

# Preparatory course of Mathematics

## 1 Target group

Beginner students in all courses of physics.

## 2 Objective

Revision of the mathematical knowledge from school which is necessary at the beginning of a study in physics. Depending on the school, gaps stemming from different curricula are filled.

## 3 Organization

The duration of the preparatory course is two weeks. There are two lectures of 90 minutes every day – one in the morning, one in the afternoon. After the lectures, exercises are worked out and discussed in smaller groups. The course is held in german.

## 4 Contents (orientation)

### 1. Basics

- Numerical systems, basic arithmetic methods, fractional arithmetics with variables
- Power laws, logarithmic laws, term conversions

### 2. Equations

- Elementary trigonometry,  $\sin(x)$ ,  $\cos(x)$ , laws of sines and cosines
- Equations, linear equations
- Quadratic equations
- Equations with roots and absolute values
- Inequalities
- Systems of linear equations, matrices, determinants, Law of Sarrus and Cramer's Law for solving systems of linear equations

### 3. Functions

- Elementary functions: polynomials,  $\sin(x)$ ,  $\cos(x)$ ,  $e^x$ ,  $\ln(x)$ ,  $\sinh(x)$ ,  $\cosh(x)$ ,  $\operatorname{arcsinh}(x)$ , ...: important properties, connections
- Functions with parameters, determination from known function values
- Functions in several variables

### 4. Vector analysis

- Vectors
- Coordinate systems
- Scalar product, vector product
- Linear maps

### 5. Complex numbers

- Calculating with complex numbers

- Representations (cartesian, polar)
- Functions with complex arguments (e.g.  $\sin(x + iy)$ )
- Connections between functions with complex arguments

#### 6. Differential calculus

- Differential quotient, physical interpretation
- Laws of derivation (sum rule, product rule, quotient rule, chain rule)
- Derivations of vectors
- Taylor expansion

#### 7. Integral calculus

- Definition of an integral, elementary properties
- Integration of elementary functions
- Laws for integration (substitution, partial integration)

## 5 Examples of exercises

1. Determine the derivative of

$$f_1(x) = \cos(x^3), \quad f_2(x) = \cos^3(x), \quad f_3(x) = x^x.$$

2. Determine an antiderivative of  $f(x) = \sin(x) \exp(-ax)$ .

3. Determine the limit

$$\lim_{x \rightarrow 0} \frac{\arcsin(x)}{x}.$$

4. Write  $\left(\frac{1-i}{1+i}\right)^{10}$  in the form  $a + ib$ .

5. Calculate

$$\det \begin{pmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \\ 1 & 1 & 1 \end{pmatrix}.$$

6. Determine a matrix  $M$  which describes a rotation of angle  $\pi/4$  around the  $z$ -axis

7. Calculate the series expansion of  $g(x) = \exp(-ax^2) \sin(x)$  up to third order.