \frac{\lambda_i}{\tilde{\lambda}} Q_i$.

Typical Loss:

$$L := \sum_{k=1}^{\min\{N,1\}} \tilde{X}_k = \begin{cases} 0, & \text{if } N = 0 \\ \tilde{X}_1, & \text{if } N > 0 \end{cases}$$

Lemma 2. Under the assumptions of Theorem 3, the *Typical Loss* distribution is given by the mixture

$$P^L = e^{-\tilde{\lambda}}\varepsilon_0 + \left(1 - e^{-\tilde{\lambda}}\right)\tilde{Q} = e^{-\tilde{\lambda}}\varepsilon_0 + \left(1 - e^{-\tilde{\lambda}}\right)\sum_{i=1}^{n}\frac{\lambda_i}{\tilde{\lambda}}Q_i, \text{ with } \tilde{\lambda} = \sum_{i=1}^{n}\lambda_i.$$

The corresponding cdf has the form

$$F_L(z) = P(L \le z) = e^{-\tilde{\lambda}} + \left(1 - e^{-\tilde{\lambda}}\right)\sum_{i=1}^{n}\frac{\lambda_i}{\tilde{\lambda}}F_i(z) = e^{-\tilde{\lambda}} + \left(1 - e^{-\tilde{\lambda}}\right)\tilde{F}(z), \ z \ge 0.$$

Lemma 3. In the classical Collective Risk Model, let

$$M := \max\{X_i \mid 1 \le i \le N\}$$

denote the *Maximum Loss*. We then have:

$$P(M \le z) = F_M(z) = \sum_{n=0}^{\infty} p_n F^n(z), \quad z \ge 0,$$

where as above, $p_n := P(N = n)$ for $n = 0, 1, \cdots$.

Remark: For the Poisson model, i.e. $P^N = \mathcal{P}(\lambda)$ with $\lambda > 0$ this means:

$$P(M \le z) = F_M(z) = \sum_{n=0}^{\infty} p_n F^n(z) = e^{-\lambda} \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} F^n(z)$$

$$= e^{-\lambda} \exp\{\lambda F(z)\} = e^{-\lambda\{1-F(z)\}} = e^{\lambda\{F(z)-1\}} = \varphi_N\big(F(z)\big), \quad z \ge 0.$$

Lemma 4. Under the conditions of Theorem 3, let

$$M := \max\left\{X_{ij} \mid 1 \le j \le N_i,\ 1 \le i \le n\right\}$$

denote the *Occurrence Loss*. Then the cdf of *M* is given by

$$P(M \le z) = \exp\left\{-\sum_{i=1}^{n}\left[\lambda_i \{1 - F_i(z)\}\right]\right\} = \exp\left\{-\tilde{\lambda}\left[1 - \tilde{F}(z)\right]\right\},\ \ z \ge 0,$$

with $\tilde{F}(z) = \sum_{i=1}^{n} \frac{\lambda_i}{\tilde{\lambda}} F_i(z),\ \ z \ge 0.$

We shall now present explicit formulas for calculating the cumulative distribution functions (cdf's) and the survival functions (sf's) of the *Typical Loss*, the *Occurrence Loss* and the *Aggregate Loss* for a basic *Event Loss Table*. Note that the sf's of the *Occurrence Loss* and the *Aggregate Loss* are usually denoted as OEP curve (Occurence Loss Exceeding Probability) and AEP curve (Aggregate Loss Exceeding Probability).

Since in the basic Event Loss Table, all scenario losses ϖ_i are deterministic, we can assume that they are ordered according to size:

$$\varpi_1 \leq \varpi_2 \leq \cdots \leq \varpi_n .$$

This can always be achieved by a proper sorting of the rows in the Event Loss Table. In particular, this ordering implies

$$F_i(\varpi_k) = \begin{cases} 0, & \text{if } i > k \\ 1, & \text{if } i \leq k, \end{cases} \quad \text{for all} \quad 1 \leq i, k \leq n.$$

For the superposed model, we thus obtain

$$P\left(\tilde{X} \le \varpi_k\right) = \tilde{F}\left(\varpi_k\right) = \sum_{i=1}^{n} \frac{\lambda_i}{\tilde{\lambda}} F_i(\varpi_k) = \sum_{i=1}^{k} \frac{\lambda_i}{\tilde{\lambda}}, \quad k = 1, \cdots, n \quad \text{and}$$

$$P\left(\tilde{X} > \varpi_k\right) = 1 - \tilde{F}\left(\varpi_k\right) = 1 - \sum_{i=1}^{k} \frac{\lambda_i}{\tilde{\lambda}} = \sum_{i=k+1}^{n} \frac{\lambda_i}{\tilde{\lambda}}, \quad k = 1, \cdots, n,$$

or, more generally,

$$P\left(\tilde{X} \le z\right) = \tilde{F}\left(z\right) = \sum_{i=1}^{n} \frac{\lambda_i}{\tilde{\lambda}} F_i(z) = \sum_{i=1}^{k} \frac{\lambda_i}{\tilde{\lambda}}, \quad \varpi_k \le z < \varpi_{k+1}, \quad k = 1, \cdots, n \quad \text{and}$$

$$P\left(\tilde{X} > z\right) = 1 - \tilde{F}\left(z\right) = 1 - \sum_{i=1}^{k} \frac{\lambda_i}{\tilde{\lambda}} = \sum_{i=k+1}^{n} \frac{\lambda_i}{\tilde{\lambda}}, \quad \varpi_k \le z < \varpi_{k+1}, \quad k = 1, \cdots, n,$$

with $\varpi_{n+1} := \infty$.

Lemma 5:

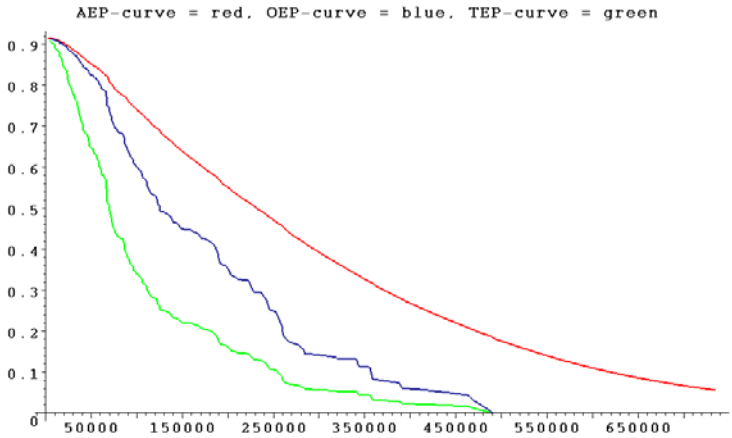
$$P(L>z)=1-e^{-\tilde{\lambda}}-\left(1-e^{-\tilde{\lambda}}\right)\tilde{F}(z)=\left(1-e^{-\tilde{\lambda}}\right)\sum_{i=k+1}^{n}\frac{\lambda_i}{\tilde{\lambda}},\qquad \varpi_k\le z<\varpi_{k+1}\quad \text{(TEP - curve)}$$

$$P(M>z)=1-\exp\left\{-\tilde{\lambda}\left[1-\tilde{F}(z)\right]\right\}=1-\exp\left\{-\sum_{i=k+1}^{n}\lambda_i\right\},\quad \varpi_k\le z<\varpi_{k+1}\quad \text{(OEP - curve)}$$

$$P(S>z)=1-e^{-\tilde{\lambda}}-e^{-\tilde{\lambda}}\sum_{k=1}^{n}\frac{\tilde{\lambda}^k}{k!}\tilde{F}^{k*}(z),\qquad z\ge 0\quad \text{(AEP - curve)}$$

Here TEP refers to Typical Loss Exceeding Probability.

The following graph shows these three curves for an artificial example with 300 scenarios and $\tilde{\lambda} = 2,465$ The maximum observed individual loss was here given by $\varpi_{300} = 489\,909$. For the calculation of the AEP-curve, a discretization with step size $\Delta = 2500$ was chosen.



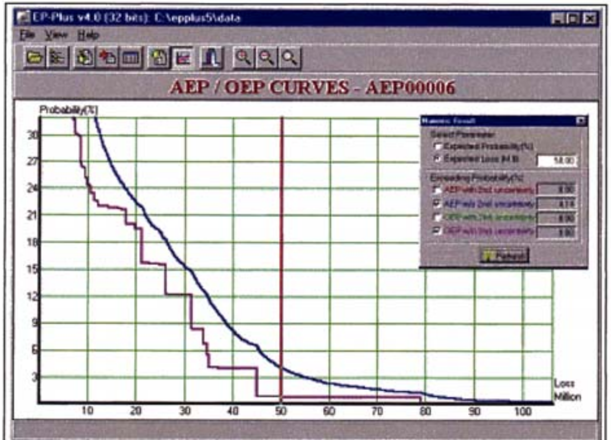


Figure 12. Comparing an aggregate loss exceeding probability (AEP) curve and an occurrence exceeding probability curve.

Source: [3], p. 18

Concerning the *Extended Event Loss Table*, where also standard deviations are given, we can proceed completely similar if the type of the individual claim size (loss) distribution is known. Suppose that we can consider Modelled Loss as location parameter $\mu > 0$ and Standard Deviation as scale parameter $\sigma > 0$ for an appropriate class of distributions (like lognormal, gamma, Fréchet, Pareto etc.), then the basic formulas in Lemma 8 remain valid, i.e. we still have, for $z \geq 0$,

$$P(L > z) = \left(1 - e^{-\tilde{\lambda}}\right)\left(1 - \tilde{F}(z)\right) \quad \text{(TEP-curve)}$$

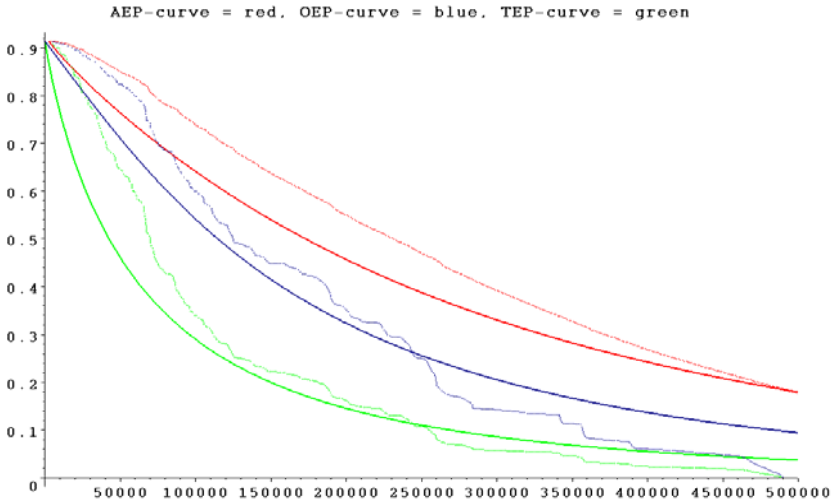
$$P(M > z) = 1 - \exp\left\{-\tilde{\lambda}\left[1 - \tilde{F}(z)\right]\right\} \quad \text{(OEP-curve)}$$

$$P(S > z) = 1 - e^{-\tilde{\lambda}} - e^{-\tilde{\lambda}} \sum_{k=1}^{n} \frac{\tilde{\lambda}^k}{k!} \tilde{F}^{k*}(z) \quad \text{(AEP-curve)}$$

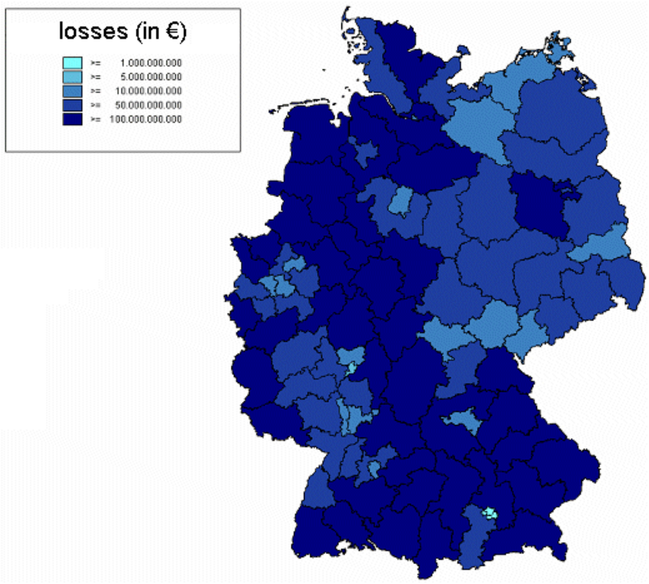
The following graph shows the corresponding result for the analysis of the virtual *Extended Event Loss Table* related to the preceding example where we assume that the individual losses are exponentially distributed, with scenario parameters mean = standard deviation = 1 / modelled loss, i.e.

$$\tilde{F}(z) = \sum_{i=1}^{n} \frac{\lambda_i}{\tilde{\lambda}} F_{\mu_i,\sigma_i}(z) = 1 - \sum_{i=1}^{n} \frac{\lambda_i}{\tilde{\lambda}} e^{-\vartheta_i z}, \; z \geq 0,$$

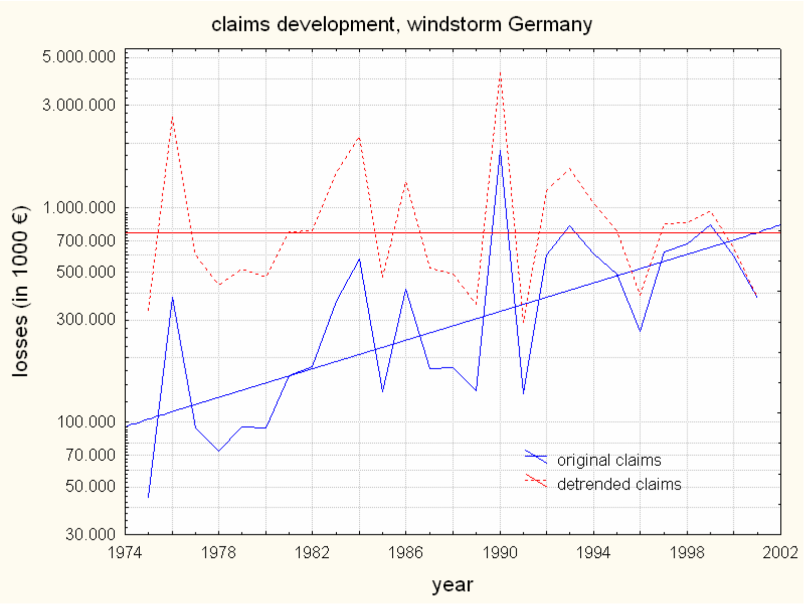
where ϑ_i is the modelled loss from scenario *i*. The dotted curves are those from the preceding graph.

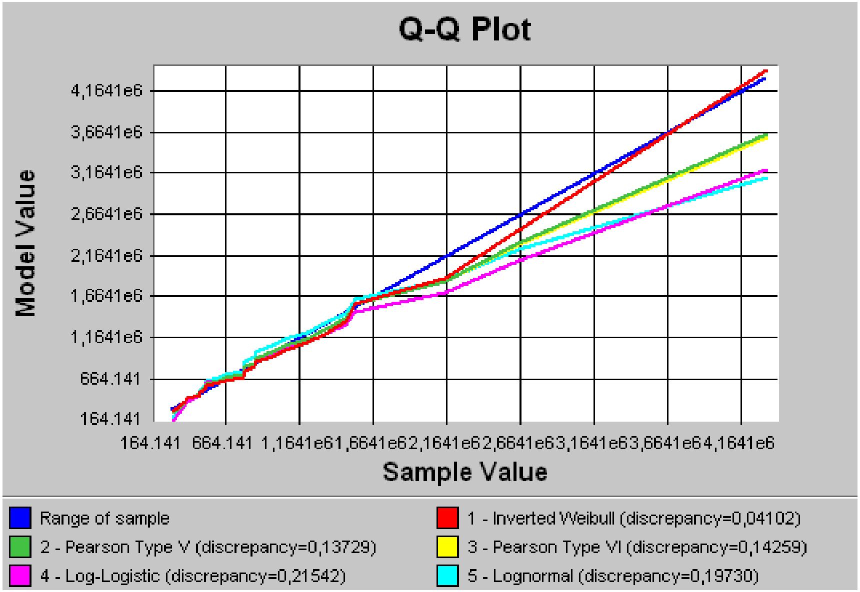

AEP-curve = red, OEP-curve = blue, TEP-curve = green
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
50000 100000 150000 200000 250000 300000 350000 400000 450000 500000

Risk Exposure Windstorm Germany

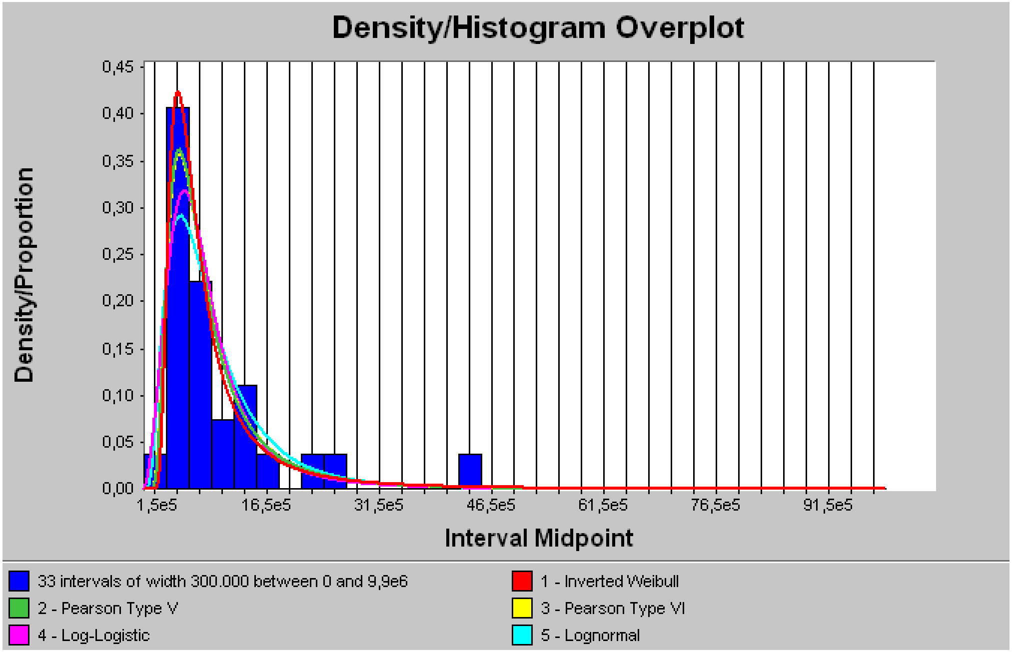


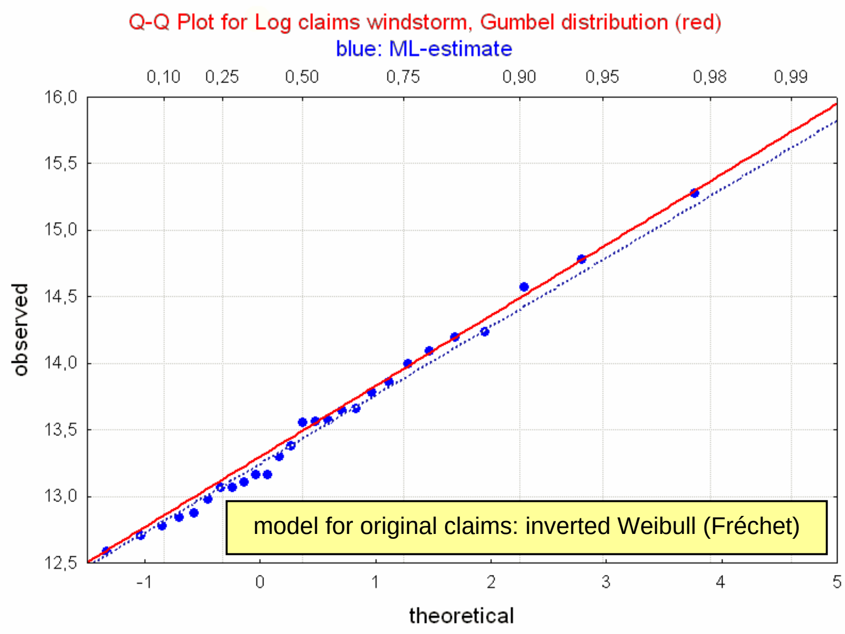
Source: Munich Re

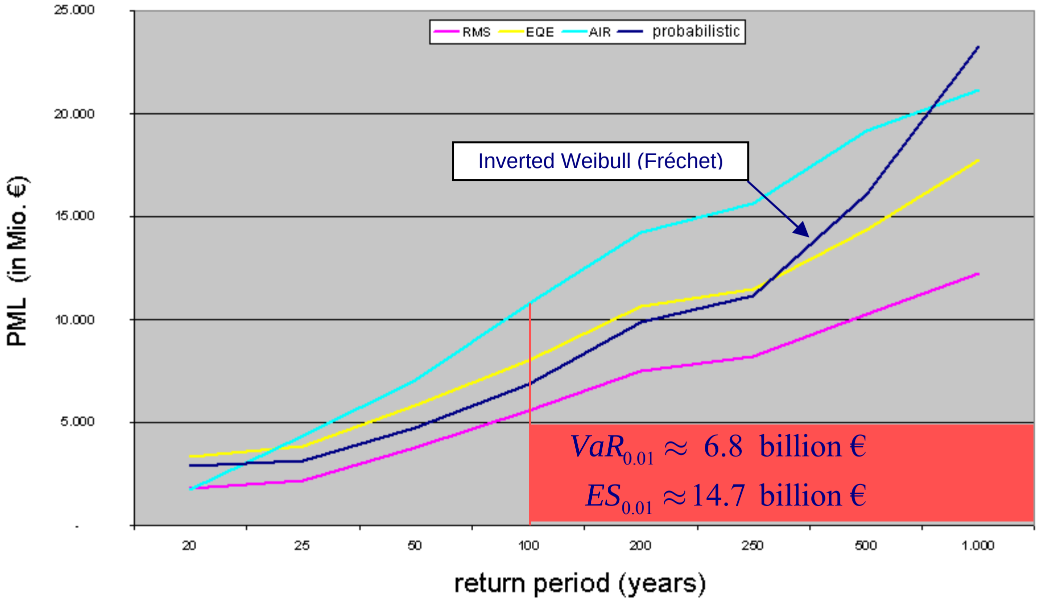

claims development, windstorm Germany
5.000.000
3.000.000
1.000.000
700.000
500.000
300.000
100.000
70.000
50.000
30.000
losses (in 1000 €)
1974
1978
1982
1986
1990
1994
1998
2002
year
original claims
detrended claims



Created with *ExpertFit* 6.00


Density/Histogram Overplot
Density/Proportion
0,45
0,40
0,35
0,30
0,25
0,20
0,15
0,10
0,05
0,00
1,5e5
16,5e5
31,5e5
46,5e5
61,5e5
76,5e5
91,5e5
Interval Midpoint
33 intervals of width 300.000 between 0 and 9,9e6
1 - Inverted Weibull
2 - Pearson Type V
3 - Pearson Type VI
4 - Log-Logistic
5 - Lognormal


Q-Q Plot for Log claims windstorm, Gumbel distribution (red)
blue: ML-estimate
0,10 0,25 0,50 0,75 0,90 0,95 0,98 0,99
16,0
15,5
15,0
14,5
14,0
13,5
13,0
12,5
observed
-1 0 1 2 3 4 5
theoretical
model for original claims: inverted Weibull (Fréchet)

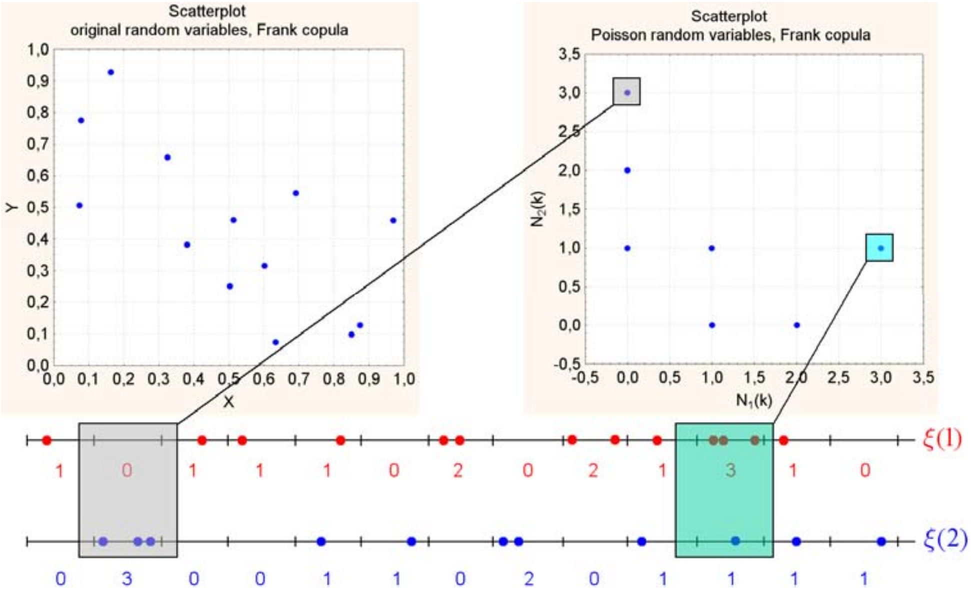


$VaR_{0.01} \approx 6.8$ billion €

$ES_{0.01} \approx 14.7$ billion €

difficulties of proprietary models vs. actuarial approach

- ***frequently no good fit of models with data (⇨ small return periods)***
- ***Poisson model not always appropriate (⇨ frequency negative binomial?)***
- ***little possibilities for simulation of individual claims (⇨ XL treaties)***
- ***models good for VaR, less for ES***
- ***inappropriate modelling of dependencies (⇨ copulas?)***
- ***mainly modelling of only individual risks (⇨ DFA, Solvency II)***



Example for copula-based construction of Poisson processes; source: [10]

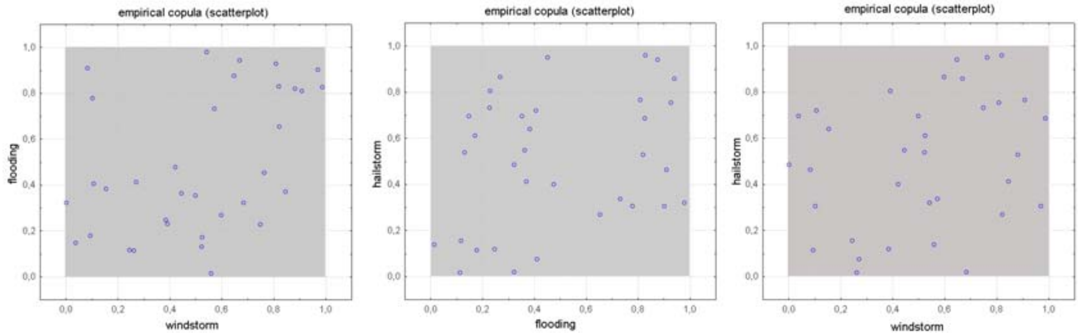
case studies

Source: Munich Re

case studies

Source: Swiss Re

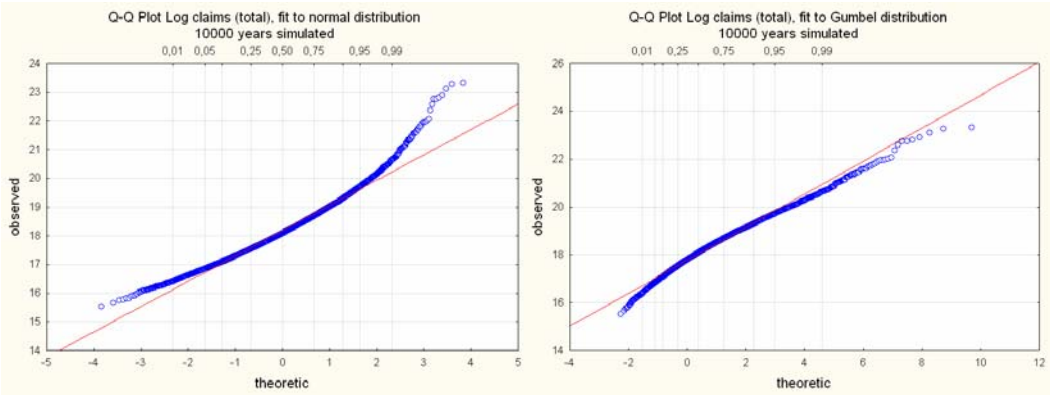
Example insurance company (private property):



correlation matrix for Gauss (??) copula with windstorm / hailstorm / flooding:

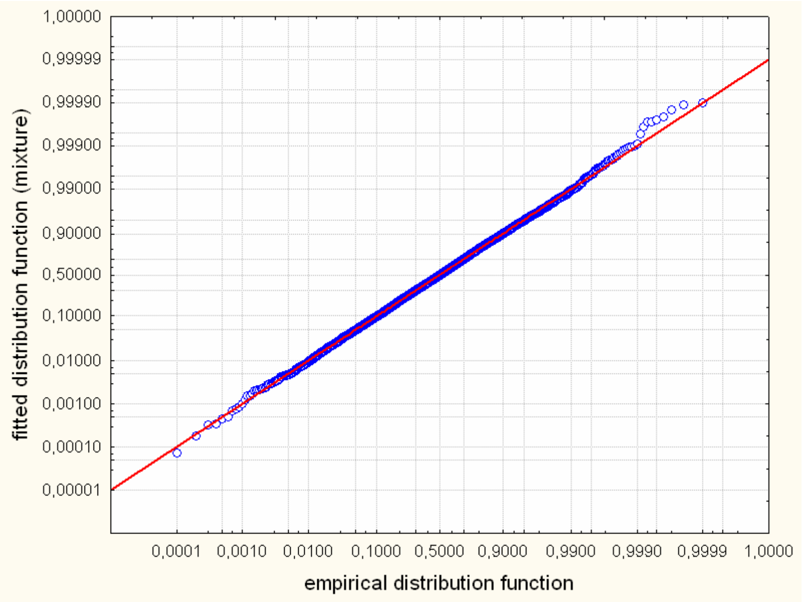
(marginal distributions: Fréchet / Lognormal / Lognormal)

$$\Sigma = \begin{bmatrix} 1 & 0{,}2226 & 0{,}3782 \\ 0{,}2226 & 1 & 0{,}3341 \\ 0{,}3782 & 0{,}3341 & 1 \end{bmatrix} = A\,A^{tr}, \qquad A = \begin{bmatrix} 1 & 0 & 0 \\ 0{,}2226 & 0{,}9749 & 0 \\ 0{,}3782 & 0{,}2563 & 0{,}8895 \end{bmatrix}$$



problem: total claims distribution is distribution of sums of dependent random variables with different types of marginal distributions!

⇨ use mixture distribution


1,00000
0,99999
0,99990
0,99900
0,99000
0,90000
0,50000
0,10000
0,01000
0,00100
0,00010
0,00001
fitted distribution function (mixture)
0,0001
0,0010
0,0100
0,1000
0,5000
0,9000
0,9900
0,9990
0,9999
1,0000
empirical distribution function

The *hard market* phase in the early nineties was triggered by the severe loss burden resulting from hurricane *Andrew* and several winter storms in Europe.
Two factors should be noted here: on the one hand, losses erode the reinsurers' capital base, meaning that less capital is available to underwrite reinsurance covers; on the other, the demand for such covers increases as a catastrophe makes both direct insurers and insureds aware of the risks to which they are exposed.
Further, their own capital base has been reduced and the necessity to minimise risks is therefore all the more acute.

Source: Swiss Re

The End