High-contrast random composites: homogenisation framework and new spectral phenomena

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We study the homogenisation problem for elliptic operators of the form $\mathcal{A}_\varepsilon = -\nabla A_\varepsilon \nabla$ with high-contrast random coefficients A_ε. In particular, we are interested in the behaviour of their spectra. We assume that on one of the components of the composite the coefficients A_ε are "of order one", the complimentary "soft" component consists of randomly distributed inclusions, whose size and spacing are of order $\varepsilon \ll 1$, and the values of A_ε on the inclusions are of order ε^2.

Our interest in high-contrast homogenisation problems is motivated by the band-gap structure of their spectra. From an intuitive point of view this phenomenon can be explained by viewing the "soft" inclusions as micro-resonators, which may dramatically amplify of completely block the propagation of waves in the medium, depending on the frequency. From a mathematically rigorous perspective, this was first analysed by Zhikov (2000, 2004) in the periodic setting.

Despite a vigorous activity in the field of periodic high-contrast homogenisation during the last two decades, the stochastic (random) high-contrast setting was largely overlooked, perhaps due to the technical challenges and more complicated intuitive picture.

In this talk I will present our recent results in this area. We analyse the homogenised operator $\mathcal{A}_{\mathrm{hom}}$ and its spectrum. We prove the convergence of the spectra of $\mathcal{A}_\varepsilon$ and describe the limit set $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}_\varepsilon)$. In contrast with the periodic setting, in the stochastic case the spectrum of the homogenised operator is, in general, a proper subset of $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}_\varepsilon)$. We analyse the "additional" part of the spectrum - the difference between $\lim_{\varepsilon\to 0} \mathrm{Sp}(\mathcal{A}_\varepsilon)$ and $\mathrm{Sp}(\mathcal{A}_{\mathrm{hom}})$, and provide its *asymptotic* characterisation.