Confinement models in relativistic quantum mechanics: spectral analysis and shape optimization

Tomas Sanz-Perela

Madrid, Spain

Abstract

Dirac operators defined on domains of the Euclidean space are used in relativistic quantum mechanics to describe particles that are confined in a bag. A remarkable example in dimension 3 is the MIT bag operator, used to model confinement of quarks in hadrons, and a fundamental topic in mathematical physics concerns the analysis of the spectral gap and its associated shape optimization problem. This consists on minimizing the first squared eigenvalue among all domains with prescribed volume, and it is conjectured that the ball is the optimal domain.

In this talk I will describe a recent work —in collaboration with N. Arrizabalaga, A. Mas, and L. Vega — in which we propose an approach towards this open problem. We have studied a family of Dirac operators $\mathcal{H}_\tau$ acting on domains of $\mathbb{R}^3$ and with boundary conditions, parametrized by a real parameter τ. This family contains the MIT bag operator (when $\tau = 0$), while some well-known operators arise in the limits as $\tau \to \pm\infty$. We parametrize the spectrum of the family $\{\mathcal{H}_\tau\}_{\tau\in\mathbb{R}}$ through a collection of increasing smooth curves, and thanks to this we manage to establish (for large values of the parameter) the optimality of the ball for the shape optimization problem involving $\mathcal{H}_\tau$. This is expected to hold for all the parametrization and thus solve the open problem for $\tau = 0$.