Electromagnetic guided waves formed by a slab of metamaterial embedded in the vacuum

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Abstract. In this work we explore the spectral properties of the time-dependent Maxwell's equations for a two dimensional infinite slab of metamaterial of finite width $2L(L > 0)$ immersed in the vacuum. In the present work, we restrict us to a metamaterial governed by the Drude model.

We choose to focus on the *transverse electric waves* (TE). By introducing the operators: $\mathbf{curl}\, U = (\partial_y\, U, -\partial_x\, U)^\top$ and $\mathrm{curl}\, \mathbf{V} = \partial_x\, \mathbf{V}_y - \partial_y\, \mathbf{V}_x$, for $U \in L^2(\mathbb{R}^2)$ and $\mathbf{V} \in (L^2(\mathbb{R}^2))^2$; and subsequently, the functional space: $\mathcal{H}_{\mathrm{curl}}(\mathbb{R}^2) := \{\mathbf{V} \in (L^2(\mathbb{R}^2))^2 \mid \mathrm{curl}\, \mathbf{V} \in L^2(\mathbb{R})\}$, and the slab domain $B := \mathbb{R} \times (-L, L)$, thus, under this basis our problem formulates as follows

$$\text{To find } (E, \mathbf{H}, J, \mathbf{K})^\top \in \mathcal{H}_{\mathbf{2D}} := H^1(\mathbb{R}^2) \times \mathcal{H}_{\mathrm{curl}}(\mathbb{R}^2) \times L^2(B) \times (L^2(B))^2, \quad \text{such that,}$$

$$\text{(Vacuum)} \begin{cases} \varepsilon_0\, \partial_t E - \mathrm{curl}\, \mathbf{H} = J_s, \\ \mu_0\, \partial_t \mathbf{H} + \mathbf{curl}\, E = 0, \end{cases} \text{in } \mathbb{R}^2 \setminus B, \quad \text{(Drude material)} \begin{cases} \varepsilon_0\, \partial_t E - \mathrm{curl}\, \mathbf{H} + J = J_s, \\ \mu_0\, \partial_t \mathbf{H} + \mathbf{curl}\, E + \mathbf{K} = 0, \\ \partial_t\, J = \varepsilon_0\, \Omega_e^2\, E, \\ \partial_t\, \mathbf{K} = \mu_0\, \Omega_m^2\, \mathbf{H}, \end{cases} \text{in } B. \tag{1}$$

Provided of the Hilbert space $\mathcal{H} := L^2(\mathbb{R}^2) \times (L^2(\mathbb{R}^2))^2 \times L^2(B) \times (L^2(B))^2$, we are able to rewrite (1) as a generalized Schrödinger evolution problem of the form

$$\text{(Abstract formulation)} \quad \frac{d}{dt}\mathbf{U} + \mathrm{i}\, \mathbb{A}\, \mathbf{U} = \mathbf{F}, \tag{2}$$

where $\mathbb{A}$ is a "*curl*"-self-adjoint operator in $\mathcal{H}$ and $\mathbf{U} := (E, \mathbf{H}, J, \mathbf{K})^\top \in \mathcal{H}_{\mathbf{2D}} = D(\mathbb{A})$. By applying the one-dimensional spatial Fourier transform on the first variable , $\mathcal{F}$, we generate a family of self-adjoint operators $(\mathbb{A}_k)_{k \in \mathbb{R}}$ (*reduced Hamiltonian family*) which satisfies

$$\mathbb{A} = \mathcal{F}^{-1} \mathbb{A}_\oplus\, \mathcal{F}, \quad \mathbb{A}_\oplus := \int_{\mathbb{R}} \mathbb{A}_k\, dk, \tag{3}$$

where the last operator is built based on the abstract tool of *direct integrals of Hilbert spaces and operators* [1].

Therefore, the spectral analysis of the full Hamiltonian $\mathbb{A}$ reduces to study the one of each reduced Hamiltonian $\mathbb{A}_k$. We focus on the point spectrum of the reduced Hamiltonian $\mathbb{A}_k$, whose existence corresponds physically to the existence of guided waves, that is solutions that propagate along the slab and decay exponentially in the transverse direction. This leads us to study the problem

$$\text{For all } k \in \mathbb{R}, \text{ find } u = u(y) \in H^1(\mathbb{R}) \text{ and } \omega \in \mathbb{R} \text{ such that}$$

$$-\frac{d^2}{dy^2} E(y) + (k^2 - \omega^2\, \varepsilon(\omega, y)\, \mu(\omega, y))\, E(y) = 0, \quad \text{where} \quad s(\omega, y) = \begin{cases} s_0, & \text{if } |y| > L \\ s(\omega) := s_0 \left(1 - \frac{\Omega_e^2}{\omega^2}\right), & \text{if } |y| < L \end{cases}, \quad \text{for } s \in \{\varepsilon, \mu\}.$$

We also compare the results to the ones obtained for the classic case of a dielectric slab, i.e., when $s(\omega) \equiv s_1 > 0$ for $s \in \{\varepsilon, \mu\}$, and provided with the neccesary condition, $c_1 := (\varepsilon_1\, \mu_1)^{-\frac{1}{2}} < (\varepsilon_0\, \mu_0)^{-\frac{1}{2}} =: c_0$ (see [2]). In particular, we emphasize the appearance of plasmonic guided waves who are exponentially confined along the interfaces between the vacuum and the Drude medium.

Keywords. Drude model; Generalized eigenfunctions; Maxwell's equations; Negative index materials; Spectral analysis of reduced Hamiltonian.

References

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