Challenges of applying a consistent Solvency II framework

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Agenda

- What is “insurance”?
- What is a “200-year event”?
- Does the SCR guarantee stability?
- Can we calculate SCR’s for aggregated risk?
- Is there a relationship between correlation and diversification?
- Conclusions and recommendations
- References

What is “insurance”?

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The trade of insurance gives great security to the fortunes of private people, and, by dividing among a great many that loss which would ruin an individual, makes it fall light and easy upon the whole society. In order to give this security, however, it is necessary that the insurers should have a very large capital.
In order to make insurance, the common premium must be sufficient to compensate the common losses, to pay the expense of management, and to afford such a profit as might have been drawn from an equal capital employed in any common trade.

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In order to make insurance, the common premium must be sufficient to compensate the common losses, to pay the expense of management, and to afford such a profit as might have been drawn from an equal capital employed in any common trade.

Adam Smith:
An Inquiry into the Nature and Causes of the Wealth of Nations (1776)

• What is "insurance"?

insurance client

insurance company

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premium payment

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risk cover

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equivalence principle of insurance:

expected (discounted) premium cash flow = expected (discounted) loss expenses cash flow

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equivalence principle of insurance:

expected (discounted) premium cash flow = expected (discounted) loss expenses cash flow

but: safety loading on premiums necessary to avoid certain ruin

• What is "insurance"?

insurance client

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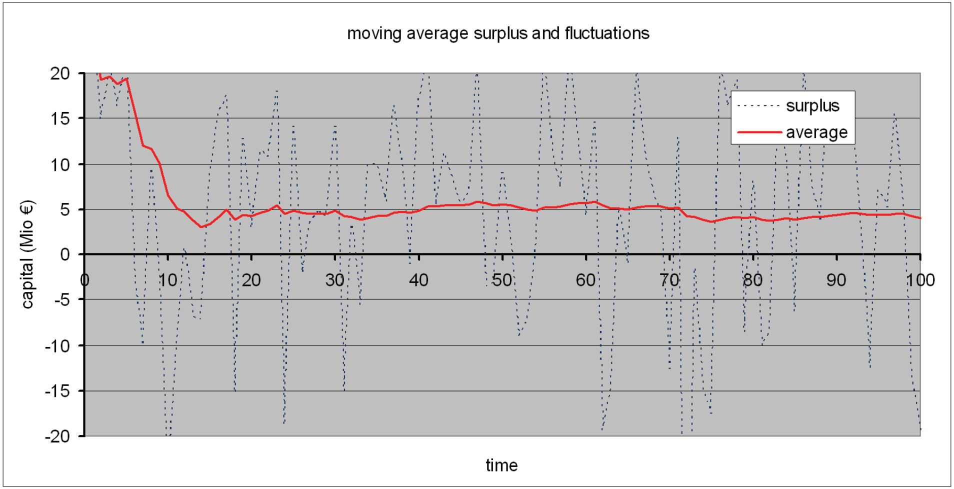
mathematical foundations:

- ► Law of Large Numbers (Jakob Bernoulli, around 1695)
  - equivalence principle, balance of risk in the collective and over time
- ► Central Limit Theorem (Abraham de Moivre, 1733)
  - ruin probabilities, aspects of solvency

• What is “insurance”?



example: average insurance surplus capital development over time

risk: lognormal distribution

mean: 55.70 Mio € standard deviation 2.27 Mio €

premium: 60 Mio € limiting surplus: 4.30 Mio €

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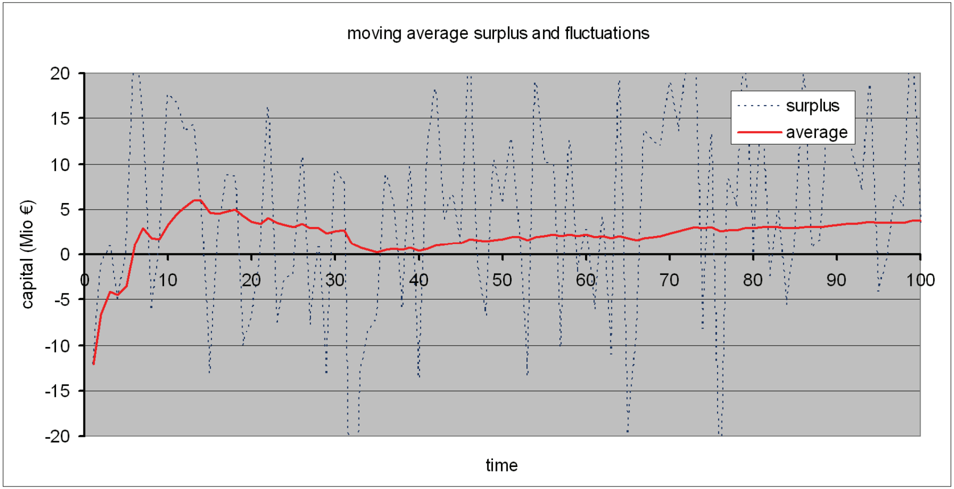

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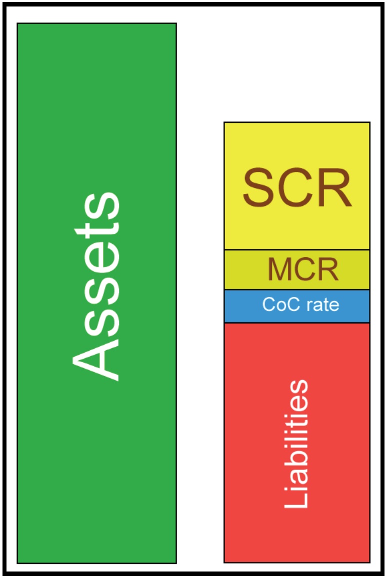

economic balance sheet view

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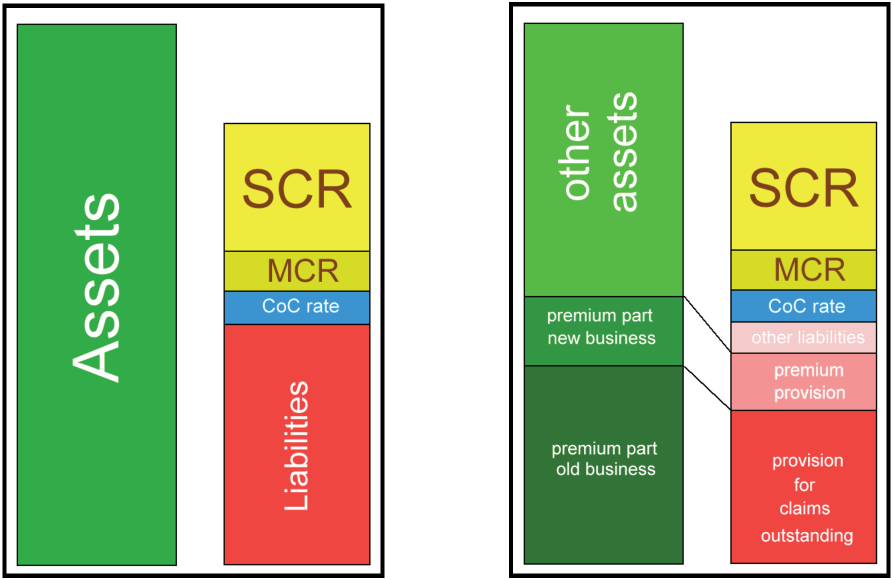

economic balance sheet view

What is a “200-year event”?

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Consider a deck of 52 playing cards:

If you draw a card every week on Sunday, put it back again und shuffle the deck, what is the average time until you draw the first queen of spades?

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Consider a deck of 52 playing cards:

If you draw a card every week on Sunday, put it back again und shuffle the deck, what is the average time until you draw the first queen of spades?

Answer:

52 weeks or 1 year. So, drawing the queen of spades is a one-year-event.

• What is a "200-year event"?

Proof: Let N denote the number of the first draw with a queen of spades. Since all cards have an equal success probability p for drawing the queen of spades with $p = \frac{1}{52}$, the expected value $E(N)$ of N is

$$E(N) = \sum_{n=0}^{\infty} P(N > n) = \sum_{n=0}^{\infty} (1-p)^n = \frac{1}{1-(1-p)} = \frac{1}{p} = 52.$$

Comment:

The number S of successes (i.e., the queen of spades is drawn) follows a binomial distribution with success probability $p = \frac{1}{52}$. Hence $E(S) = 52 \cdot p = 1$, i.e. on average the queen of spades is drawn once during the year.

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- What is a "200-year event"?

  - For the insurance problem, this means:

    If p denotes the probability of a ruin during a single year, then on average a ruin occurs exactly once during $m = \frac{1}{p}$ years. Hence a ruin is a m-year-event.

    For $p = 0.005$ (Solvency II standard), this means $m = 200$.

  - But: a ruin can potentially occur in any year! The following table gives probabilities p_k for a ruin occurring already during the first k years:

k	1	10	25	50	75	100	150	200
p_k	0.0050	0.0489	0.1178	0.2217	0.3134	0.3942	0.5285	0.6330

• What is a "200-year event"?

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For $p = 0.005$ (Solvency II standard), this means $m = 200$.

➢ But: a ruin can potentially occur in any year! The following table gives probabilities q_k for a ruin occurring at least k times during 200 years:

k	1	2	3	4	5
q_k	0.63304	0.2642	0.07984	0.01868	0.00355

Does the SCR guarantee stability?

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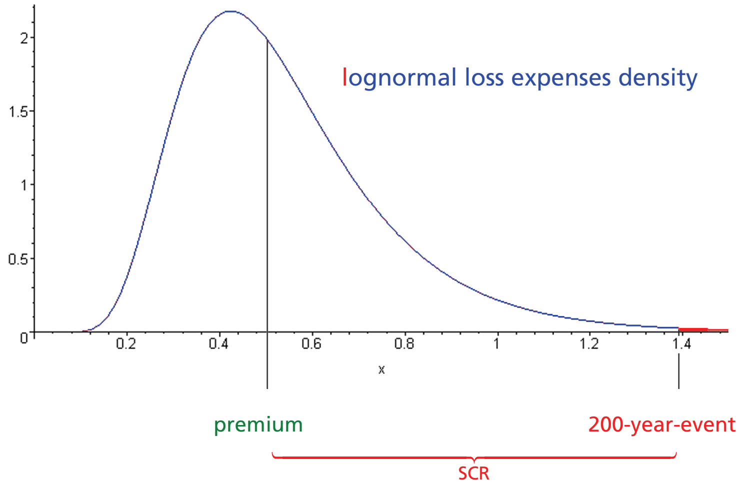
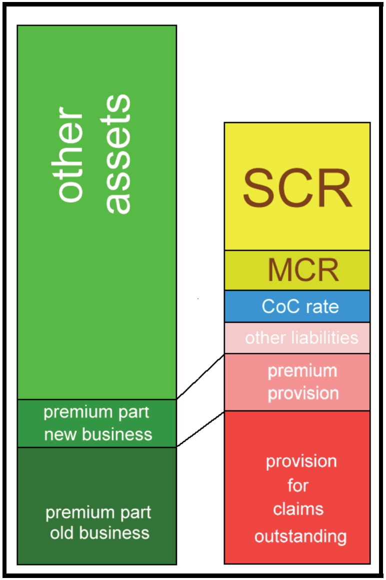
- ➢ The basis of premium calculation in insurance is the yearly average amount of loss expenses including external and internal cost
- ➢ The basis of the SCR is the 200-year-event (Value@Risk)



- Does the SCR guarantee stability?

  ➢ A stable development of the company requires an average combined ratio (cr) of less than 100%



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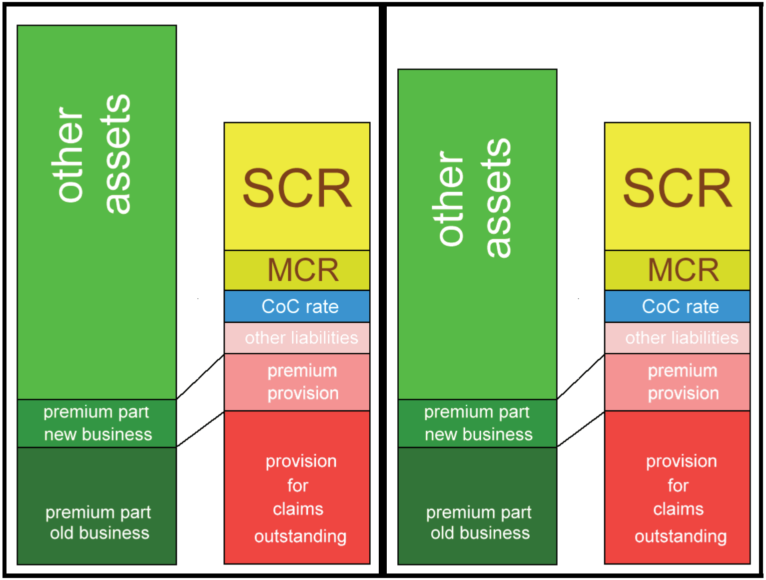

unstable development with average cr > 100%

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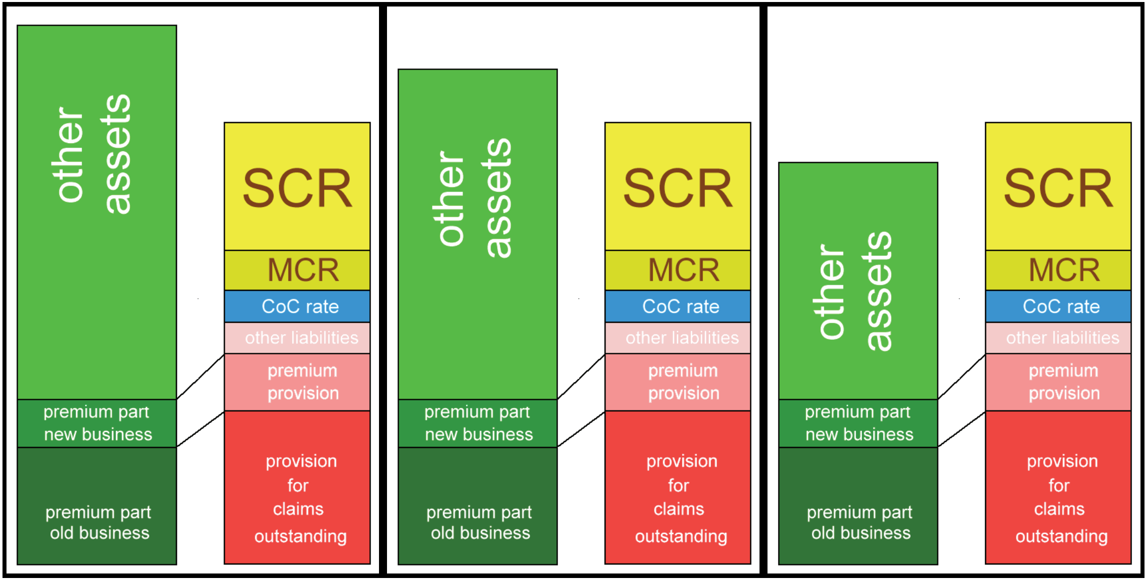

unstable development with average cr > 100%

• Does the SCR guarantee stability?

➢ Example: lognormal combined ratio cr (non-life) with different mean values, but the same initial capital and the same true yearly SCR

case	1	2	3
initial capital	300,000 €	300,000 €	300,000 €
cr mean	100%	110%	90%
cr standard deviation	30.35%	31.90%	28.72%
true SCR	250,000 €	250,000 €	250,000 €
3-sigma-rule[1] SCR	264,413 €	242,318 €	290,965 €

➢ Observation: in the "bad" case 2, the 3-sigma-rule SCR underestimates the true SCR, while in the "good" cases, the 3-sigma-rule SCR overestimates the true SCR [HAMPEL AND PFEIFER (2011)]

[1] According to COMMISSION DELEGATED REGULATION (EU) 2015/35 of 10 October 2014, Article 115

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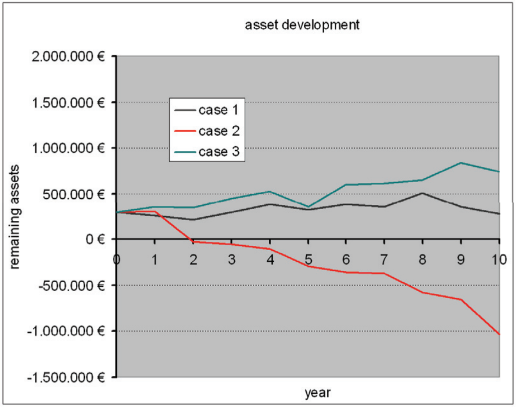
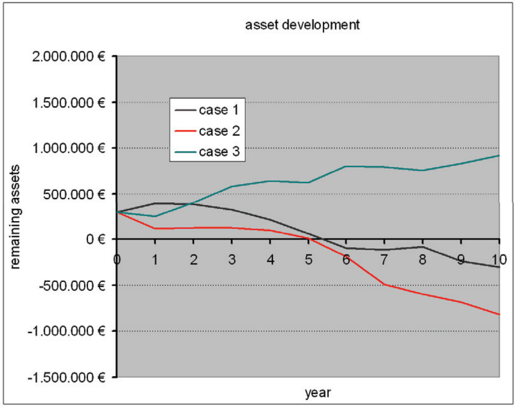
- Example: lognormal combined ratio cr (non-life) with different mean values, but the same initial capital and the same true yearly SCR





- No contradiction to the interpretation of a 200-year-event!
- The one-year SCR is no guaranty for stability (assets may go down)
- The ORSA is a necessary add-on to achieve stability

Can we calculate SCR’s for aggregated risk?

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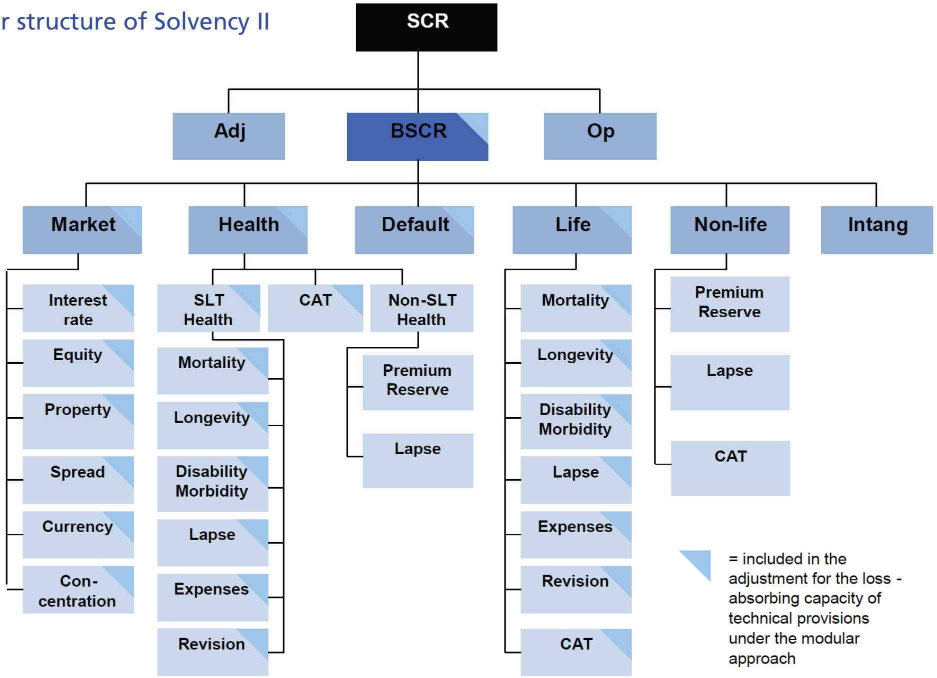
modular structure of Solvency II



• Can we calculate SCR's for aggregated risk?

➢ In the world of normally distributed risks X with mean μ and standard deviation σ, there holds:

$$\text{SCR}(X) = u_{0.995} \cdot \sigma$$

with the 99.5%-quantile $u_{0.995} = 2.5758...$ of the standard normal distribution

• Can we calculate SCR's for aggregated risk?

- In the world of (jointly) normally distributed risks $X_1,\cdots,X_n$ with standard deviations $\sigma_1,\cdots,\sigma_n$, the total SCR can therefore be calculated via individual SCR's and the pairwise correlations ρ_{ij} of risks:

$$\mathrm{SCR}_{\mathrm{total}} = \mathrm{SCR}\left(\sum_{k=1}^{n} X_k\right) = u_{0.995}\cdot\sigma_{\mathrm{total}} = u_{0.995}\cdot\sqrt{\sum_{k=1}^{n}\sigma_k^2 + \sum\sum_{1\le i,j\le n}\rho_{ij}\cdot\sigma_i\cdot\sigma_j}$$

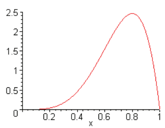
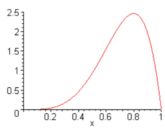
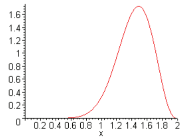
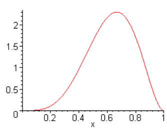
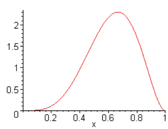
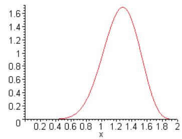
$$= \sqrt{\sum_{k=1}^{n}\mathrm{SCR}_k^2 + \sum\sum_{1\le i,j\le n}\rho_{ij}\cdot\mathrm{SCR}_i\cdot\mathrm{SCR}_j} \le \sum_{k=1}^{n}\mathrm{SCR}_k$$

- This is the basis for the DELEGATED REGULATION (EU) 2015/35, Article 114 and the assumption that there is a relationship between diversification and correlation (which is true in the normal world)

• Can we calculate SCR's for aggregated risk?

- In the world of non-normally distributed risks X, this can be completely different

- Example: independent beta-distributed combined ratios, identical premium volume 10 Mio. €, risks X and Y [PFEIFER AND STRAßBURGER (2008)]

risk	X	Y	$S = X + Y$	true SCR	SCR_{total}	error
density [4141]				4.531	3.776	-16.66%
density [4242]				5.092	4.521	-11.21%

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- Can we calculate SCR's for aggregated risk?

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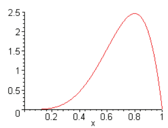
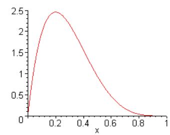
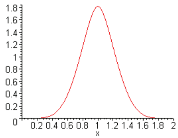
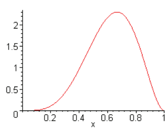
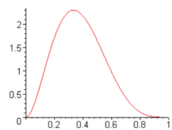
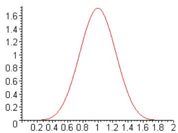
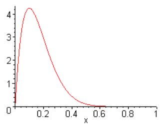
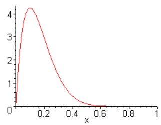
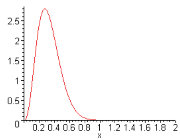
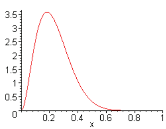
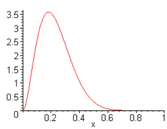
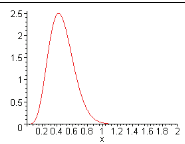
risk	X	Y	$S = X + Y$	true SCR	SCR_{total}	error
density [4114]				5.760	5.321	-7.63%
density [4224]				5.672	5.294	-6.66%

- Can we calculate SCR's for aggregated risk?

  - In the world of non-normally distributed risks *X*, this can be completely different

  - Example: independent beta-distributed combined ratios, identical premium volume 10 Mio. €, risks *X* and *Y* [PFEIFER AND STRAßBURGER (2008)]

risk	X	Y	$S = X + Y$	true SCR	SCR_{total}	error
density [1919]				4.549	4.835	+6.30%
density [2929]				4.665	4.839	+3.73%

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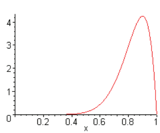
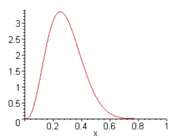
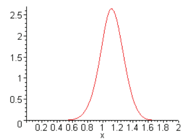
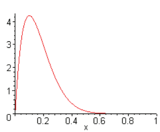
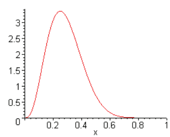
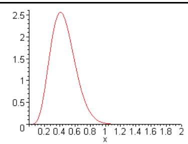
➢ Example: independent beta-distributed combined ratios, identical premium volume 10 Mio. €, risks *X* and *Y* [PFEIFER AND STRAßBURGER (2008)]

risk	X	Y	$S = X + Y$	true SCR	SCR_{total}	error
density [9139]				3.967	3.698	-6.77%
density [1939]				4.597	4.786	+4.12%

• Can we calculate SCR's for aggregated risk?

- In the world of non-normally distributed risks *X*, this can be completely different

- Example: independent beta-distributed combined ratios, identical premium volume, risks *X* and *Y* [PFEIFER AND STRAßBURGER (2008)]

- Observation: the SCR_{total} according to the aggregation formula (DELEGATED REGULATION (EU) 2015/35, Article 114) overestimates the true SCR for less dangerous risks, underestimates the true SCR for more dangerous risks

- Similar results hold in the presence of pairwise correlation or, more generally, stochastic dependence

Is there a relationship between correlation and diversification?

• Is there a relationship between correlation and diversification?

- In the world of (jointly) normally distributed risks $X_1,\cdots,X_n$, there is a strict relationship between pairwise correlation of risks and risk diversification, because the Value@Risk and hence also the total SCR is subadditive in this case.

- In the world of non-normally distributed risks X, this can be completely different.

- Is there a relationship between correlation and diversification?

  - Example A (Pfeifer [2013]): Joint distribution of risks X and Y:

$P(X=x, Y=y)$		x: 0	50	100	$P(Y=y)$	$P(Y \le y)$
y	0	β	$0.440-\beta$	0.000	0.440	0.440
	40	$0.554-\beta$	β	0.001	0.555	0.995
	50	0.000	0.001	0.004	0.005	1.000
	$P(X=x)$	0.554	0.441	0.005		
	$P(X \le x)$	0.554	0.995	1.000		

with $0 \le \beta \le 0.440.$ Mean, range of correlation and SCR of X and Y:

$E(X)$	$E(Y)$	$\rho(X,Y)$	$\mathrm{SCR}(X)$	$\mathrm{SCR}(Y)$
22.55	22.45	$-0.9494 \le 3.9579\beta - 0.9494 \le 0.7921$	27.45	17.55

• Is there a relationship between correlation and diversification?

➢ Example A (Pfeifer [2013]): Joint distribution of risks X and Y:

$P(X=x, Y=y)$		x				
		0	50	100	$P(Y=y)$	$P(Y\le y)$
y	0	β	$0.440-\beta$	0.000	0.440	0.440
	40	$0.554-\beta$	β	0.001	0.555	0.995
	50	0.000	0.001	0.004	0.005	1.000
	$P(X=x)$	0.554	0.441	0.005		
	$P(X\le x)$	0.554	0.995	1.000		

with $0\le\beta\le 0.440.$ Distribution of the aggregate risk $S=X+Y$:

s	0	40	50	90	100	140	150
$P(S=s)$	β	$0.554-\beta$	$0.440-\beta$	β	0.001	0.001	0.004
$P(S\le s)$	β	0.554	$0.994-\beta$	0.994	0.995	0.996	1.000

- Is there a relationship between correlation and diversification?

➢ Example A (Pfeifer [2013]): Joint distribution of risks *X* and *Y*:

$P(X=x, Y=y)$		x = 0	x = 50	x = 100	$P(Y=y)$	$P(Y \le y)$
y	0	β	$0.440-\beta$	0.000	0.440	0.440
	40	$0.554-\beta$	β	0.001	0.555	0.995
	50	0.000	0.001	0.004	0.005	1.000
	$P(X=x)$	0.554	0.441	0.005		
	$P(X \le x)$	0.554	0.995	1.000		

We have: $-0.9494 \le \rho(X,Y) \le 0.7921$, but in any case

$$\text{true SCR}(X+Y) = 55 > 45 = \text{SCR}(X) + \text{SCR}(Y) > \text{SCR}_{\text{total}} \le 37.99$$

► risk aggregation, no diversification; independent of correlation!

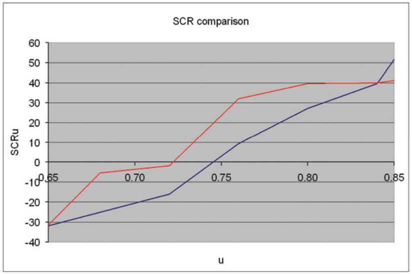
- Is there a relationship between correlation and diversification?

  - ➢ Example B (Pfeifer [2016]): a real-world example:

year	X	Y	$S=X+Y$	u	SCRu(X)	SCRu(Y)	SCRu(X)+SCRu(Y)	SCRu(S)
1	40.513	44.650	85.163	0.04	-58.835	-51.017	-109.852	-92.897
2	16.968	28.874	45.842	0.08	-57.802	-44.674	-102.476	-91.885
3	45.337	51.018	96.355	0.12	-55.829	-36.220	-92.049	-90.637
4	57.120	19.016	76.136	0.16	-55.821	-36.056	-91.877	-85.669
5	41.480	27.470	68.950	0.20	-55.300	-35.095	-90.395	-85.656
6	14.987	28.595	43.582	0.24	-50.039	-34.816	-84.855	-79.137
7	74.524	101.544	176.068	0.28	-48.215	-30.430	-78.645	-78.645
8	64.578	111.933	176.511	0.32	-42.775	-26.834	-69.609	-67.529
9	42.072	92.727	134.799	0.36	-42.044	-25.438	-67.482	-65.268
10	24.574	33.260	57.834	0.40	-39.734	-23.837	-63.571	-60.343
11	177.842	81.139	258.981	0.44	-34.639	-19.040	-53.679	-54.759
12	17.489	39.853	57.342	0.48	-32.276	-15.229	-47.505	-51.316
13	70.719	60.297	131.016	0.52	-32.122	-12.715	-44.837	-49.480
14	30.014	56.985	86.999	0.56	-31.309	-12.672	-43.981	-47.036
15	40.667	140.794	181.461	0.60	-30.717	-8.027	-38.744	-40.124
16	112.692	55.663	168.355	0.64	-27.452	-6.705	-34.157	-40.062
17	13.954	36.856	50.810	0.68	-21.598	-3.393	-24.991	-5.463
18	30.745	50.975	81.720	0.72	-15.669	-0.328	-15.997	-1.680
19	38.150	12.673	50.823	0.76	-8.211	17.449	9.238	31.876
20	668.552	276.521	945.073	0.80	-2.070	29.037	26.967	39.589
21	22.750	48.461	71.211	0.84	1.735	37.854	39.589	40.032
22	16.960	27.634	44.594	0.88	39.903	48.243	88.146	44.982
23	33.055	63.362	96.417	0.92	105.053	77.104	182.157	122.502
24	51.191	38.252	89.443	0.96	595.763	212.831	808.594	808.594

risk concentration

no diversification



red: SCRu(S),

blue: SCRu(X) + SCRu(Y)

$$\rho(X,Y) = 0.8481$$

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- A possible discrepancy between the Solvency II framework assumptions and "reality" can neither be mathematically explained nor quantified. However, the Solvency II framework is a very good practical compromise between the necessity of insurance regulation and risk modelling, based on experience reasoning.

- The mathematical parts of the Solvency II framework should, in any case, be carefully observed with the option of gradual improvements motivated by practice.

• Conclusions and recommendations

➢ A crucial point is the assumed dependence between correlations and risk diversification, which might in practice lead to a severe under-estimation of the appropriate SCR. A possible diversification effect is only justified by a detailed investigation of the true joint dependence structure of risks, which cannot be described by a few simple parameters.

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- In the light of the fact that the Solvency II framework might suffer from potentially large deviations from "reality", it could be wise to reduce the bureaucratic complexity in favour of more transparency while maintaining the overall goal of a good insurance supervision.

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> - Every user of mathematics should understand precisely what he does.

[translated from Topsøe (1990): Spontane Phänomene]

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Thank you for your attention!