# Asymptotics of the number of endpoints of a random walk on a directed Hamiltonian metric graph

Vsevolod Chernyshev

02.08.2022

Let us consider a directed metric graph (one-dimensional cell complex, see the book [2] and references therein). Each of its edges is a smooth regular curve and has length, as well as permitted direction of movement. We will consider a situation in general position and assume that all edge lengths are linearly independent over the field of rational numbers. We will also fix the vertex, which we will call the starting one. A point leaves it at the initial moment of time (see [1], [3]). At each vertex with non-zero probability, we can select an outgoing edge for further movement. Reversals on the edges are prohibited. To analyze the number of possible endpoints of such a walk, it is useful to assume that all possibilities are realized. Thus, we arrive at the following dynamical system. At the initial moment of time, points begin to move with unit speed from the starting vertex along all the edges that start from it. As soon as one of the points is at the vertex, a new point appears at each incident vertex, which begins to move towards the end of the edge, and the old one disappears. If several points simultaneously come to one vertex at once, then all of them disappear, while new points appear as if one point arrived at the vertex. Our main task is to study an asymptotics iof $N(T)$, i.e. the number of moving points at time $T$ on various finite compact graphs.

This dynamical system has already been studied for the case of undirected graphs (see [5], [6]). For the number of moving points, a polynomial approximation was obtained, that is, a description of a polynomial of degree $E-1$ was given, where $E$ is the number of edges of the graph that approximates $N(T)$ up to a certain power of the logarithm. Consideration of such dynamical systems was motivated by problems of propagation of narrow wave packets on metric graphs and hybrid manifolds (see [4] and references there).

In my talk I will consider a case of an arbitrary directed Hamiltonian graph.

## 1 References


[1] L. Lovasz, Random walks on graphs: A survey, Combinatorics, Paul Erdos is Eighty, 1993, pp. 1-46.

[2] G. Berkolaiko, P. Kuchment, Introduction to Quantum Graphs, Mathematical Surveys and Monographs, 2014, vol. 186 AMS.

[3] V. L. Chernyshev, A. A. Tolchennikov, Polynomial approximation for the number of all possible endpoints of a random walk on a metric graph, Electronic Notes in Discrete Mathematics, Elsevier, 2018, vol. 70, pp. 31-35.

[4] V. L. Chernyshev, A. I. Shafarevich, Statistics of Gaussian packets on metric and decorated graphs, Philosophical transactions of the Royal Society A, 2014, vol. 372, issue 2007.

[5] V. L. Chernyshev, A. A. Tolchennikov, Correction to the leading term of asymptotics in the problem of counting the number of points moving on a metric tree, Russian Journal of Mathematical Physics, 2017, vol. 24, issue 3, pp. 290-298.

[6] V. L. Chernyshev, A. A. Tolchennikov, The Second Term in the Asymptotics for the Number of Points Moving Along a Metric Graph, Regular and Chaotic Dynamics, 2017, vol. 22, issue 8, pp. 937-948.