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CHIMERA STATES IN DYNAMICAL NETWORKS: SPONTANEOUS SYMMETRY-BREAKING

Eckehard Schöll
Institut für Theoretische Physik
und
Sonderforschungsbereich SFB 910
Control of Self-Organizing Nonlinear Systems
Technische Universität Berlin
Germany

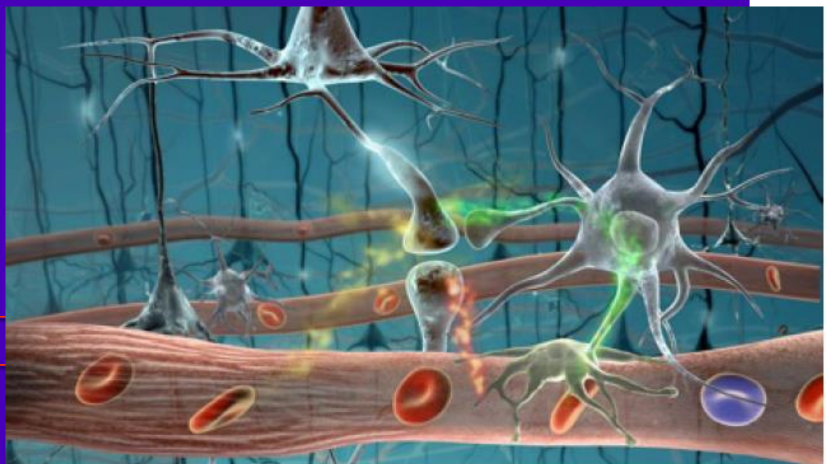
http://www.itp.tu-berlin.de/schoell

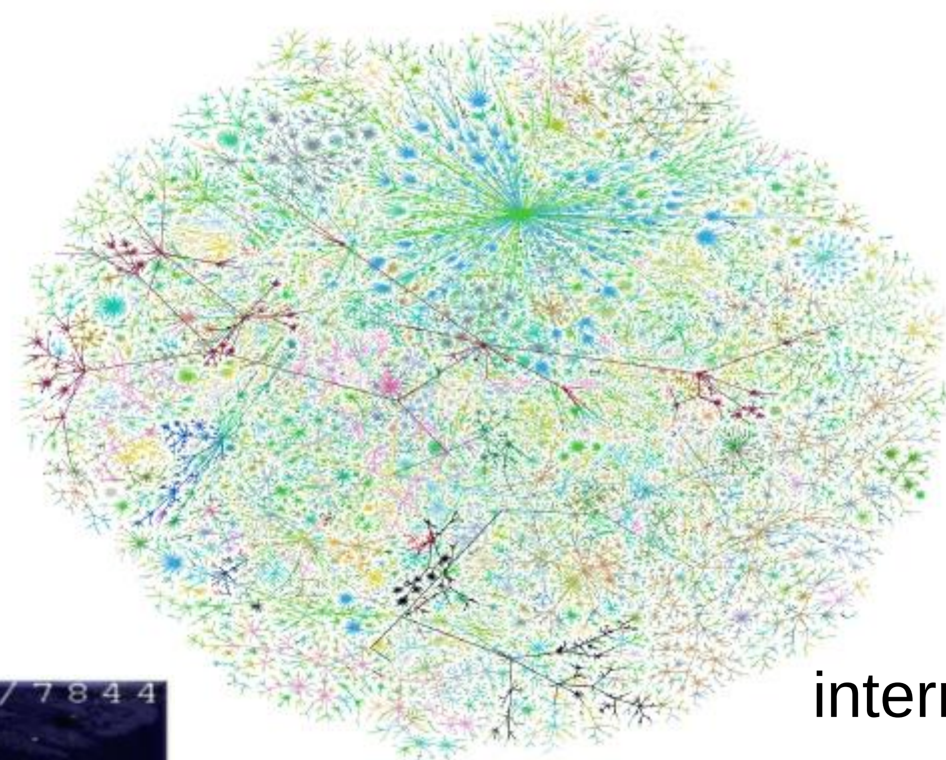
Outline

- Chimera states in dynamical networks
  - Motivation
  - Coherence-incoherence transitions in coupled maps
  - Experiment with liquid crystal spatial light modulator
  - Time-continuous systems
  - Multi-chimera states in a neuronal model

Examples of complex networks

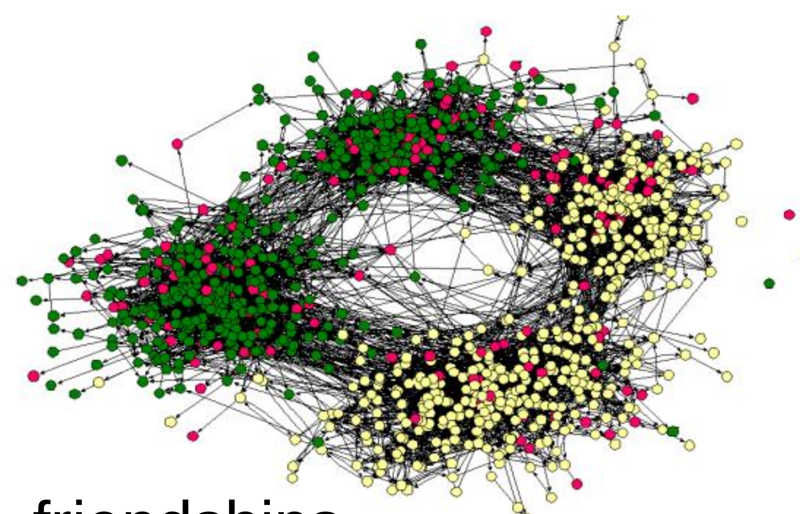
brain





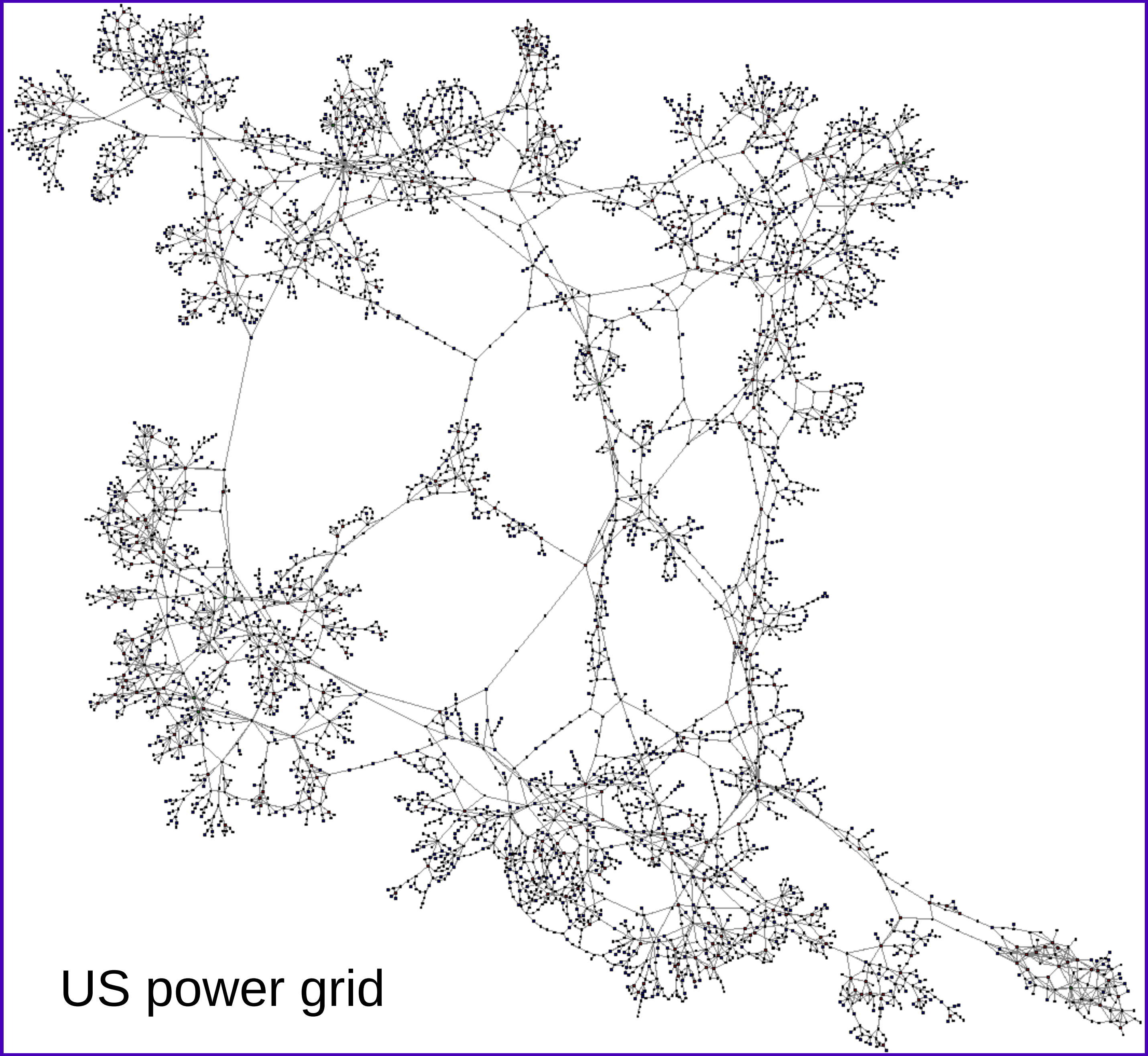
internet

power grid



friendships

Complex networks



Synchronization in complex networks

- Synchronization and Desynchronization
  - Constructive role for strongly coherent fields:
    - Laser system, …

      → **Synchronization**

      - **A. Pikovsky, *et al.*, *Synchronization*, Cambridge, 2001**
  - On occasion, undesirable phenomenon:
    - Parkinsonian tremor
    - Swaying motion of London's Millennium Bridge

      → **Desynchronization**

Tass, *Biol. Cybern.*, 89, 81 (2003)

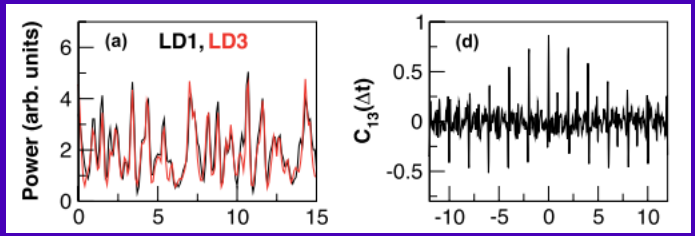
. Rosenblum *et al.*, *Phys. Rev. Lett.* 92, 114102 (2004)

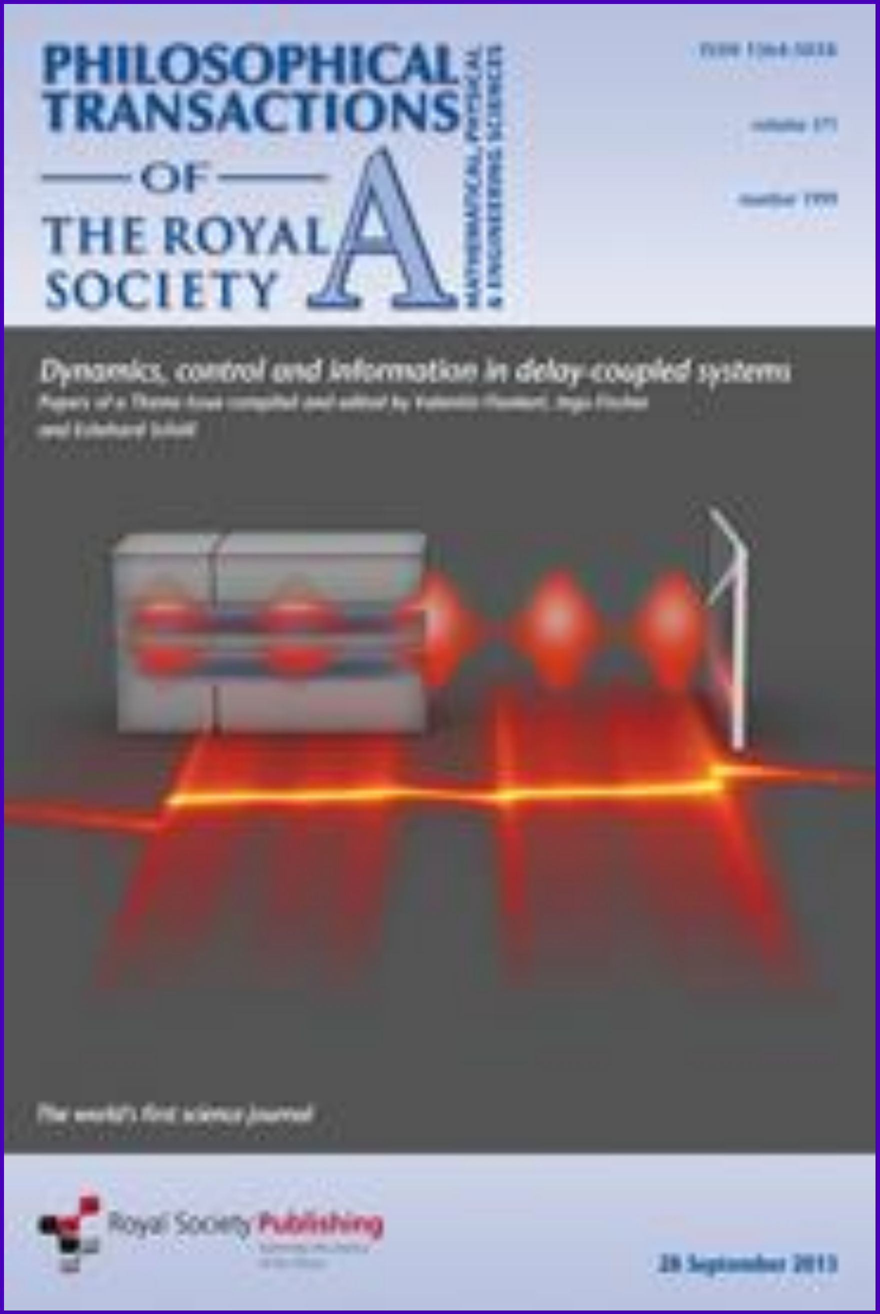
Popovych *et al.*, *Phys. Rev. Lett.* 94, 164102 (2005)

Synchronization in delay-coupled networks

* M. Soriano, J. Garcia-Ojalvo, C. Mirasso, I. Fischer: Rev. Mod. Phys. 85, 421 (2013).
* I. Fischer, R. Vicente, J.M. Buldu, M. Peil, C. Mirasso, M. Torrent, J. Garcia-Ojalvo: PRL 97, 123902 (2006)

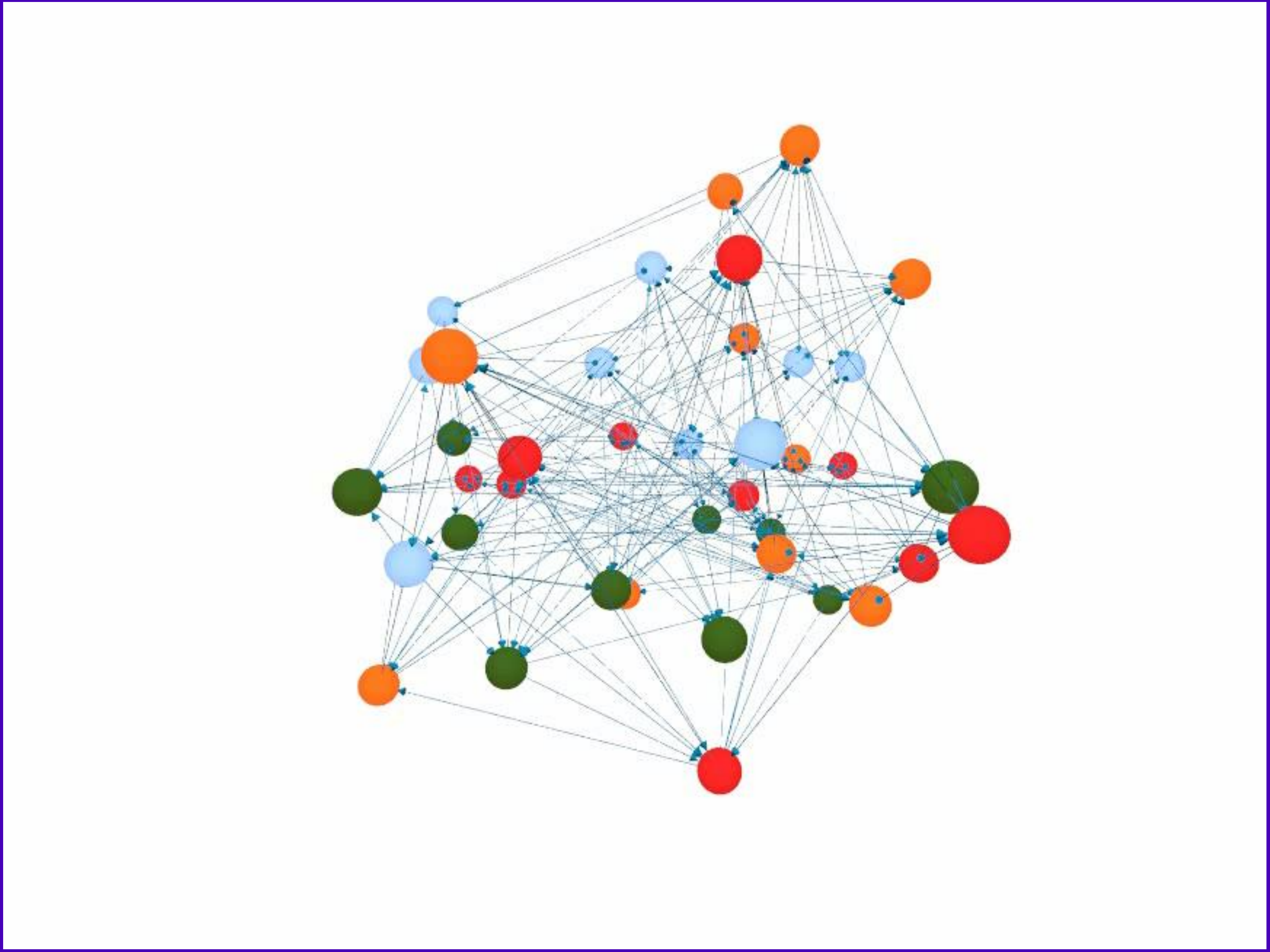
Chaotic synchronization of lasers:





Theme Issue on Dynamics, Control and Information in Delay-Coupled Systems
V. Flunkert, I. Fischer, and E. Schöll (Eds.): Phil. Trans. Royal Soc. A 371, 28 Sept. (2013)

Group synchrony



T. Dahms, J. Lehnert, E. Schöll: Phys. Rev. E 86, 016202 (2012)

Symmetry-breaking in neuronal systems

- Unihemispheric sleep: some birds and dolphins sleep with one half of their brain, while the other half remains awake

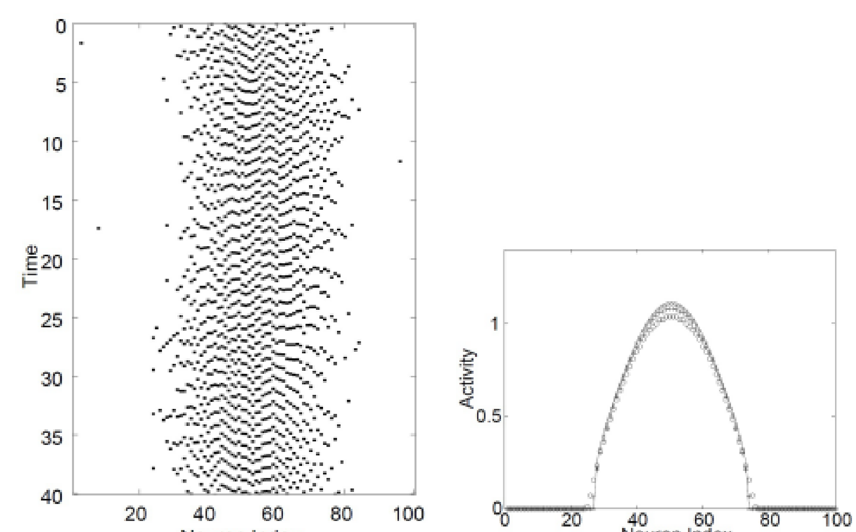
- Bump states in neural networks
  Localized patch of incoherent asynchronous firing
  - partial synchronization

• C.R. Laing, C.C. Chow.

Neural Comput. **13**, 1473 (2001).

Nonlocally coupled Integrate-and-fire neurons

$$\frac{dv_i}{dt} = I_i - v_i + \sum_{j,m} \frac{J_{ij}}{N}\alpha(t - t_j^m) - \sum_{l} \delta(t - t_i^l).$$



Chimera states in networks of identical oscillators with nonlocal coupling

- Spatially coexisting domains of coherent/phase-locked and incoherent/desynchronized oscillators
- Chimera in Greek mythology: fire-breathing monster with three heads: lion's head, goat's head, serpent's head
- Prototype behavior of system on the transition from complete coherence to complete incoherence
- Essential: nonlocal coupling of range r between local and global coupling

Chimera states in networks of identical oscillators

- Theory: Kuramoto and Battogtokh 2002
  . Abrams and Strogatz 2004

2002 Nonlinear Phenomena in Complex Systems

Coexistence of Coherence and Incoherence in Nonlocally Coupled Phase Oscillators

Y. Kuramoto[1] and D. Battogtokh[2]

PHYSICAL REVIEW LETTERS — week ending 22 OCTOBER 2004

Chimera States for Coupled Oscillators

Daniel M. Abrams* and Steven H. Strogatz†

Chimera states in networks of identical oscillators

- Theory: Kuramoto and Battogtokh 2002
  . Abrams and Strogatz 2004

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PHYSICAL REVIEW LETTERS week ending 22 OCTOBER 2004

Chimera States for Coupled Oscillators

Daniel M. Abrams* and Steven H. Strogatz†

- Experimentally verified only recently (2012/2013):

nature physics LETTERS
PUBLISHED ONLINE: 15 JULY 2012 | DOI: 10.1038/NPHYS2371

Chimera and phase-cluster states in populations of coupled chemical oscillators

Mark R. Tinsley, Simbarashe Nkomo and Kenneth Showalter*

nature physics LETTERS
PUBLISHED ONLINE: 15 JULY 2012 | DOI: 10.1038/NPHYS2372

Experimental observation of chimeras in coupled-map lattices

Aaron M. Hagerstrom[1,2]*, Thomas E. Murphy[1,3], Rajarshi Roy[1,2,4], Philipp Hövel[5,6], Iryna Omelchenko[5,6] and Eckehard Schöll[5]

PNAS May 2013 PNAS Early Edition

Chimera states in mechanical oscillator networks

Erik Andreas Martens[a,b,1,2], Shashi Thutupalli[c,d,1,2], Antoine Fourrière[c], and Oskar Hallatschek[a,e]

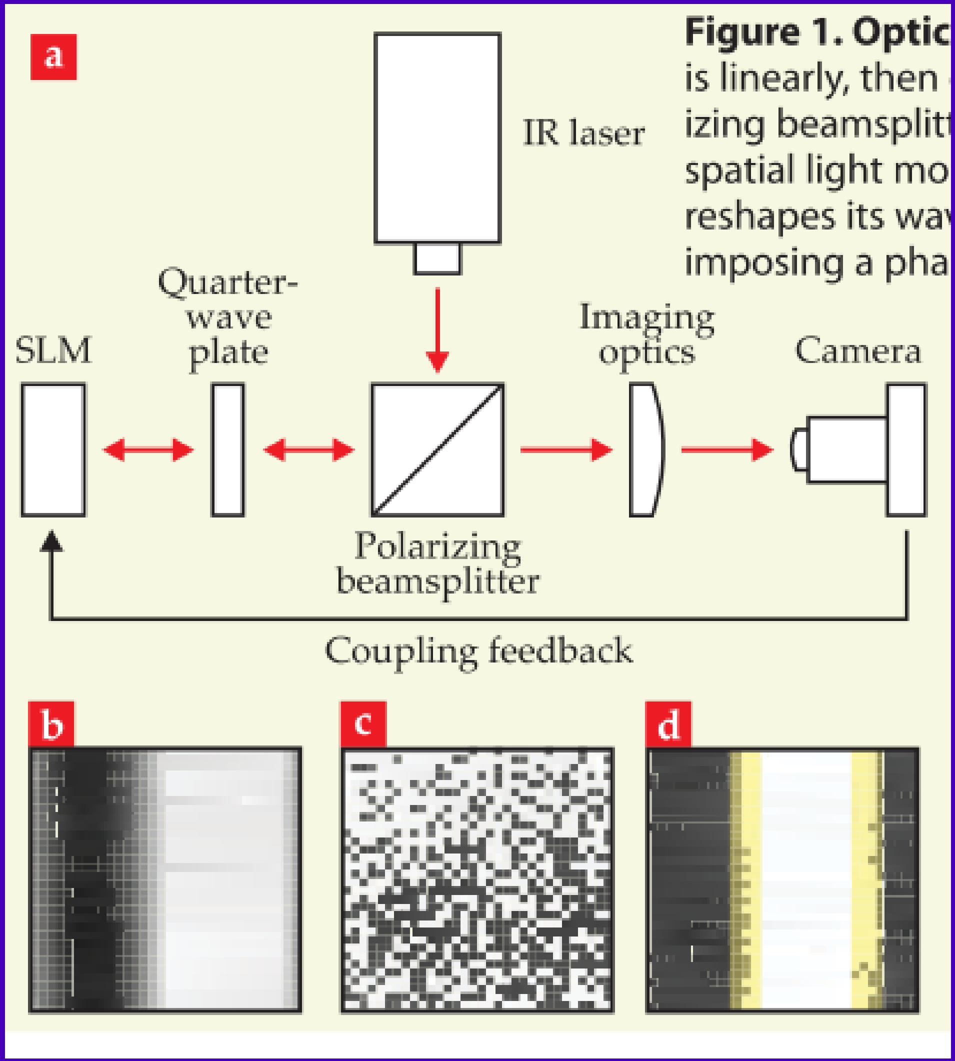
PHYSICAL REVIEW LETTERS week ending 2 AUGUST 2013

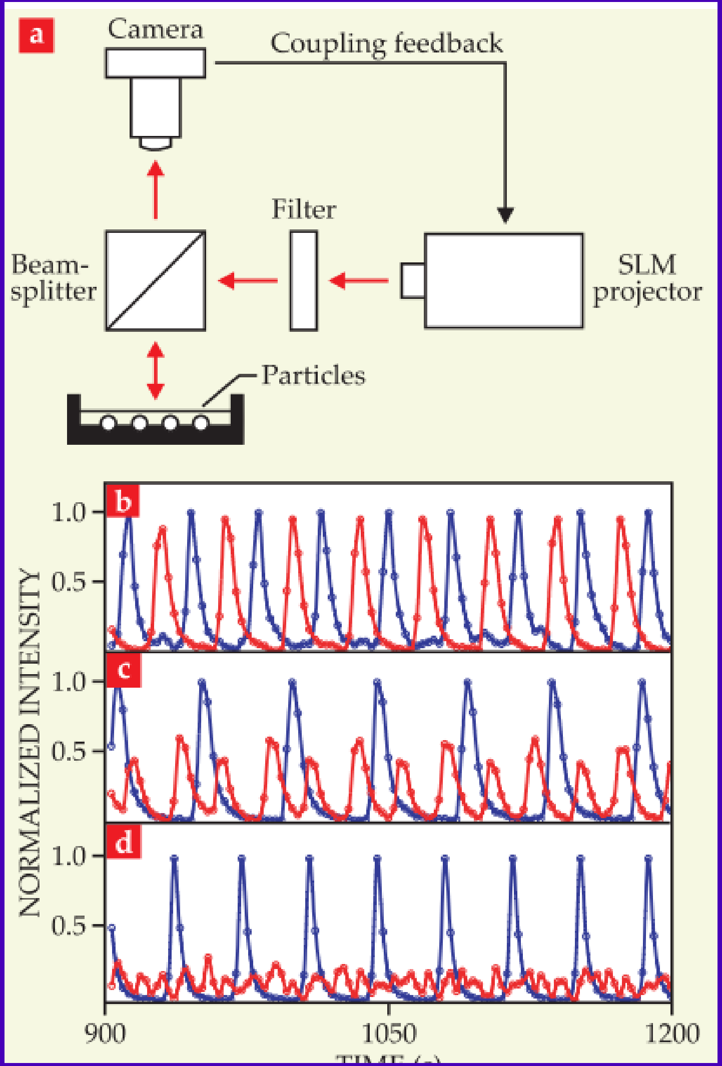
Virtual Chimera States for Delayed-Feedback Systems

Laurent Larger,[1] Bogdan Penkovsky,[1,2] and Yuri Maistrenko[1,3]

Experiments on chimera states

- Optical experiment:
  Spatial light modulator
- Chemical experiment:
  Light-sensitive BZ reaction



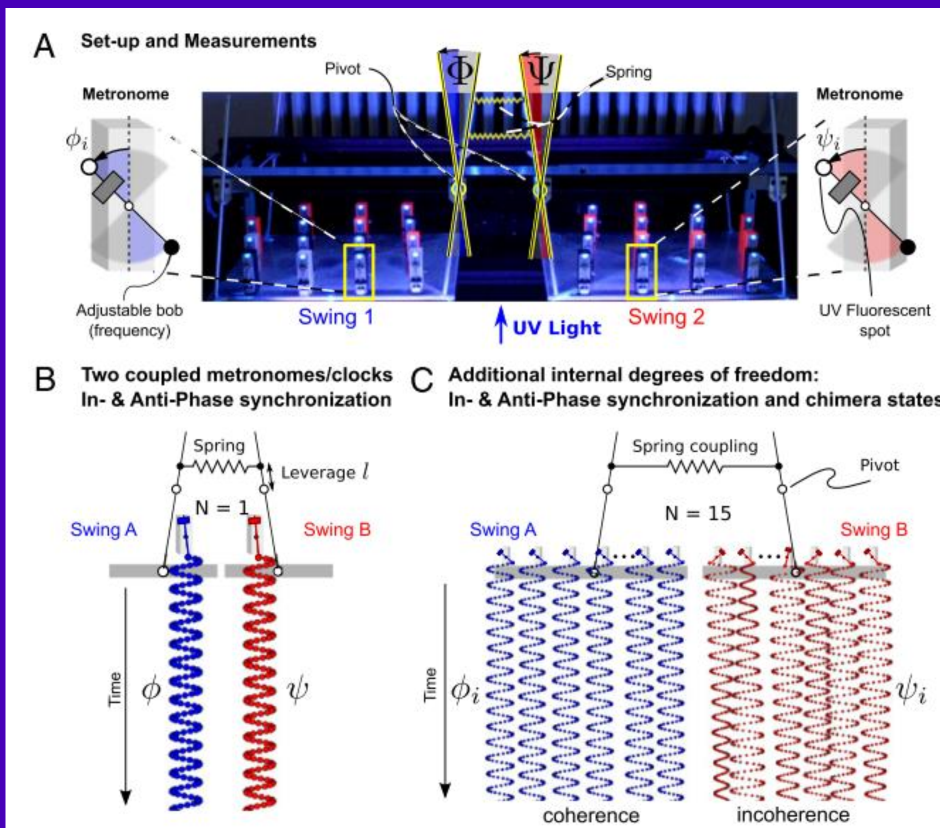


Hagerstrom, Murphy, Roy, Hövel, Omelchenko, Schöll:
Nature Phys. 8, 658 (2012)

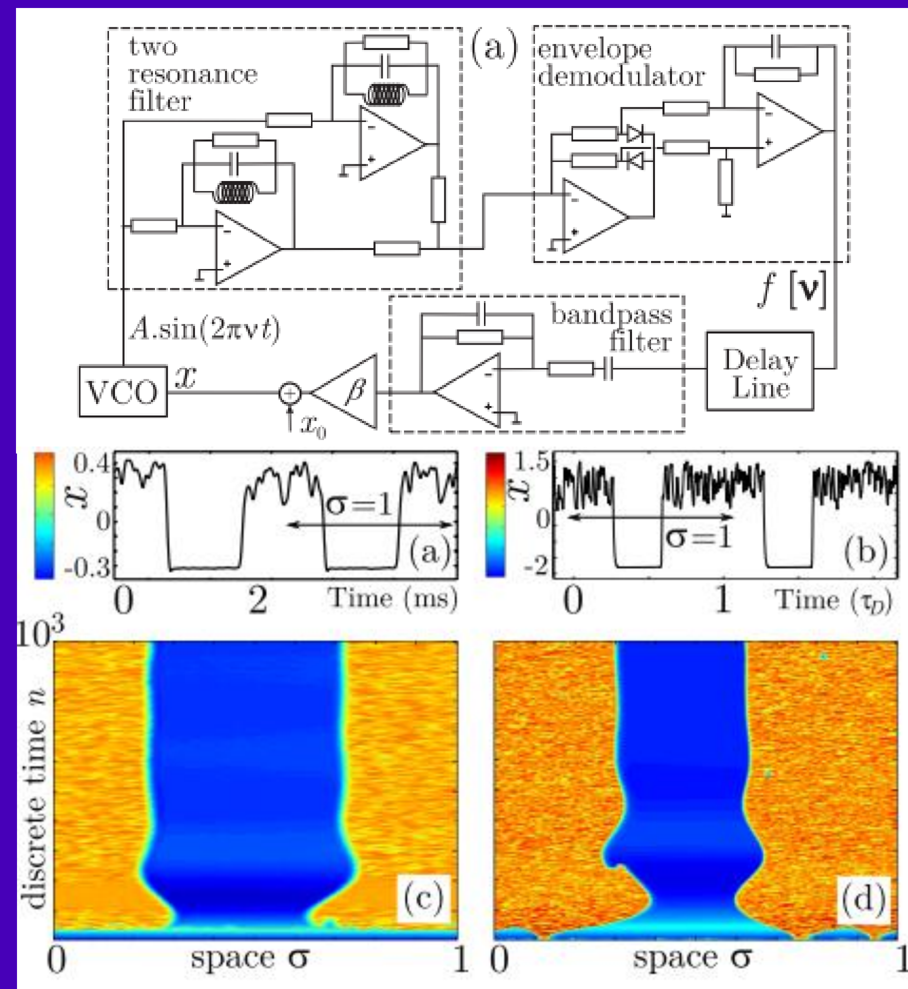
Tinsley, Nkomo, Showalter:
Nature Phys. 8, 662 (2012)

Experiments on chimera states

- Mechanical experiment: coupled pendula
- Electronic experiment: frequ. modul. delay oscillator

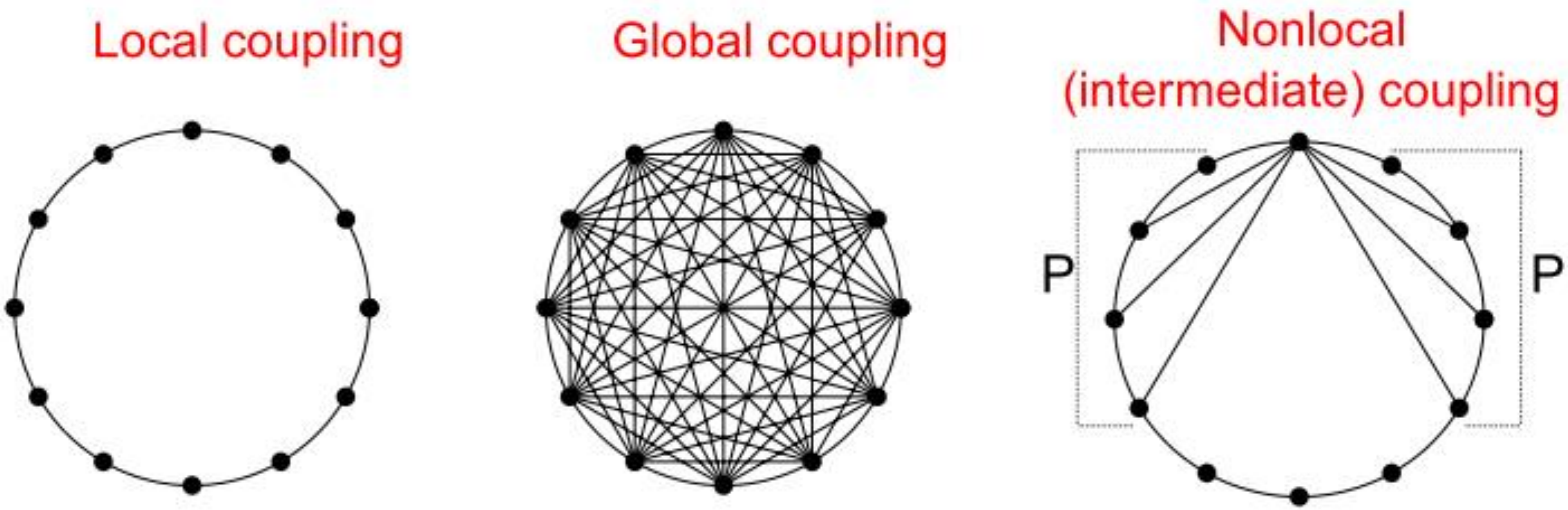


Martens, Thutupalli, Fourriere, Hallatschek, 110, 10563 (2013)



Larger, Penkovsky, Maistrenko, PRL 111, 054103 (2013)

Networks with nonlocal coupling



Coupling radius

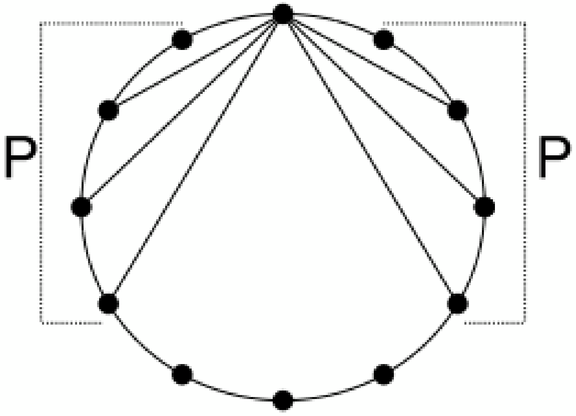
$r = P/N$

P – number of coupled nearest neighbors

N – total number of elements in network

Dynamics of networks with nonlocal coupling of range r

$$\dot{\mathbf{X}} = \mathbf{F}(\mathbf{X}) + \frac{\sigma}{2P}(\mathbf{G} \otimes \mathbf{H})\mathbf{X}$$



Coupling radius

$r = P/N$

$\mathbf{X} = (X_1, \ldots, X_N)$ – state vector

$\mathbf{F}$ – dynamics of individual element

$\mathbf{H}$ – local interaction matrix

$\mathbf{G}$ – coupling matrix (network topology)

Here $\mathbf{G}$ – circulant matrix with rows

$$(-2P, \underbrace{1, \ldots, 1}_{P}, 0, \ldots, 0, \underbrace{1, \ldots, 1}_{P}),$$

$$g_{ii} = -2P$$

σ – coupling strength

P – number of coupled neighbors (in each direction)

N – total number of elements

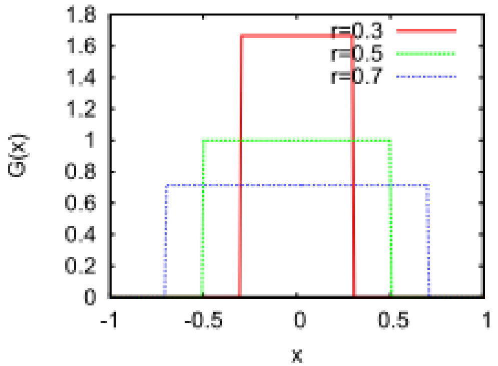
Generalization of nonlocal coupling in the continuum limit of large N (space x)

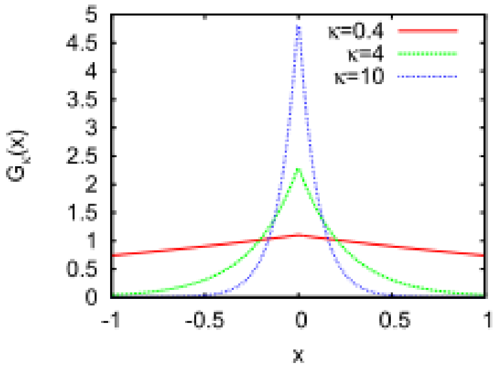
Kuramoto phase oscillator model: phase lag α

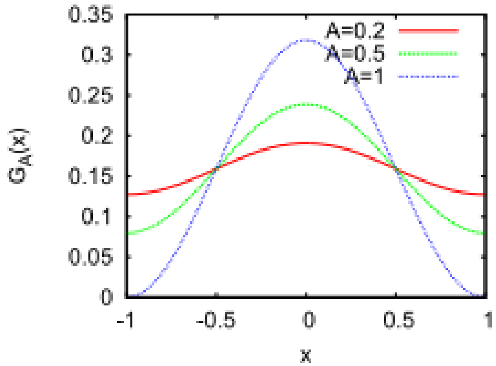
$$\frac{\partial \varphi(x,t)}{\partial t} = \omega - \int_{-1}^{1} G(x-x')\sin[\varphi(x,t)-\varphi(x',t)+\alpha]dx'$$

Spatial coupling functions (integral kernels):

$$G(x) = \begin{cases} 1/2r & |x| \le r \\ 0 & |x| > r, \end{cases} \quad G_{\exp}(x) = \frac{\kappa e^{-\kappa|x|}}{2\left(1-e^{-\kappa}\right)}, \quad G_{\cos}(x) = \frac{1+A\cos(x\pi)}{2\pi}$$

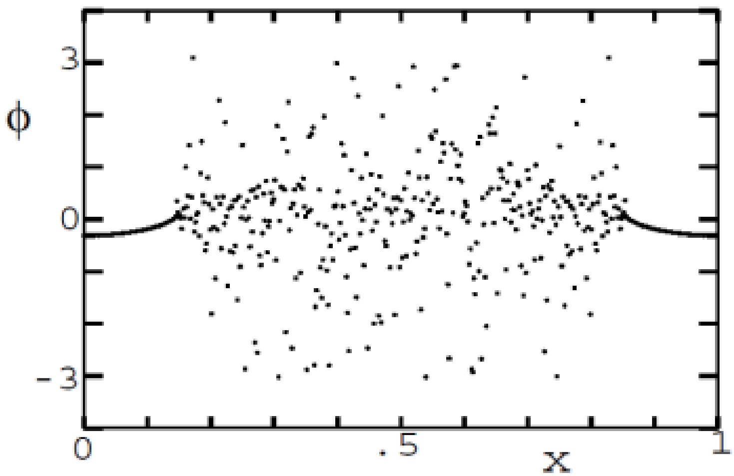






Exponential coupling function:

specially prepared initial condition (high multistability)



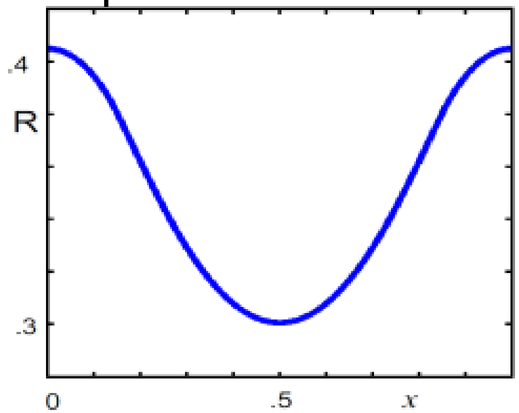
Coupling kernel:

$$G_{\exp}(x) = \frac{\kappa}{2} e^{-\kappa|x|}$$

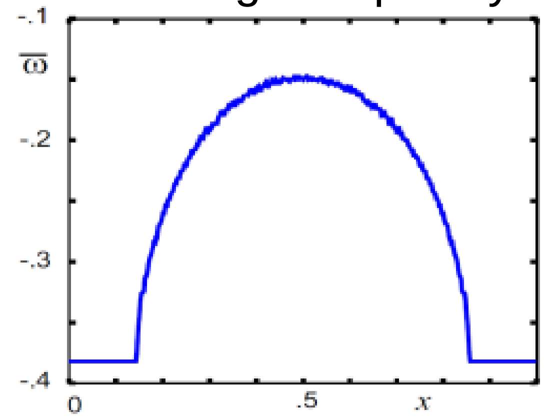
Local order parameter:

$$R e^{i\Theta} = \int_0^1 G_{\exp}(x - x') e^{i\phi(x',t)} dx'$$

spatial phase cohererence



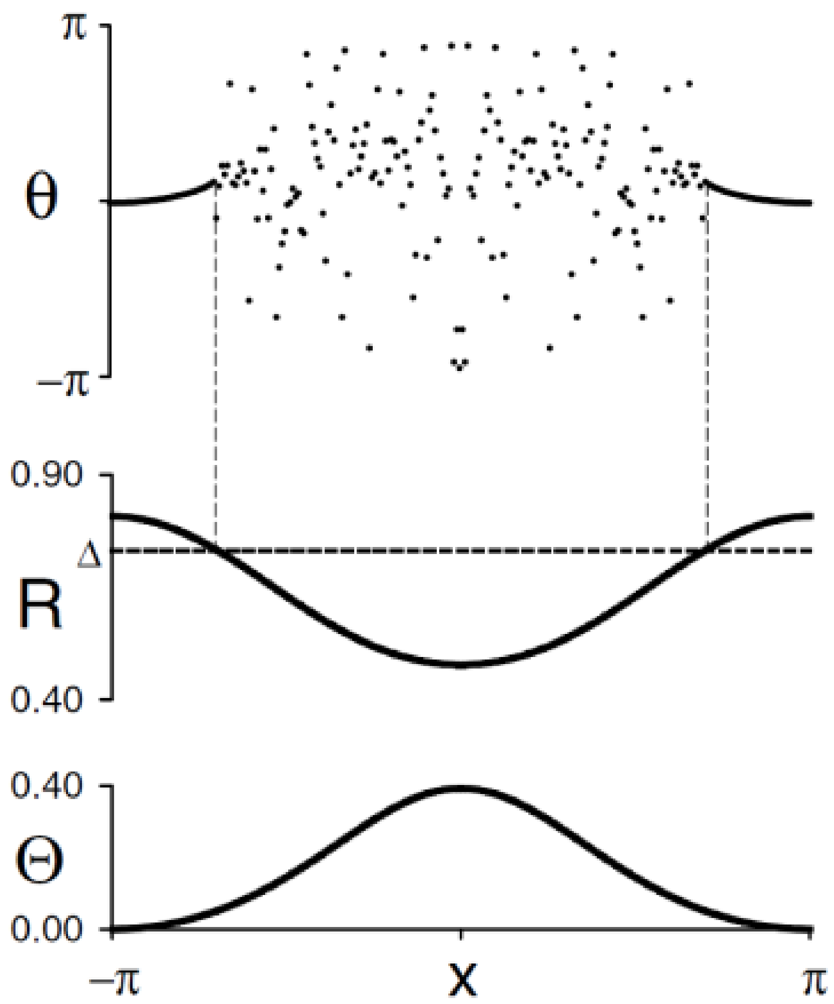
average frequency



Y. Kuramoto and D. Battogtokh: *Coexistence of Coherence and Incoherence in Nonlocally Coupled Phase Oscillators*, Nonlin. Phen. in Complex Sys. **5**, 380 (2002).

Cosine coupling function

specially prepared initial condition (high multistability)



Coupling kernel:

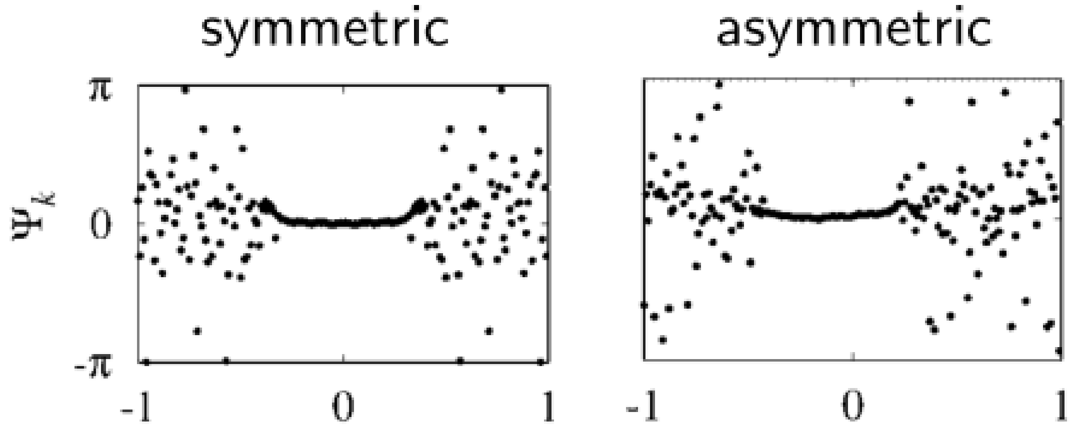
$$G_{\cos}(x) = \frac{1 + A\cos(x\pi)}{2\pi}$$

Local order parameter:

$$Re^{i\Theta} = \int_{-\pi}^{\pi} G_{\cos}(x - x')e^{i\theta(x',t)}dx'$$

D. M. Abrams and S. H. Strogatz: *Chimera States for Coupled Oscillators*, Phys. Rev. Lett. **93**, 174102 (2004).

Step-like coupling function:



unstable with respect to
asymmetric perturbations

Coupling kernel:

$$G(x) = \begin{cases} 1/2r & |x| \leq r \\ 0 & |x| > r, \end{cases}$$

O. E. Omel'chenko, M. Wolfrum, and Yu. Maistrenko: *Chimera states as chaotic spatiotemporal patterns*, Phys. Rev. E **81**, 065201(R) (2010).

Time-discrete maps (logistic map) with step-like coupling function

$$z_i^{t+1} = f\left(z_i^t\right) + \frac{\sigma}{2P} \sum_{j=i-P}^{i+P} \left[f\left(z_j^t\right) - f\left(z_i^t\right) \right]$$

z_i – state variables, $i = 1, ..., N$

N - number of elements

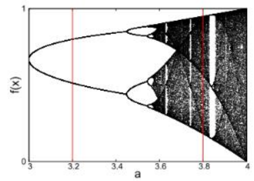
t - discrete time

P - number of coupled nearest neighbors (in each direction)

σ - coupling strength

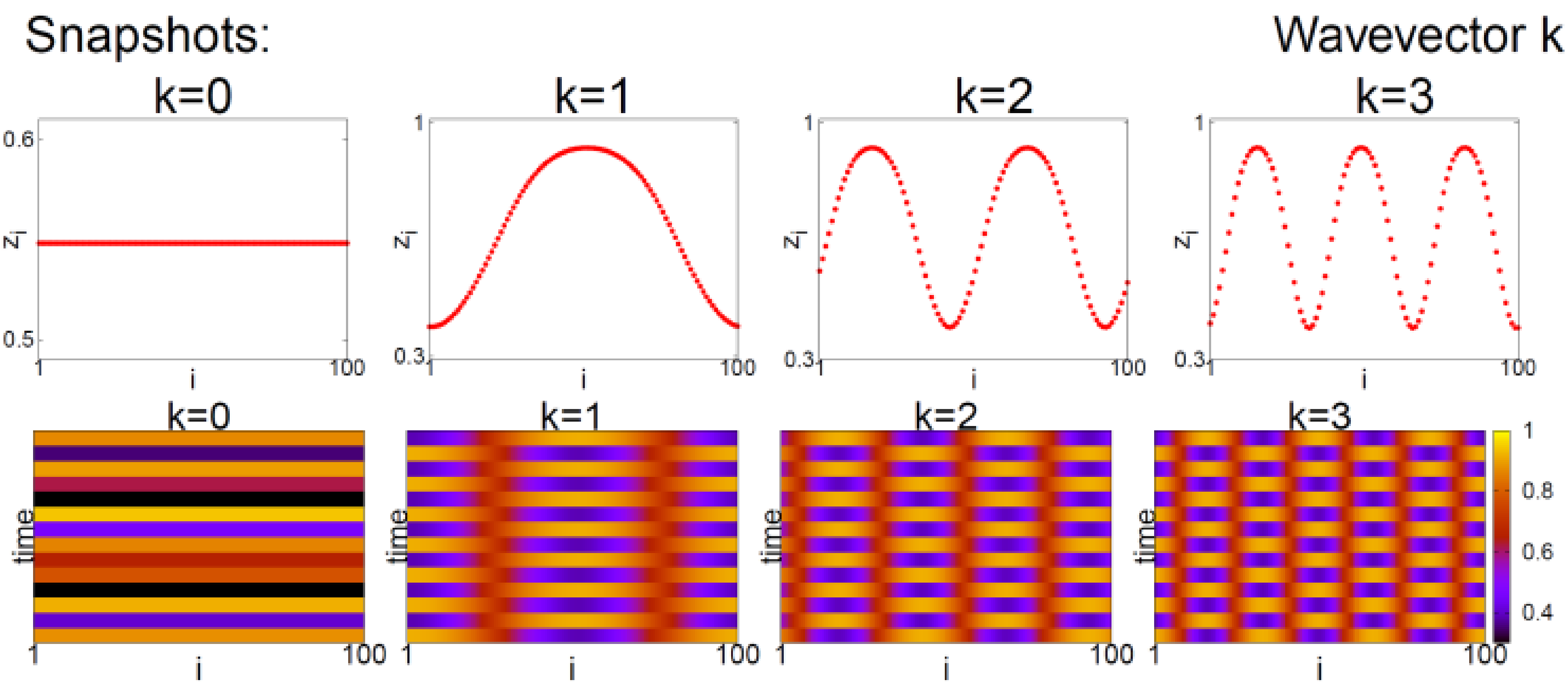
Periodic boundary conditions: $z_{N+1} = z_1$

Local dynamics: $f(z) = az(1-z)$, $a = 3.8$ – chaotic



I. Omelchenko, Yu. Maistrenko, P. Hövel, and E. Schöll: *Loss of coherence in dynamical networks: spatial chaos and chimera states*, Phys. Rev. Lett. **106**, 234102 (2011).

Spatially coherent states



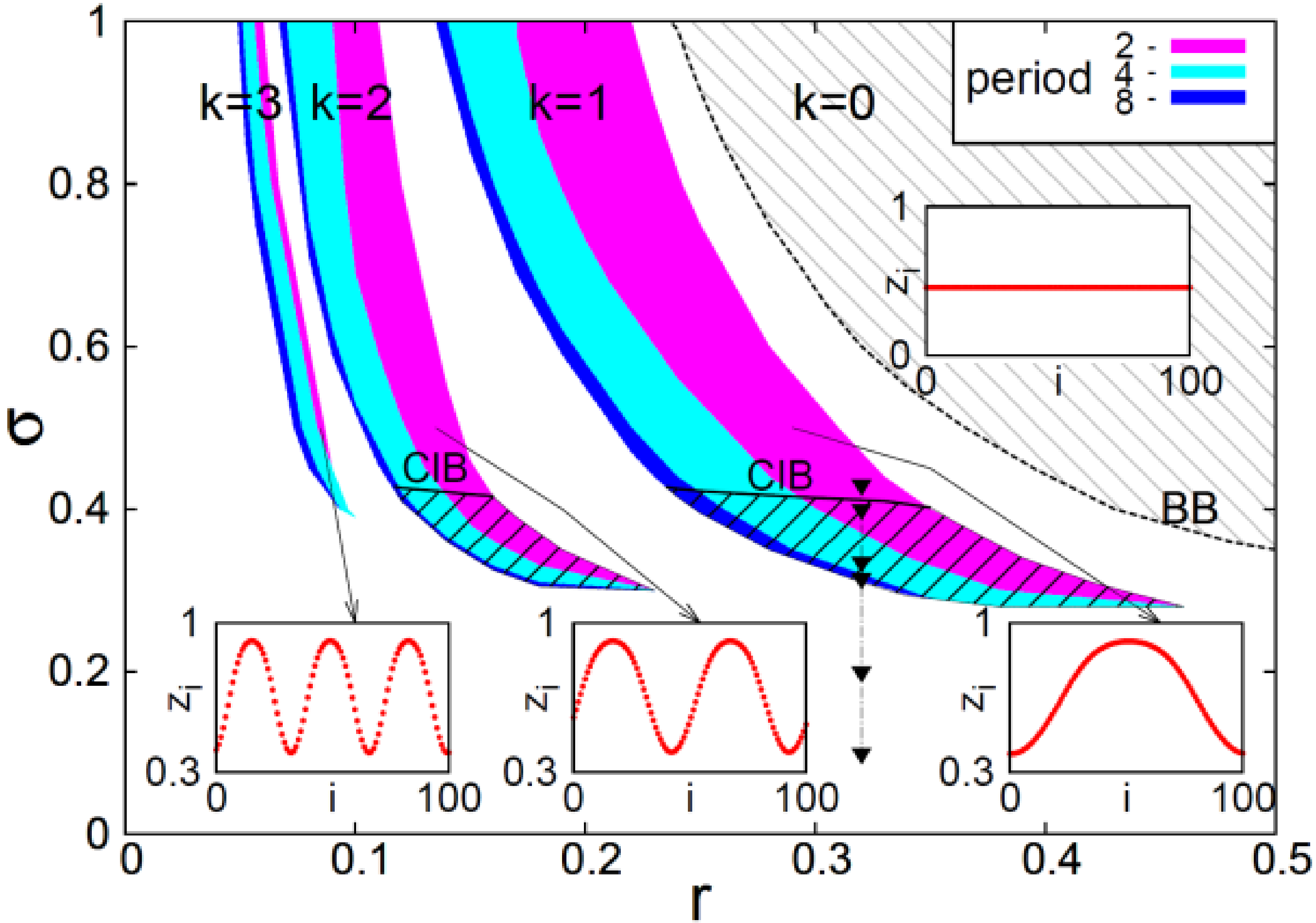
z_i^t $(i = 1, ..., N)$– *coherent* on the ring $\mathcal{S}^1$ as $N \to \infty$ if for any point $x \in \mathcal{S}^1$

$$\lim_{N\to\infty} \lim_{t\to\infty} \sup_{i,j\in U_\delta^N(x)} \left| z_i^t - z_j^t \right| \to 0, \quad \text{for} \quad \delta \to 0,$$

$\longrightarrow$ scan (σ, r)-plane

Bifurcation diagram

coherence-incoherence tongues:

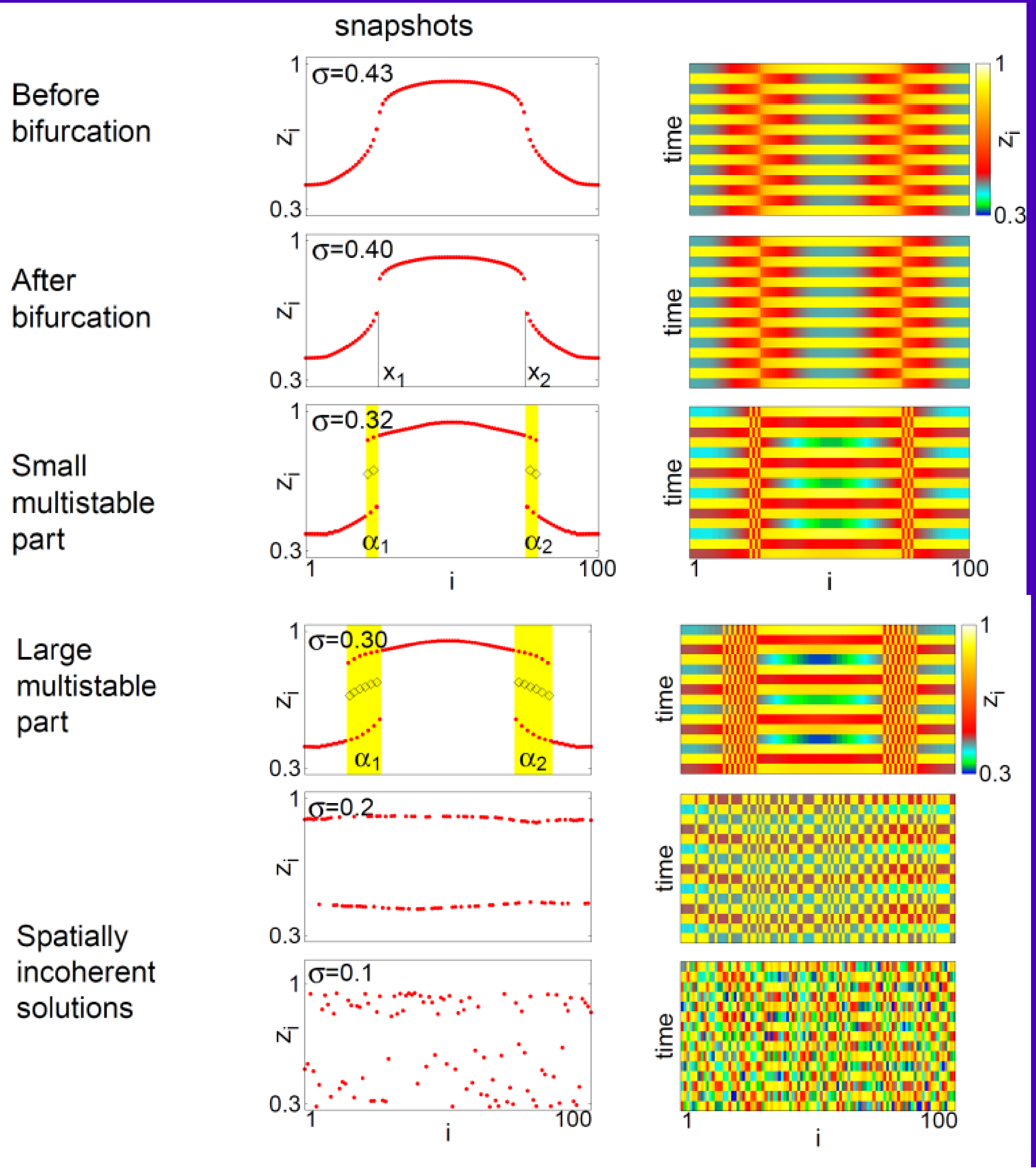


Bifurcation parameters:

$r = P/N$ - coupling radius, σ - coupling strength

CIB = Coherence-Incoherence Bifurcation

Coherence-incoherence transition (r=0.32)



I. Omelchenko, Yu. Maistrenko, P. Hövel, E. Schöll, Phys. Rev. Lett. **106**, 234102 (2011).

Analytical results: critical coupling strength

Continuum limit (large N), period-2 dynamics:

$$z_{1-j}(x) = (1-\sigma) f(z_j(x)) + \frac{\sigma}{2r}\int_{x-r}^{x+r} f(z_j(y))dy,$$

Transition from coherence to incoherence:
Profile becomes discontinuous (infinite slope)
at some point x –> neglect coupling term

$$z'_{1-j}(x) = (1-\sigma) f'(z_j(x)) z'_j(x) + \frac{\sigma}{2r}[f(z_j(x+r)) - f(z_j(x-r))].$$

Multiplying the eqs for even and odd time steps:

$$z'_0(x) z'_1(x) = [(1-\sigma)^2 f'(z_0(x)) f'(z_1(x))] z'_0(x) z'_1(x)$$

$$1 = (1-\sigma)^2 f'(z_0(x)) f'(z_1(x))$$

Logistic map f(z)=az(1-z), f'(z)=a(1-2z)
G(z)=0 at turning points x_c –> σ_c

$$G(x) = (1-\sigma)^2 a^2 [1 - 2z_0(x)][1 - 2z_1(x)] - 1$$

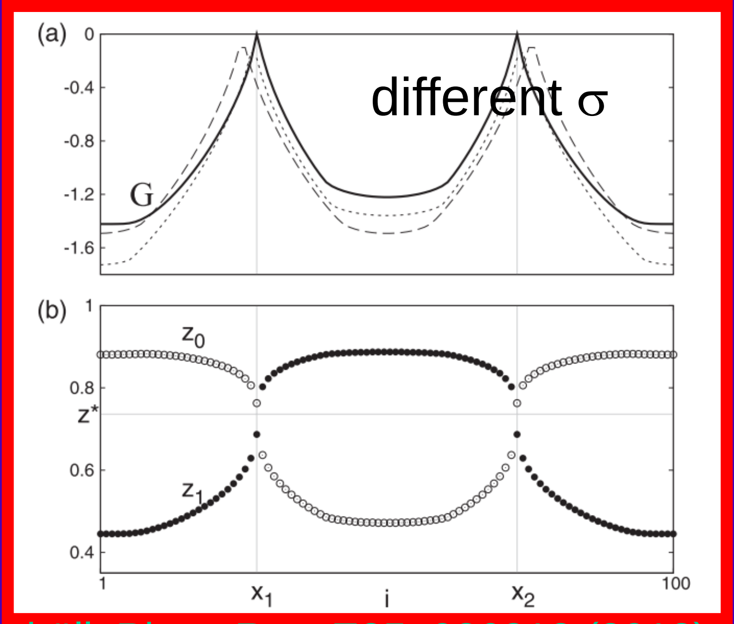
Analytical approximation: $z_0(x)=z_1(x)=z^*$ if G=0
with fixed point of map $z^*=f(z^*)=1-1/a$

$z^* \approx 0.737$. Under the assumption $z_0(x) = z_1(x) = z^*$ if $G = 0$, we obtain an approximation for σ:

$$G(x) = [a(1-\sigma)(1-2z^*)]^2 - 1 = 0 \qquad (11)$$

$$\Rightarrow \sigma \approx 1 - \frac{1}{a-2}. \qquad (12)$$

For $a = 3.8$ we get $\sigma \approx 0.44$.



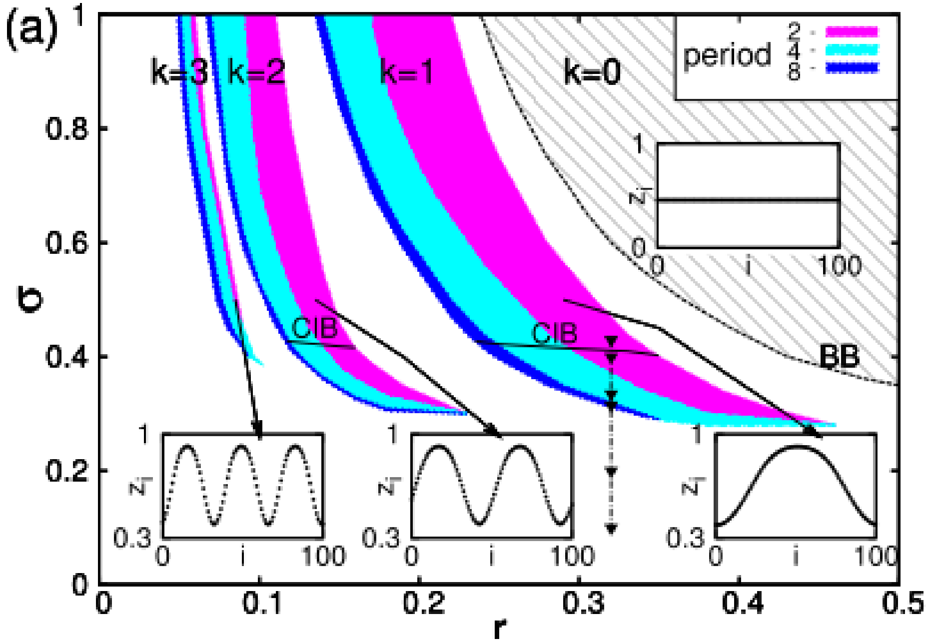
I. Omelchenko, B. Riemenschneider, P. Hövel, Y. Maistrenko, E. Schöll: Phys. Rev. E85, 026212 (2012)

Experimental realization

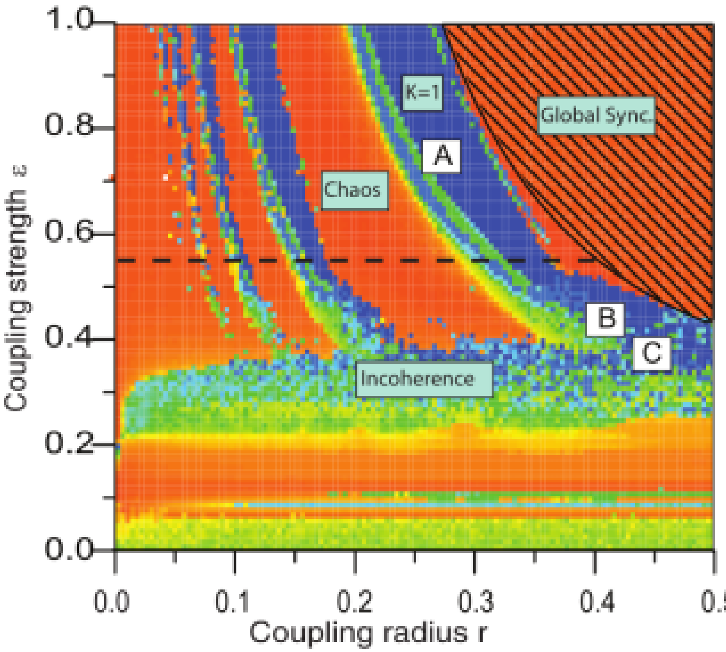
Experimental realization

Liquid crystal spatial light modulator

Simulation



Experiment

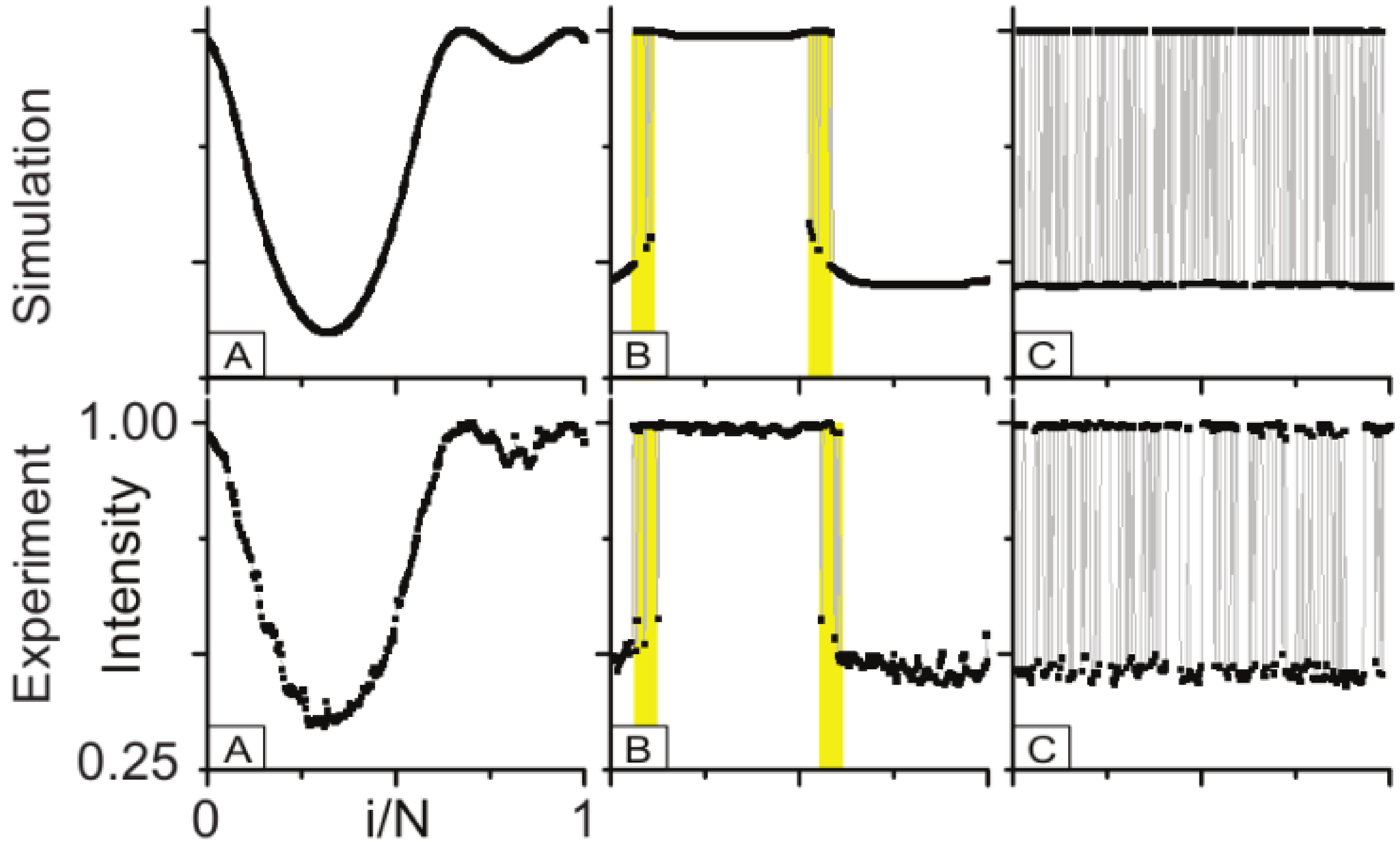


$$z_i^{t+1} = a z_i^t \left(1 - z_i^t\right)$$

$$z_i^{t+1} = \pi a \left(1 - \cos z_i^t\right)$$

A. M. Hagerstrom, T. E. Murphy, R. Roy, P. Hövel, I. Omelchenko, and E. Schöll: *Experimental Observation of Chimeras in Coupled-Map Lattices*, Nature Physics **8**, 658 (2012).

Comparison between experiments and simulation



A. M. Hagerstrom, T. E. Murphy, R. Roy, P. Hövel, I. Omelchenko, and E. Schöll:
Experimental Observation of Chimeras in Coupled-Map Lattices, Nature Physics **8**, 658 (2012).

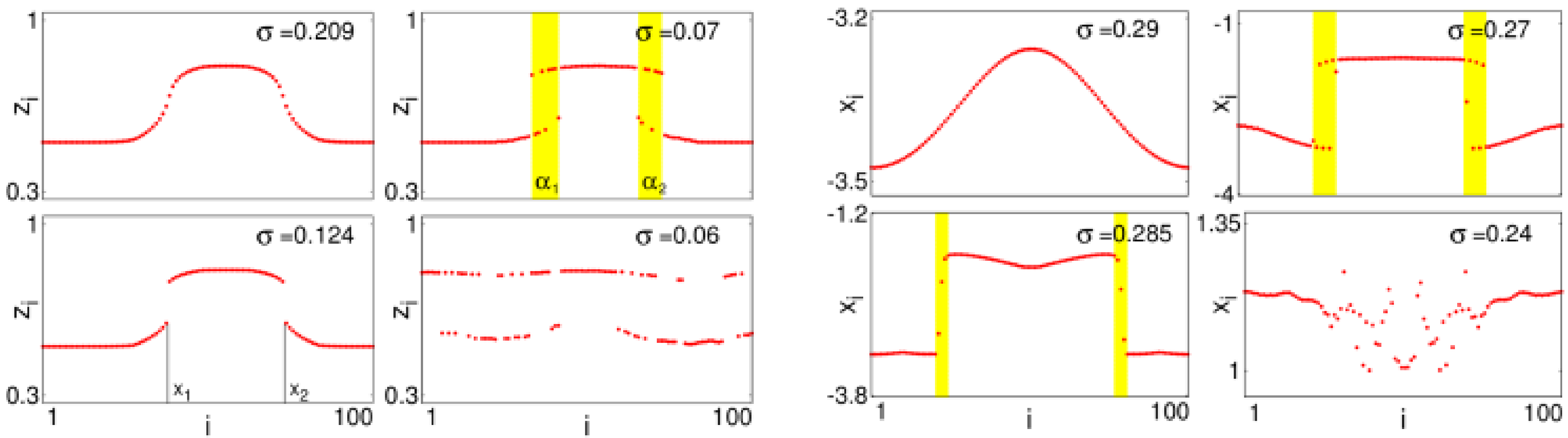
Comparison with time-continuous systems

Logistic map

Rössler model

$$z_i^{t+1} = a z_i^t \left(1 - z_i^t\right)$$

$$\dot{x}_i = -y_i - z_i$$
$$\dot{y}_i = x_i + a y_i$$
$$\dot{z}_i = b + z_i(x_i - c)$$



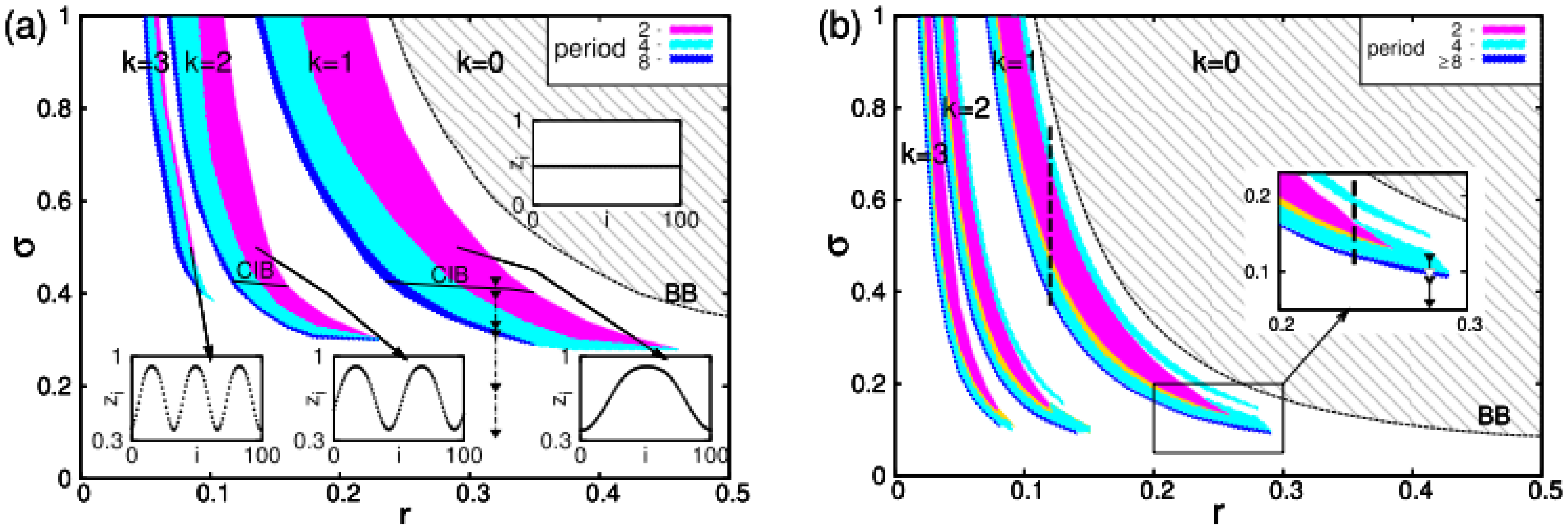
$a = 3.2$ (periodic), $r = 0.1$

$a = 0.42$, $b = 2$, $c = 4$, $r = 0.3$

I. Omelchenko, Yu. Maistrenko, P. Hövel, E. Schöll, Phys. Rev. Lett. **106**, 234102 (2011).

Structure of coherence-incoherence tongues

Logistic map ($a = 3.8$) | Rössler model



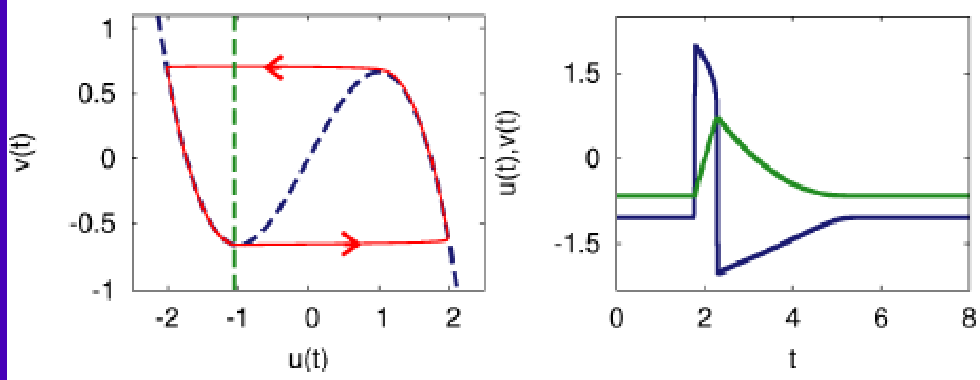
I. Omelchenko, B. Riemenschneider, P. Hövel, Yu. Maistrenko, and E. Schöll: *Transition from spatial coherence to incoherence in coupled chaotic systems*, Phys. Rev. E **85**, 026212 (2012).

Neural networks: FitzHugh-Nagumo system

The FitzHugh-Nagumo model for neuronal activity

with activator u, inhibitor v: $\mathbf{x} = (u, v)^T$

$$\mathbf{F}(\mathbf{x}) = \begin{pmatrix} \frac{1}{\epsilon}(u - \frac{u^3}{3} - v) \\ u + a \end{pmatrix}$$



- operation in the excitable regime
- uncoupled neurons rest in fixed point
- operation in the oscillatory regime ($a < 1$)
- uncoupled: oscillates periodically

FitzHugh-Nagumo (FHN) network

$$\varepsilon \frac{du_k}{dt} = u_k - \frac{u_k^3}{3} - v_k + \frac{\sigma}{2P} \sum_{j=k-P}^{k+P} \left[b_{\mathrm{uu}}(u_j - u_k) + b_{\mathrm{uv}}(v_j - v_k) \right]$$

$$\frac{dv_k}{dt} = u_k + a_k + \frac{\sigma}{2P} \sum_{j=k-P}^{k+P} \left[b_{\mathrm{vu}}(u_j - u_k) + b_{\mathrm{vv}}(v_j - v_k) \right].$$

σ – coupling strength (control parameter!)

$r = P/N$ – coupling radius (control parameter!)

ε – small parameter

$a_k,\ k = 1, ..., N$ – threshold parameters, $a_k \equiv a \in (-1, 1)$

Local interaction matrix:

$$B = \begin{pmatrix} b_{\mathrm{uu}} & b_{\mathrm{uv}} \\ b_{\mathrm{vu}} & b_{\mathrm{vv}} \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}, \quad \phi \in [-\pi, \pi)$$

For what parameters expect chimeras?

$$\varepsilon \frac{du_k}{dt} = u_k - \frac{u_k^3}{3} - v_k + \frac{\sigma}{2P} \sum_{j=k-P}^{k+P} \left[b_{uu}(u_j - u_k) + b_{uv}(v_j - v_k) \right],$$

$$\frac{dv_k}{dt} = u_k + a_k + \frac{\sigma}{2P} \sum_{j=k-P}^{k+P} \left[b_{vu}(u_j - u_k) + b_{vv}(v_j - v_k) \right]$$

⇓ ⇓ ⇓

Phase reduction of FitzHugh-Nagumo model

$$\frac{d\theta_k}{dt} = -\frac{1}{2R} \sum_{j=k-R}^{k+R} \left[H(\theta_k - \theta_j) - H(0) \right], \quad k = 1, ..., N,$$

$H(\Psi)-$ is a $T-$periodic function

Phase oscillator model

Nonlocally coupled phase oscillators:

$$\frac{d\theta_k}{dt} = \omega - \frac{1}{2R} \sum_{j=k-R}^{k+R} \sin(\theta_k(t) - \theta_j(t) + \alpha), \quad k = 1, ..., N,$$

θ_k- phases, $\alpha-$ phase lag parameter.

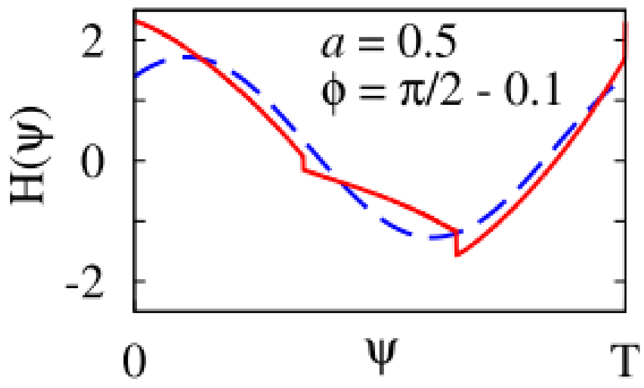
Chimera states found for α close to but less then $\pi/2$.

• O. Omel'chenko, M. Wolfrum, Yu. Maistrenko. *Phys. Rev. E* **81**, 065201(**R**) (2010).

Compare with phase oscillator model: find appropriate value of ϕ

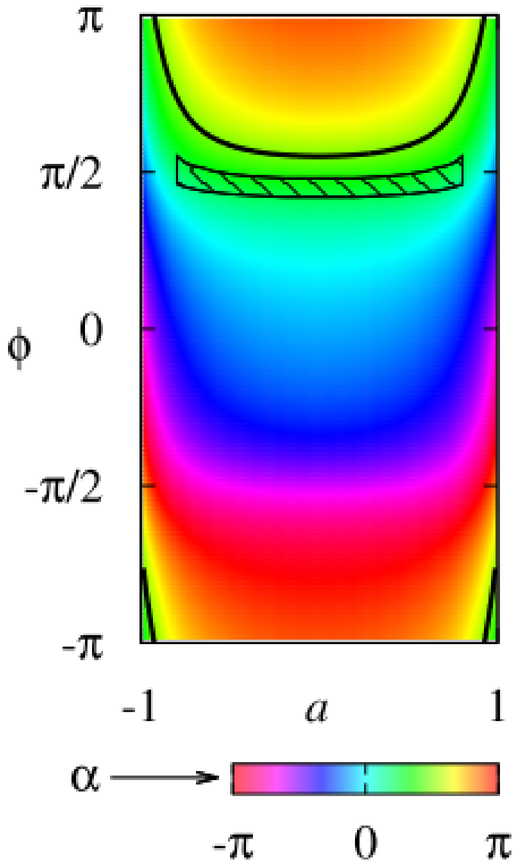
Approximation:

$$H(\Psi) \approx \frac{h_0}{2} + h_1 \sin\left(\frac{2\pi}{T}\Psi + \alpha\right)$$

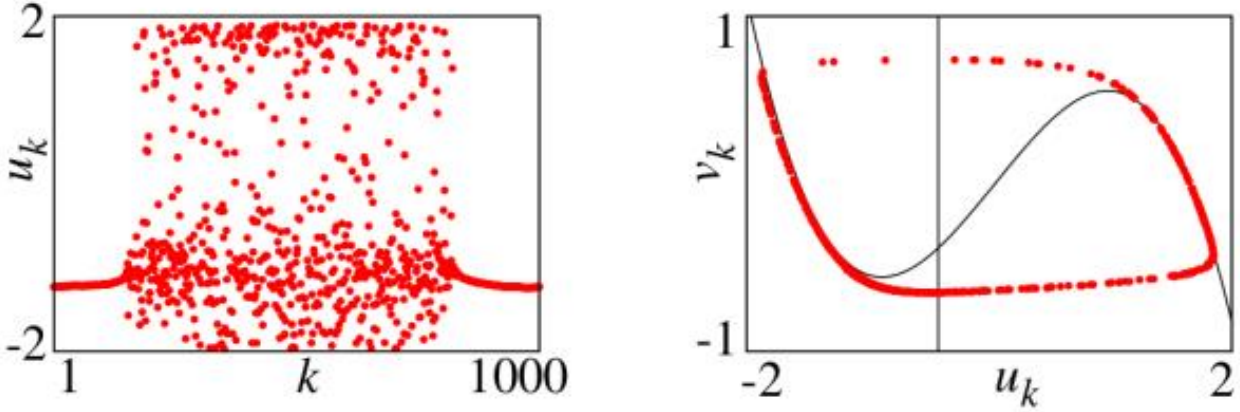


Chimera states can be observed for pronounced off-diagonal coupling ($\phi \approx \pi/2$).

$$\alpha = \alpha(a, \phi)$$



Chimera states in FHN networks



System parameters:

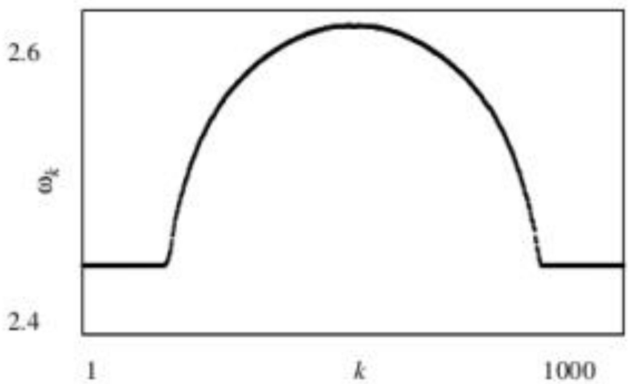
$N = 1000$ – large system
$r = 0.35$ – intermediate coupling radius
$\sigma = 0.1$ – small coupling strength
$a = 0.5, \quad \phi = \pi/2 - 0.1$

I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **110**, 224101 (2013).

Chimera states in FHN networks

FitzHugh-Nagumo system

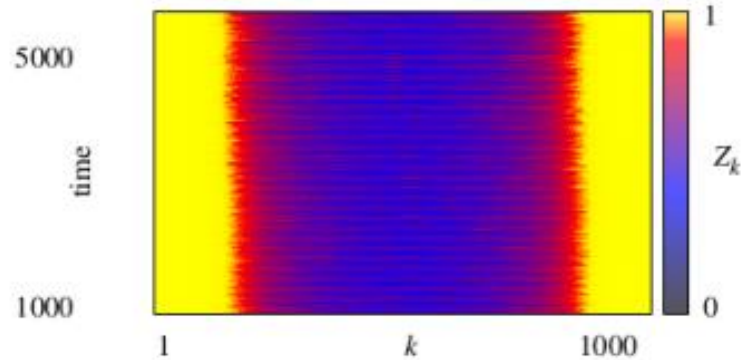
Mean phase velocities



$$\omega_k = 2\pi M_k / \Delta T, \quad k = 1, \ldots, N$$

M_k – number of complete rotations of k-th unit during the time interval ΔT.

Local order parameter



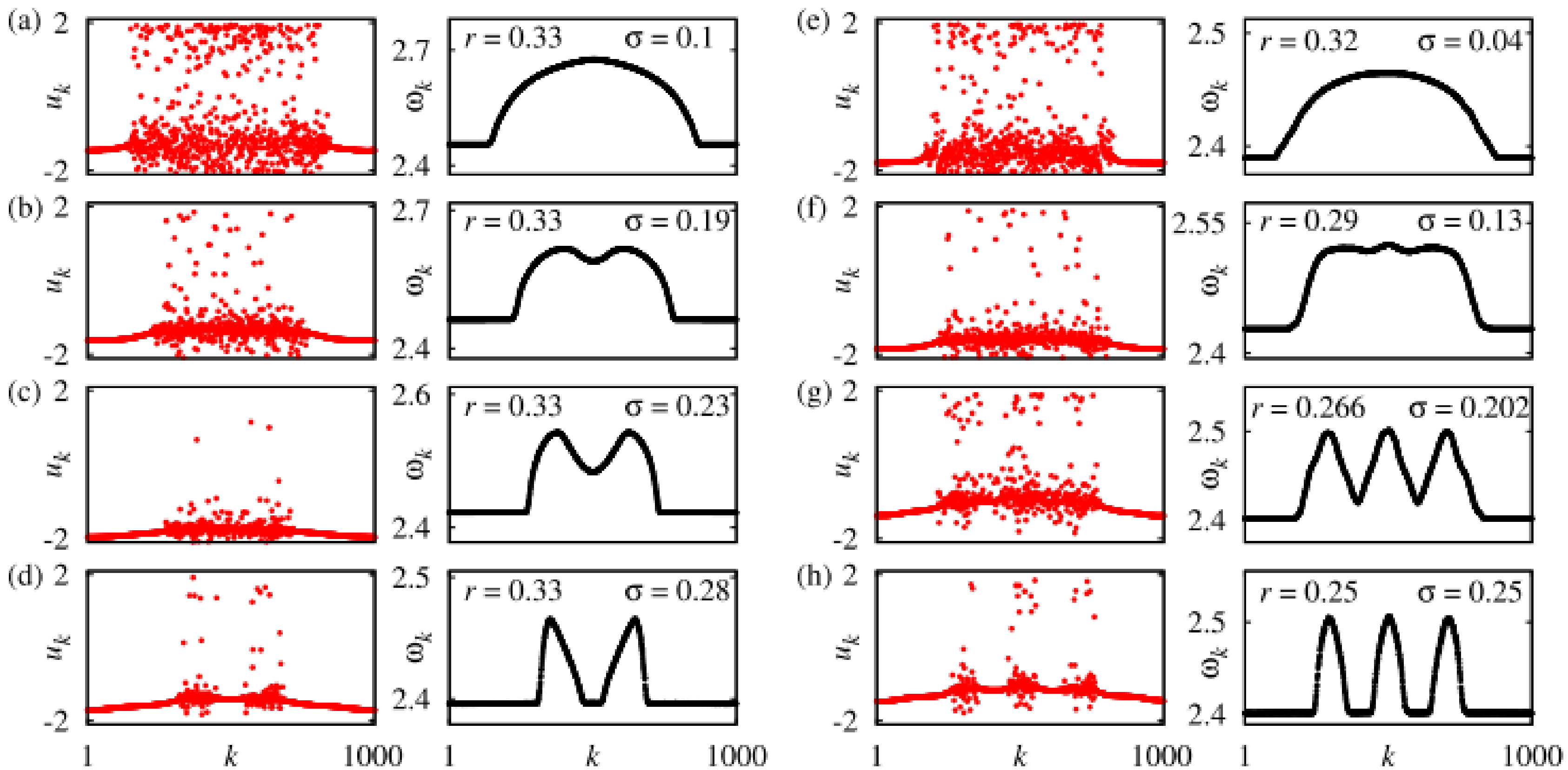
$$Z_k = \left| \frac{1}{2\delta} \sum_{|j-k| \le \delta} e^{i\Theta_j} \right|, \quad k = 1, \ldots, N$$

$\Theta_j = \arctan(v_j / u_j)$

Spatial average window $\delta = 25$

I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **110**, 224101 (2013).

Multi-chimera states for strong coupling

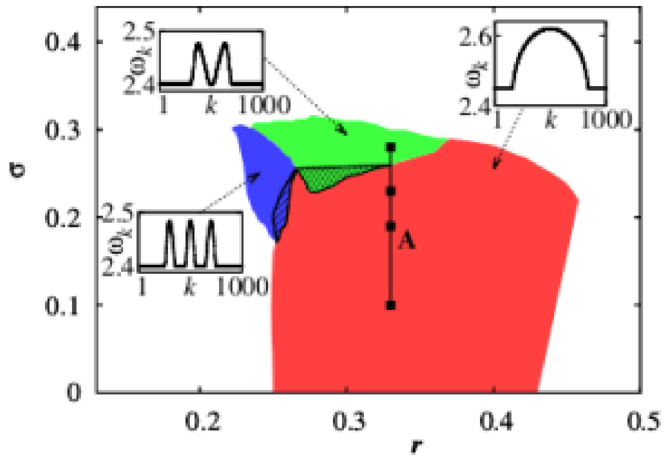


I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll,, Phys. Rev. Lett. **110**, 224101 (2013).

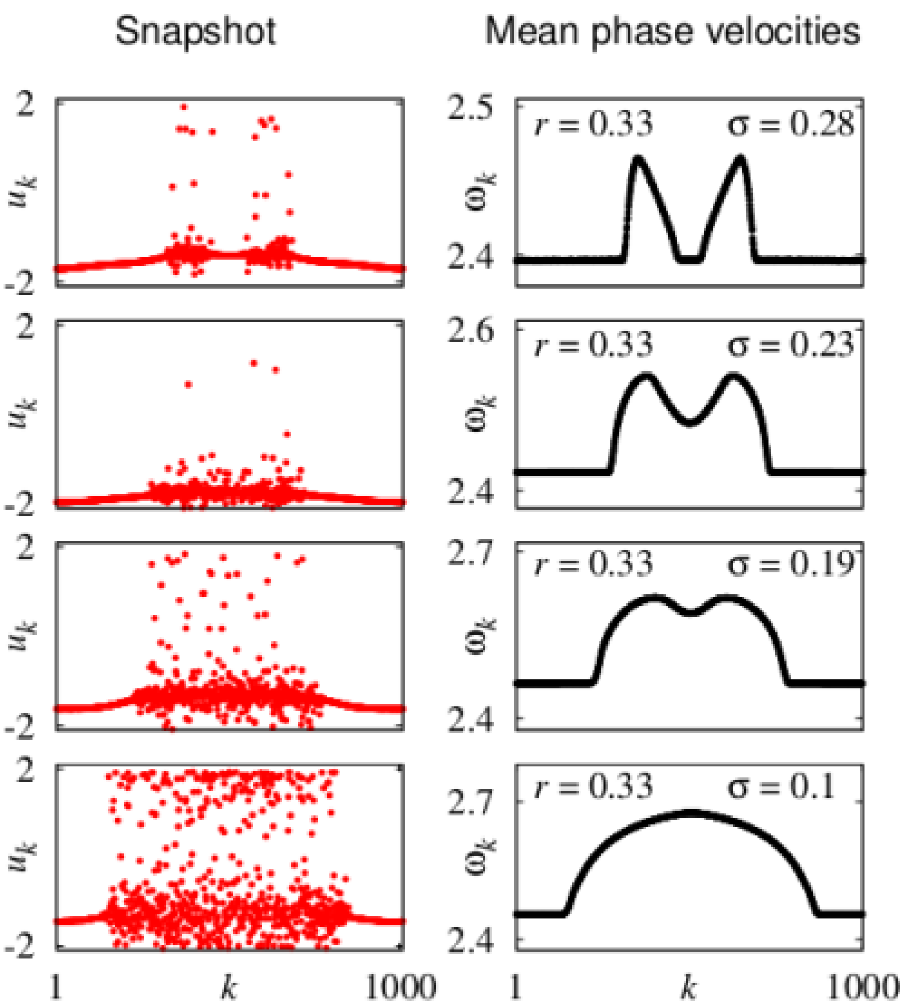
Two-chimera states

FitzHugh-Nagumo system (line A)

Transition from chimera with one incoherent part to multi-chimera with two incoherent parts (along line A, from bottom to top)



$a = 0.5, \quad \phi = \pi/2 - 0.1$

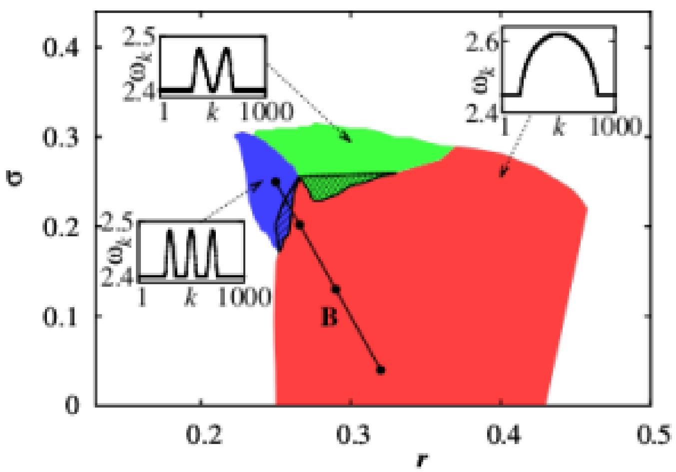


I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **110**, 224101 (2013).

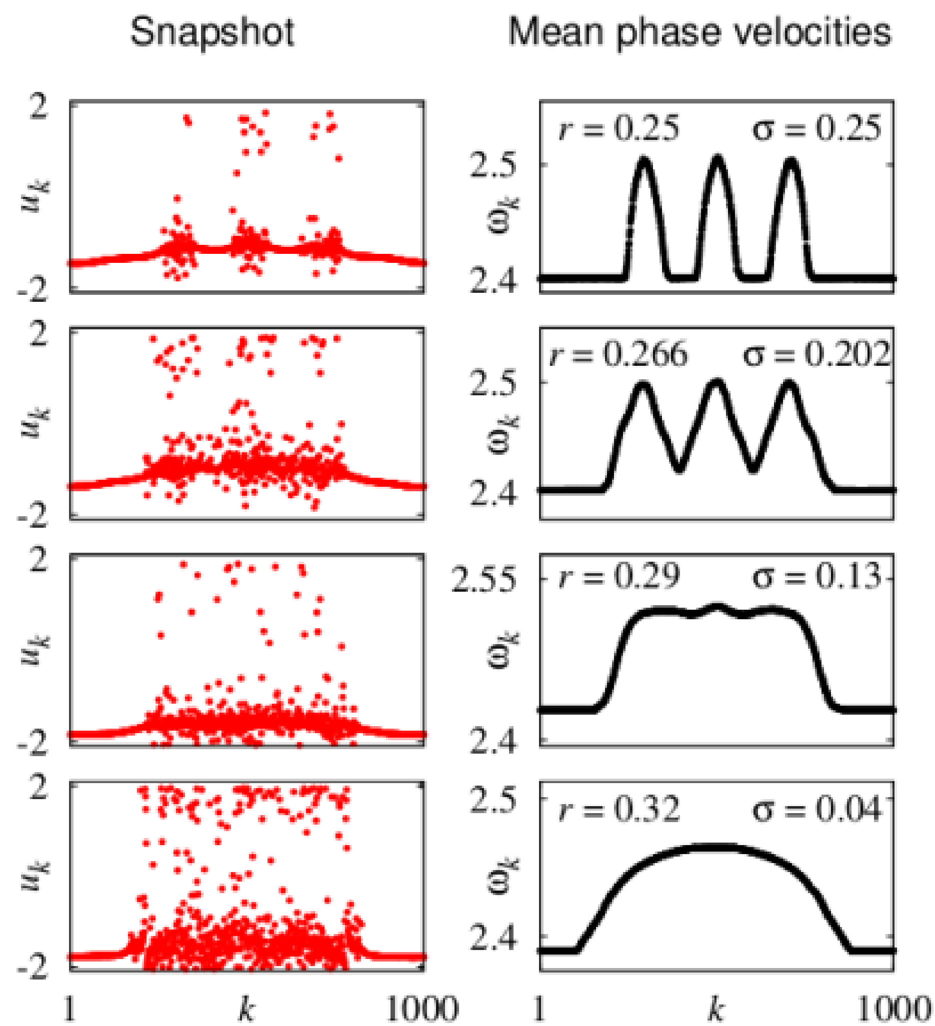
Three-chimera states

FitzHugh-Nagumo system (line B)

Transition from chimera with one incoherent part to multi-chimera with three incoherent parts (along line B, from bottom to top)



$a = 0.5, \quad \phi = \pi/2 - 0.1$



I. Omelchenko, O. E. Omel'chenko, P. Hövel, and E. Schöll, Phys. Rev. Lett. **110**, 224101 (2013).

Conclusions

- Chimera states in nonlocally coupled networks
  - Spontaneous synchrony breaking in networks of identical oscillators: splitting in spatially coherent and incoherent domains
  - Transition from coherence to incoherence via chimera states: logistic map, Rössler oscillator
  - Experiment with liquid crystal spatial light modulator
  - Multi-chimera states in the FitzHugh-Nagumo model
  - Application to neurosystems: some dolphins and birds sleep with one half of their brain

In collaboration with:

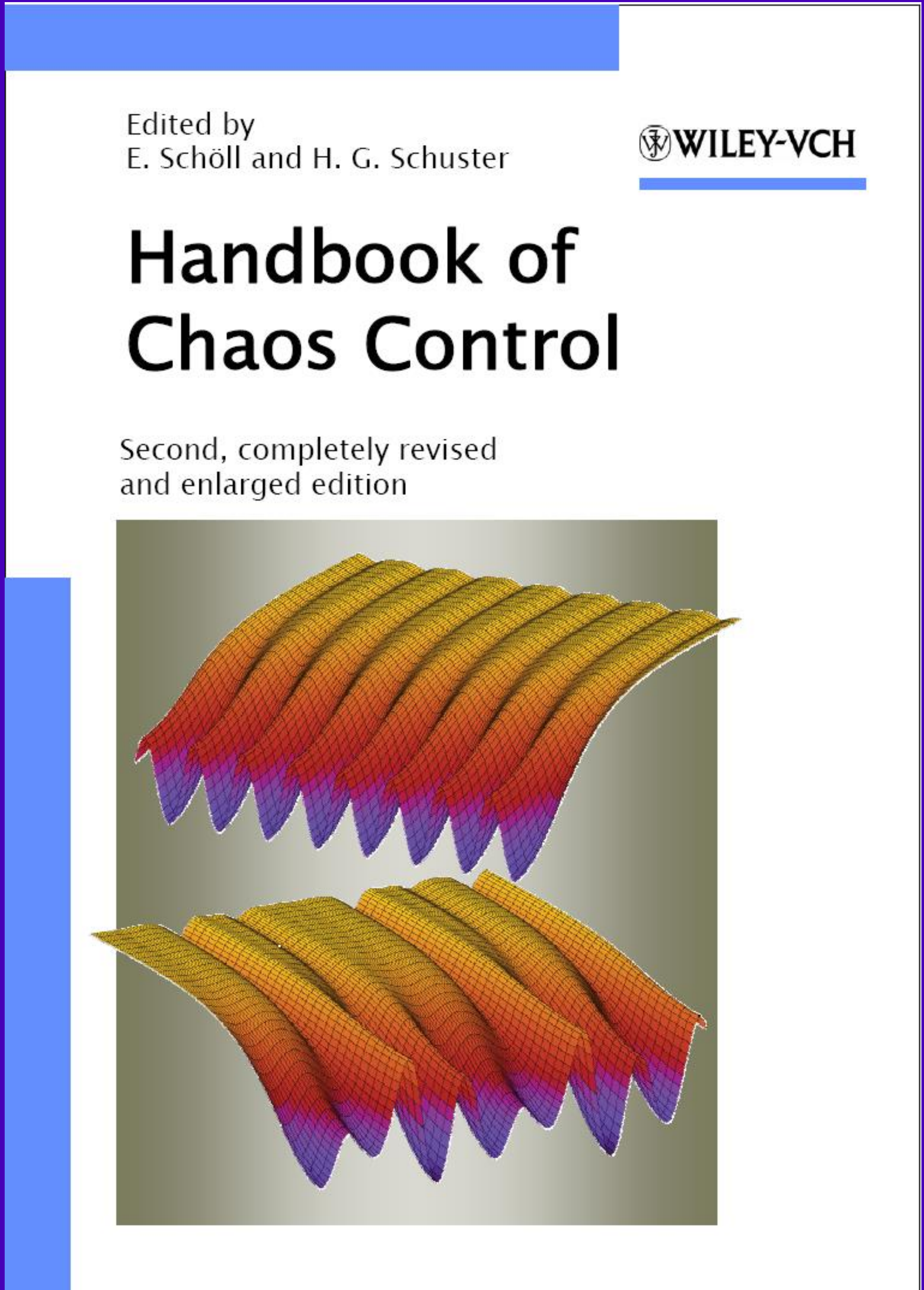
Philipp Hövel   Iryna Omelchenko   Anna Zakharova

Further collaborators

Yuri Maistrenko (Kiev)
Oleh Omel‘chenko (WIAS Berlin)
Aaron Hagerstrom (Univ of Maryland, USA)
Thomas Murphy (Univ. of Maryland, USA)
Rajarshi Roy (Univ. of Maryland, USA)

Students

Thomas Dahms   Andrew Keane
Thomas Isele   Judith Lehnert
Marie Kapeller   Winnie Poel
David Rosin   Alice Schwarze
Andrea Vüllings   Carolin Wille



Published 2008

Suppression of chaos, stabilization of unstable states: Steady states, periodic states, spatio-temporal patterns

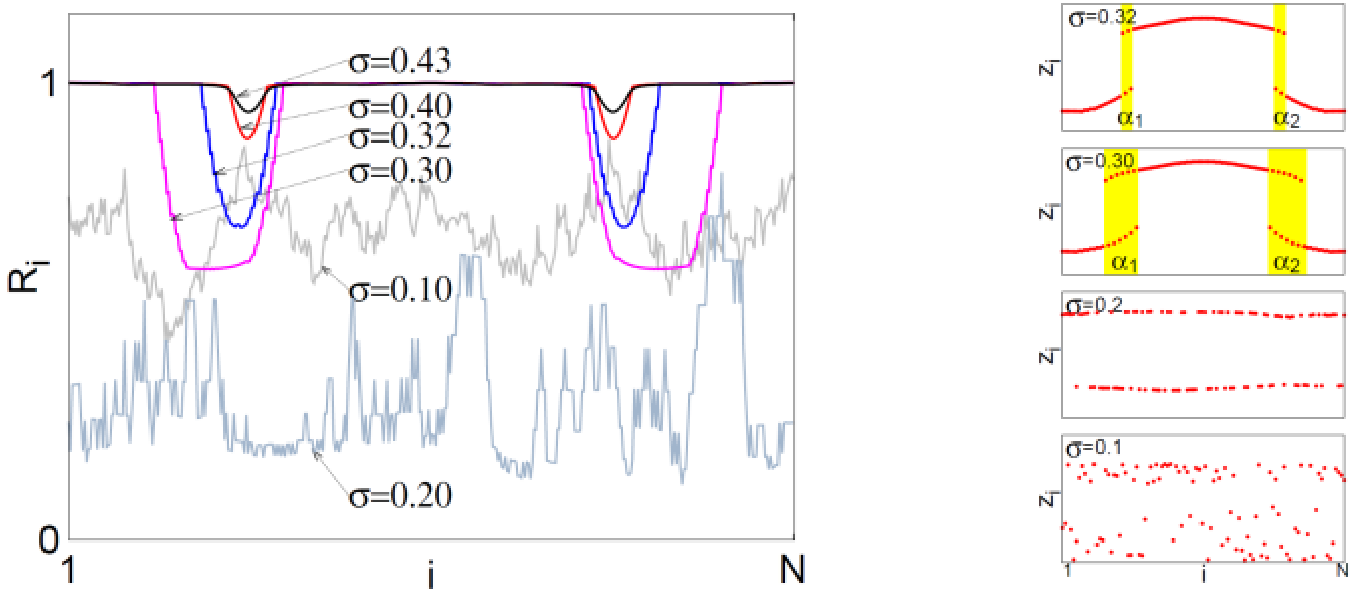
Local order parameter als measure for spatial coherence

Local order parameter

$$R_i = \lim_{N\to\infty} \frac{1}{2\delta(N)} \left| \sum_{|j/N - i/N| \le \delta} e^{i\psi_j} \right|, \quad i = 1, \ldots, N$$

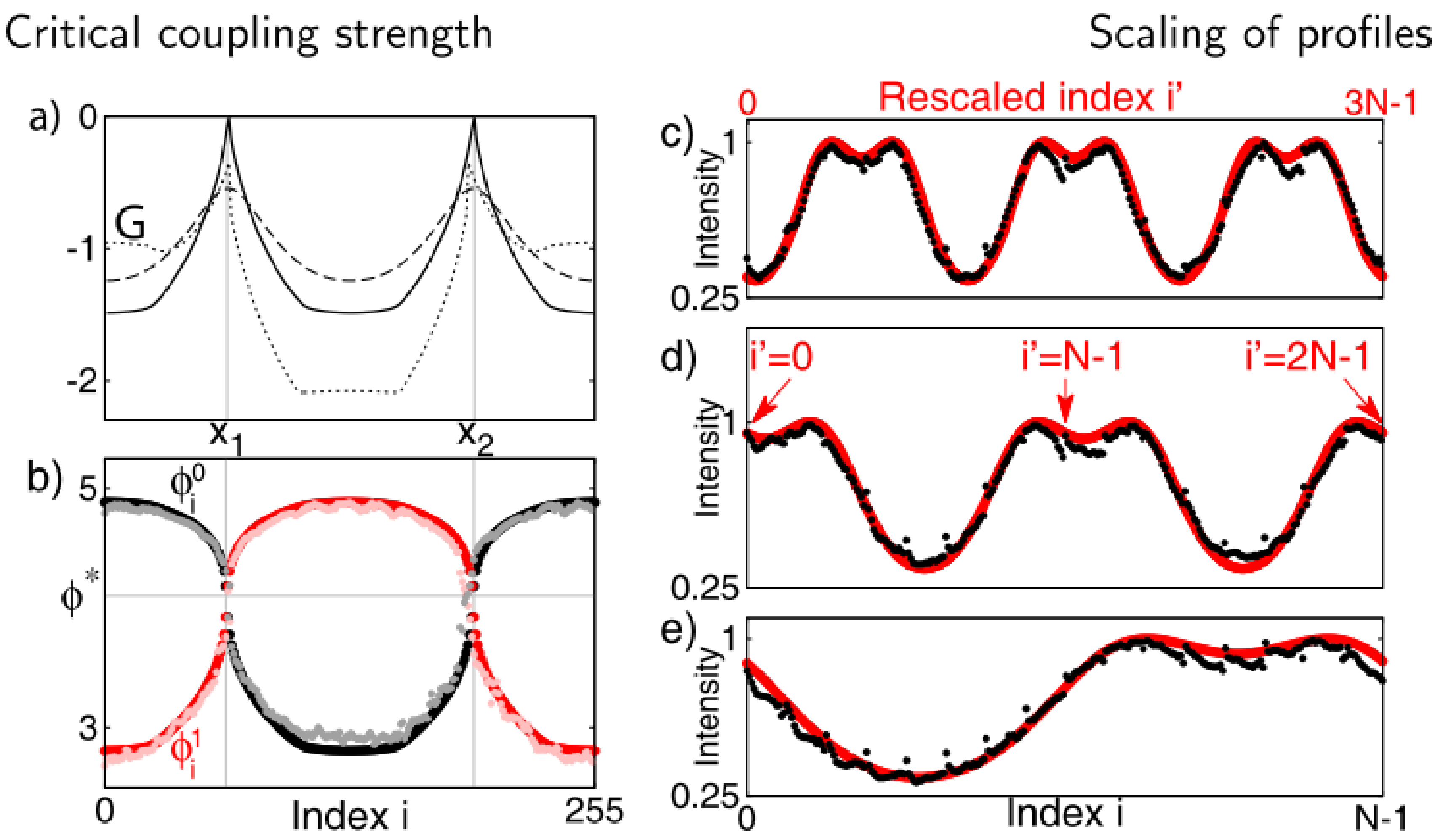
$$\psi_j \quad \Rightarrow \quad \sin\psi_j = (2z_j - \max_j z_j - \min_j z_j)/(\max_j z_j - \min_j z_j),$$

$$\delta(N) \to 0 \text{ for } N \to \infty.$$



I. Omelchenko, Yu. Maistrenko, P. Hövel, E. Schöll, Phys. Rev. Lett. **106**, 234102 (2011).

Analytical results for spatial light modulator



A. M. Hagerstrom, T. E. Murphy, R. Roy, P. Hövel, I. Omelchenko, and E. Schöll: *Experimental Observation of Chimeras in Coupled-Map Lattices*, Nature Physics **8**, 658 (2012).

Experimental setup: spatial light modulator

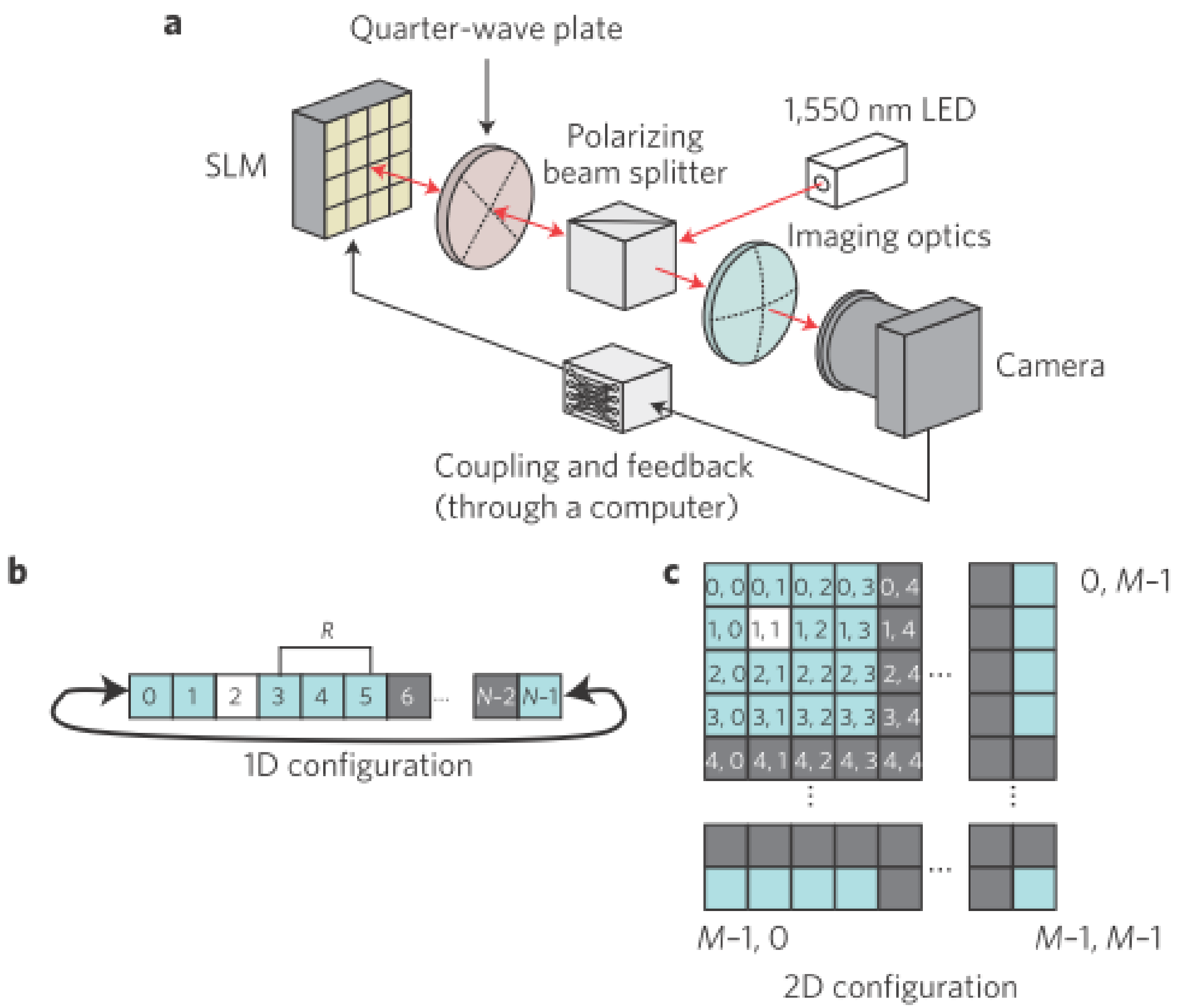


Figure 1 | Experimental apparatus. a, Optical configuration. Polarization optics create a nonlinear relationship between the spatially dependent phase shift applied by the SLM and the intensity of the light falling on the camera. Feedback and coupling are implemented using a computer.