SZEGŐ TYPE ASYMPTOTICS OF THE THREE DIMENSIONAL LANDAU HAMILTONIAN

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Abstract. We study the Szegő type asymptotics associated to the three dimensional Landau Hamiltonian H, which describes a free electron subject to a constant magnetic field. To that end, let $\Lambda \subset \mathbb{R}^3$ be a bounded Lipschitz domain and $f\colon [0,1] \to \mathbb{R}$ be a function with $f(0) = 0$. Under some assumptions, we show that, for the Wiener–Hopf operator $1_{L\Lambda}\mathbb{1}_{\leqslant\mu}(H)1_{L\Lambda}$, we have the asymptotic expansion $\operatorname{tr} f(1_{L\Lambda}\mathbb{1}_{\leqslant\mu}(H)1_{L\Lambda}) = CL^d f(1) + C(f)L^{d-1}\ln(L) + o(L^{d-1}\ln(L))$ with explicitly known constants, depending on μ, the strength of the magnetic field, f and the set Λ. As for the case with no magnetic field we find, to sub-leading order $L^2\ln(L)$, a logarithmically enhanced area law. This is in contrast to the two-dimensional case since the eigenprojector has only a weak polynomial decay in the direction of the magnetic field, which causes the extra $\ln(L)$ factor. The explicit expression for the coefficient of the leading order contains a surface integral similar to the Sobolev–Widom formula in the non-magnetic case. It differs however in the sense that the dependence on the boundary is not solely on its area but on the "area perpendicular to the direction of the magnetic field". On the way we prove an improved 2-term asymptotic expansion (up to an error term of order one) of certain traces of one-dimensional Wiener–Hopf operators with a discontinuous symbol.

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