POINCARÉ-STEKLOV OPERATOR FOR THE MIT BAG MODEL

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ABSTRACT. In this talk, I will discuss the pseudodifferentiel properties of the Poincaré-Steklov (PS) operator associated to the MIT bag operator on a smooth domain $\Omega \subset \mathbb{R}^3$ with a compact boundary $\partial\Omega$. More precisely, this operator is the analogue of the Dirichlet-to-Neumann operator for the Laplace operator, and is associated with the following boundary value problem

$$\begin{cases} (D_m - z)\psi = 0, & \text{in } \Omega, \\ P_\pm t_{\partial\Omega}\psi = g \in H^{1/2}(\partial\Omega)^4, \end{cases}$$

where $D_m = -i\alpha \cdot \nabla + m\beta$ is the free Dirac operator and $P_\pm$ are projections along the boundary $\partial\Omega$.

In the first part of this talk, I will first explain how the PS operator fits well into the framework of classical pseudodifferential operators and determine its principal symbol. In the second part, I will discuss the properties of the PS operator when the mass m becomes large enough. I will show that it is a $1/m$-pseudodifferential operator and I will give its main properties, in particular its semiclassical principal symbol. Then we apply these results to establish a Krein-type resolvent formula for the Dirac operator $H_M = D_m + M\beta 1_{\mathbb{R}^3\setminus\overline{\Omega}}$ in terms of the resolvent of the MIT bag operator when $M > 0$ is large enough. With its help, we show that in the large coupling limit $M \to \infty$, the operator H_M convergences toward the MIT bag operator in the norm-resolvent sense with a convergence rate of $\mathcal{O}(M^{-1})$.

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