PERMUTATION ENTROPY FOR GRAPHS AND THE CARTESIAN GRAPH PRODUCT.

JOHN STEWART FABILA-CARRASCO

Entropy metrics are nonlinear measures to quantify the complexity of time series. Among them, permutation entropy [1] is a common metric due to its robustness and fast computation. Multivariate entropy metrics techniques are needed to analyse data consisting of more than one time series. To this end, we present a multivariate permutation entropy, $\mathrm{MPE_G}$, using a graph-based approach.

Given a multivariate signal, the algorithm $\mathrm{MPE_G}$ introduced in [2] involves two main steps:

1) Graph construction: we construct an underlying graph G as the Cartesian product of two graphs G_1 and G_2, i.e., $G := G_1 \square G_2$, where G_1 preserves temporal information of each times series together with G_2 that models the relations between different channels.

2) Permutation entropy for graph signals: we consider the multivariate signal as samples defined on the regular graph G and apply the recently introduced permutation entropy for graphs $\mathrm{PE_G}$ [3].

$\mathrm{PE_G}$ is an entropy metric to analyse signals measured over irregular graphs by generalising permutation entropy, a metric based on the comparison of neighbouring values within patterns in a time series. The algorithm $\mathrm{PE_G}$ is based on comparing signal values on neighbouring vertices, using the adjacency matrix, and it has important relations with the sign of the Laplacian matrix. This generalisation preserves the properties of classical permutation for time series and the recent permutation entropy for images, and it can be applied to any graph structure with synthetic and real signals.

Our graph-based approach gives the flexibility to consider diverse types of cross channel relationships and signals, and it overcomes with the limitations of current multivariate permutation entropy.

This is joint work with Javier Escudero (University of Edinburgh) and Chao Tan (Tianjin University).

Supported by the Leverhulme Trust via a Research Project Grant (RPG-2020-158).

Keywords. – Entropy metric, permutation entropy, graph signal, multivariate signal, dynamic on graphs, non-linear analysis on graphs.

References

[1] C. Bandt, and B. Pompe, "Permutation Entropy: A Natural Complexity Measure for Time Series", *Physical Review Letters*, 2002, 88(17), pp. 174102.

[2] J.S. Fabila-Carrasco, C. Tan, and J. Escudero, "Multivariate permutation entropy, a Cartesian graph product approach". *arXiv preprint* arXiv:2203.00550.

[3] J.S. Fabila-Carrasco, C. Tan, and J. Escudero,"Permutation Entropy for Graph Signal", *IEEE Transactions on Signal and Information Processing over Networks*, 2022, vol. 8, pp. 288-300.

School of Engineering, Institute for Digital Communications, University of Edinburgh, West Mains Rd, Edinburgh EH9 3FB, U.K

Email address: John.Fabila@ed.ac.uk