COMPLEX NETWORK DYNAMICS

Physikzentrum Bad Honnef

Bar-Ilan University
אוניברסיטת בר-אילן

Introduction to Network Science

Baruch Barzel

Macroscopic

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Macroscopic

Microscopic

Complex Systems

Macroscopic

Microscopic

Complex systems

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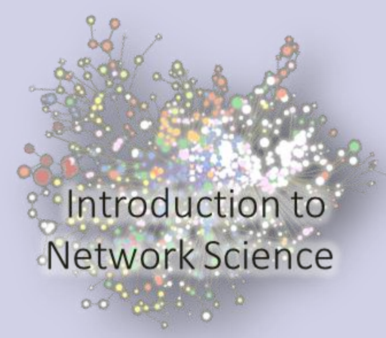
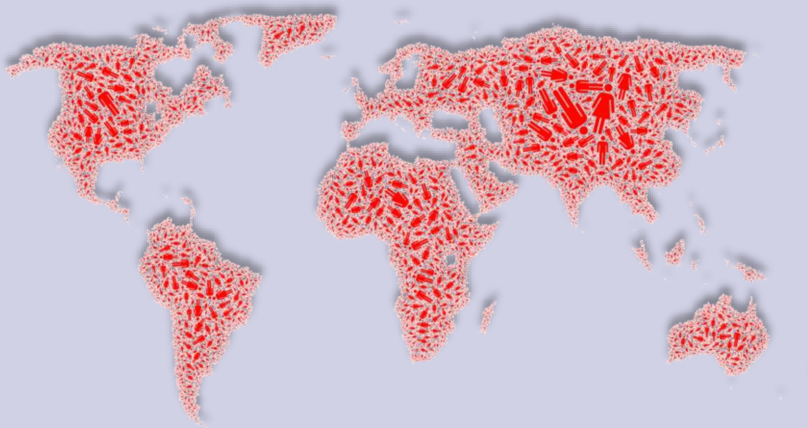


Complex systems

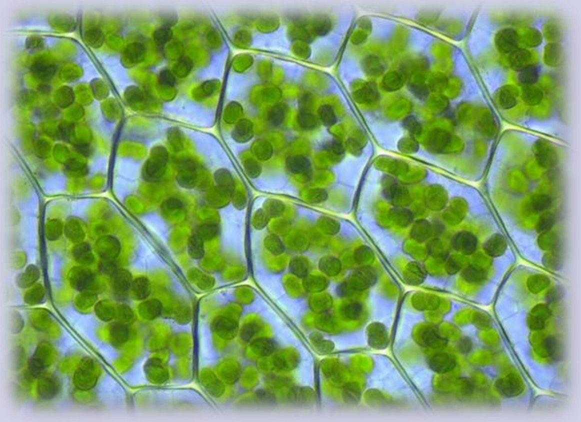
Macroscopic

Microscopic



Complex systems

Macroscopic



Microscopic

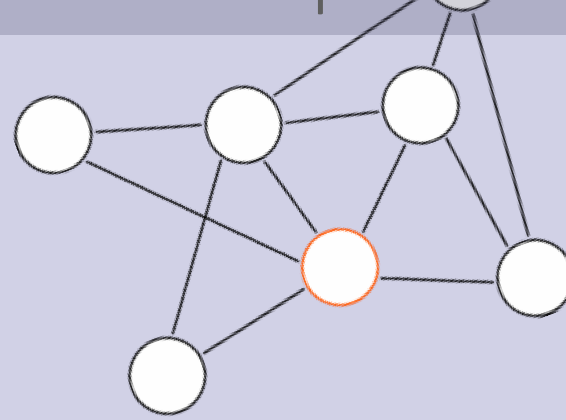
Complex systems

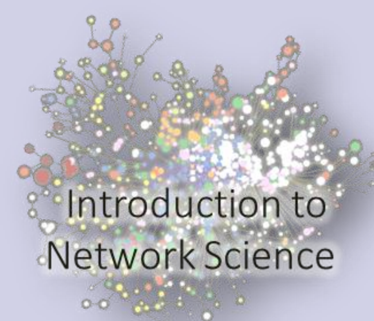
More Is Different

Complex systems and Networks



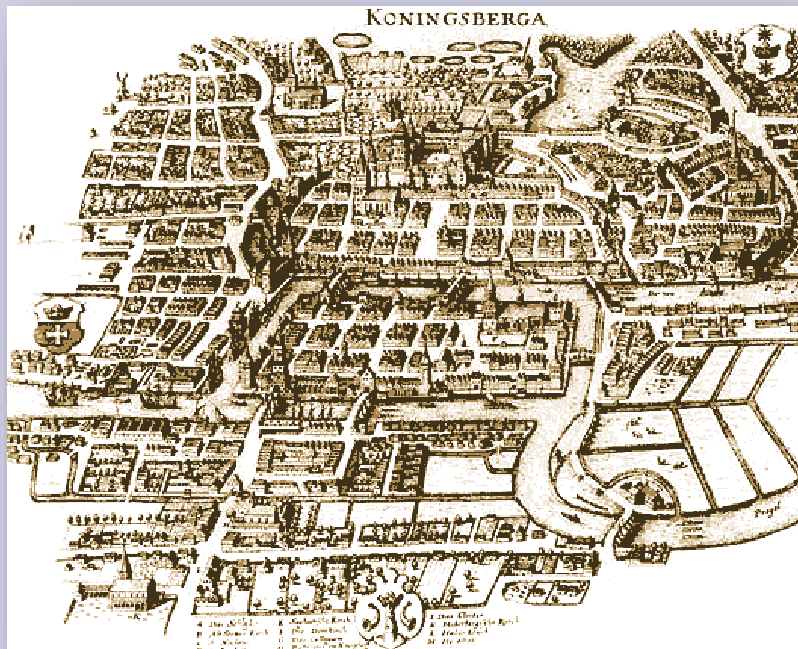
Behind each complex system there is an underlying network that describes the interactions between the microscopic building blocks





Where it all began – back to 1735

Can one walk across the seven bridges and never cross the same bridge twice?



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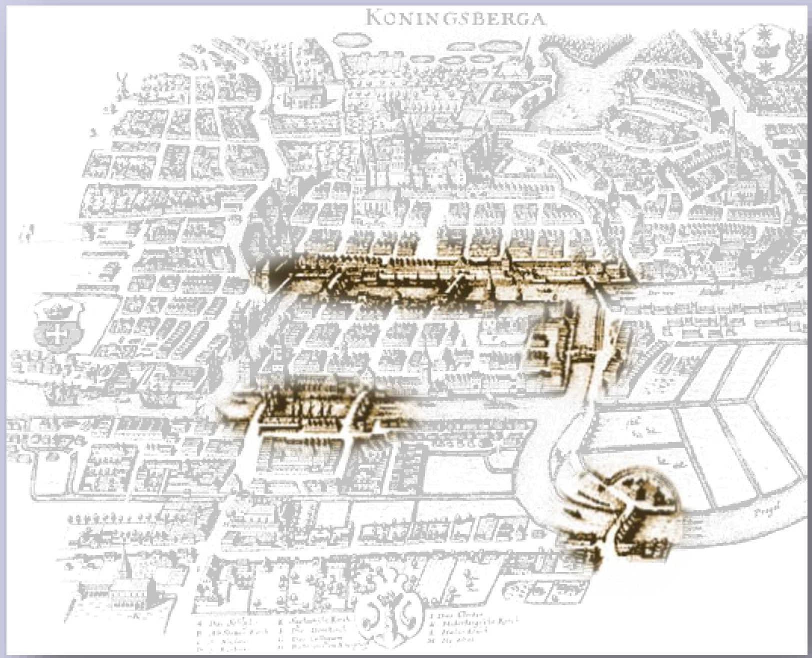
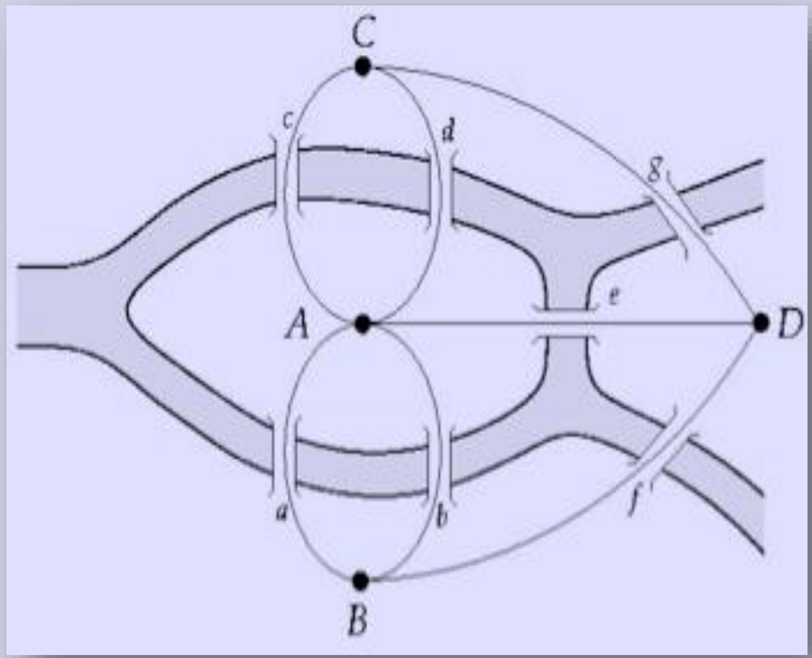


Introduction to Network Science

Where it all began – back to 1735



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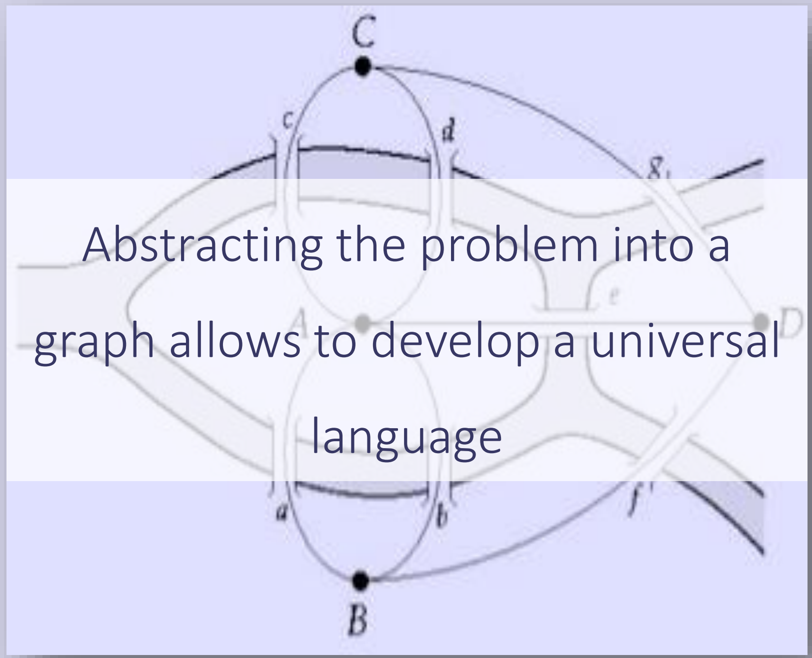


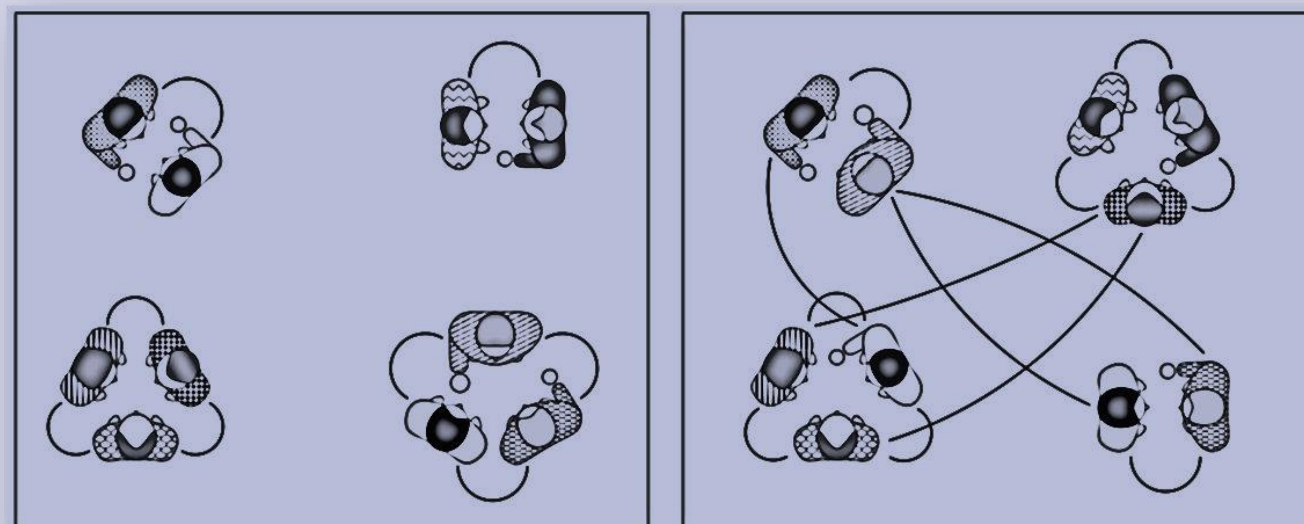
Abstracting the problem into a graph allows to develop a universal language

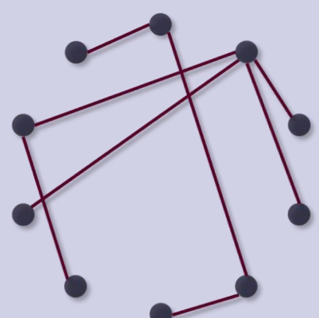
The Erdős-Rényi Random Graph

$G(N,p)$ - Begin with N nodes.

Connect each pair with probability p.

Obtain L links.





$N = 10$

$p = {}^{1}/_{6}$

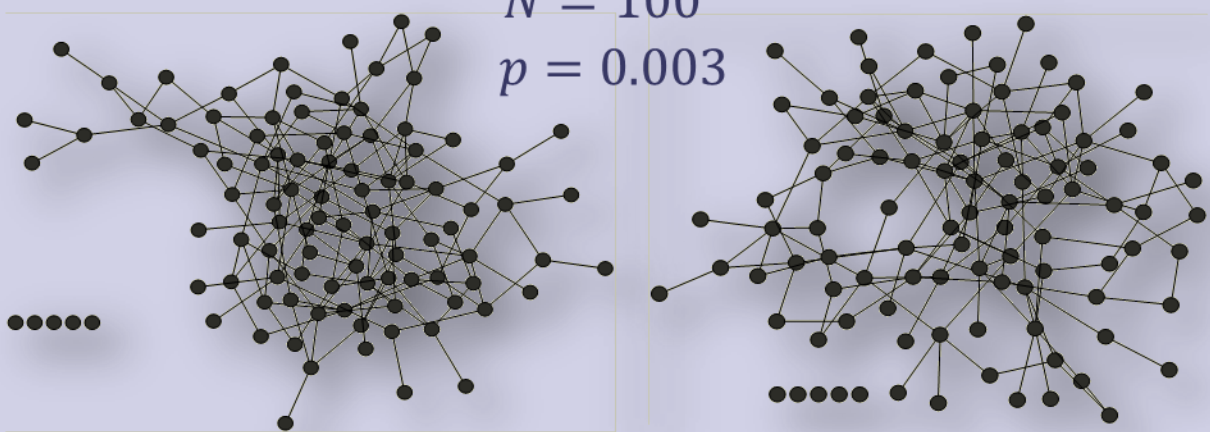
$L = 8$

Introduction to Network Science

The Erdős-Rényi Random Graph

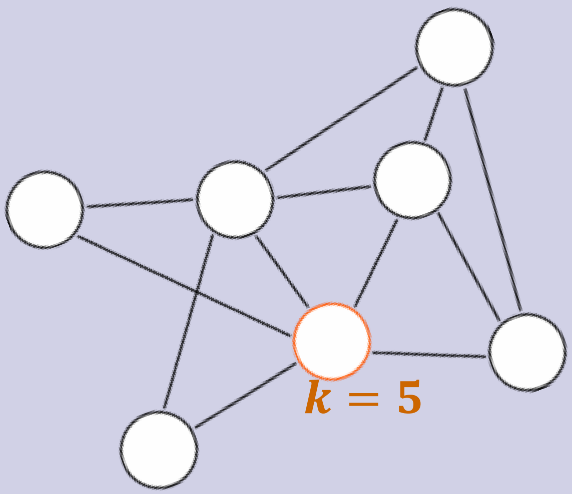
$N = 100$

$p = 0.003$



$$L_{ER} = \binom{N}{2} p = \frac{N(N-1)}{2} p$$

The Erdős-Rényi Random Graph

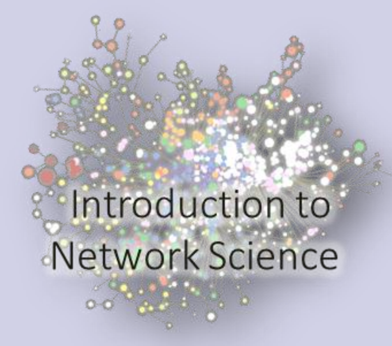


$$L_{ER} = \binom{N}{2} p = \frac{N(N-1)}{2} p$$

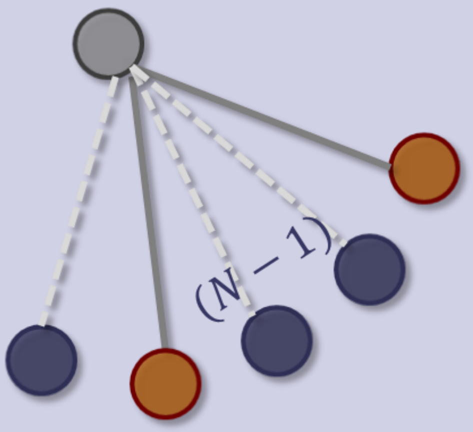
$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i = \frac{2L}{N}$$

$$\langle k \rangle_{ER} = p(N-1) \approx pN$$

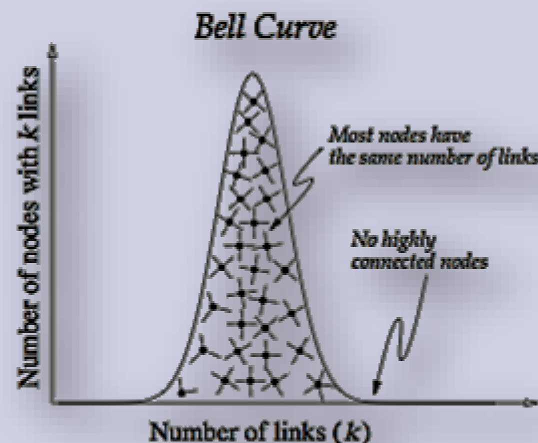
Degree – The number of links of a node

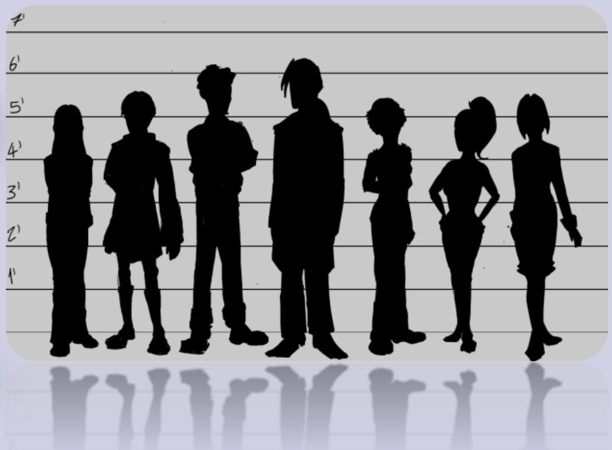


The Erdős-Rényi Random Graph



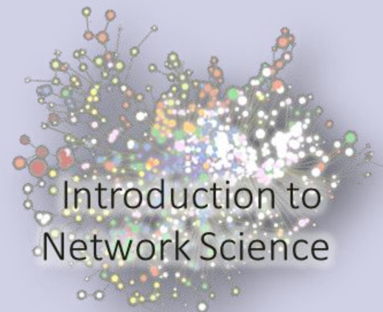
$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-k-1}$$



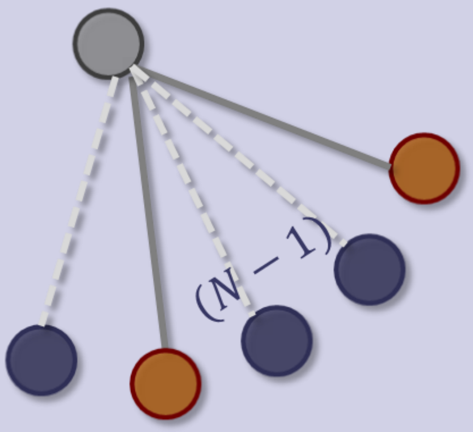


Degree – The number of links of a node

Degree distribution – the probability that a randomly selected node has degree k



The Erdős-Rényi Random Graph

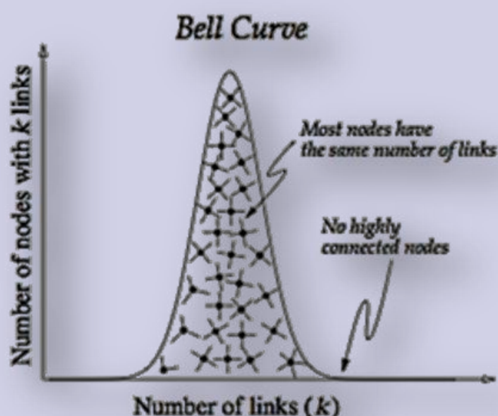


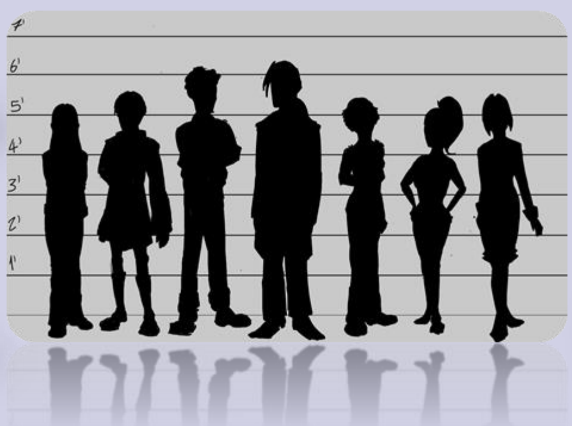
$$P(k) = \binom{N-1}{k} p^k (1-p)^{N-k-1}$$

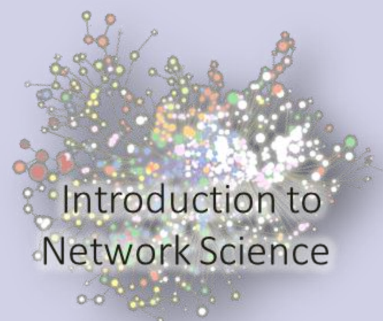
Binomial Distribution

$$P(k) \approx e^{-p(N-1)} \frac{\left(p(N-1)\right)^k}{k!} = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

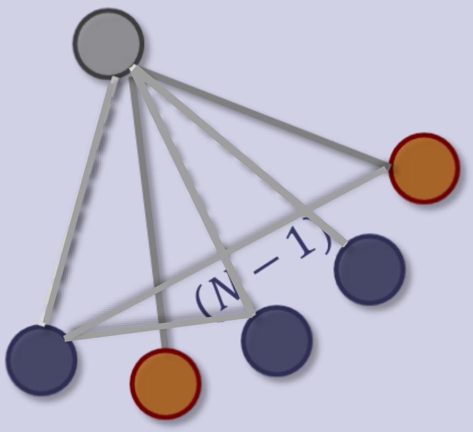
Poisson Distribution







The Erdős-Rényi Random Graph



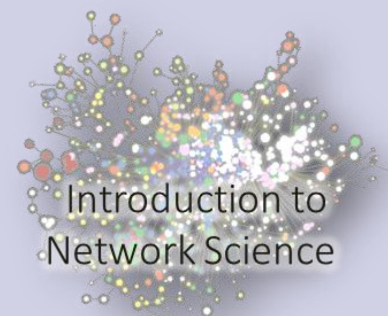
$$C_i = \frac{E_i}{\frac{1}{2}k_i(k_i-1)} = \frac{2}{10} = \frac{1}{5}$$

$$\langle C \rangle = \frac{1}{N}\sum_{i=1}^{N} C_i$$

$$\langle C \rangle_{ER} = p$$

How loopy is your network?

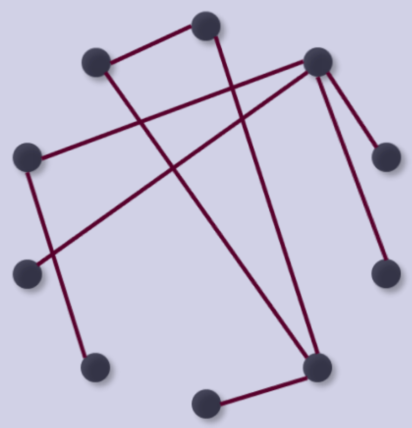
Clustering – the average density of triangles in the network



Types of Graphs

Undirected

- Protein interaction networks
- Collaboration networks
- Actor co-stardom networks
- Internet



$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Types of Graphs

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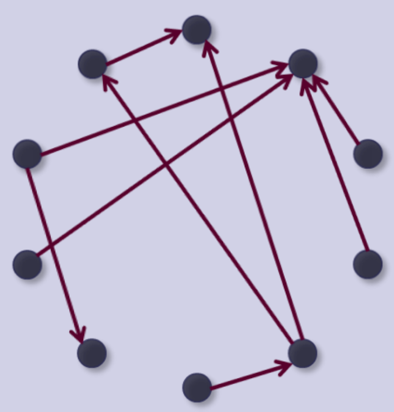
Directed

- Metabolic
- Citation networks
- World Wide Web



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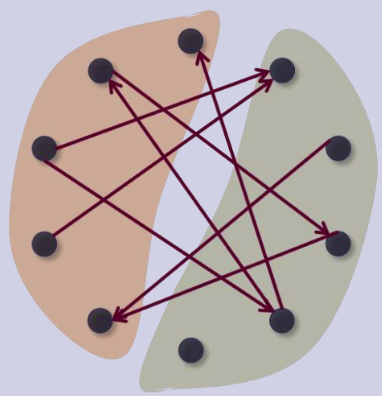
Bipartite

- Collaboration networks
- Actor co-stardom network
- Disease network



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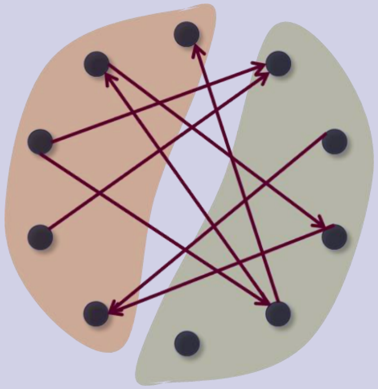

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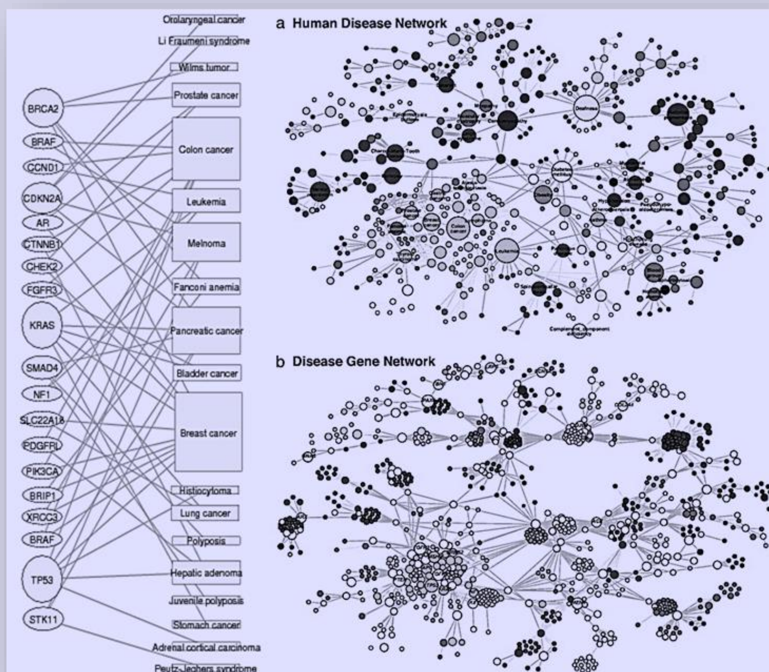

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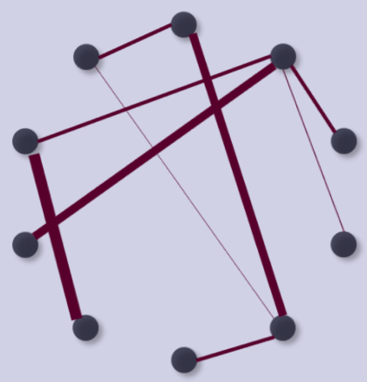


$$A_{ij} = \begin{pmatrix} 0 & 0.2 & 0 & 0 & 1.3 & 0 \\ 0.8 & 0 & 0 & 0.9 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 1.1 & 0 \\ 0 & 3.1 & 0.1 & 0 & 2.5 & 0 \\ 1.8 & 0 & 0.6 & 0.5 & 0 & 0.8 \\ 0 & 0 & 0 & 0 & 0.7 & 0 \end{pmatrix}$$

Bipartite

- Collaboration networks
- Actor co-stardom network
- Disease network

Weighted

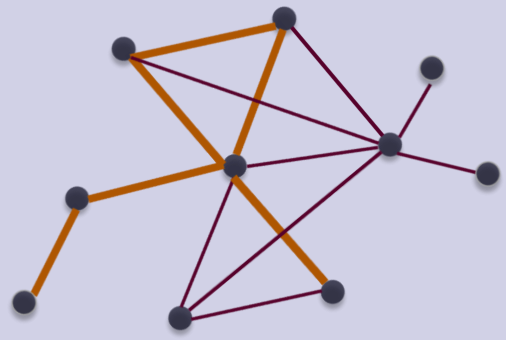
- Metabolic networks
- Collaboration networks
- Actor co-stardom networks
- Social networks

The Metric of Paths

$$P_{ij} = i \xrightarrow{A_{ik}} k \xrightarrow{A_{km}} m \xrightarrow{A_{ml}} l \cdots q \xrightarrow{A_{qj}} j$$

$$N_{ij}^{l} = \sum_{k,m\cdots q} A_{ik} A_{km} \cdots A_{qj} = [A^{l}]_{ij}$$

$$D_{ij} = \begin{pmatrix} 0 & 1 & 2 & 3 & 2 & 1 \\ 2 & 0 & 4 & 1 & 4 & 6 \\ 2 & 4 & 0 & 1 & 1 & 2 \\ 5 & 1 & 1 & 0 & 1 & 3 \\ 1 & 3 & 1 & 4 & 0 & 1 \\ 5 & 2 & 3 & 2 & 1 & 0 \end{pmatrix}$$



Path – a set of consecutive edges

Network Distance – the shortest path linking a pair of nodes

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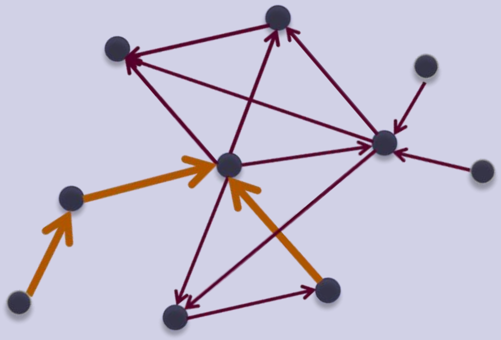


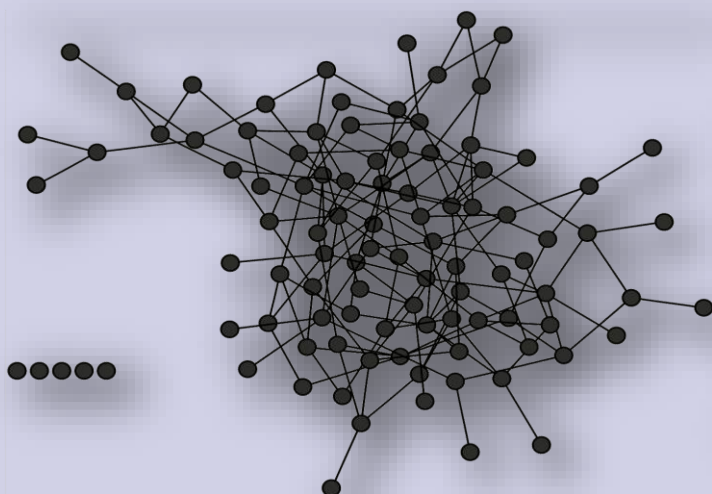
$$D_{ij} = \infty$$

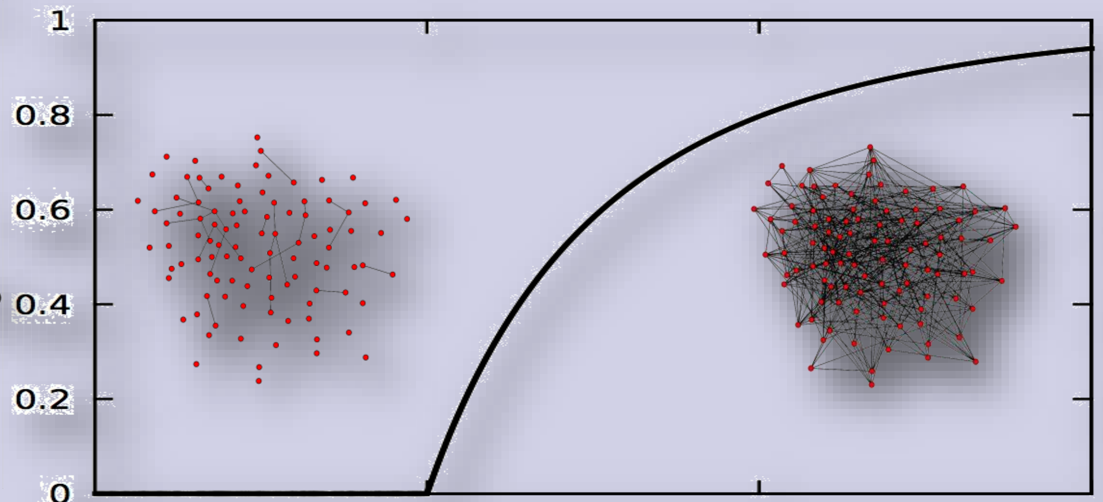
$$D_{ji} = 4$$

Path – a set of consecutive edges

Network Distance – the shortest path linking a pair of nodes

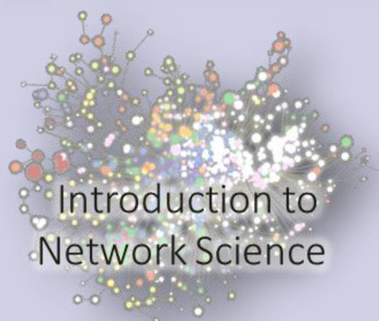
Connectivity





$$\langle k \rangle_{critical} = 1$$

Connected component – a subset of nodes linked through finite paths



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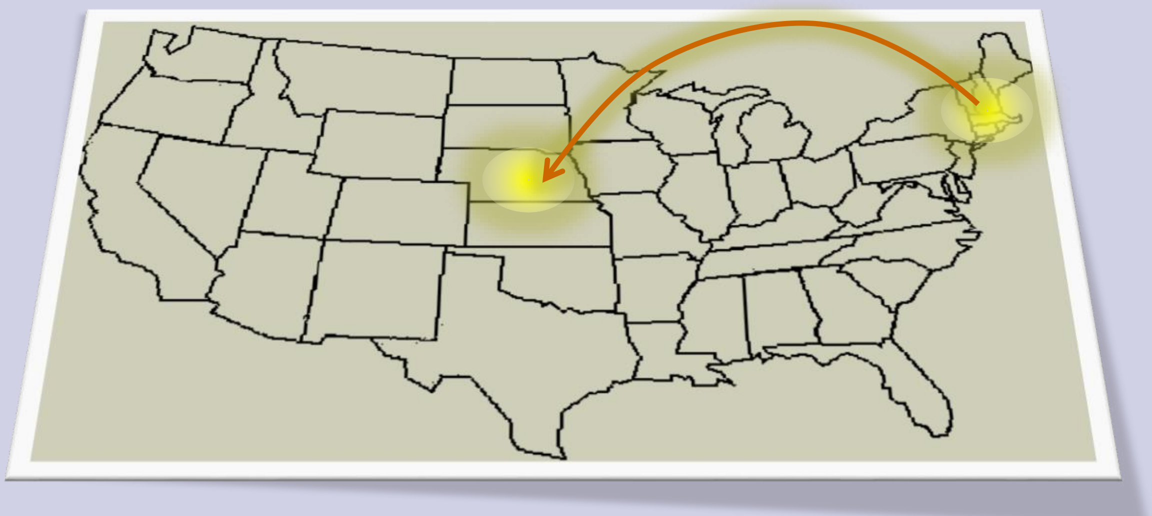
Strange Letter Arrives in Omaha

Introduction to Network Science

Strange Letter Arrives in Omaha



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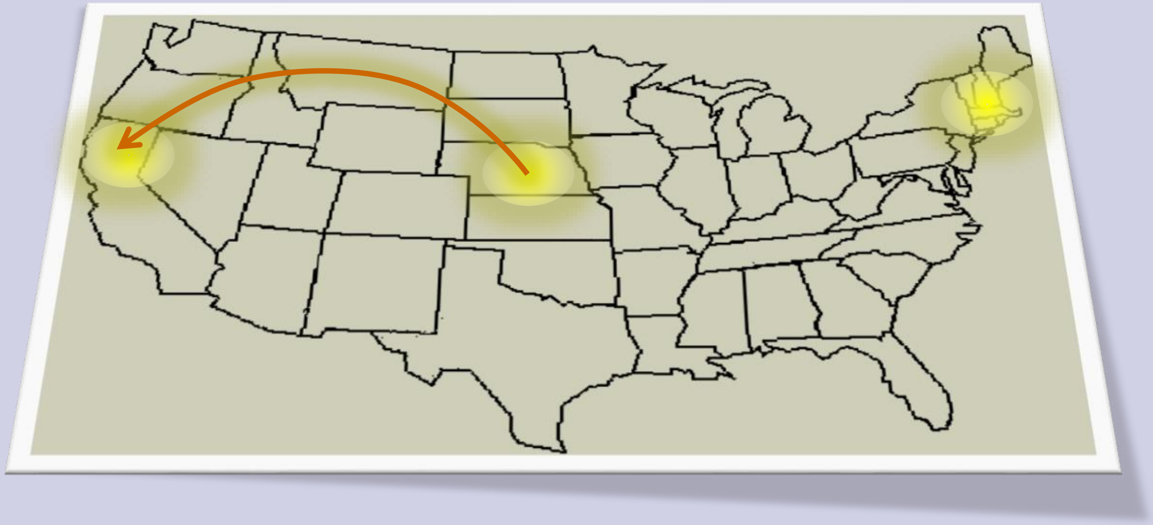

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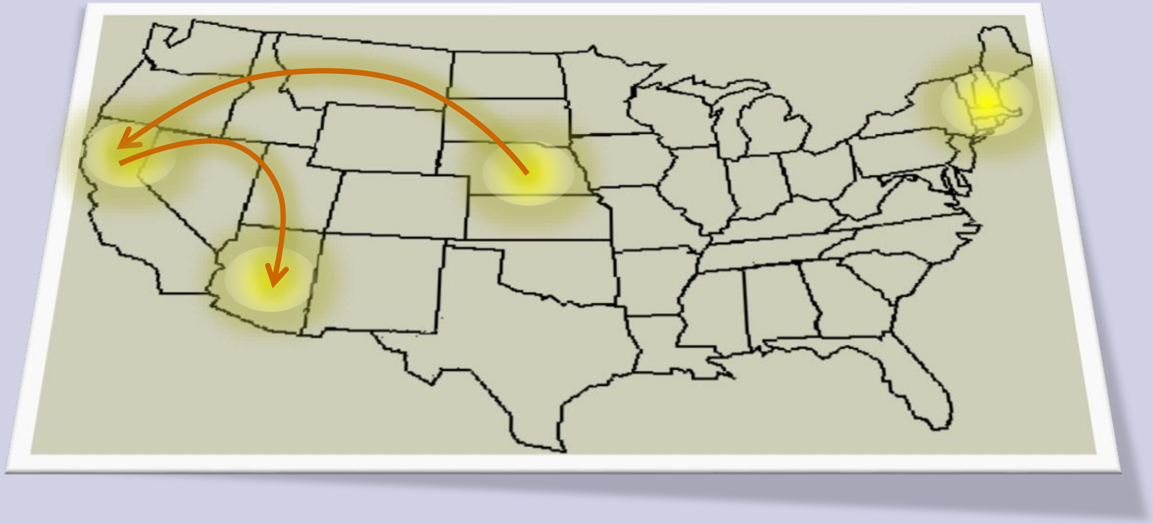

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Introduction to
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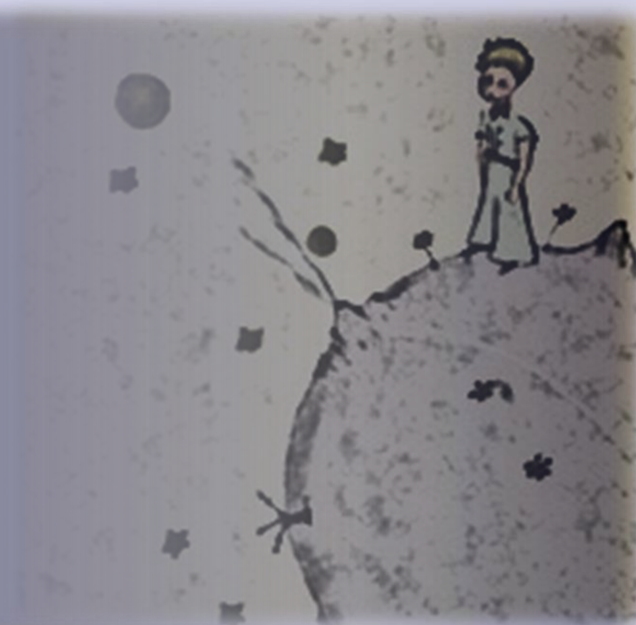
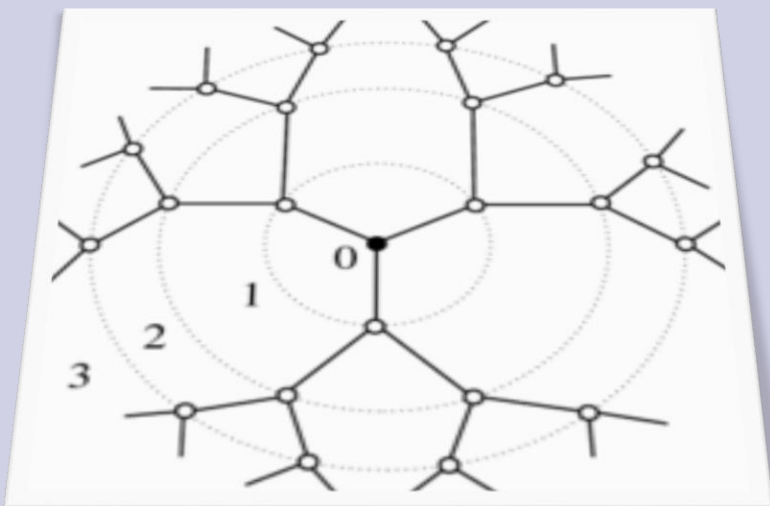
It's a Small World After All

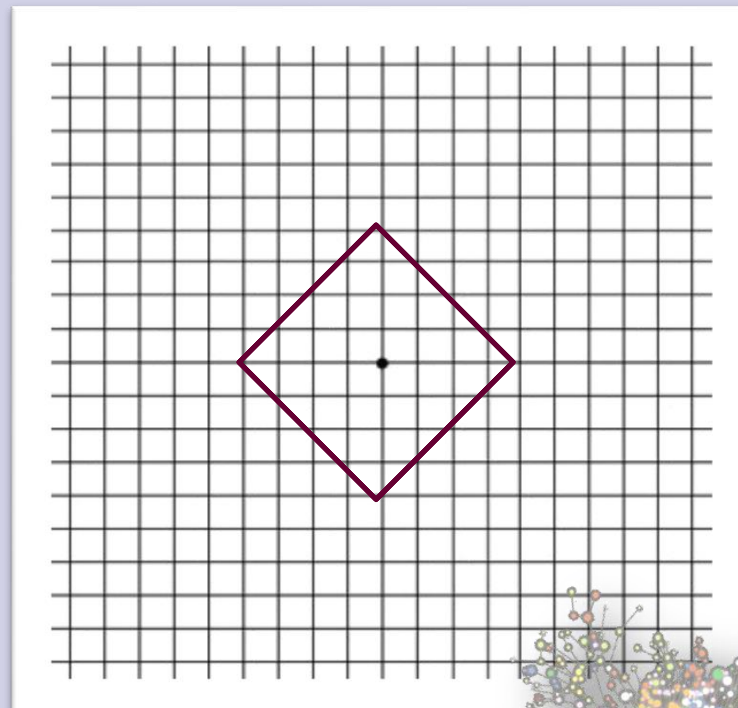
5.73 –**Facebook**
4.67 –**Twitter**
3 –**Metabolism**
5 – **Protein interactions**
3.87 – **Internet**
19 – **WWW**
2.5 – **Neuronal**
3 – **Food webs**



Exploding Volume of Networks

$$S(d) = 4d$$

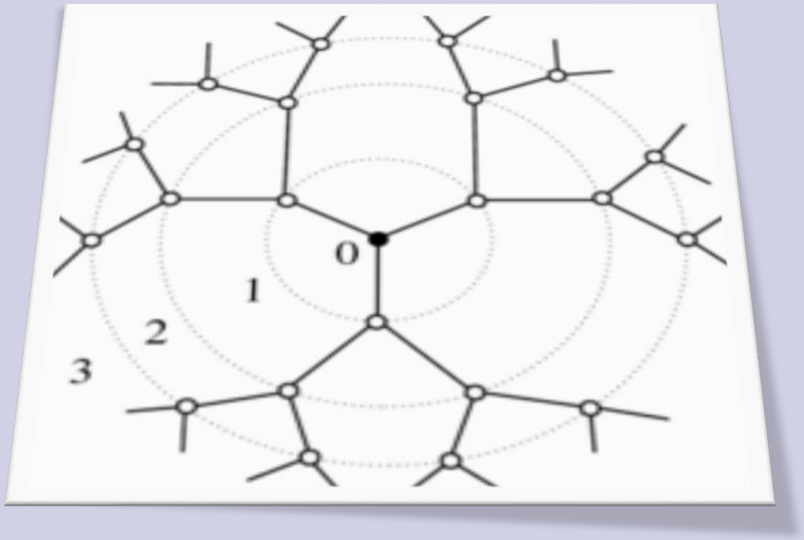


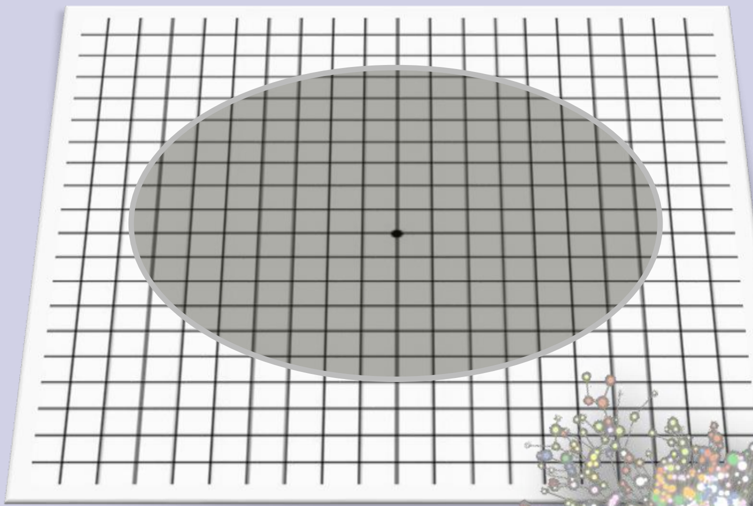


Exploding Volume of Networks

$$N(d) = \sum_{x=1}^{d} 4x = 2d(d+1) \sim d^2$$

Polynomial growth





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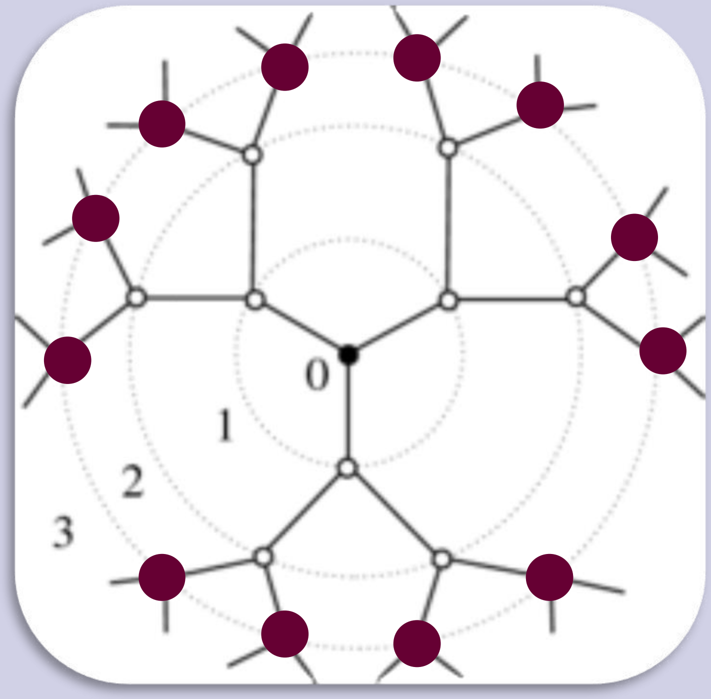

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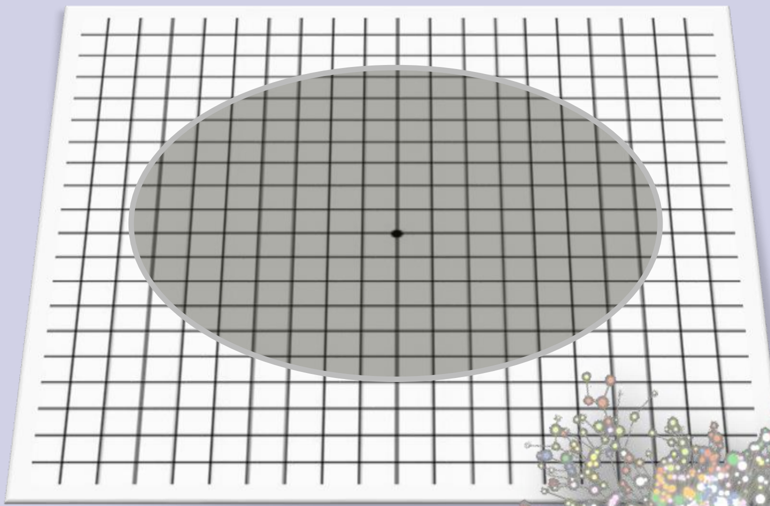
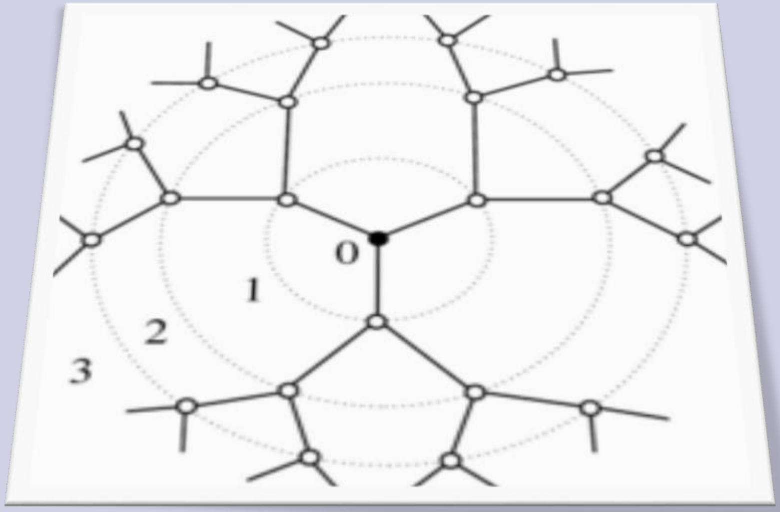


Exploding Volume of Networks

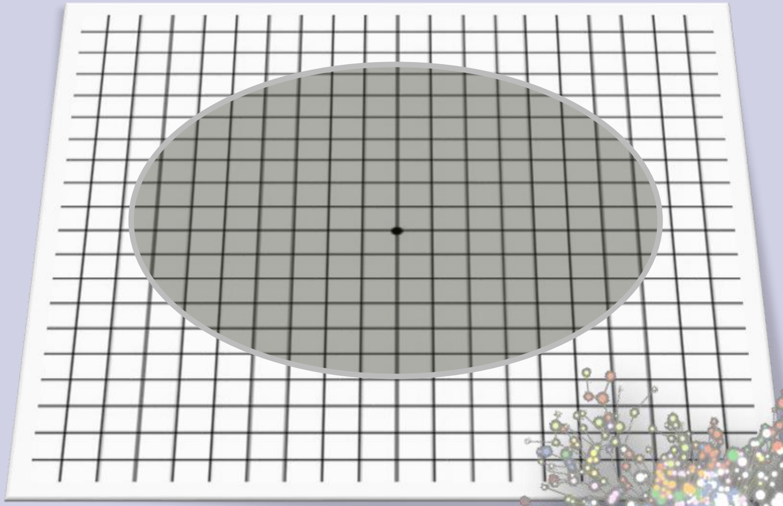
$$N(d)=\sum_{x=1}^{d}k^{x}=\frac{k^{d+1}-1}{k-1}\sim k^{d}$$

Exponential growth



$$N(d)=\sum_{x=1}^{d}4x=2d(d+1)\sim d^{2}$$

Polynomial growth



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Exponential growth

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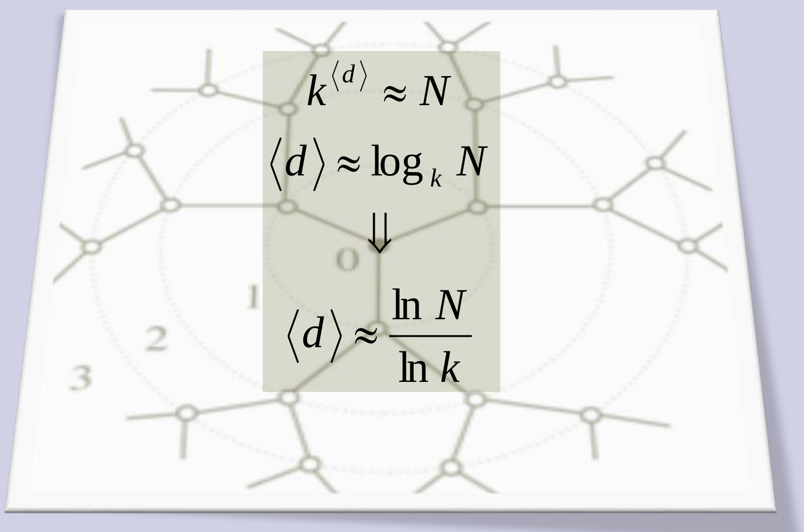
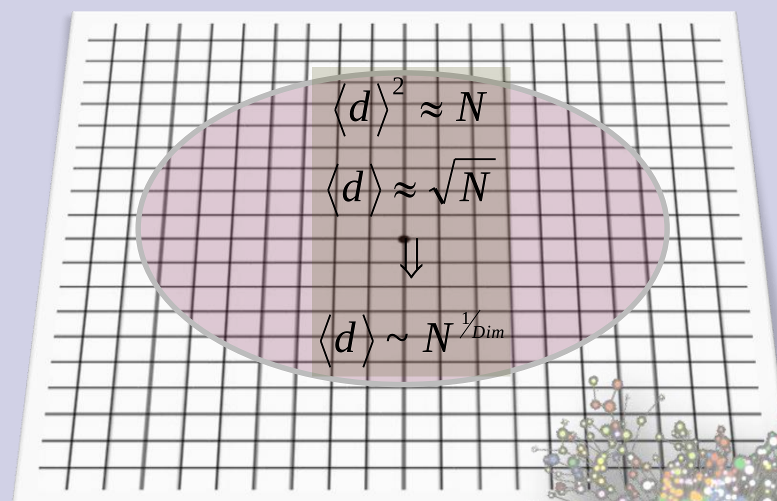
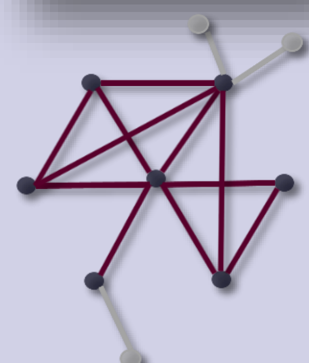




The Erdős-Rényi Graph Model

Poisson – narrow distribution around the mean

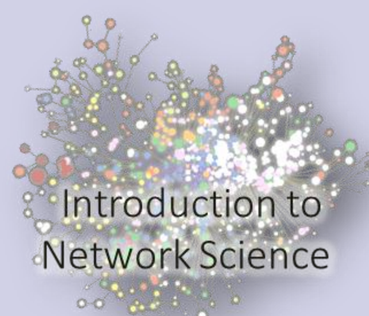


Clustering – vanishes for large networks.

Almost no loops. ($p = \frac{1}{N}$)

Small world –

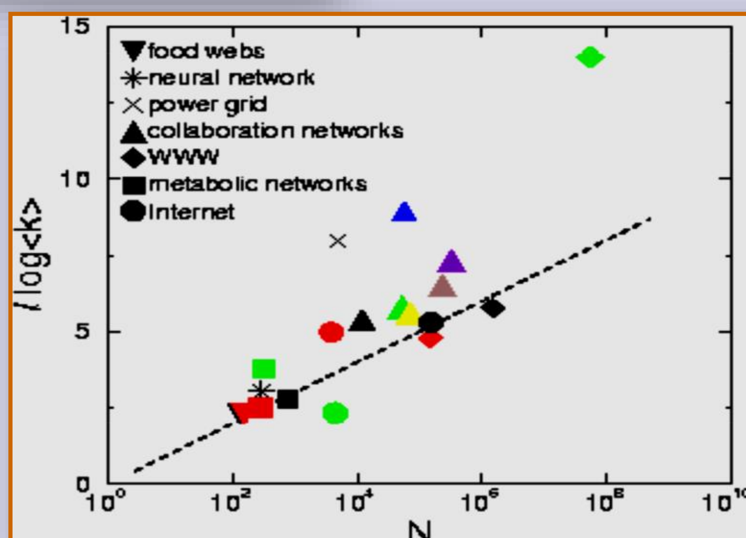
radius scales logarithmically with volume



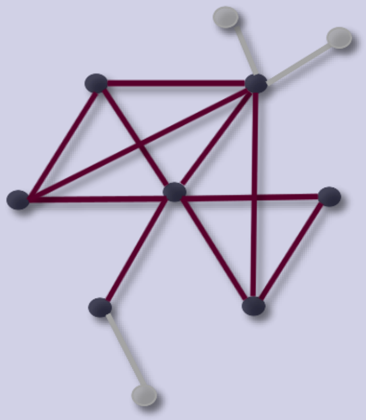
Erdős-Rényi vs. Reality

Small world –

radius scales logarithmically with volume



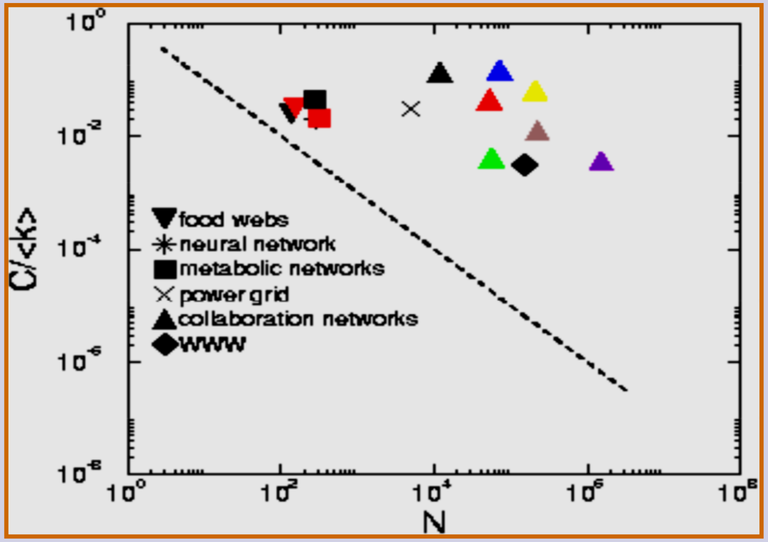
Erdős-Rényi vs. Reality



Clustering –

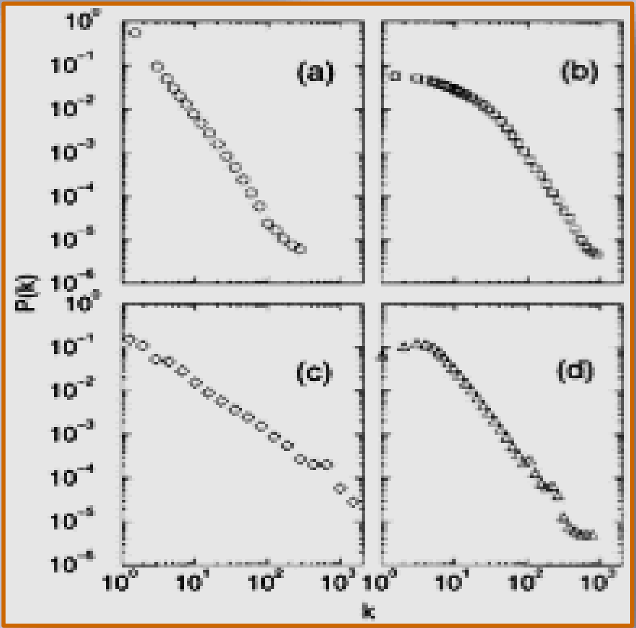
vanishes for large networks.

$\langle C \rangle_{ER} = p = \langle k \rangle / N$



Erdős-Rényi vs. Reality

Poisson – narrow distribution around the mean



Erdős-Rényi vs. Reality

We do not observe a single network in nature that follows this model