Spectra of Dirichlet fractional Laplacians in domains with cylindrical outlets to infinity

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Abstract

The standard positive Laplacian $-\Delta$ in a domain $\Omega \subset \mathbb{R}^n$ corresponds, up to a multiplicative constant, to the quantization of the kinetic energy p^2/m of a free particle with momentum p and mass m confined in Ω. This is because the quantization procedure maps the classical momentum p to the operator $-i\nabla$. The Dirichlet condition in this case means the hard walls of the domain. However, the relativity theory tells that the choice of kinetic energy as above is not appropriate for high energies and for a massive relativistic particle should be replaced by $\sqrt{p^2 + m^2}$. Thus, the corresponding quantum Hamiltonian should be chosen as $\sqrt{-\Delta + m^2}$. This gives an inspiration of study fractional powers of the Helmholtz operator, especially their spectral properties. Notice that such powers are **non-local** operators.

We discuss mainly the **fractional Laplacian** though our results can be transferred to fractional Helmholtz operator.

As in case of non-relativistic particle, the important impact to the statement of the problem is brought by the boundary condition. In contrast to the local case, we have non-unique procedure to impose the Dirichlet condition. The first one is to take the spectral power of the conventional Dirichlet Laplacian in Ω. In this case the analysis of spectrum of such a problem reduces to the analysis of the standard Dirichlet Laplacian. Thus we will consider the so-called **restricted** Dirichlet fractional Laplacian.

We assume $s \in (0, 1)$. This case has a strong connection to the theory of stochastic processes. While the Laplacian Δ can be considered as a generator of the standard Brownian semigroup $\exp(t\Delta)$, the fractional Laplacian, or more carefully the operator $-(-\Delta)^s$ for $s \in (0, 1)$ stands for the generator of the Lévi-stable motion semigroup. In both cases restricting to the domain Ω and posing the Dirichlet conditions means posing the killing or absorbing boundary condition for the process.

In the talk we will discuss the spectral properties of the operator mentioned above in unbounded domains with cylindrical outlets to infinity. The results will be quiet similar to the local case, however the non-local behaviour of the operator brings some new effects and requires new approaches in proofs.