Large scale Gaussian upper bounds for heat kernels on graphs with unbounded geometry

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The heat kernel as the minimal fundamental solution of the heat equation is one of the most important objects studied in geometric analysis and encodes the geometry of the underlying space in analytic terms. On graphs it is nowadays known that the short time behaviour of the heat kernel is very different from the short time behaviour of heat kernels on manifolds due to the non-locality of the graph Laplacian. However, on large scales, i.e., large times or distances, heat kernels on graphs satisfying certain regularity properties are expected to behave like heat kernels on manifolds with corresponding properties. Our main result is an optimal Gaussian decay estimate for large times and distances for any graph satisfying volume doubling and Sobolev inequalities on large balls encoded via intrinsic metrics. The explicit estimates depend on means of the vertex degree and inverted vertex measure which vanish on large scales, and the Sobolev dimension. I will illustrate our main result for the important cases of normalizing and counting measures. Our estimates particularly yield estimates for heat kernels on antitrees, for which such bounds had been unknown so far. This is joint work with Matthias Keller.