SPECTRAL PROPERTIES OF THE LAPLACIAN ON A DOMAIN PERTURBED BY SMALL RESONATORS

ANDRII KHRABUSTOVSKYI

University Hradec Kralove, Czech Republic

It is widely known that the spectrum of the Dirichlet Laplacian is stable under small perturbations of a domain, while in the case of the Neumann or mixed boundary conditions the spectrum may abruptly change. In the talk we discuss an example of such a domain perturbation.

Let Ω be a (not necessary bounded) domain in $\mathbb{R}^n$. We perturb it to

$$\Omega_\varepsilon = \Omega \setminus \left(\bigcup_{k=1}^{m} S_{k,\varepsilon} \right),$$

where $S_{k,\varepsilon}$ are closed surfaces with small suitably scaled holes ("windows") through which the bounded domains enclosed by these surfaces ("resonators") are connected to the outer domain. When ε goes to zero, the resonators shrink to points.

We prove that in the limit $\varepsilon \to 0$ the spectrum of the Laplacian on Ω_ε with the Neumann boundary conditions on $S_{k,\varepsilon}$ and the Dirichlet boundary conditions on the outer boundary converges to the union of the spectrum of the Dirichlet Laplacian on Ω and the numbers γ_k, $k = 1, \dots, m$, being equal $1/4$ times the limit of the ratio between the capacity of the kth window and the volume of the kth resonator. We obtain an estimate on the rate of this convergence with respect to the Hausdorff-type metrics.

Our proofs are based on abstract results for studying the convergence of operators in varying Hilbert spaces developed in [2,3].

Also, we present an application of the above result: we construct an unbounded waveguide-like domain with inserted resonators such that the eigenvalues of the Laplacian on this domain lying below the essential spectrum threshold do coincide with prescribed numbers.

This is a joint work with G. Cardone (University of Naples Federico II) [1].

References

[1] G. Cardone, A. Khrabustovskyi, Spectrum of the Laplacian on a domain perturbed by small resonators, submitted (2022); `https://arxiv.org/abs/2203.01971`

[2] A. Khrabustovskyi and O. Post, A geometric approximation of δ-interactions by Neumann Laplacians, J. Phys. A: Math. Theor. 54 (2021), 465201.

[3] O. Post, Spectral analysis on graph-like spaces, Springer, Berlin, 2012.