Spin glass and quasiperiodicity?

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Institute for Basic Science

- Government research institution
- $\sim$ 40 research centers: math, physics, biology
- equivalent of Max Planck society - approximately
- PCS: Center for Theoretical Physics of Complex Systems
- CTIQS: Center for Trapped Ions Quantum Science

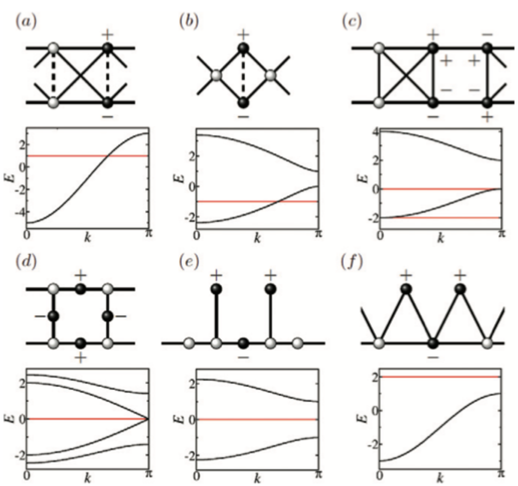
My interests

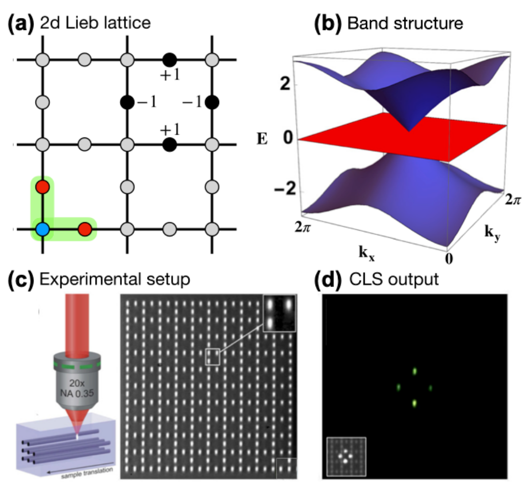
- frustration: geometry and randomness - spin glass/frustrated magnetism, sphere packing
- degeneracy: destructive interference - **flat bands**

- quantum fluctuations - localisation, thermalisation, many-body localisation

- PhD: statistical physics - structural glasses

- transitioned to condensed matter
- but kept interested in statistical mechanics





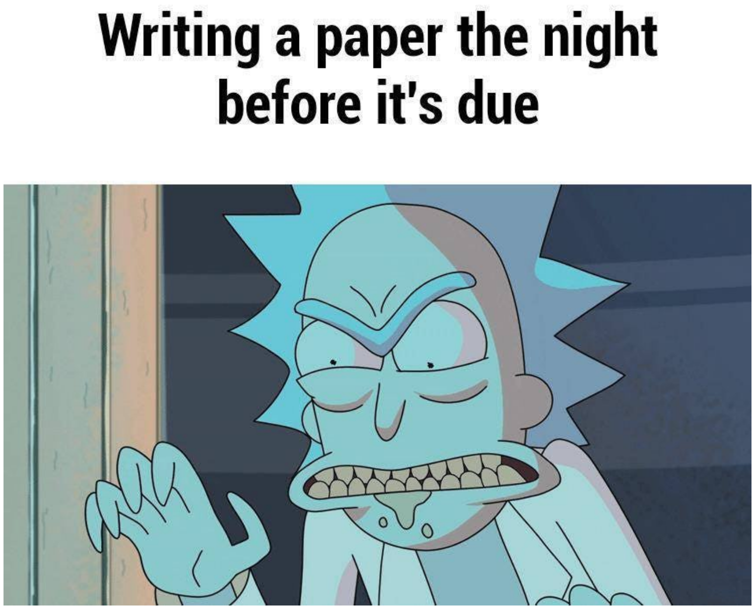
Flat bands

1. What are *perfect* flatbands and why are they interesting?
2. How do we find them *systematically*?
    - various settings, e.g. 1D, 2D, 3D
    - geometries -- d.c. fields, etc?
    - Floquet?
    - many-body?
3. What happens in presence of perturbations?
    - disorder
    - nonlinearity
    - interactions
    - dissipation?

People involved

- **Alexander Hartmann**, University of Oldenburg, Germany

Frustration



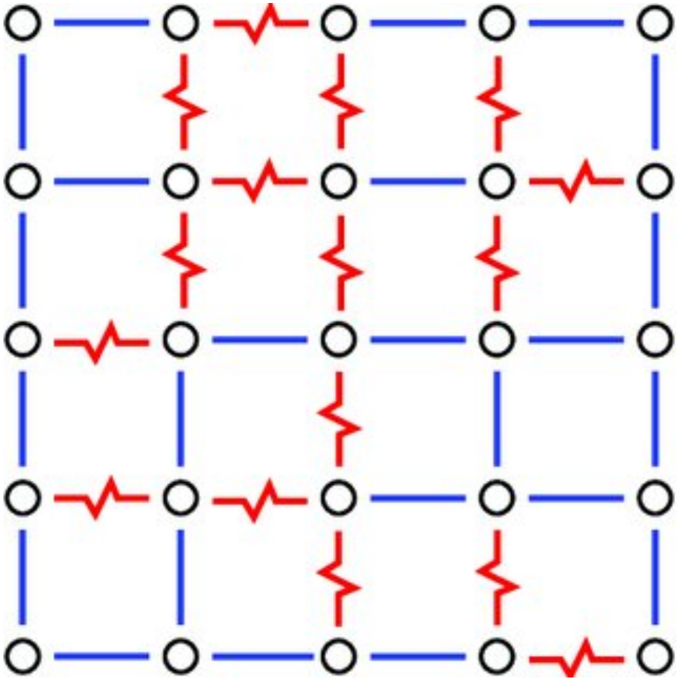
Not that one!

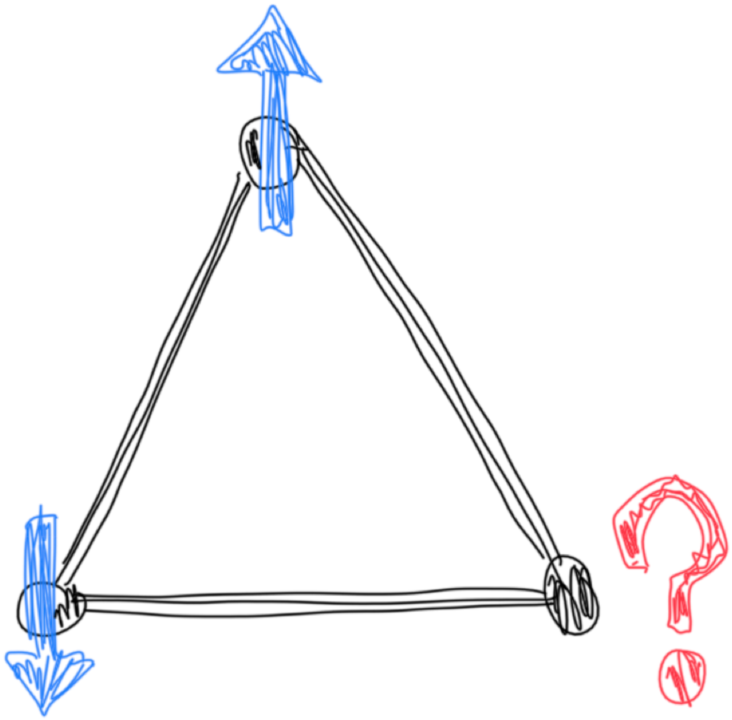
Frustration

Definition: **Inability to satisfy individual interactions simultaneously**

Origin:

- *competing interactions*: mix of FM and AF couplings
- *geometrical frustration*: geometry of the model

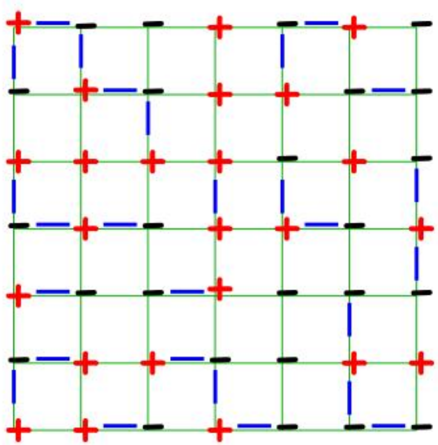




What is a spin glass?

Prototypical complex system

- Spins + RKKY interaction (long-range)
- Edwards-Anderson, 1975: $\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} s_i s_j$
- Sherrington-Kirkpatrick, 1975: $\mathcal{H} = -\frac{1}{\sqrt{N}} \sum_{i \leq j} J_{ij} s_i s_j$
- High-T: paramagnet
- Low-T: broken ergodicity, order -- spin glass

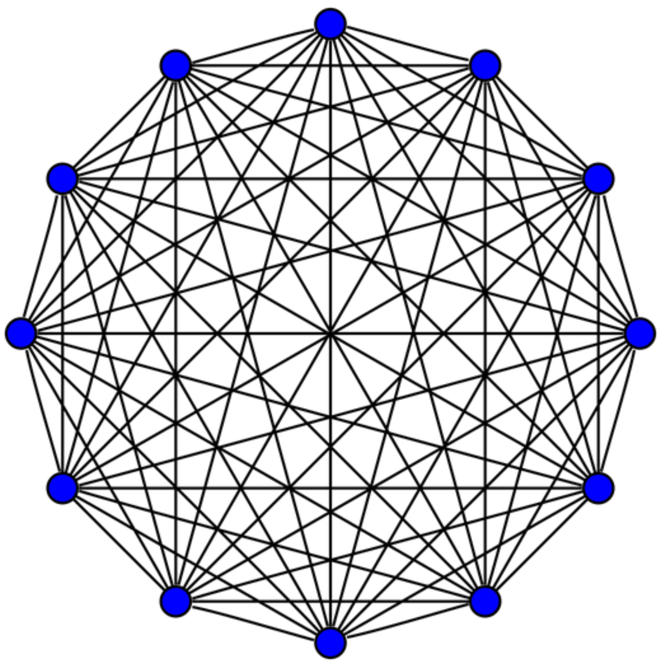


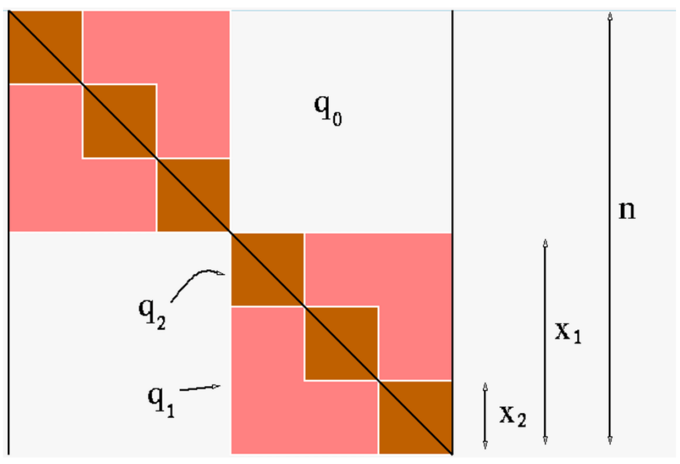
$$E(\sigma) = -\sum_{\langle ij \rangle} J_{ij} \sigma_i \sigma_j$$

A random interacting Ising model - two types of random, but fixed coupling constants (ferro $J_{ij} > 0$) and (anti-ferro $J_{ij} < 0$)

Mean-field

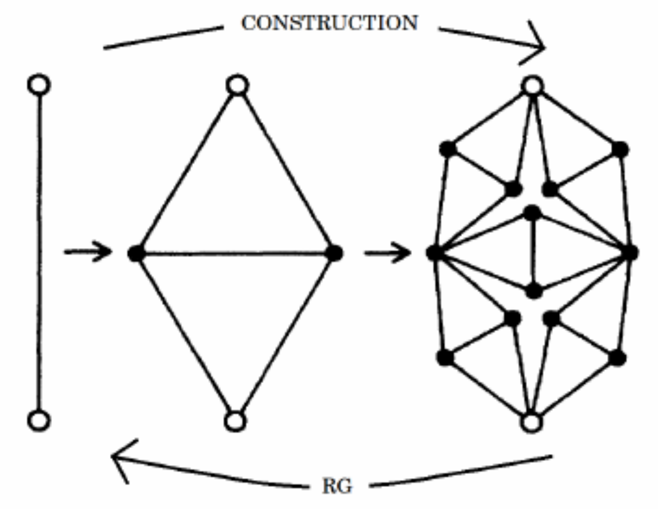
- Mean-field:
  - high-d geometry: all-to-all, random and hierarchical/expander graphs
  - Complex low-T phase
- Replica symmetry breaking (Parisi, 1979)
- Proof for SK model: Guerra-Talagrand, 2003

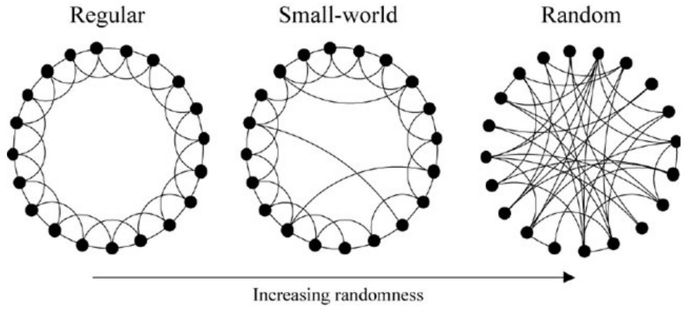




Combinatorial optimisation

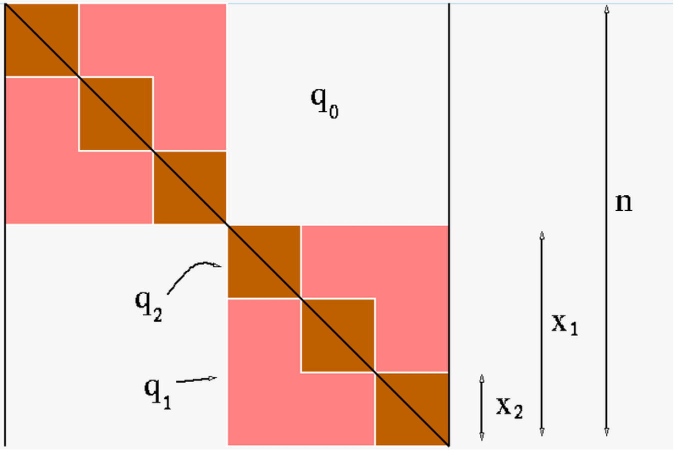
- Random (tree-like) graphs: Migdal-Cadanoff, random regular, small world
- Problems: XORSAT, k-SAT, travelling salesman, …
- Cavity method
- Message passing and belief propagation
- LDPC - proved RSB? (Placke et al, 2025)
- no (easy) extension of Parisi solution!





Finite dimensions

- Open problem:
  - replicas: (Parisi, 1979) - RSB
  - droplets (Fisher and Huse, 1987)
- RG: (De Dominicis and Giardina, 2000)
  - based on replicas
  - mean-field fixed point down, $d \geq 8$
  - no non-perturbative fixed point, $d < 6$?
- rigorous results: Read, Stein, ...

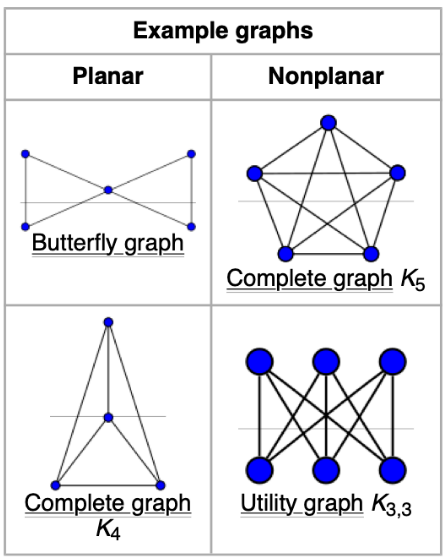


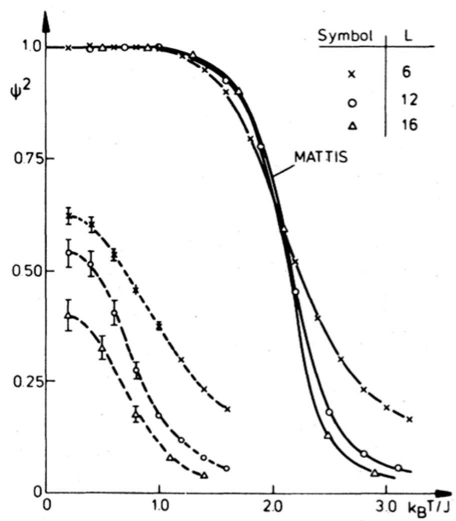
Numerics

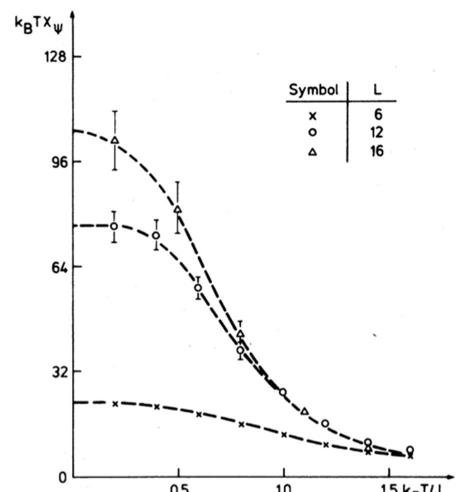
- 2D:
  - computationally feasible: polynomial time algorithms (Onsager solution)
  - uncorrelated disorder: spin glass only at $T = 0$ (Read, 2000)
  - correlated disorder: fragmentation into domains (Muenster, 2021)
- 3D: Spin glass GS -- NP-hard problem (Barahona, 1982)
- Simulations:
  - classical system: Monte-Carlo?
  - rough energy landscape with many local minima
  - Metropolis/single spin updates: inefficient
  - parallel tempering: limited system sizes (especially in 3D) + averaging
  - dedicated supercomputer: Janus collaboration - Baity-Jesi et al (2013-2014)
  - tensor networks (Chen et al, 2025)?

2D Ising models

- Onsager solution: any planar interaction
- Bond disorder: disordered fermions
- But: local fields break planarity
- Partition function $Z(\beta) = \det(M(\beta))$: efficient polynomial sampling (Creighton, 2009)
- Uncorrelated Gaussian disorder: No $T > 0$ glass (Morgenstern, 1980; Read, 2000)
- Uncorrelated binary disorder $\pm J$: marginal case? (Hartmann, 2001)

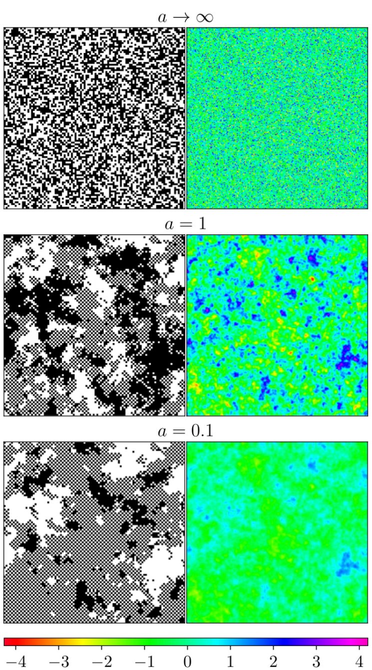


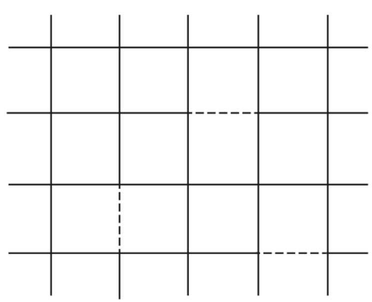
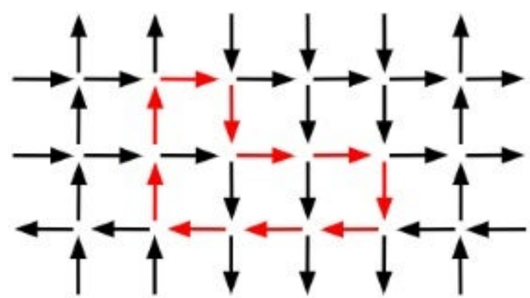




Correlated planar Ising models

- uncorrelated disorder: $T = 0$ spin-glass
- correlated disorder?: **How**?
  - direct correlations $\langle J_x J_y \rangle = f(x, y)$
- uncorrelated disorder, but correlated spin configurations:
  - $\langle J_x J_y \rangle = J^2 \delta_{xy}$
  - a subset of all 2^N spin configurations



Nishimori models

Nishimori, 2024:

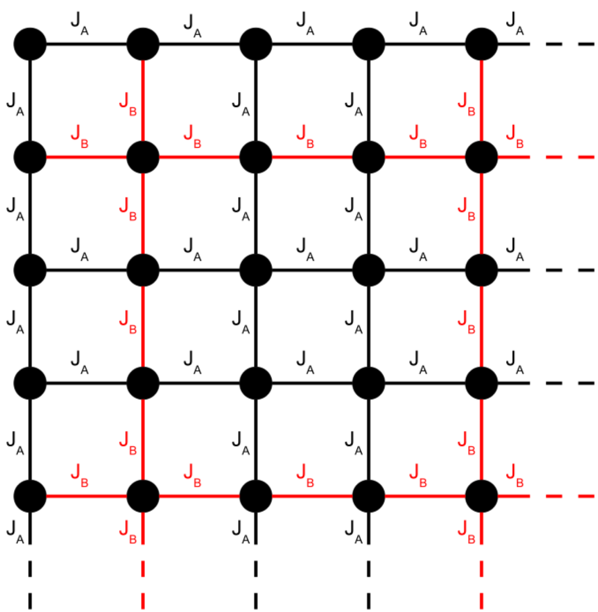
The effect of correlations in disorder variables is a largely unexplored topic in spin glass theory. We study this problem through a specific example of correlated disorder introduced in the Ising spin glass model. We prove

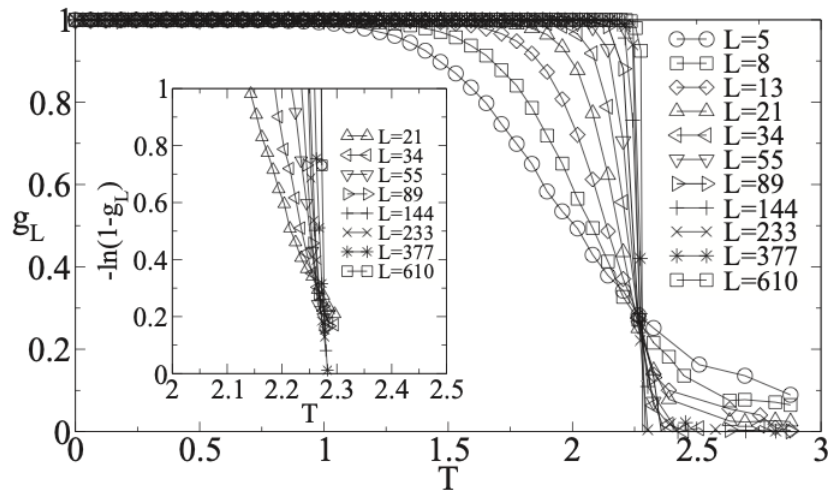
- Model: $\mathcal{H} = -J \sum_{x,y} \tau_{x,y} s_x s_y$
- Correlated interaction: $P(\tau) = \frac{1}{A} \frac{\exp(\beta_p \sum_{xy} \tau_{xy})}{Z_\tau(\beta_p)}$
- $Z_\tau(\beta_p) = \mathrm{Tr}_\sigma \exp(\beta_p \sum_{xy} \tau_{xy} \sigma_x \sigma_y)$
- Analytical results on the Nishimori line (Nishimori, 2024)

Fibonacci glass?

Alves, 2016:

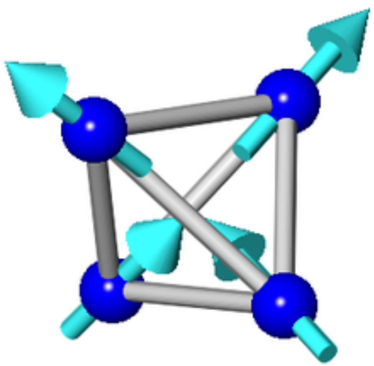
- square lattice, 2D Fibonacci potential
- $T_c > 0$ claimed
- Metropolis Monte-Carlo: single flip and parallel tempering
- **No**: ferrimagnetic order

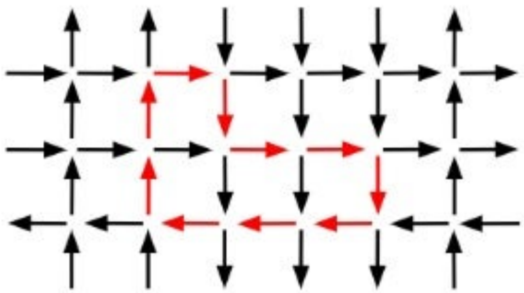


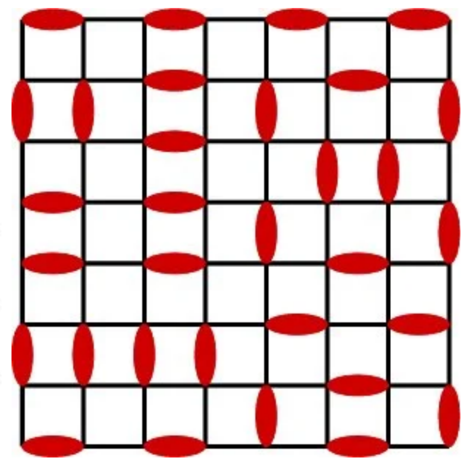


Spin-ice and dimers

- correlated spin configurations:
    - spin ice
    - dimer coverings
- random dimer model (Caracciolo, 2021)
- disordered spin ice (Ludovic Jaubert (U Bordeaux), ongoing)

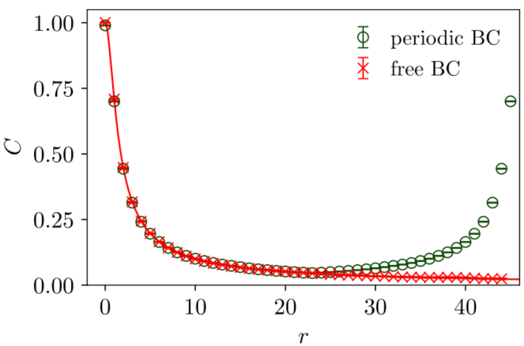


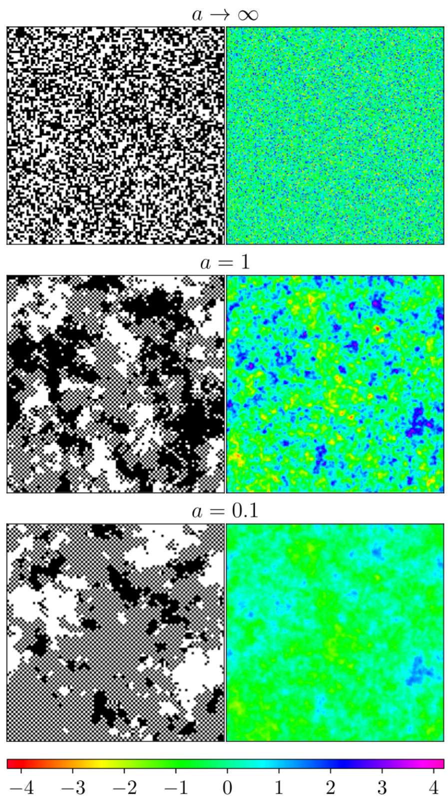


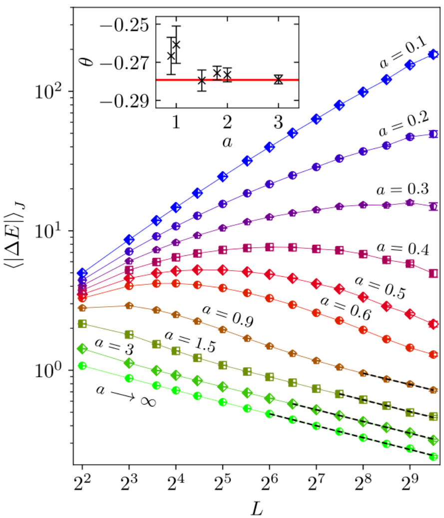


2D glass - Gaussian correlations

- Model: $\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} s_i s_j$ (Muenster, 2021)
- Gaussian covariance: $\langle J(\mathbf{x}) J(\mathbf{x}+\mathbf{r}) \rangle = (1+r^2)^{-a/2}$







Other correlations?

Ingredients required for a putative $T_c > 0$ glass phase:

- Strong correlations?
- FM/AF alternation on short scales?

Quasiperiodic patterns:

- not regular
- strongly correlated
- have the short scale FM/AF alternation

Examples:

- Aubry-Andre model of localisation
- Fibonacci chains

Quasiperiodic glass? quantum models (Chandran, 2017)

2D Aubry-Andre-Ising

Model (inspired by 2D Aubry-Andre model - Devakul and Huse, 2017):

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} s_i s_j$$

$$J_{ij} = \cos(2\pi \mathbf{n}_1 \cdot \mathbf{r}_{ij} + \phi_1) + \cos(2\pi \mathbf{n}_2 \cdot \mathbf{r}_{ij} + \phi_2)$$

- triangular lattice $N = L \times L$: increase frustration
- open b.c.: quasiperiodicity
- random unit vectors $\mathbf{n}_{1,2}$
- 2 angles: suppress periodicity in the transverse direction
- $N \to \infty, N_{\text{angle}} \to \infty$: Gaussian disorder

```
0 +++ 0 +++ 0 +++ 0 +++ 0 +++
+ -   + -   + -   + -   + -
+  -  +  -  +  -  +  -  +  -
+   - +   - +   - +   - +   -
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```

T=0 GS stability

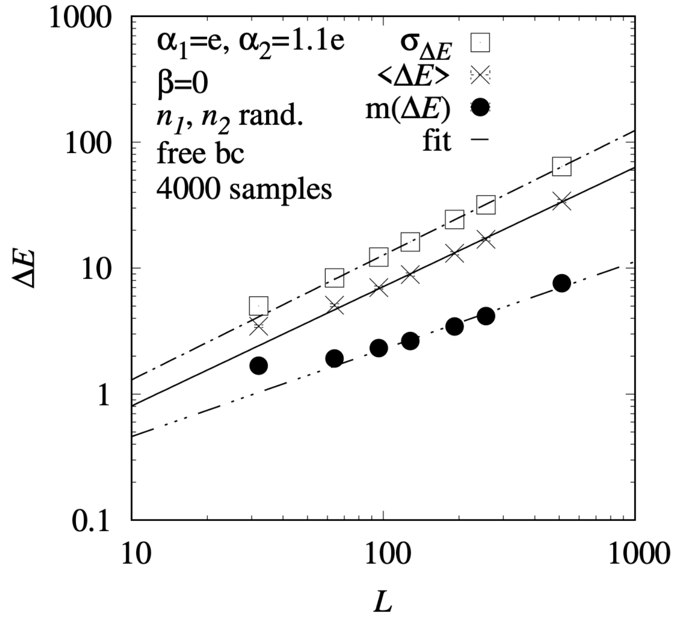
- Exact GS: perfect matchings - complex but polynomial complexity
- Compute exact GS: open b.c. and twisted/frustrating b.c.
- Domain wall energy:

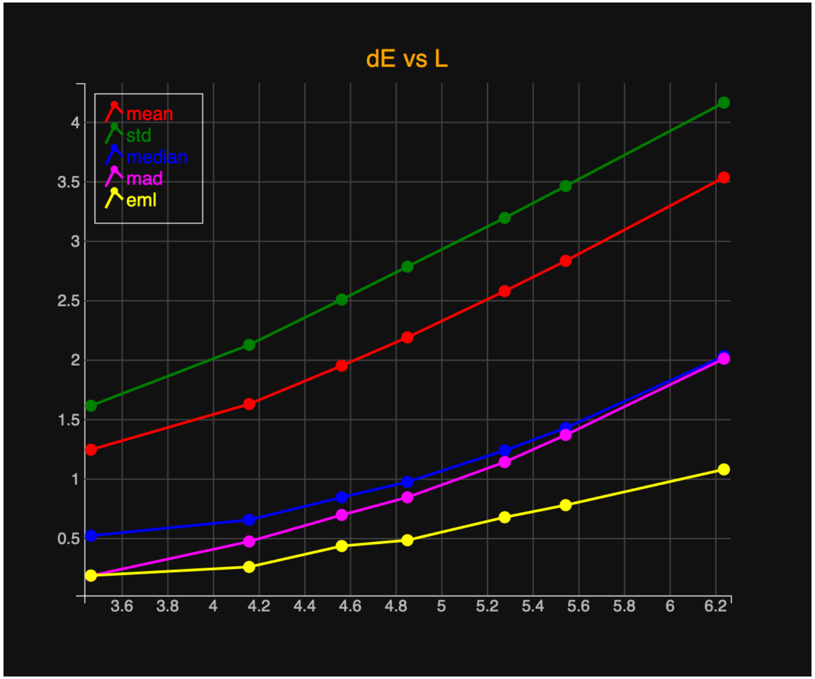
$$\Delta E = E_{\text{twisted}} - E_{\text{open}} \sim L^{\theta}$$

- Finite-T order test for ΔE:
    - Increases, $\theta > 0$: stable $T_c > 0$ order
    - Decreases, $\theta < 0$: $T_c = 0$ order
- Spin-glass: $\overline{\Delta E} = 0$, $\sigma_{\Delta} \sim T^{\theta}$
- Available sizes: $L = 2^5 \dots 2^9$, could be pushed to $L = 10^4$ (Khoshbahkt, 2018)
- Realisations: $1 - 4 \cdot 10^3$

Moments of ΔE

- $\overline{\Delta E}, \sigma_\Delta \sim L^\theta \quad \theta \approx 1$
- Strong fluctuations

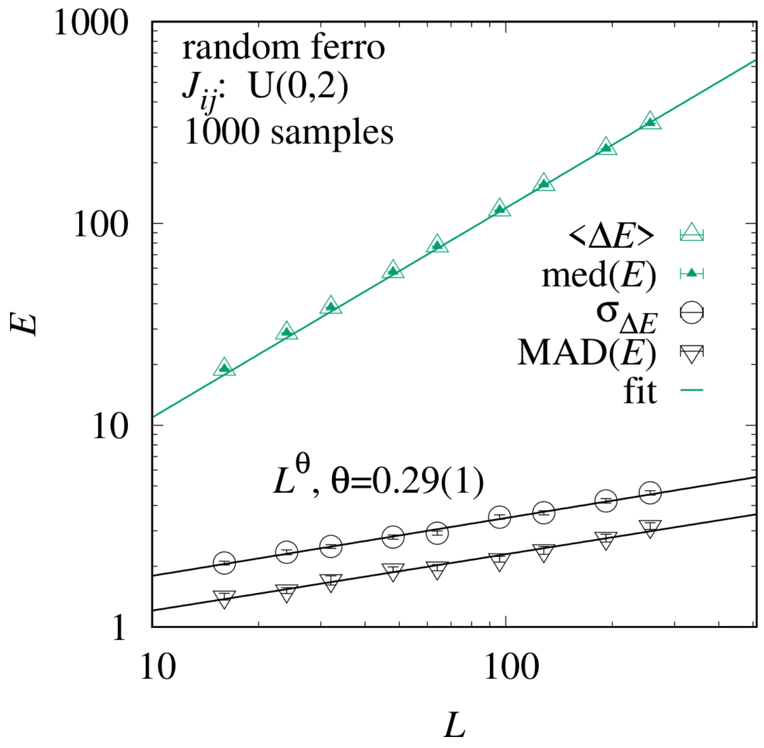




Random FM

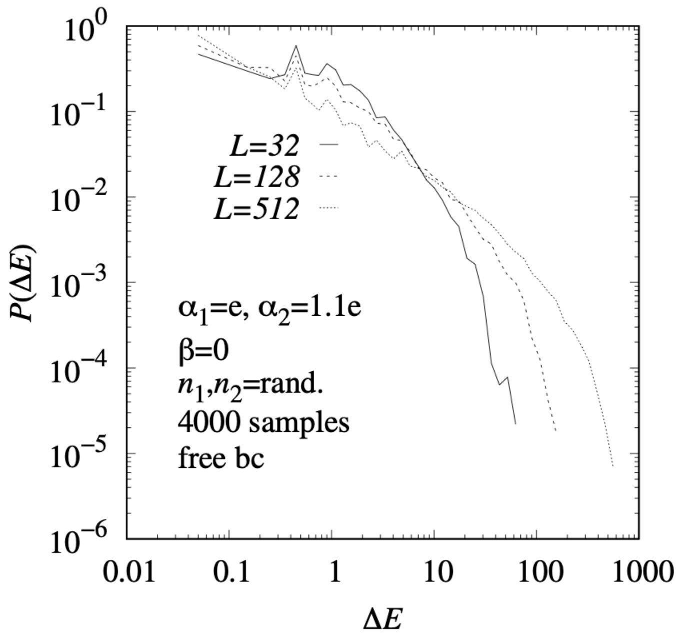
$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} s_i s_j$$

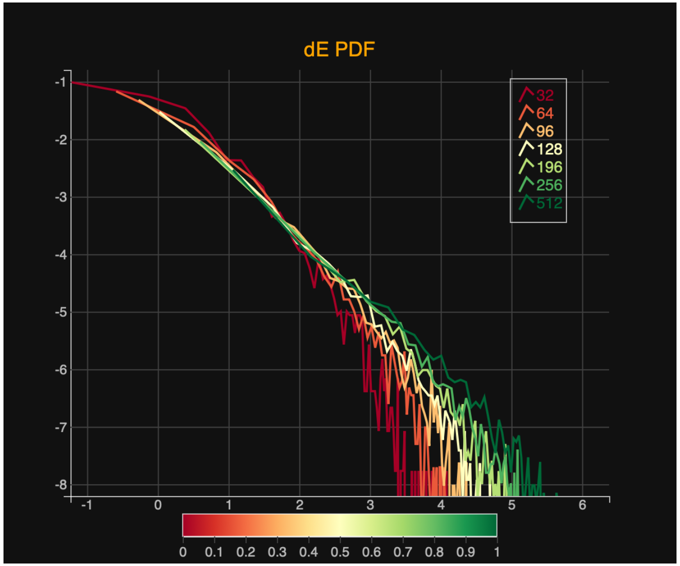
- Same lattice
- $J_{ij} \in U(0,2)$ - random uniform



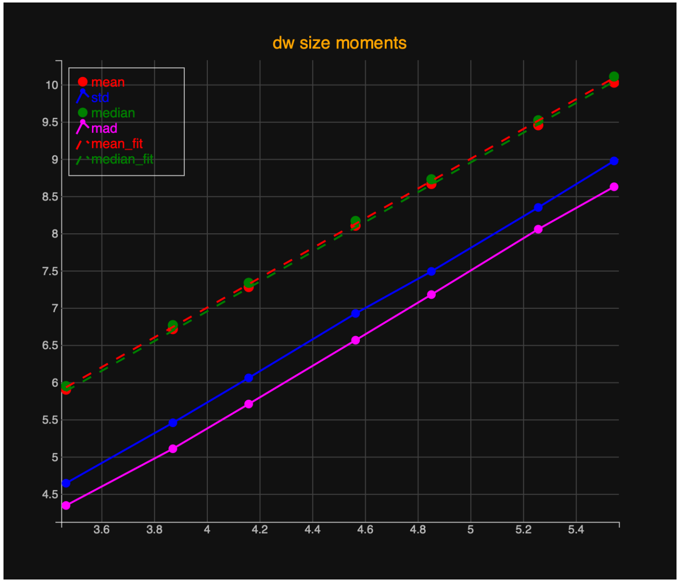
PDF of ΔE

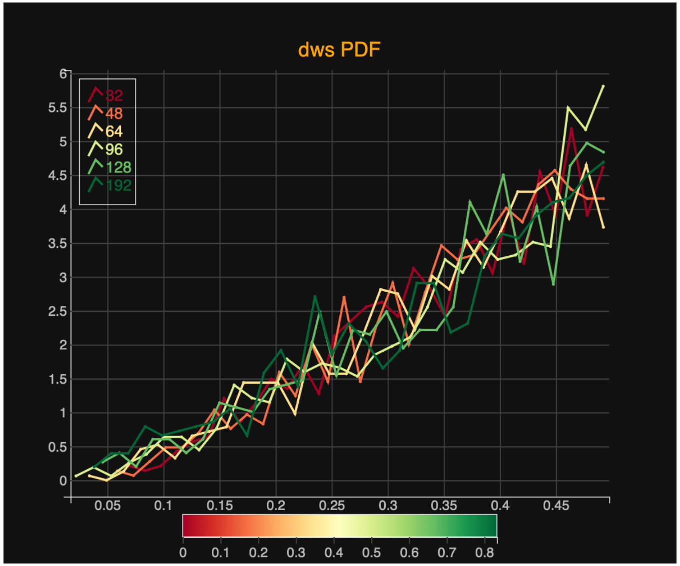
- PDF broadens with L
- Stable $T > 0$ order? **What kind**?





Flipped spins



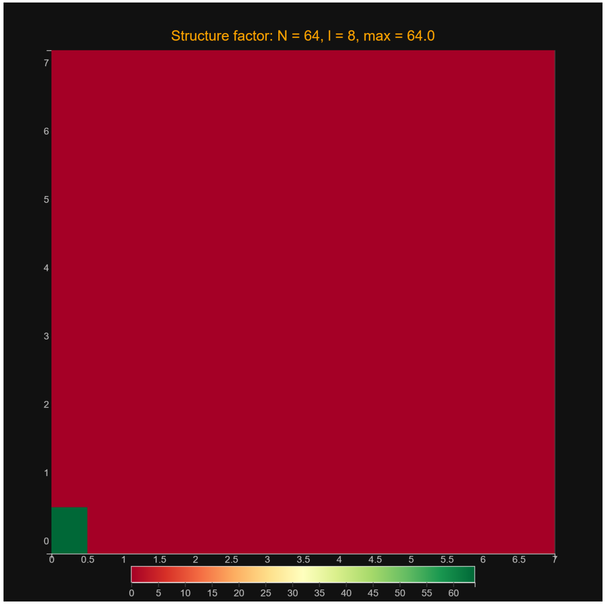


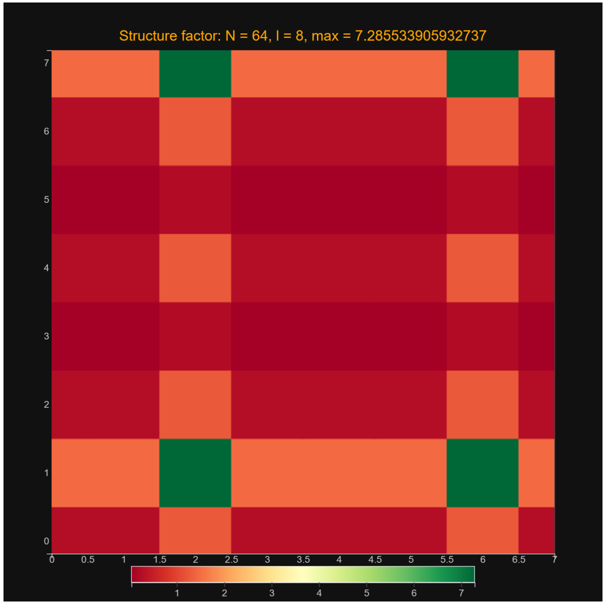
Structure factor

- Definition:

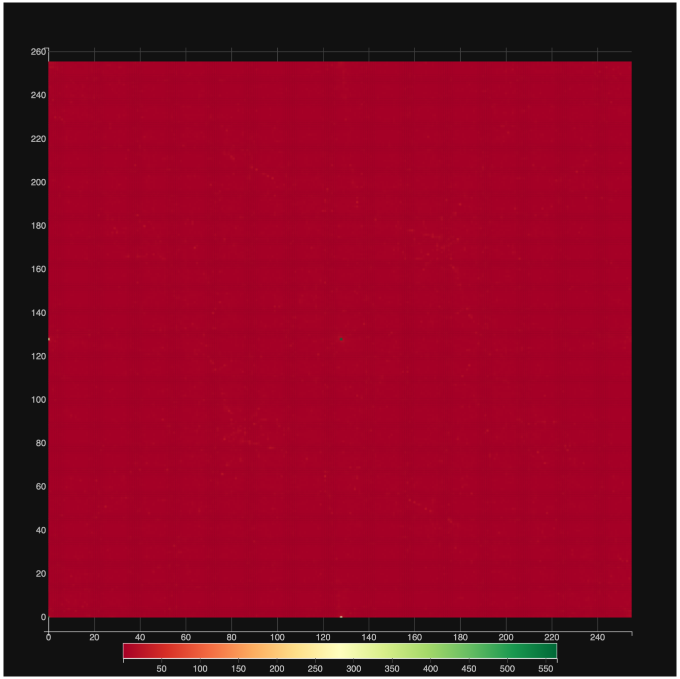
$$S_{\mathbf{q}} = \left| \frac{1}{\sqrt{N}} \sum_x e^{-i\mathbf{q}\cdot\mathbf{r}_x} \boldsymbol{s}_x \right|^2$$

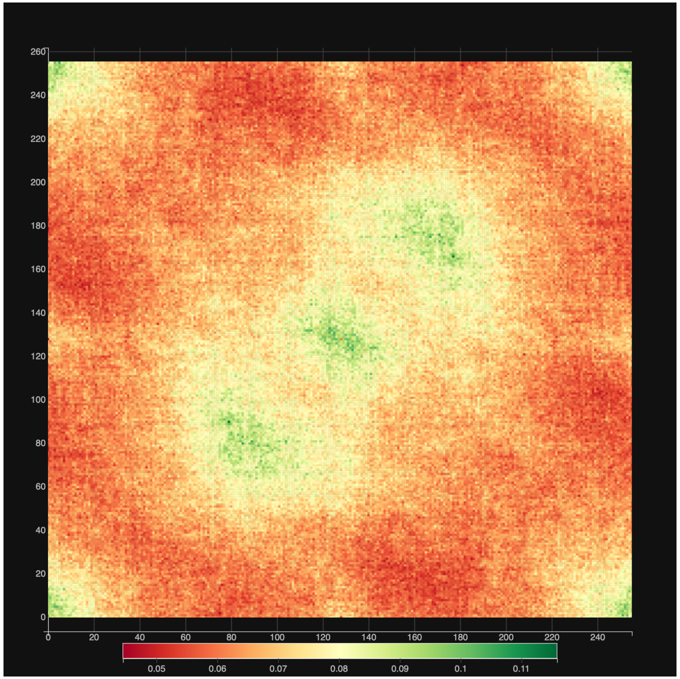
- Detects any order encoded in any pair correlation functions, e.g. $C(x,y) = \langle \boldsymbol{c}_x \boldsymbol{s}_x \boldsymbol{c}_y \boldsymbol{s}_y \rangle$





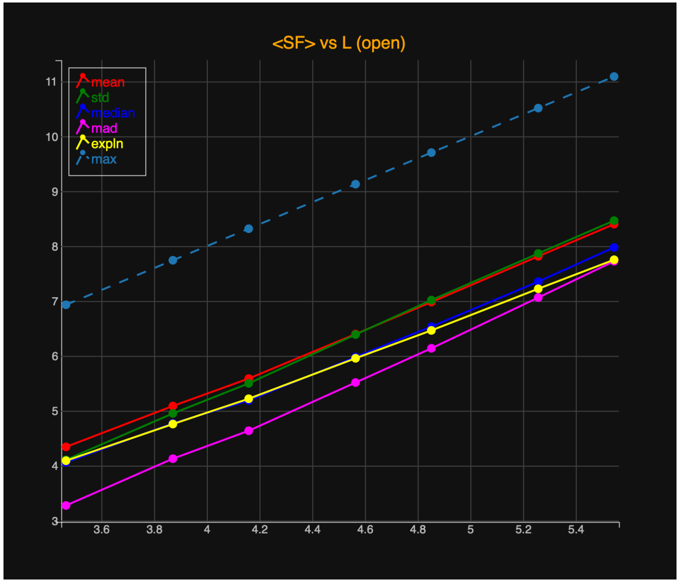
SF - average vs median

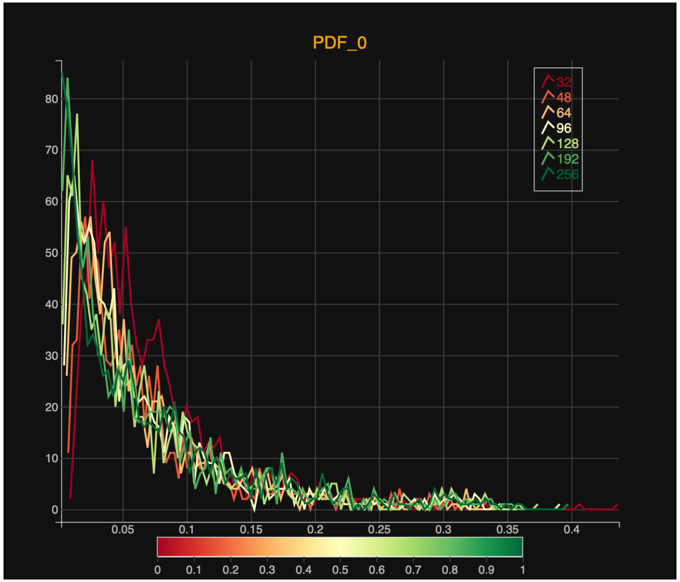




SF - moments and PDF

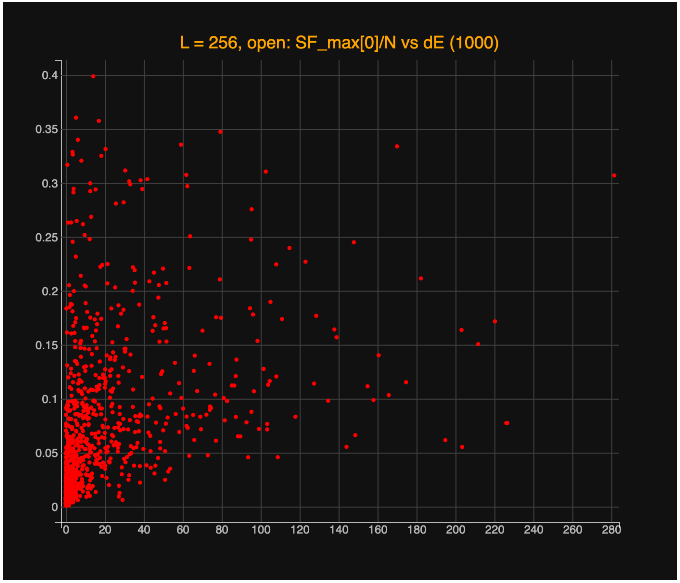
Moments of the maximum of the structure factor:

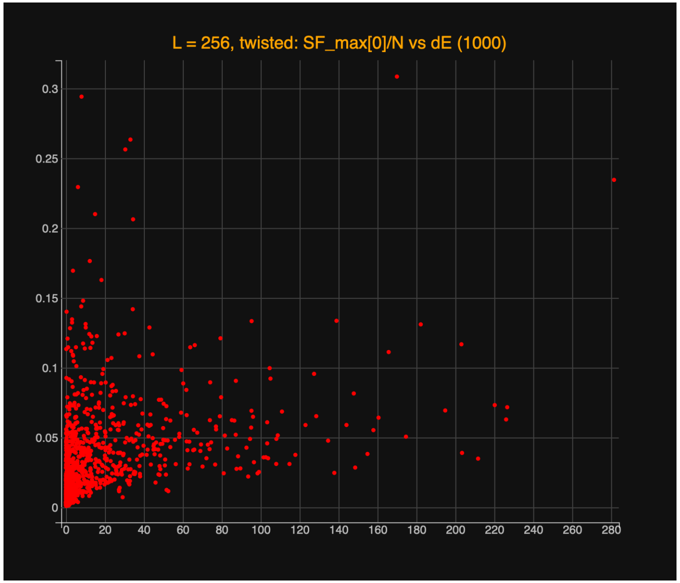




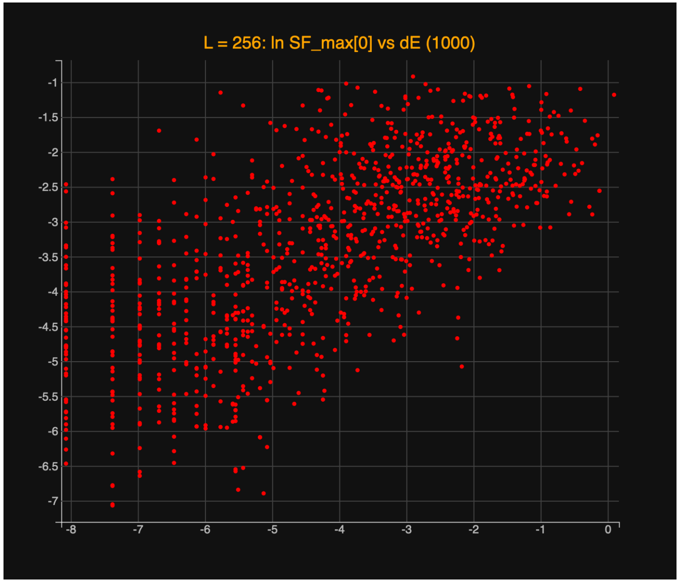
Structure factor

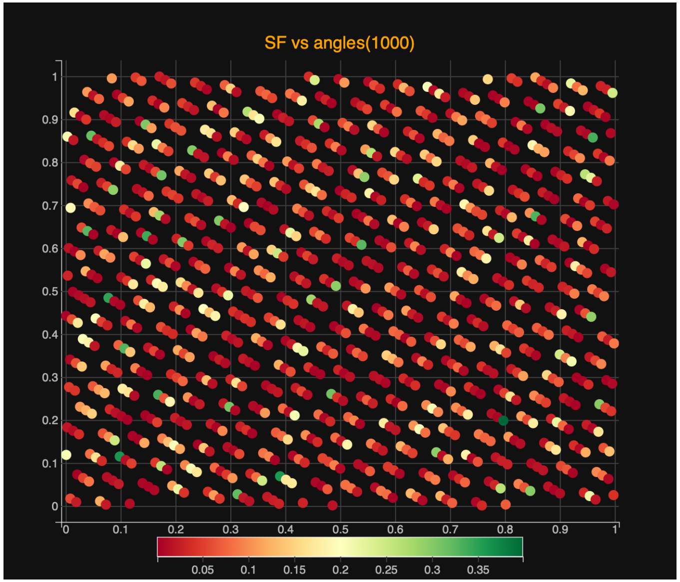
- Domain wall energy vs maximum structure factor: open and twisted b.c.
- No clear correlation





Peaks of SF vs domain wall





Preliminary summary

- Growing moments of domain wall energiy: mean/std/median/MAD
- Stable order? $L_{\mathrm{max}} = 512$
- Structure factor: no signs of *simple* order
- Glass?
- Hidden simple order?
- Something else?
- What?