CND
COMPLEX NETWORK DYNAMICS

Physikzentrum Bad Honnef

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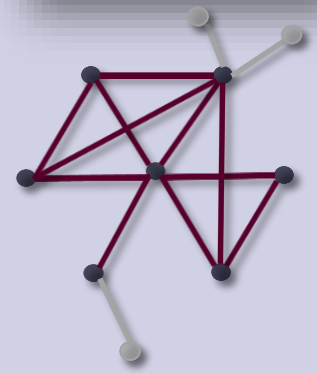
# Introduction to Network Science

Baruch Barzel

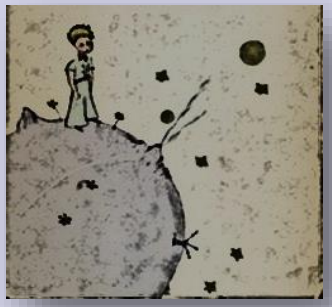
# Recap

Poisson – narrow distribution around the mean



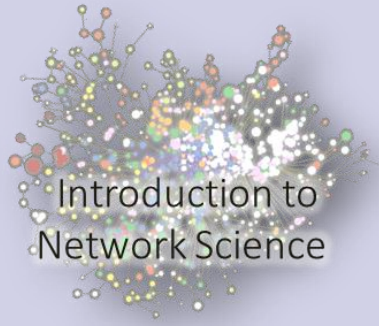
Clustering – vanishes for large networks.

Almost no loops. ($p = \frac{1}{N}$)



Small world –

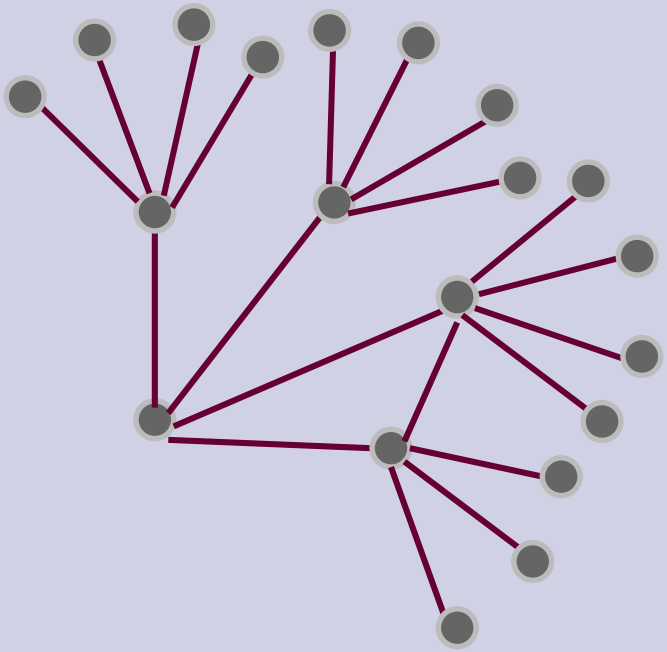
radius scales logarithmically with volume



# Clustering vs. Small World-ness

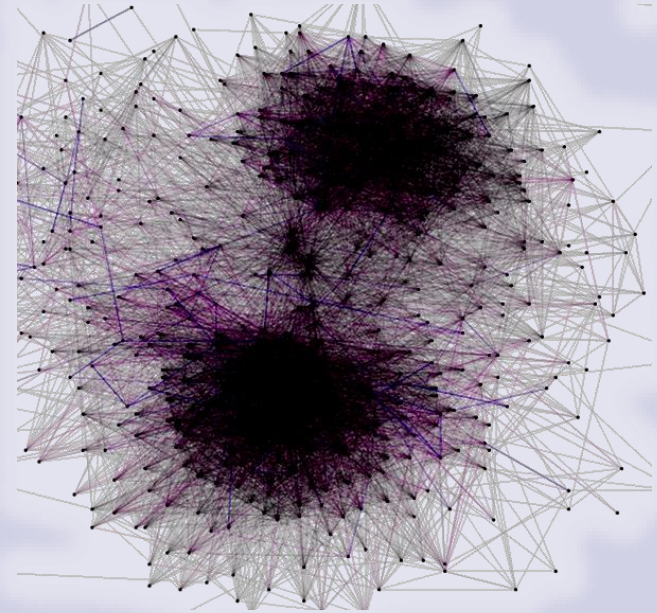
Tree-like expansion is the secret of small world-ness

Loops inhibit the exponential growth

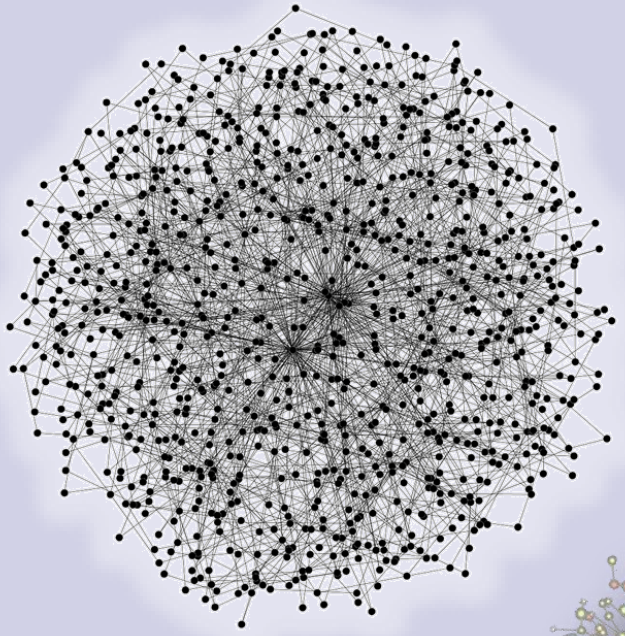


# Clustering vs. Small World-ness

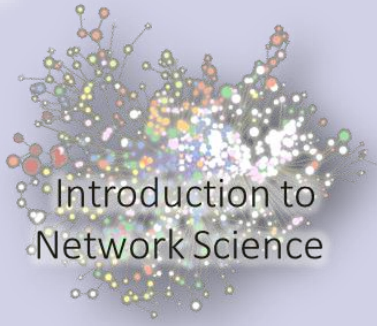
Where should we place the social network?



Clustered

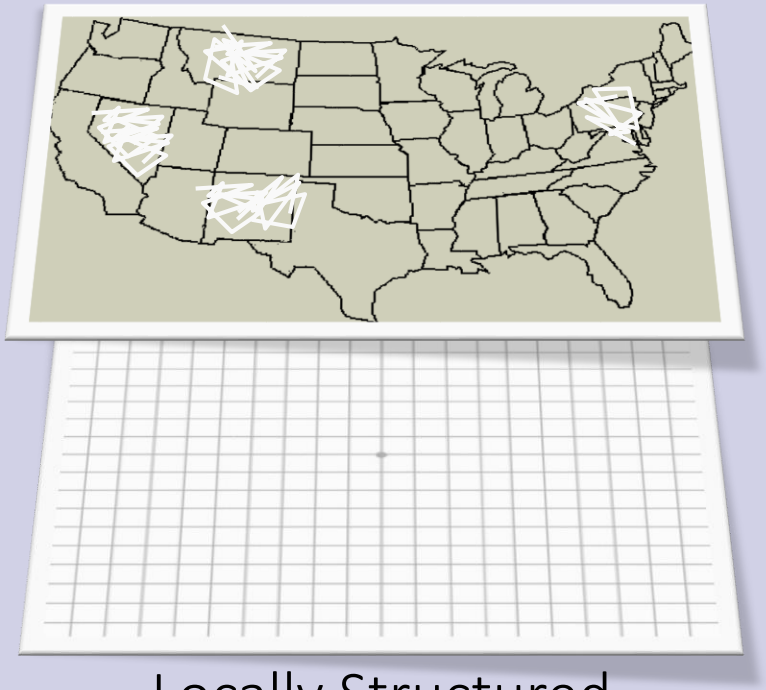


Random



# Clustering vs. Small World-ness

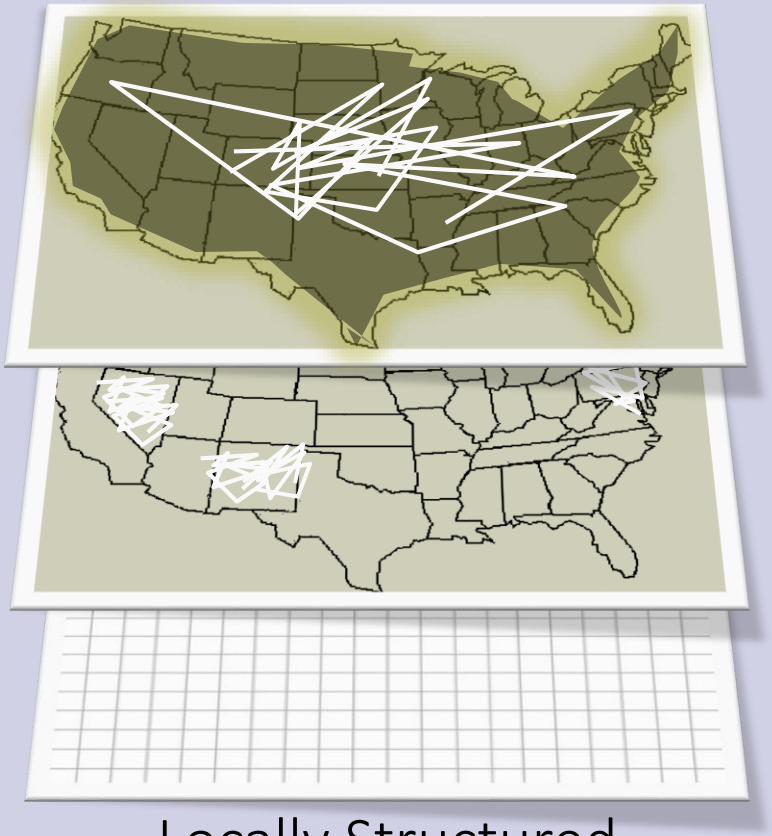
## Clustering Implies Structure



Locally Structured

# Clustering vs. Small World-ness

Randomness kills locality



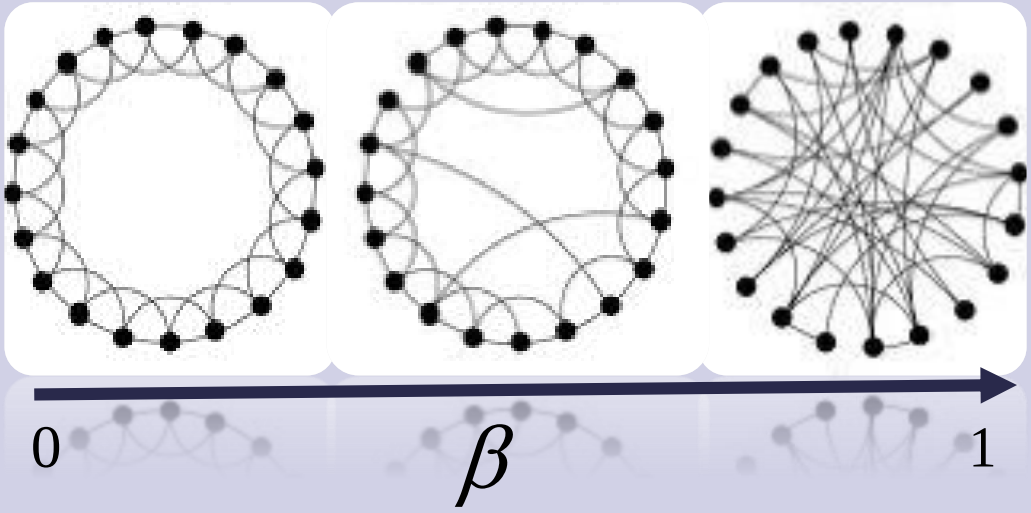
Locally Structured

# Watts Going on with Social Networks

The Watts-Strogatz Model:

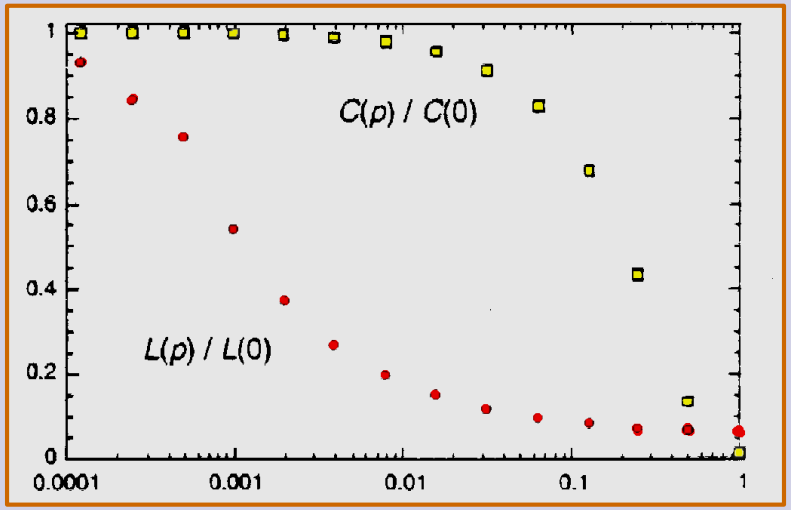
1. Start with a lattice network.

2. For every edge rewire with a probability $\beta$.




Watts and Strogatz, *Nature* **393**,409 (1998)

# Watts Going on with Social Networks




The Watts Strogatz Model: It takes a lot of randomness to ruin the clustering, but a very small amount to overcome locality

Watts and Strogatz, *Nature* **393**,409 (1998)

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# Coexistence of Clustering and Randomness

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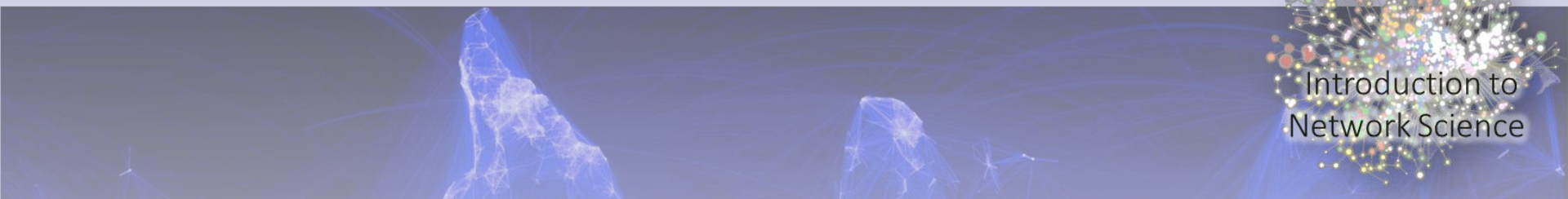
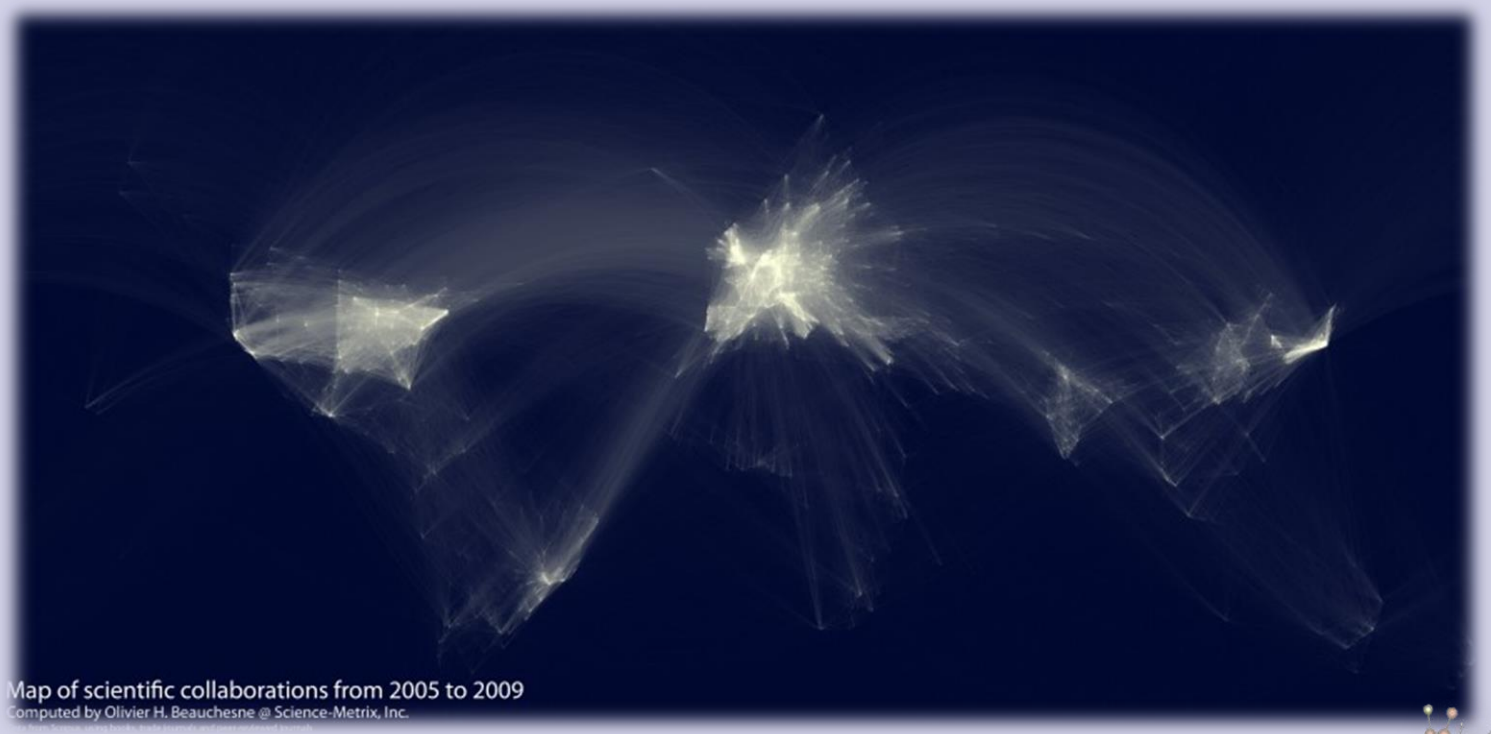
# Coexistence of Clustering and Randomness



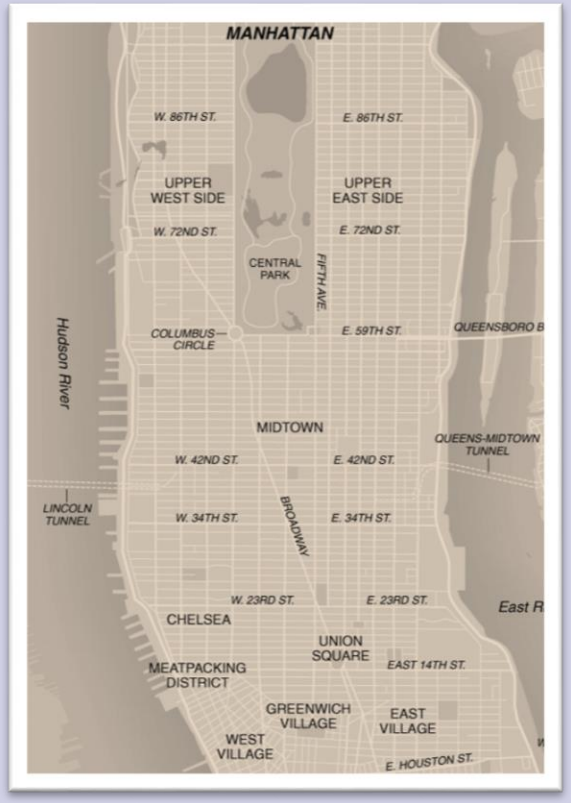
Introduction to
Network Science

# Coexistence of Clustering and Randomness

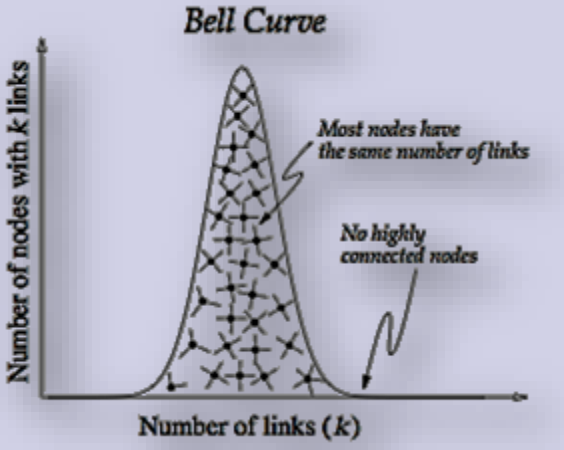


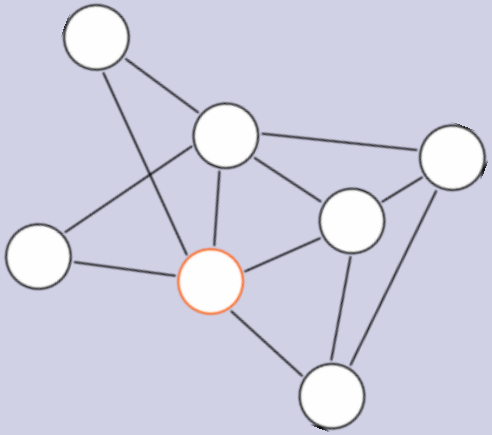
Map of scientific collaborations from 2005 to 2009
Computed by Olivier H. Beauchesne @ Science-Metrix, Inc.

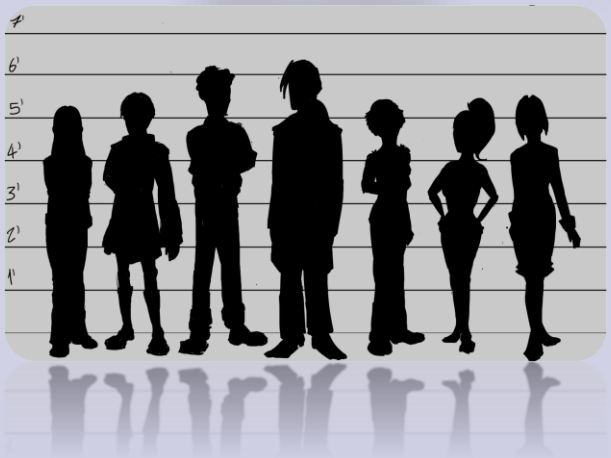
# Revisiting Milgram's Experiment



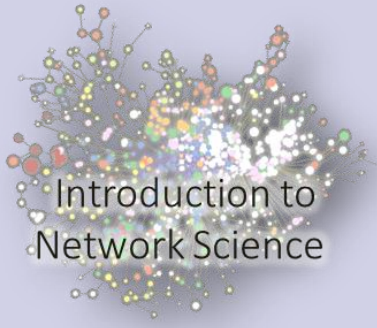

# Networks Continue to Surprise





AL Barabási, R Albert, H Jeong *Physica A* **281**, 69



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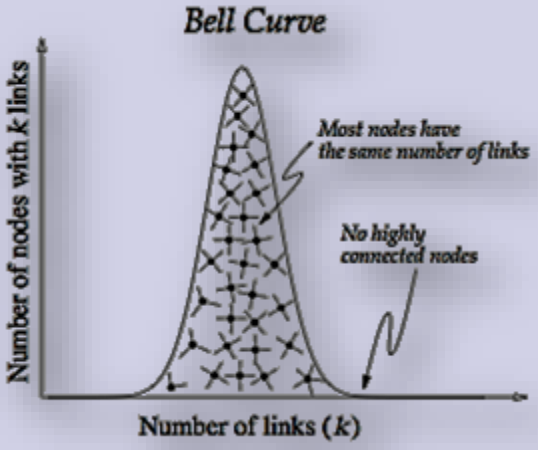


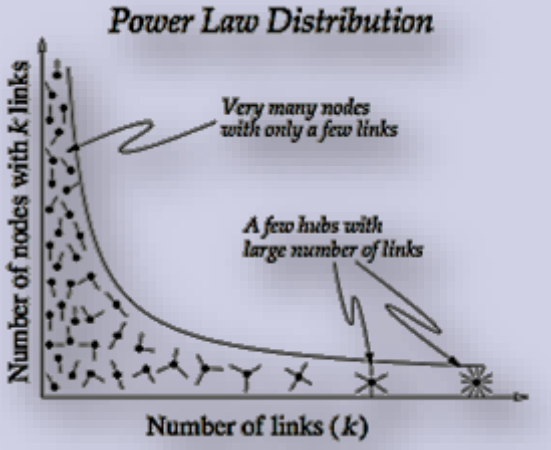


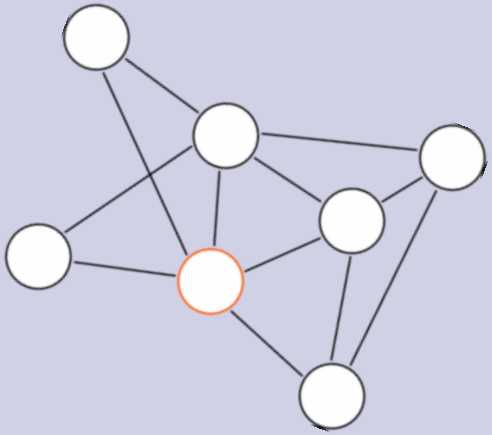

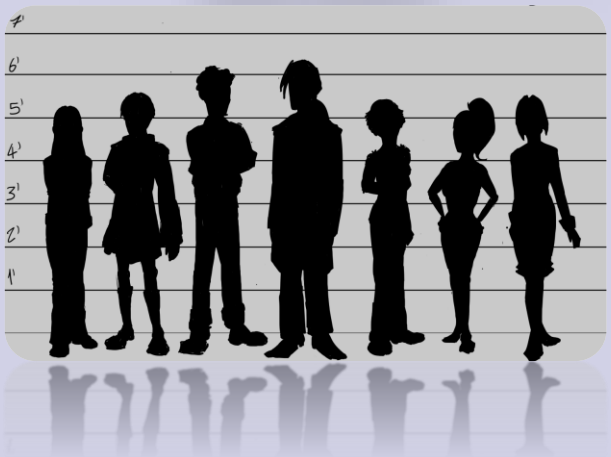


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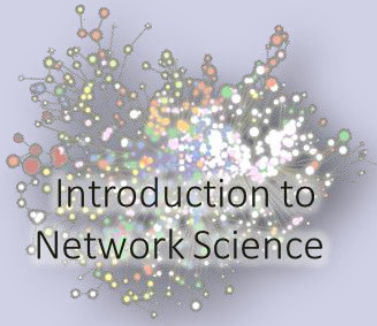

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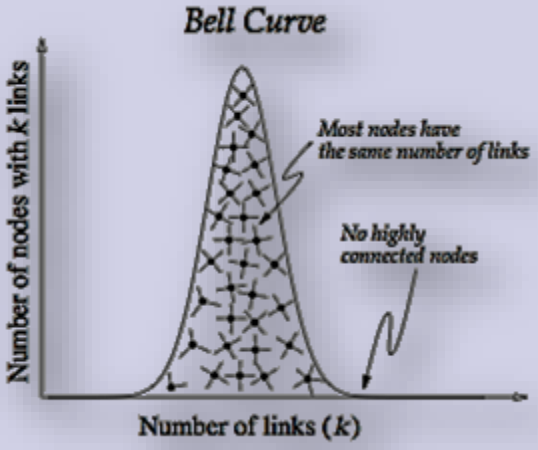


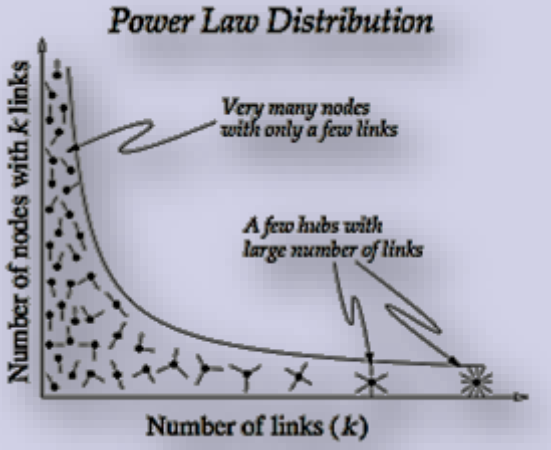


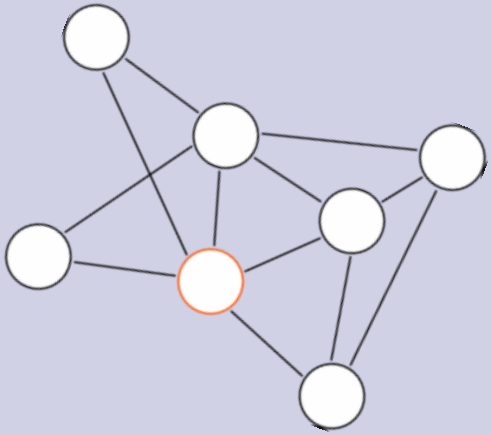

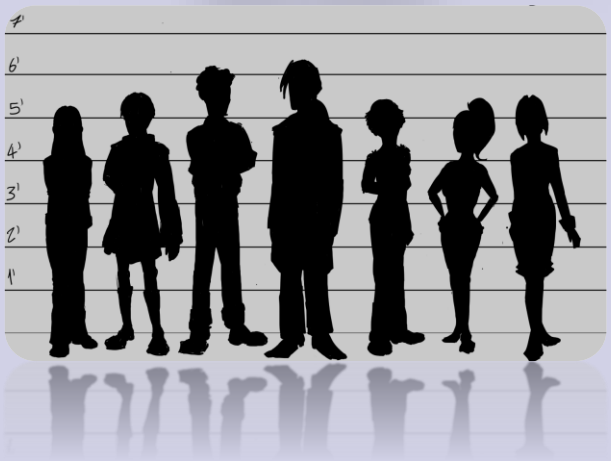


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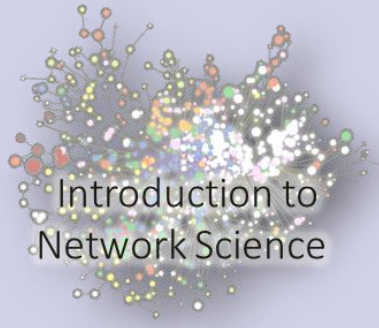

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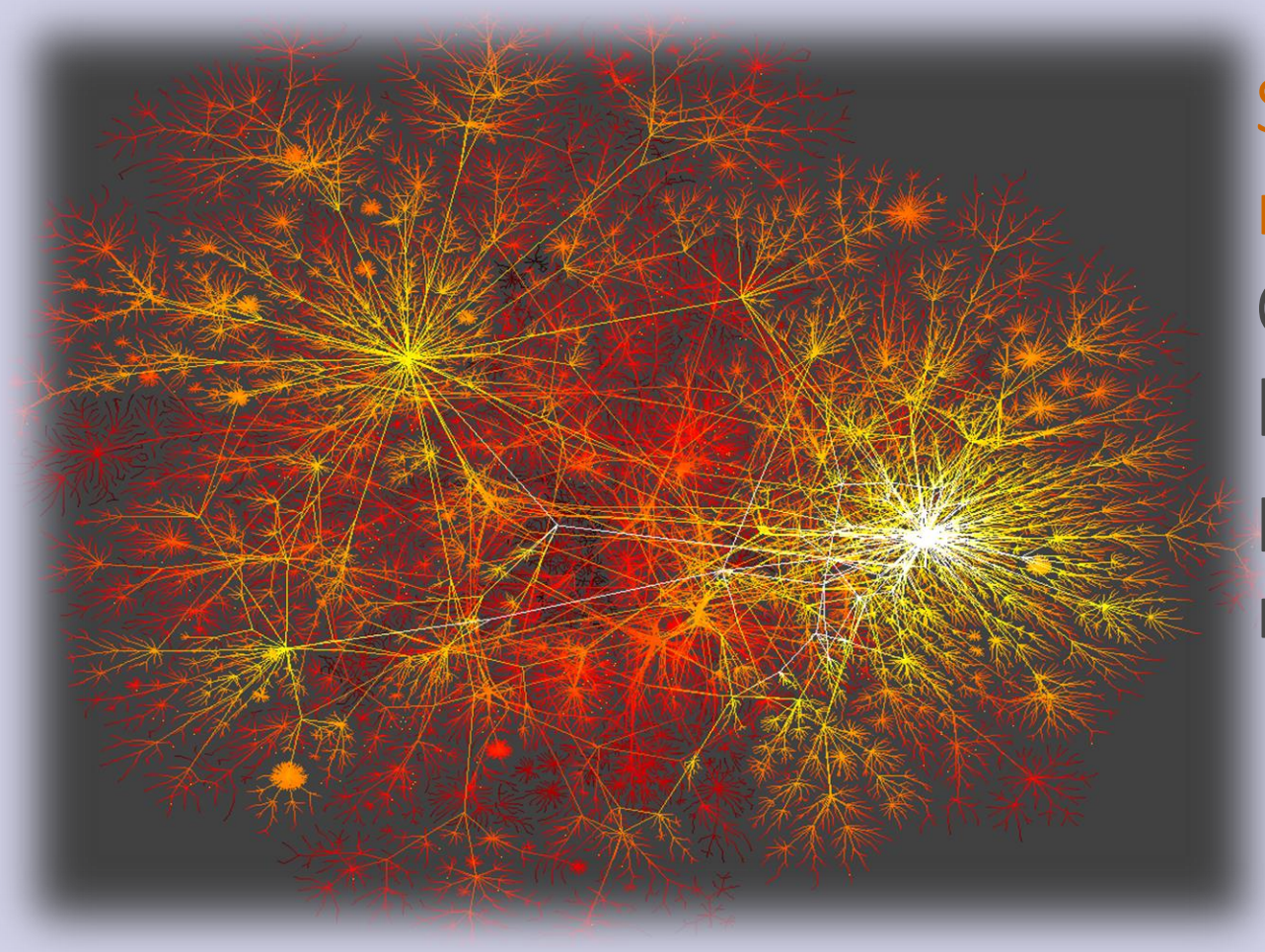
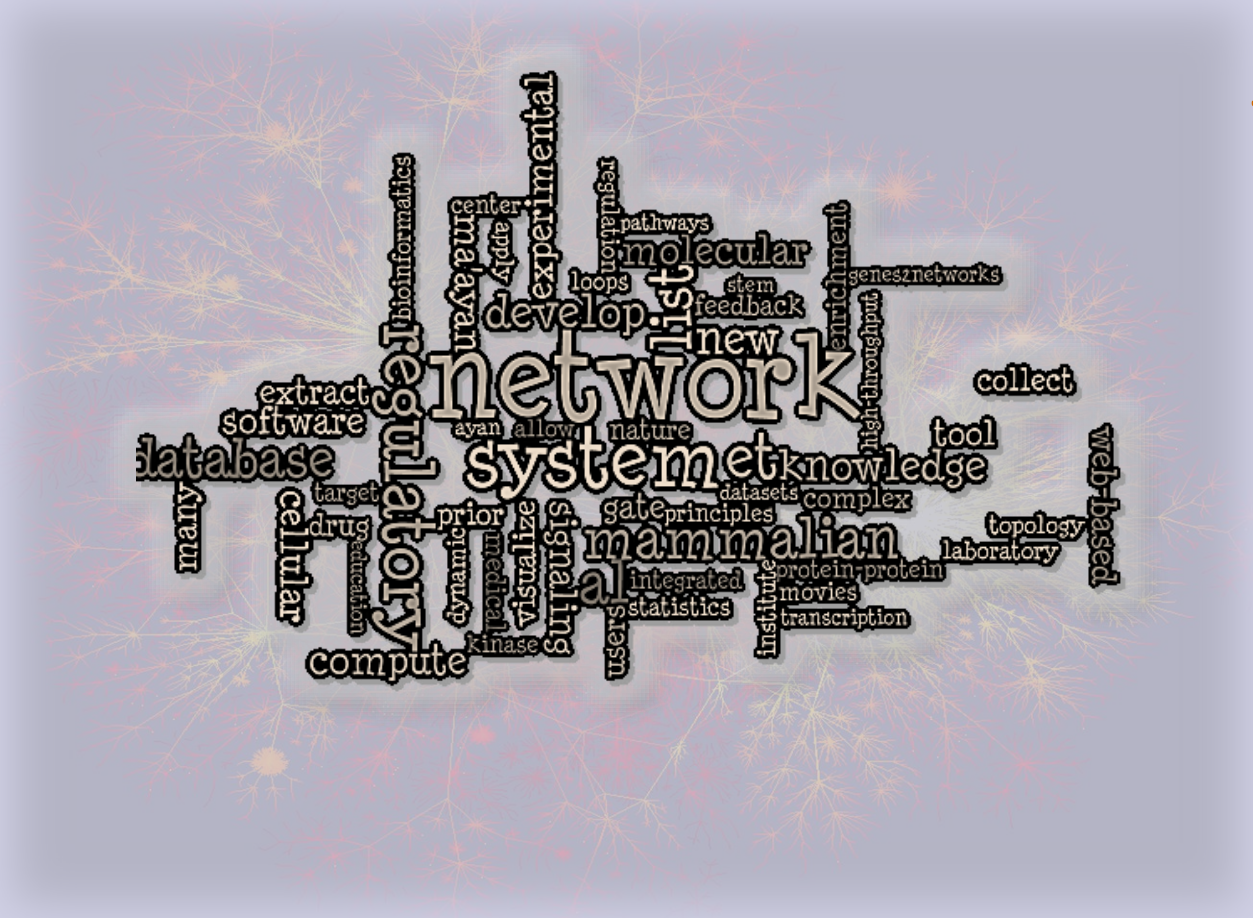


Scale-free networks

Coexistence of hubs and peripheral nodes

# Networks Continue to Surprise

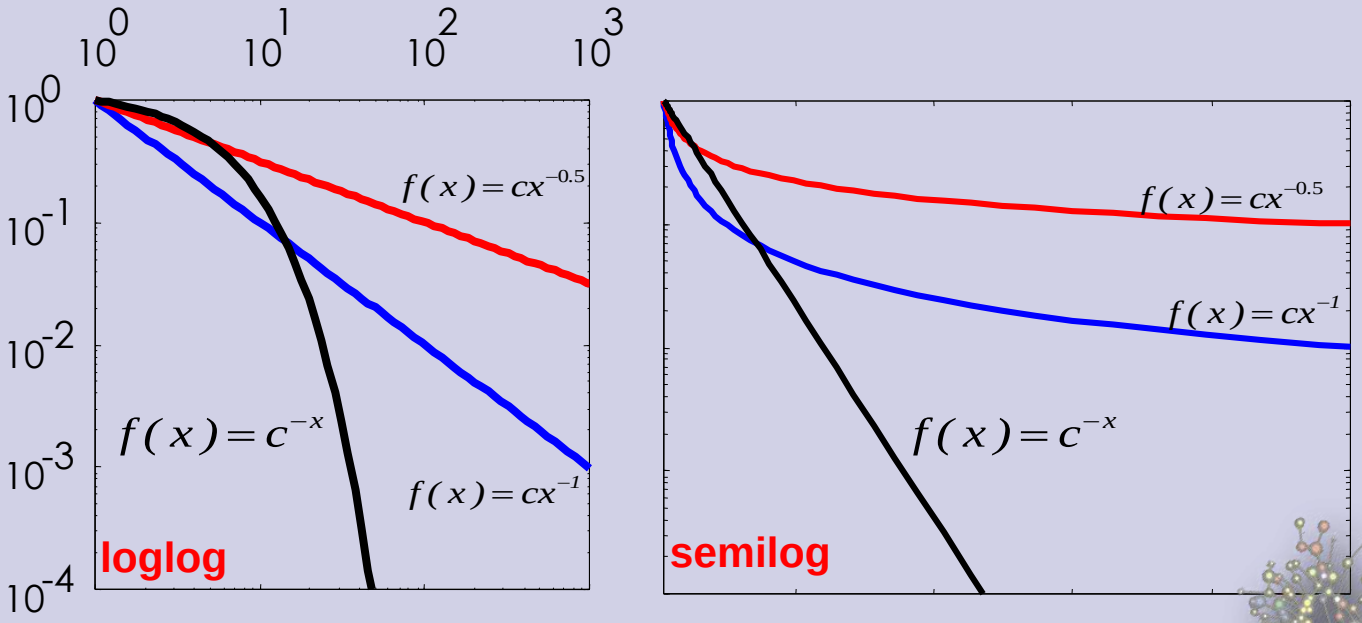



Scale-free networks

Coexistence of hubs and peripheral nodes

# Power-Law vs. Exponential

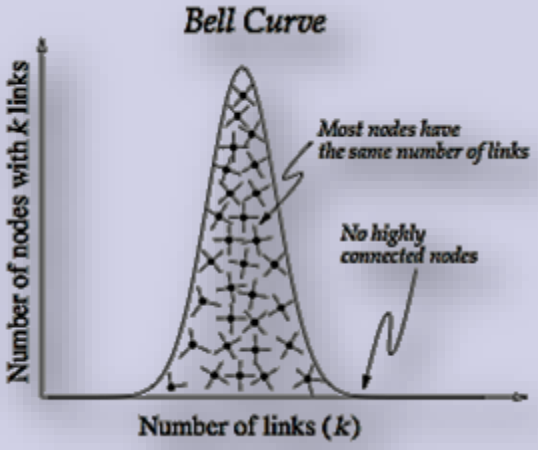
The probability of large *x* is orders of magnitude higher in a power-law than in an exponential

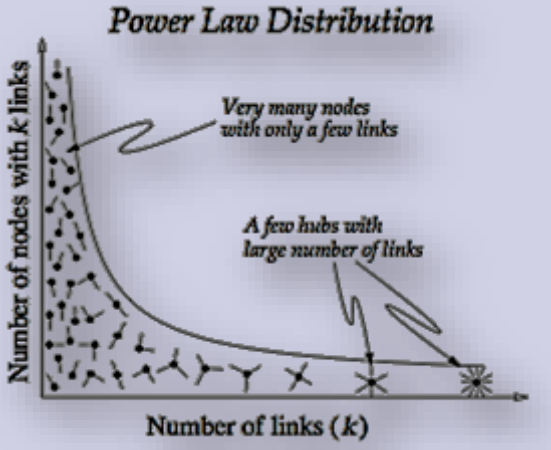


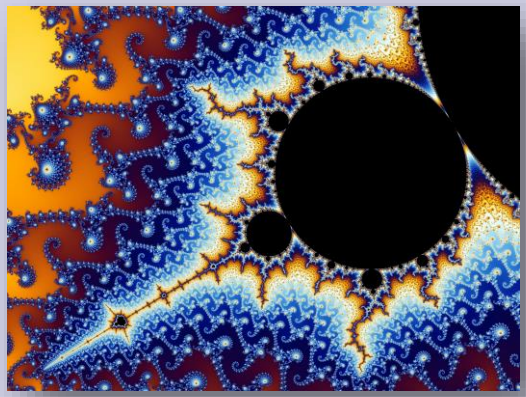

Power-Law $P(x) \sim x^{-\alpha}$

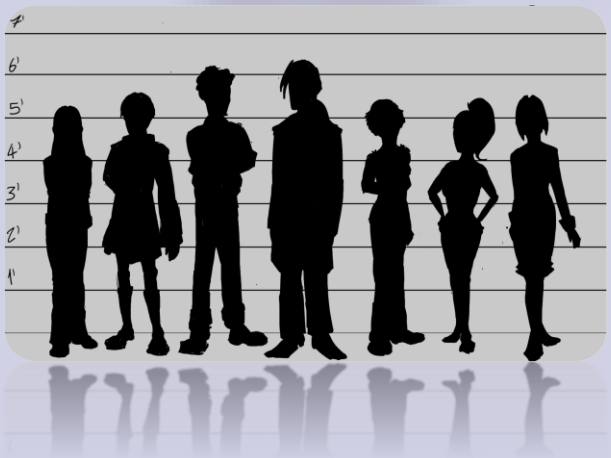
Exponential $P(x) \sim \alpha^{-x}$

# Scale Free

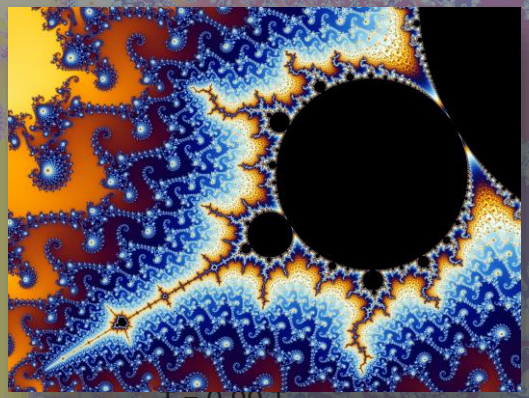

Bell Curve
Number of nodes with k links
Most nodes have the same number of links
No highly connected nodes
Number of links (k)


Power Law Distribution
Number of nodes with k links
Very many nodes with only a few links
A few hubs with large number of links
Number of links (k)



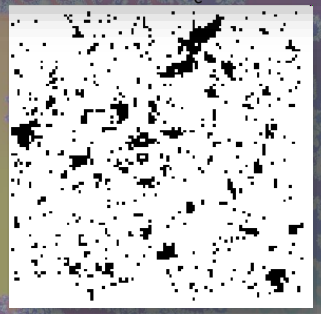
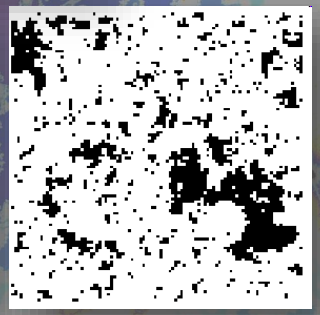
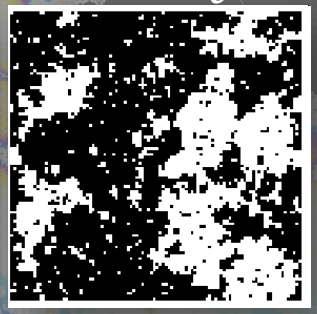
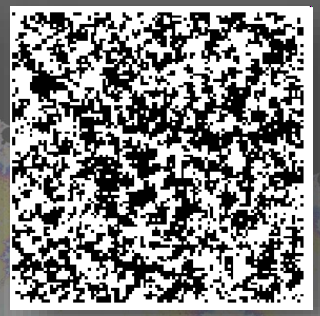

7'
6'
5'
4'
3'
2'
1'

# Scale Free

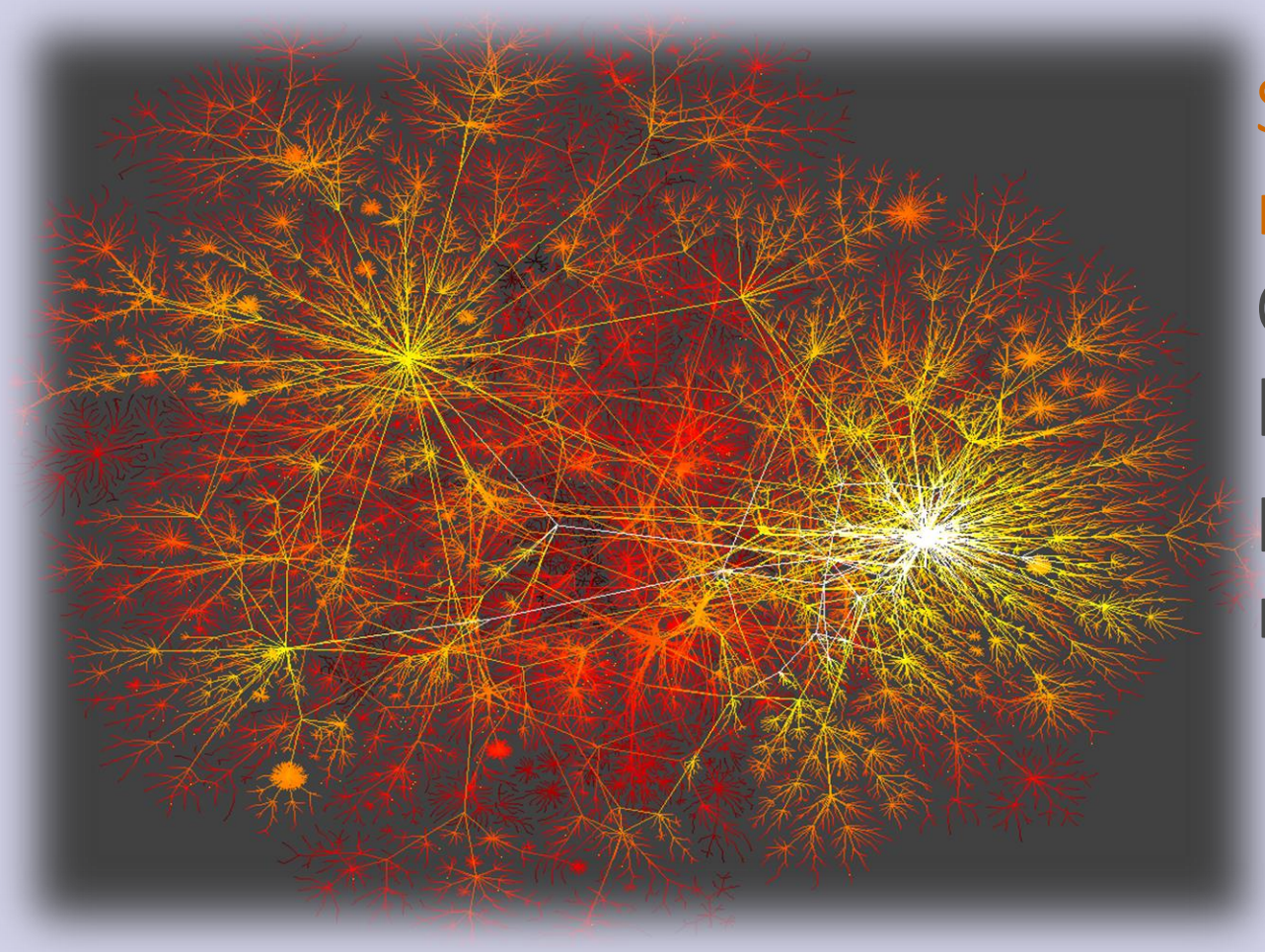


$T = 0.99\ T_c$

$T = T_c$

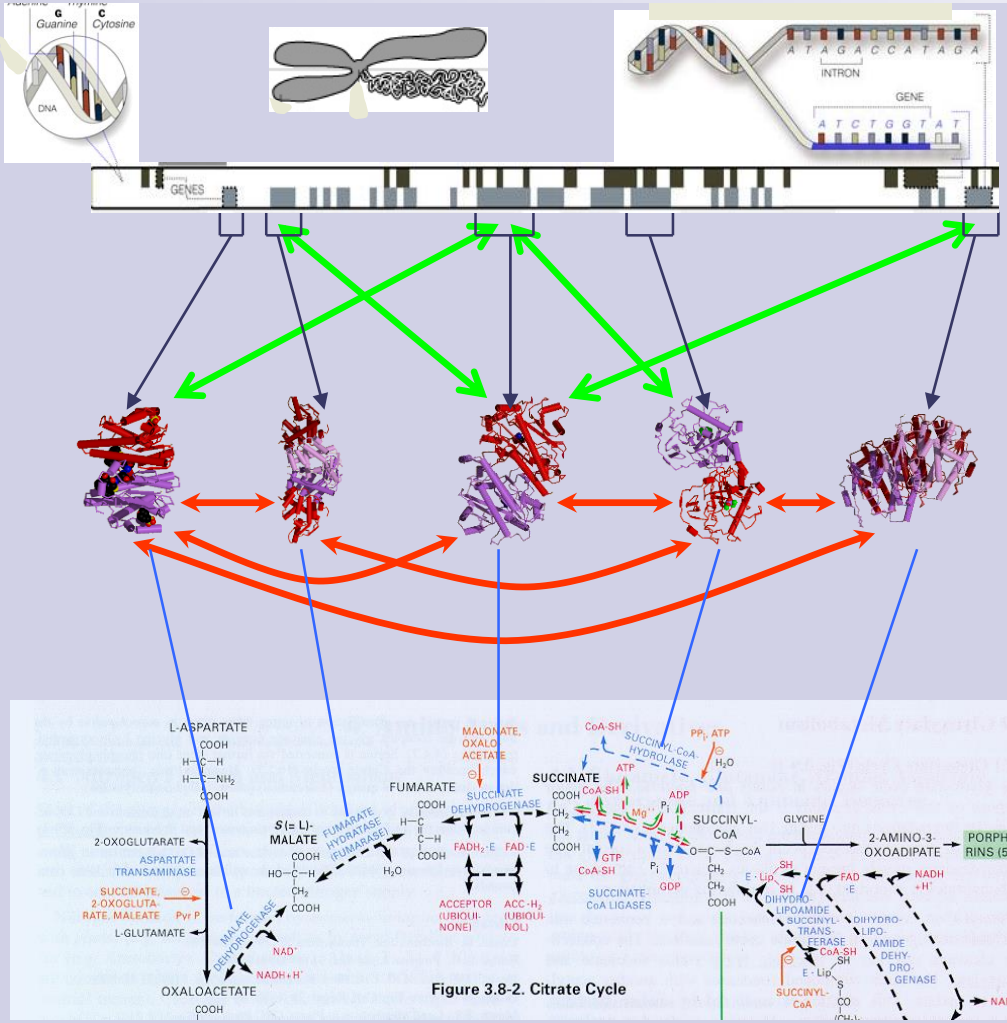

# Networks Without a Typical Scale



Scale-free networks

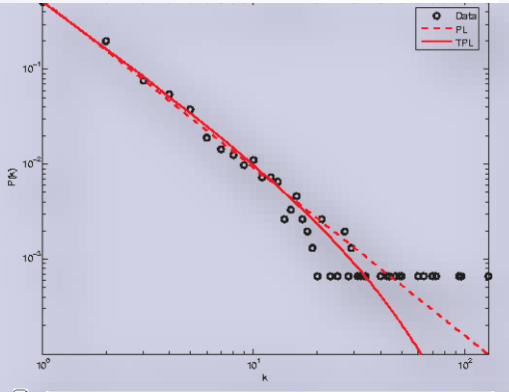
Coexistence of hubs and peripheral nodes

# Universality



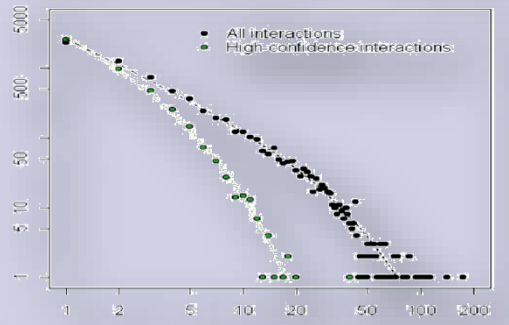

GENOME

protein-gene interactions



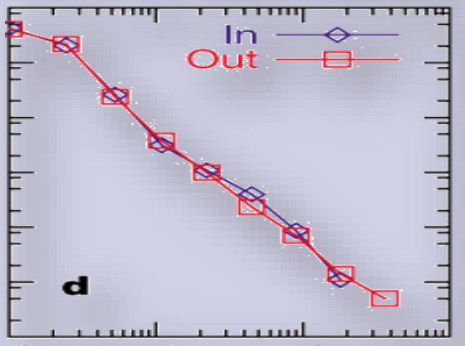

PROTEOME

protein-protein interactions

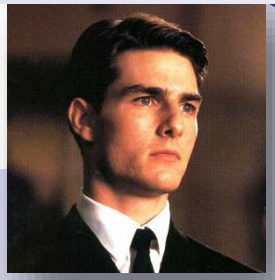



METABOLISM

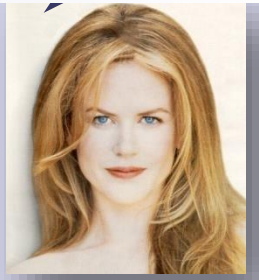
Bio-chemical reactions

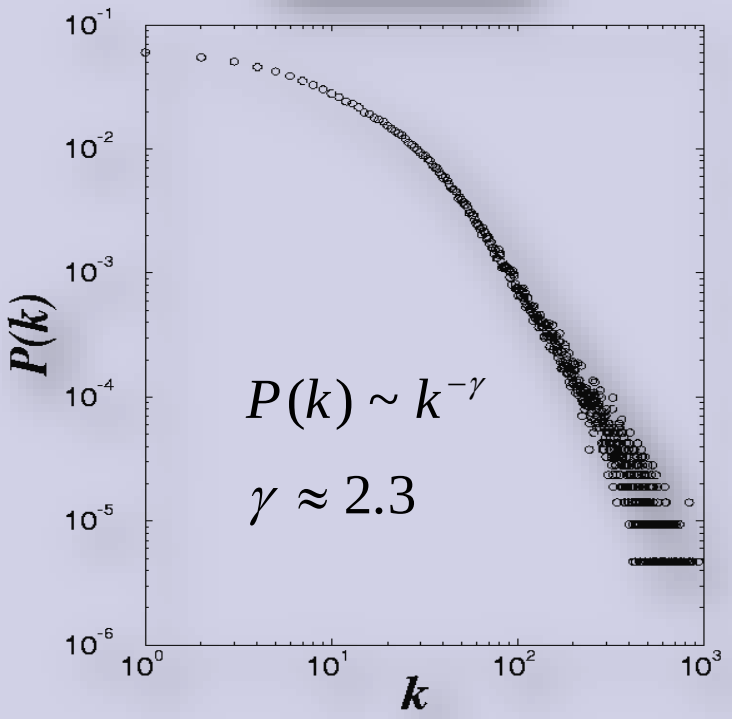


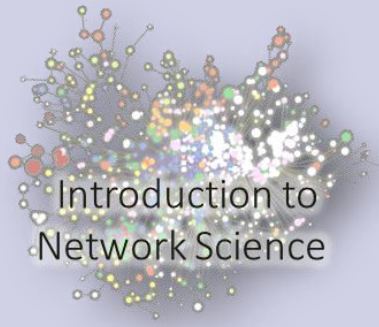
# Universality

Days of Thunder (1990)
Far and Away (1992)
Eyes Wide Shut (1999)

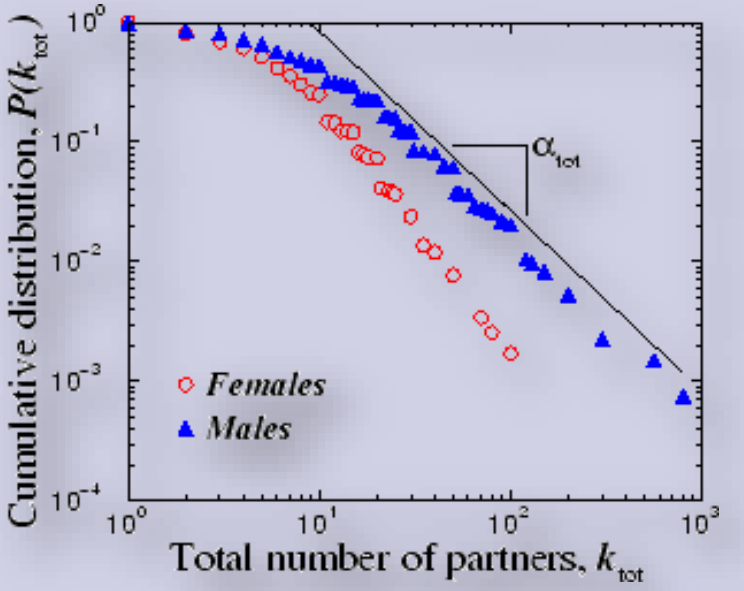




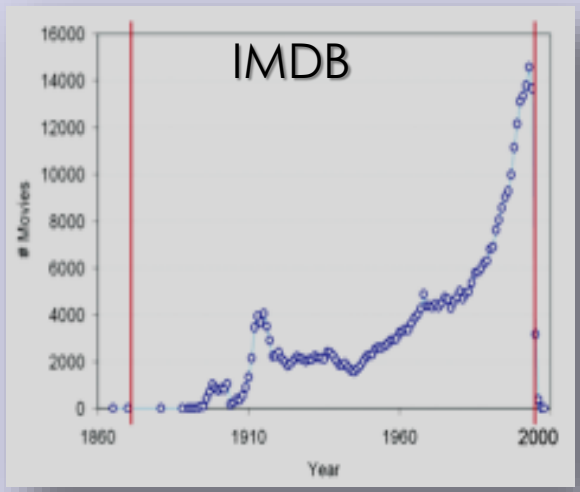
# Universality

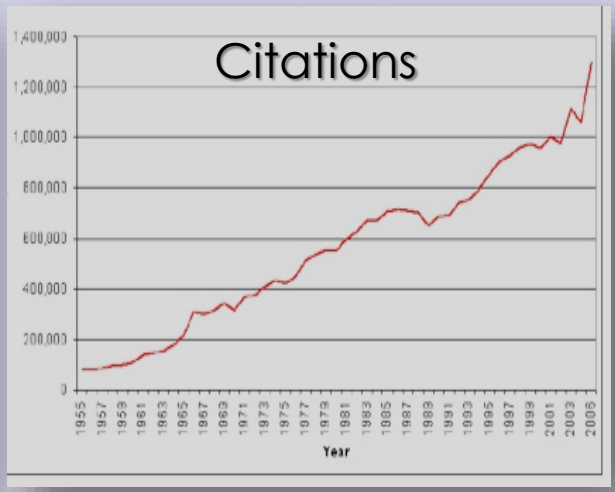
Nodes: people (Females; Males)
Links: sexual relationships

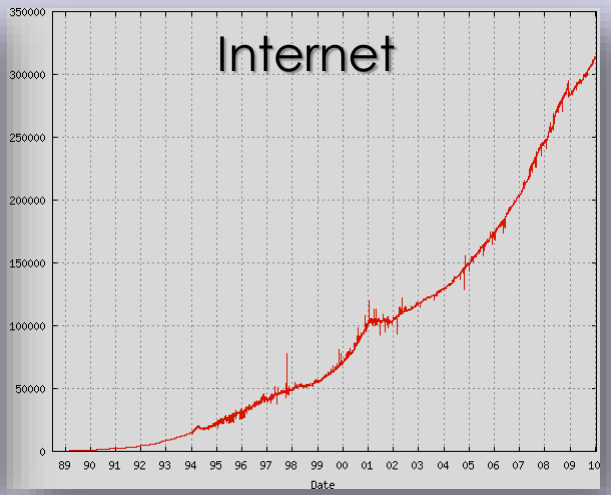



Liljeros et al. Nature 2001

# Barabási-Albert Model



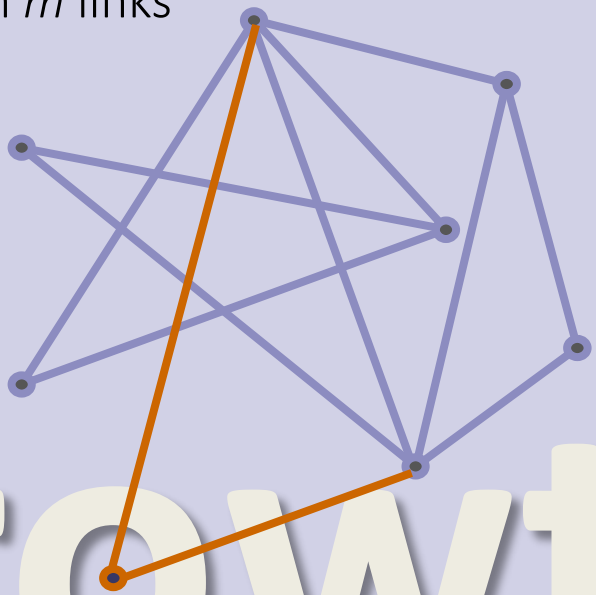

# Dynamics

Barabási & Albert, *Science* **286,** 509 (1999)

# Barabási-Albert Model

Networks continuously expand by the addition of new nodes

Add a new node with *m* links



**Growth**

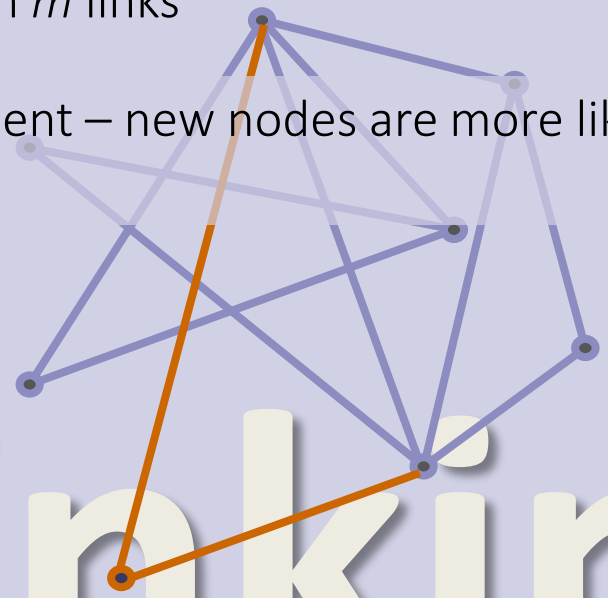
# Barabási-Albert Model

Networks continuously expand by the addition of new nodes

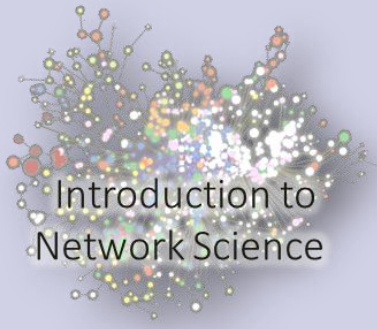
Add a new node with *m* links

$$\Pi(k_i) = \frac{k_i}{\Sigma_j k_j}$$

Preferential attachment – new nodes are more likely to link to well connected nodes

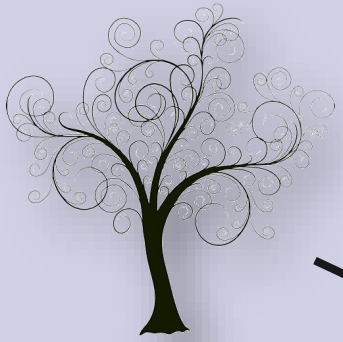


Linking

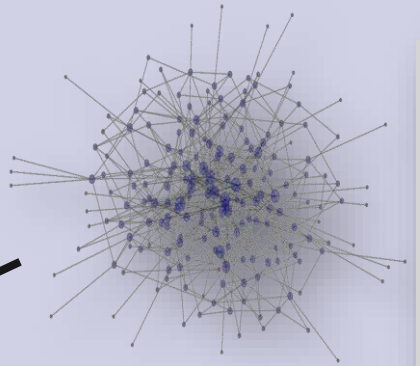


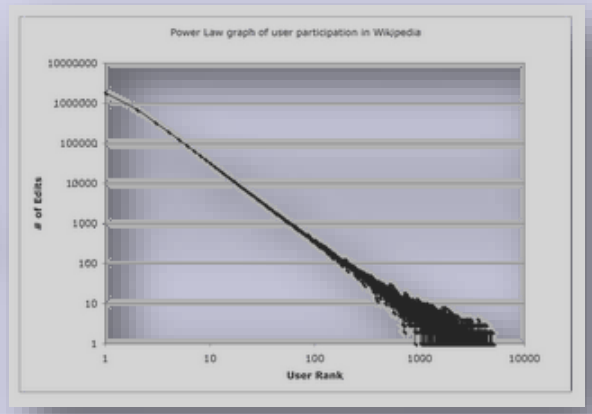
# Rich Get Richer

Growth



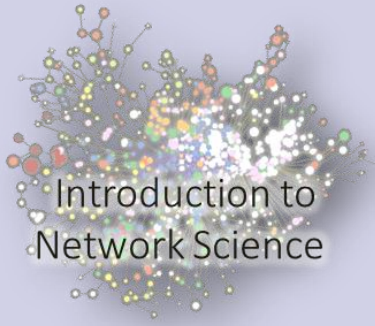
Preferential Attachment





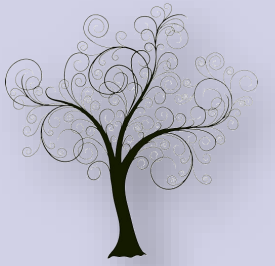

Richer nodes accumulate links more rapidly

Degree Heterogeneity

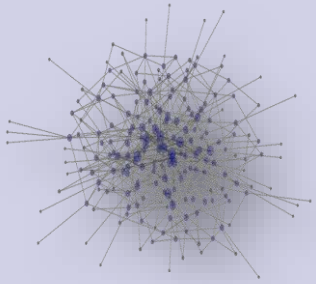


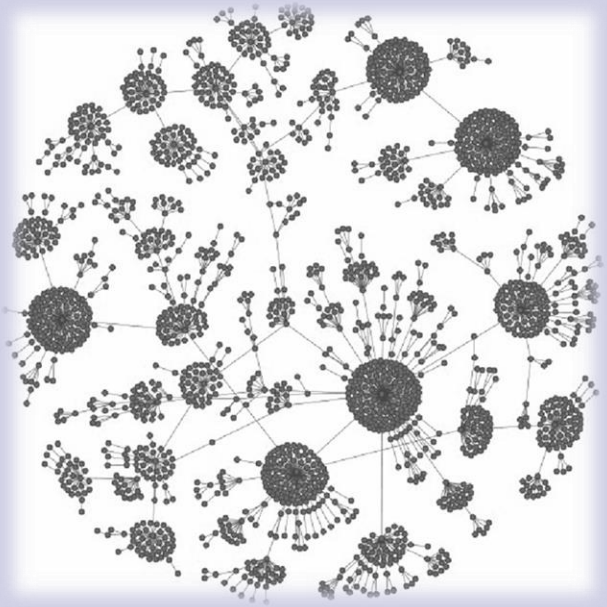
# Dynamic Network Models

Growth

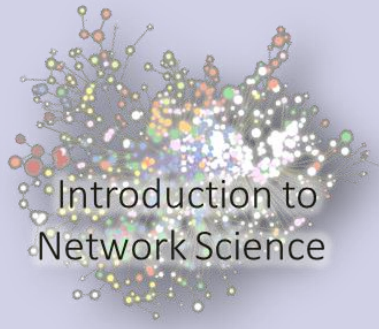


Preferential Attachment





- Non-linear preferential attachment $\Pi(k_i) = \frac{k_i^{\alpha}}{\sum_j k_j^{\alpha}}$
- Fitness model $\Pi(k_i) = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$
- Initial Attractiveness $\Pi(k_i) \sim A + k_i^{\alpha}$
- Models with different exponent *g*



# Properties of Scale Free Networks

$$\langle d \rangle \sim \begin{cases} Const & \gamma = 2 \\ \dfrac{\ln \ln N}{\ln(\gamma - 1)} & 2 < \gamma < 3 \\ \dfrac{\ln N}{\ln \ln N} & \gamma = 3 \\ \ln N & \gamma > 3 \end{cases}$$

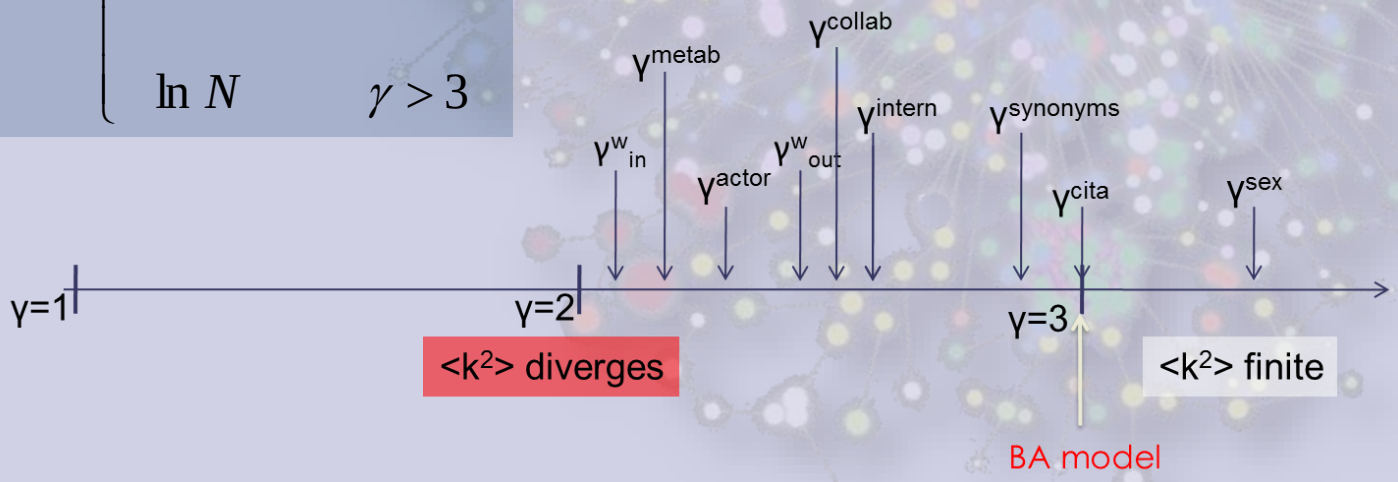
$\gamma < 2$ $\langle k \rangle$ diverges

$2 < \gamma < 3$ $\langle k^2 \rangle$ diverges

$\gamma \geq 3$ Finite Variance

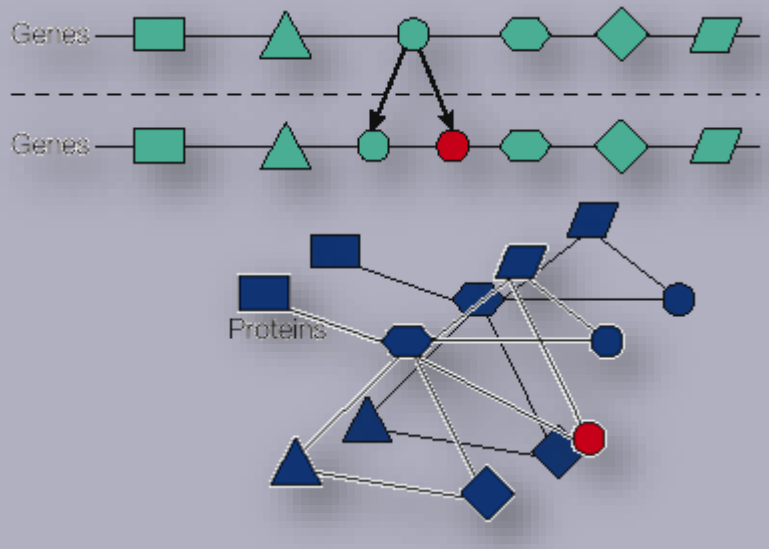
Maximum degree

$$K_{max} = K_{min} N^{\frac{1}{\gamma - 1}}$$



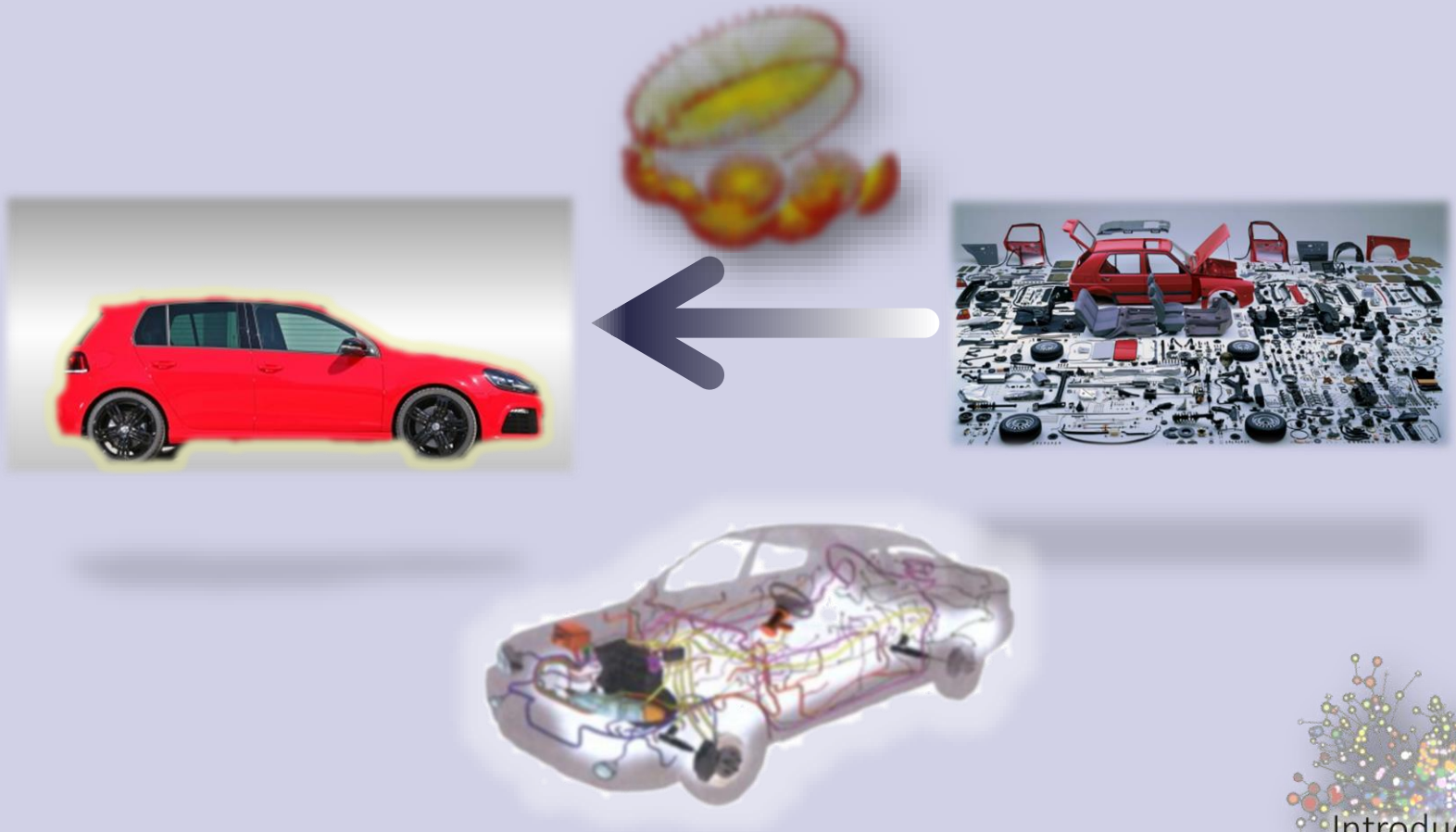
# Mechanisms of Preferential Attachment

1. Copying mechanism
   - directed network
   - select a node and an edge of this node
   - attach to the endpoint of this edge
2. Walking on a network
   - directed network
   - the new node connects to a node
   - then to every first, second… neighbor of this node
3. Attaching to edges
   - select an edge
   - attach to both endpoints of this edge
4. Node duplication
   - duplicate a node with all its edges
   - randomly prune edges of new node




A node with k neighbors is k times as likely to have one of its neighbors duplicated

# From Structure to Dynamics

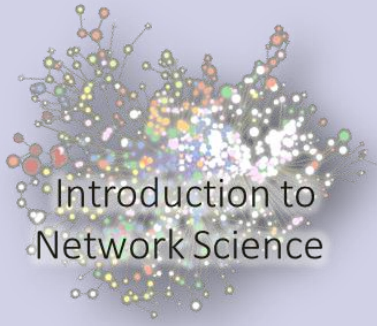


# Patterns of Influence

# Spread of Information

# Spread of Information

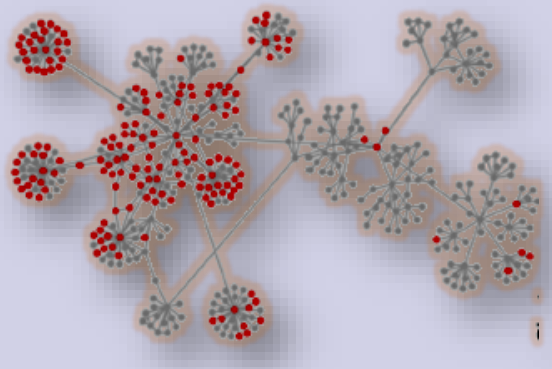
Viruses
Ideas
Memes
Fads



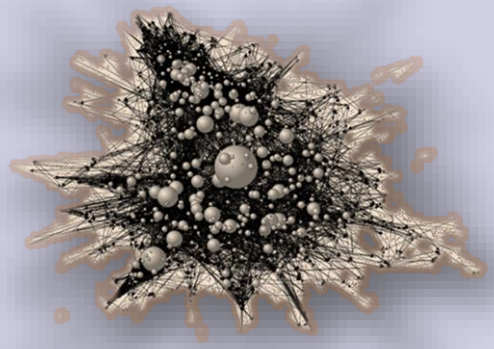
# Spread of Information

Electric signals
Chemical signals

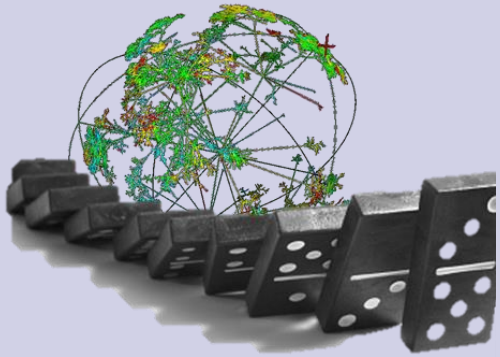
# Translating Topology into Dynamics

How does influence spread in space and time?

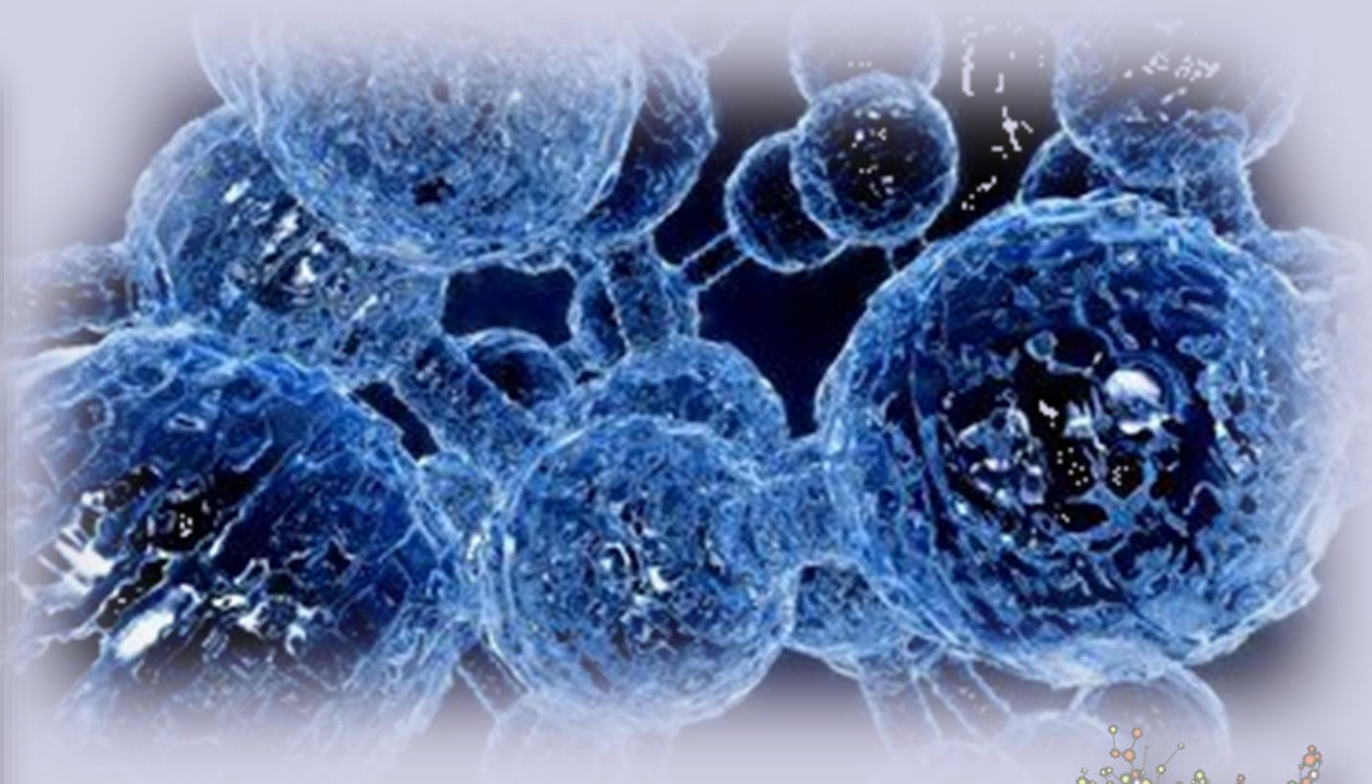


Do the hubs dominate the dynamics?



The Small World generate instability?

# Universality



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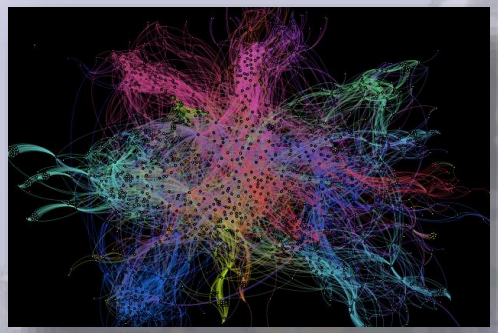

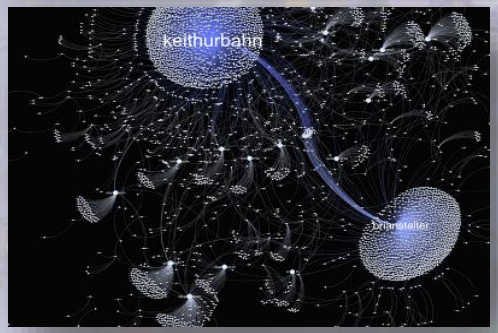


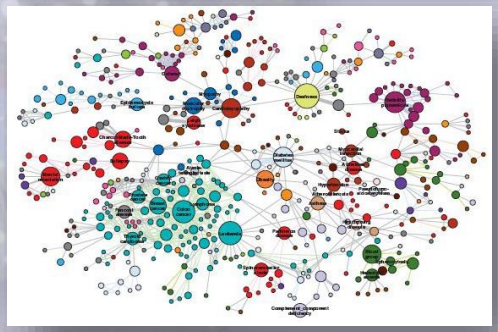

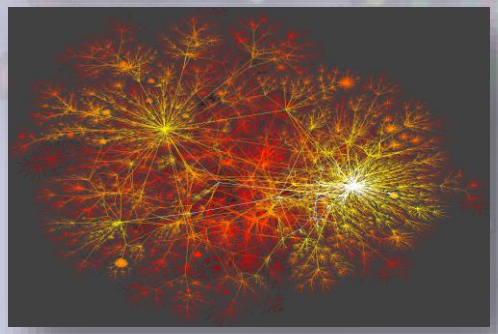

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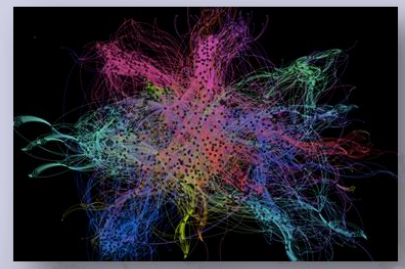


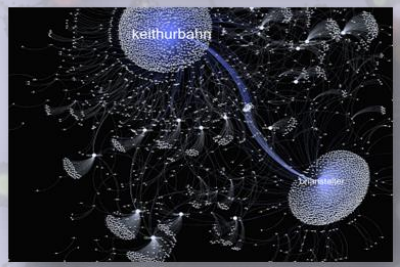

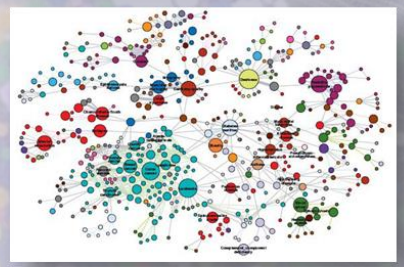

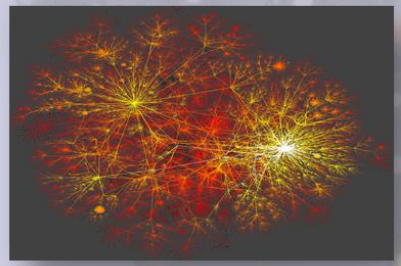

# Introduction to Network Science