UNIVERSITÄT DUISBURG ESSEN

Magnetic Friction in Ising Systems

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Overview

Uni Oldenburg 25.06.09

The Ising model

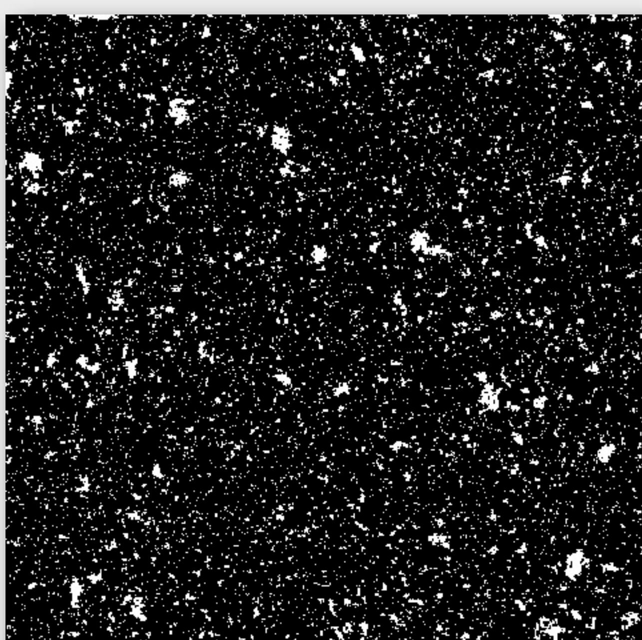
- Model for magnets, liquid/gas or other binary mixtures, epidemics, neural networks, ...

- Consider spin variables $\sigma_i = \pm 1$ on a two dimensional lattice
  Hamiltonian gives energy of configuration (J: coupling, B: magnetic field)

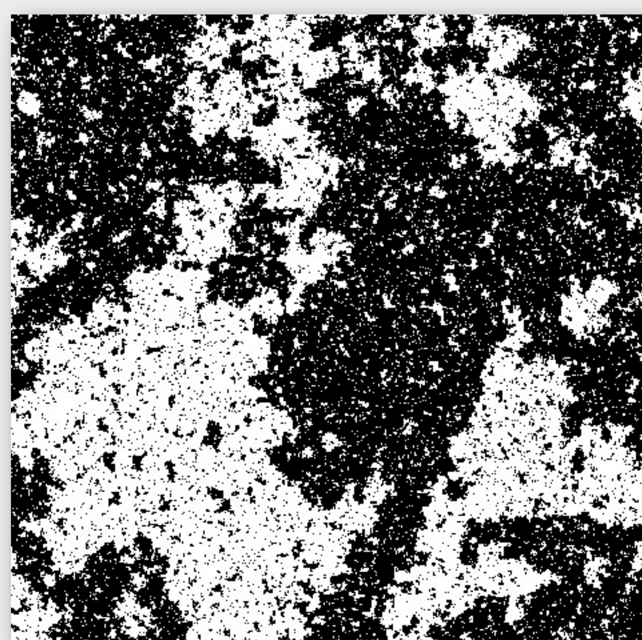
$$\mathcal{H} = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - B \sum_i \sigma_i$$

- Ising model shows continuous phase transition at critical temperature

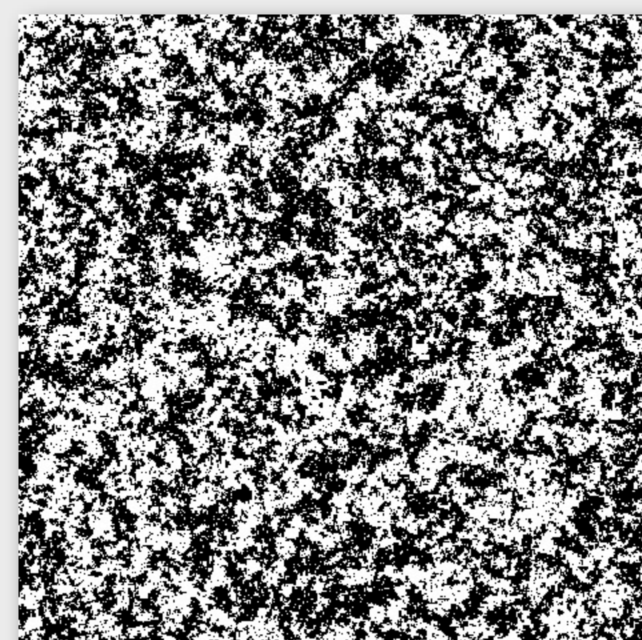
$T = 0.96\ T_c$



$T = T_c$



$T = 1.1\ T_c$



Universality at continuous phase transitions

- Universality: many quantities independent on details
  E.g. bulk quantities: critical exponents

- Universality hypothesis [1]: only important are
  OP dimension n and space dimension d

  - Ising ($n = 1$): uniaxial ferromagnets, liquid-gas transition, binary liquid mixtures

  - XY ($n = 2$): planar ferromagnet, superconductors, liquid crystals, superfluid ^{4}He

  - Heisenberg ($n = 3$): isotropic ferromagnets

[1] L. P. Kadanoff, Proceedings of the Varenna summer school on critical phenomena (1970)

Monte Carlo method

- Use computer simulations to calculate expectation values
- Generate independent equilibrium configurations with Metropolis algorithm:
  - flip random spin and calculate ΔE
  - accept spin flip with, e. g.,

$$p_{\text{flip}}^{\text{MP}}(\Delta E) = \min(1, e^{-\beta\,\Delta E})$$

- Detailed balance guarantees that we reach steady state:

$$0 = \dot{P}_i(t) = \sum_j P_j(t) w_{j\to i} - P_i(t) w_{i\to j} \quad \Leftarrow \quad \frac{w_{j\to i}}{w_{i\to j}} = \frac{P_i(t)}{P_j(t)} = e^{-\beta \Delta E_{ij}}$$

What is magnetic friction?

- Energy dissipation in driven system through magnetic degrees of freedom

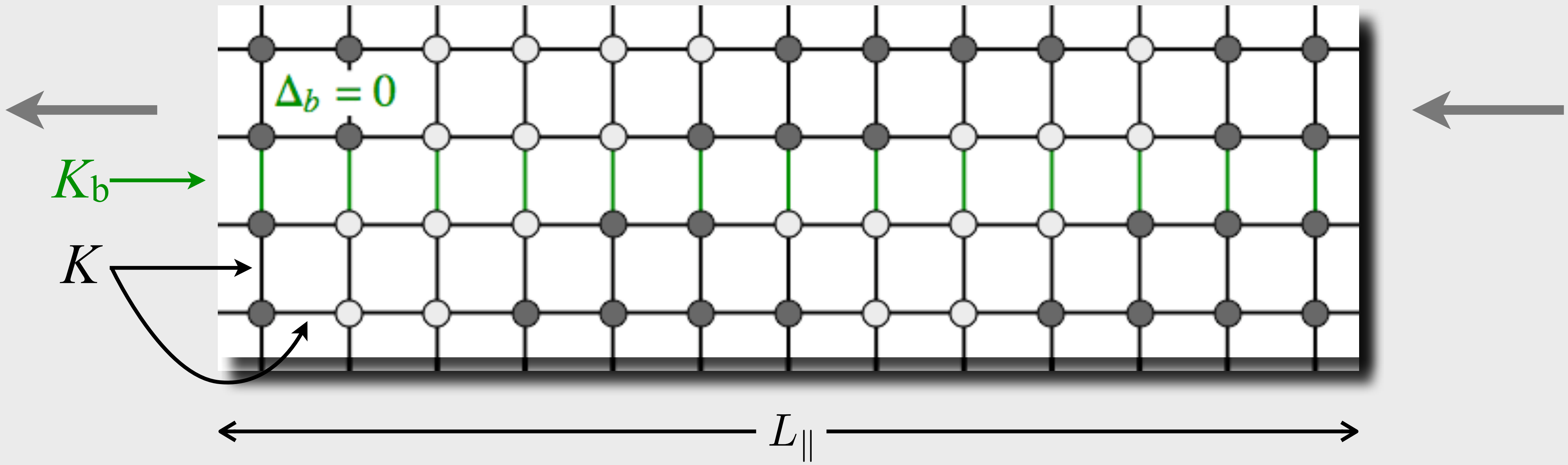
motion → exchange/dipole interaction → spins → heat bath

- Note:
  Eddy currents (Wirbelströme) + Joule heating in
  also denoted "magnetic friction"
  - eddy current
  - ...

no spin degrees of freedom!

motion → magnetic field → electrons → phonons

Ising model with moving boundary conditions

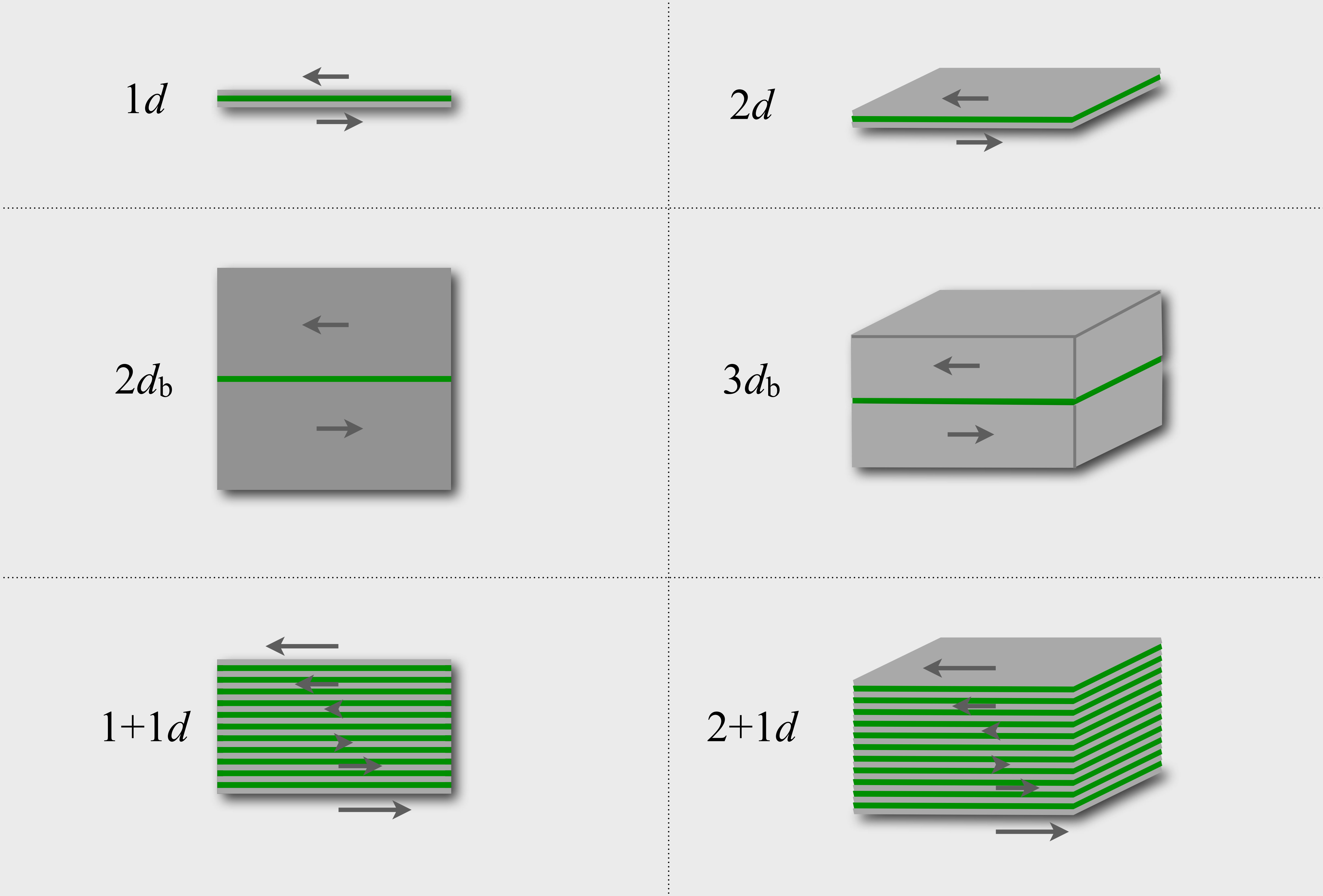


- Time–dependent Hamiltonian
  $\beta = 1/k_\mathrm{B}T,\ K = \beta J$

$$\beta\mathcal{H}(t) = -K \sum_{\langle ij \rangle} \sigma_i \sigma_j - K_\mathrm{b} \sum_{\langle ij \rangle_\mathrm{b}(t)} \sigma_i \sigma_j$$

- $\Delta_\mathrm{b}(t) = vt$
- $v = 1 \quad \widehat{=} \quad 1\,\mathrm{cm/s}$

Geometries



Non–equilibrium quantities: Monte Carlo results

- Energy dissipation rate

$$P(v) = \frac{\langle \Delta E_\mathrm{b} \rangle}{\Delta t}$$

- Friction force

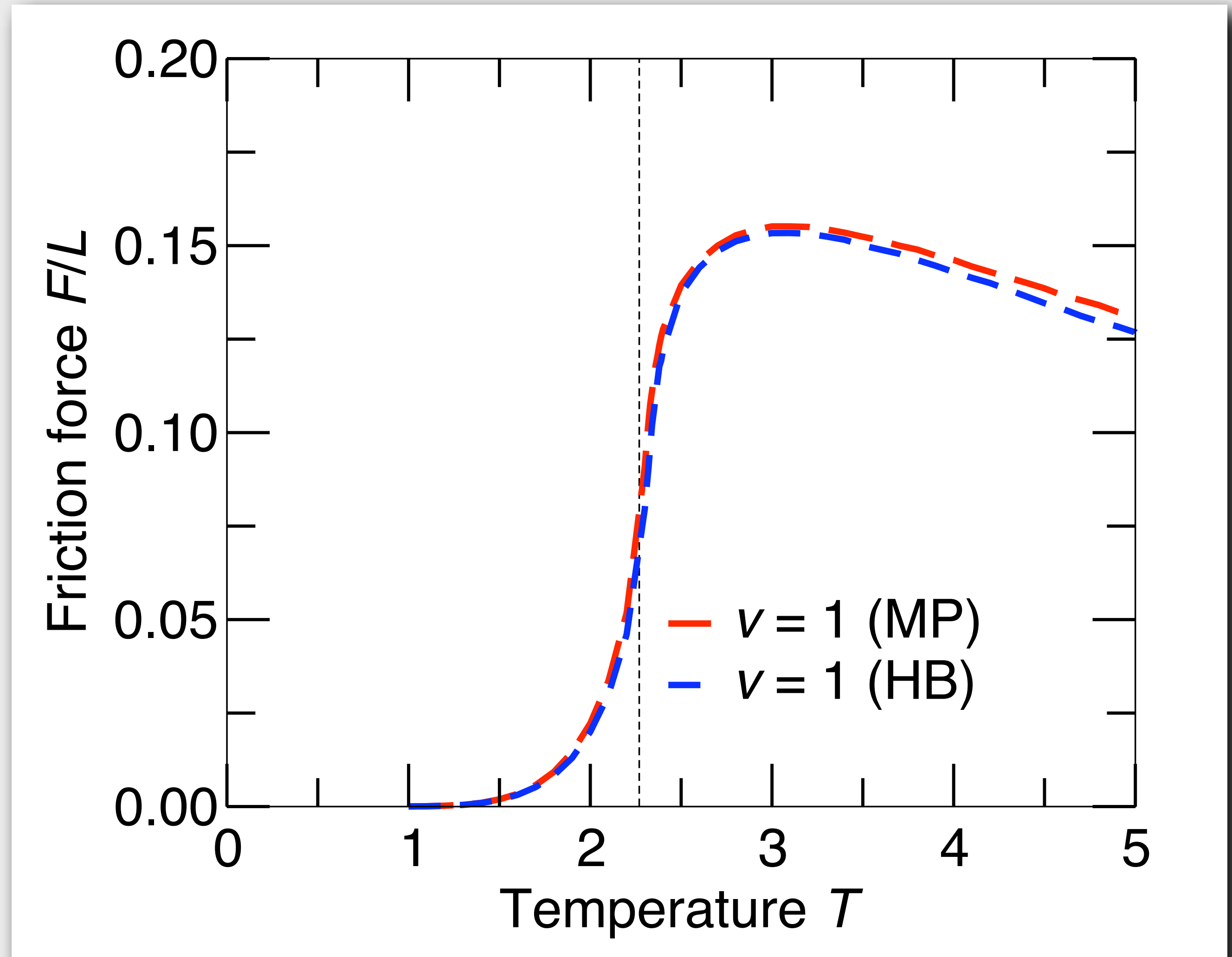
$$F(v) = \frac{P(v)}{v}$$

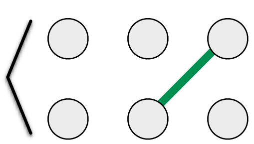
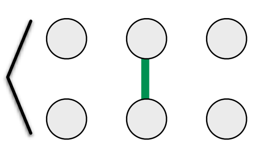
- Shear stress τ in $3d_\mathrm{b}$ case:

Scherspannung

$$\tau(v) = \frac{F(v)}{L^2} \approx 10\,\mathrm{MPa}$$

- Exact solution for small v



$$F(v \to 0) = \frac{\langle E_\mathrm{b}^{(f)} - E_\mathrm{b}^{(i)} \rangle}{\Delta s} = -LJ_\mathrm{b}\Big(\Big\langle$$  $$\Big\rangle_0 - \Big\langle$$  $$\Big\rangle_0\Big)$$

$$= -LJ_\mathrm{b}\big(\langle \sigma_{0,0}\sigma_{1,1}\rangle_0 - \langle \sigma_{0,0}\sigma_{0,1}\rangle_0\big)$$

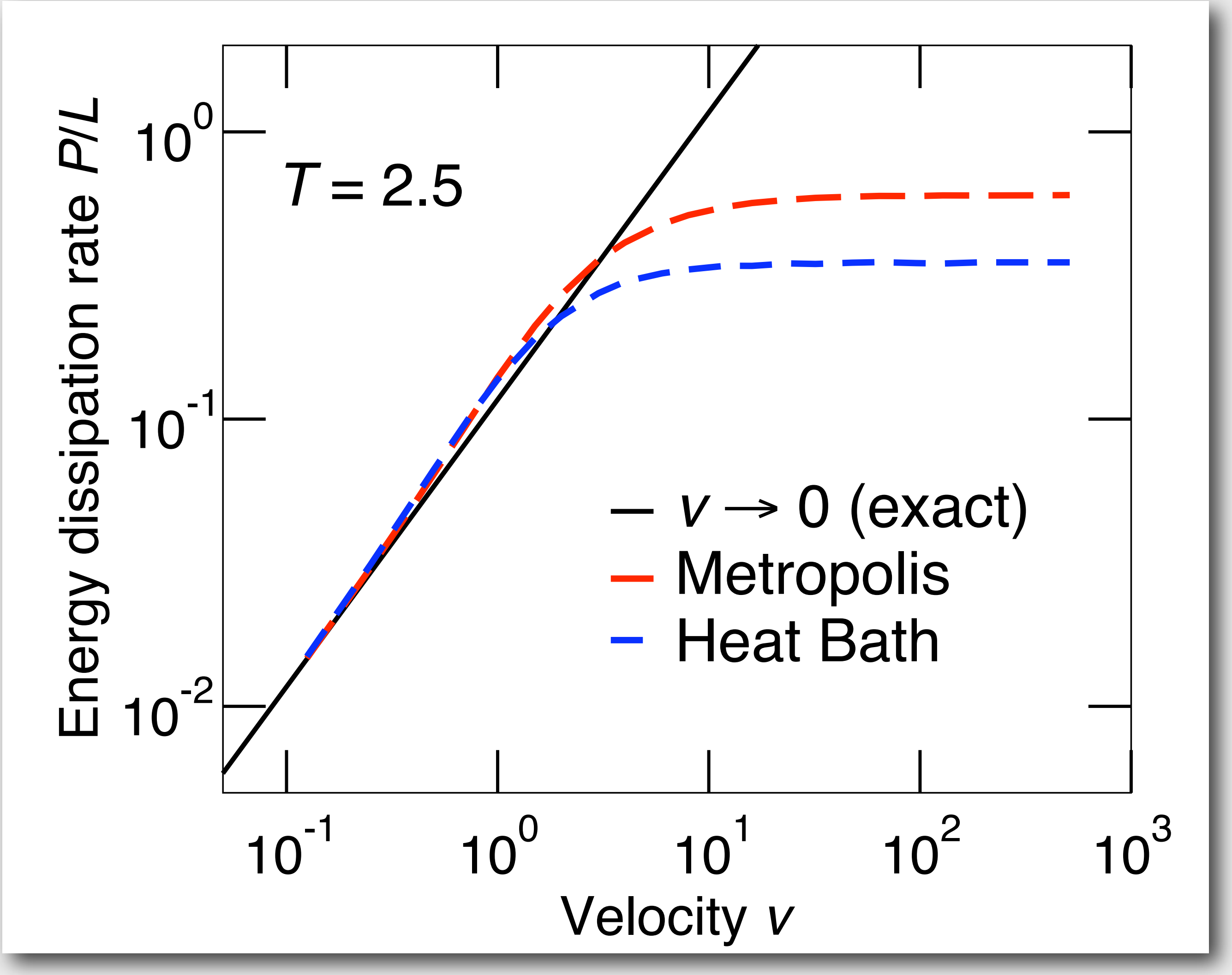
D. Kadau, A.H. & D.E. Wolf, Phys. Rev. Lett. 101, 137205 (2008)

Influence of finite driving velocity

- Dissipation P saturates at high velocities $v \gg 1$
- $3d_\mathrm{b}$ case:

$$P(v \gg 1) \approx 100\,\mathrm{W cm^{-2}}$$

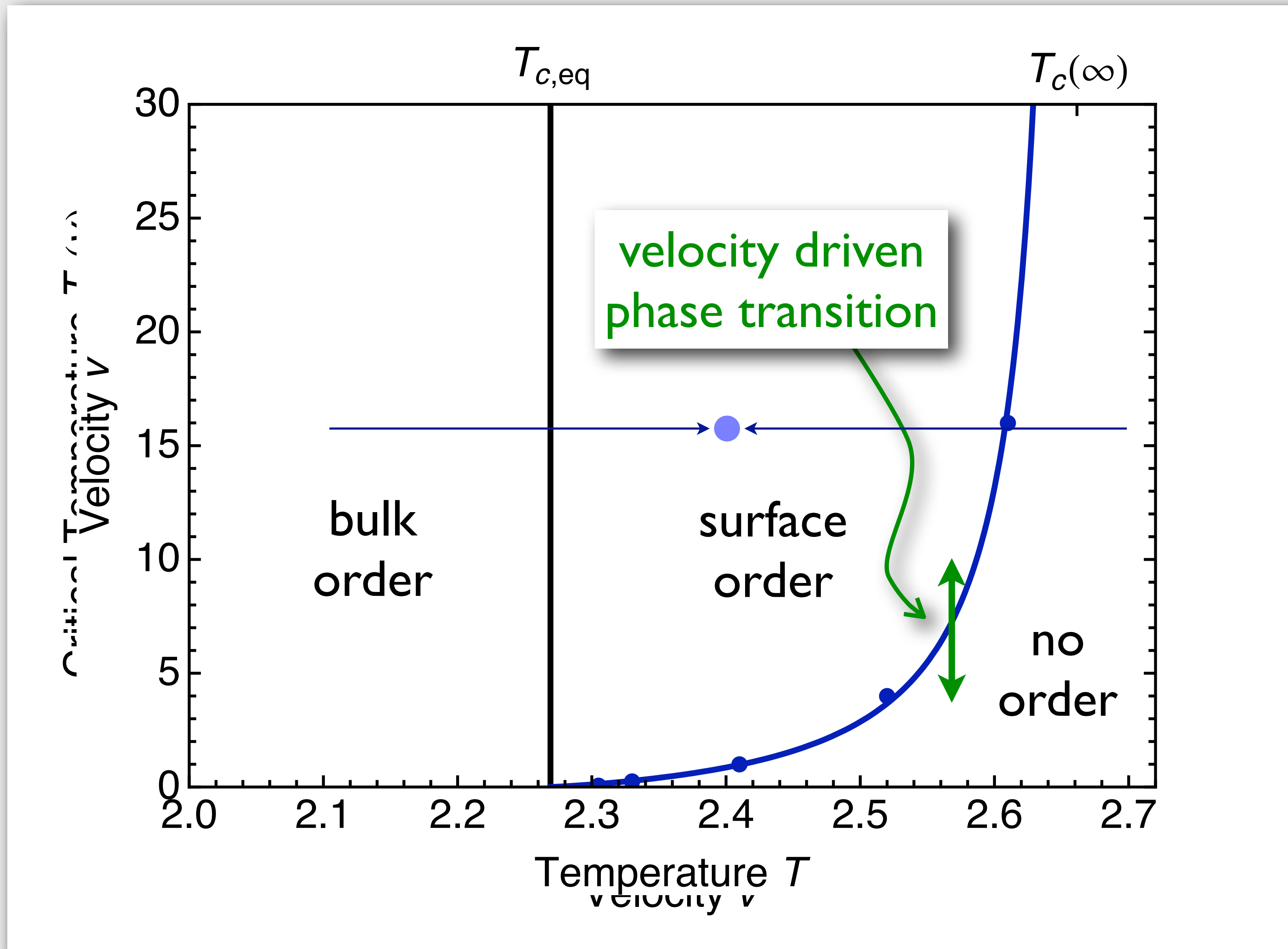
- Quantities depend on Monte Carlo algorithm



D. Kadau, A.H. & D.E. Wolf, Phys. Rev. Lett. 101, 137205 (2008)

The phase diagram

- Critical temperature T_c depends on velocity v
- Phase diagram with 3 phases:
  - bulk order
  - surface order
  - no order
- Velocity driven phase transition



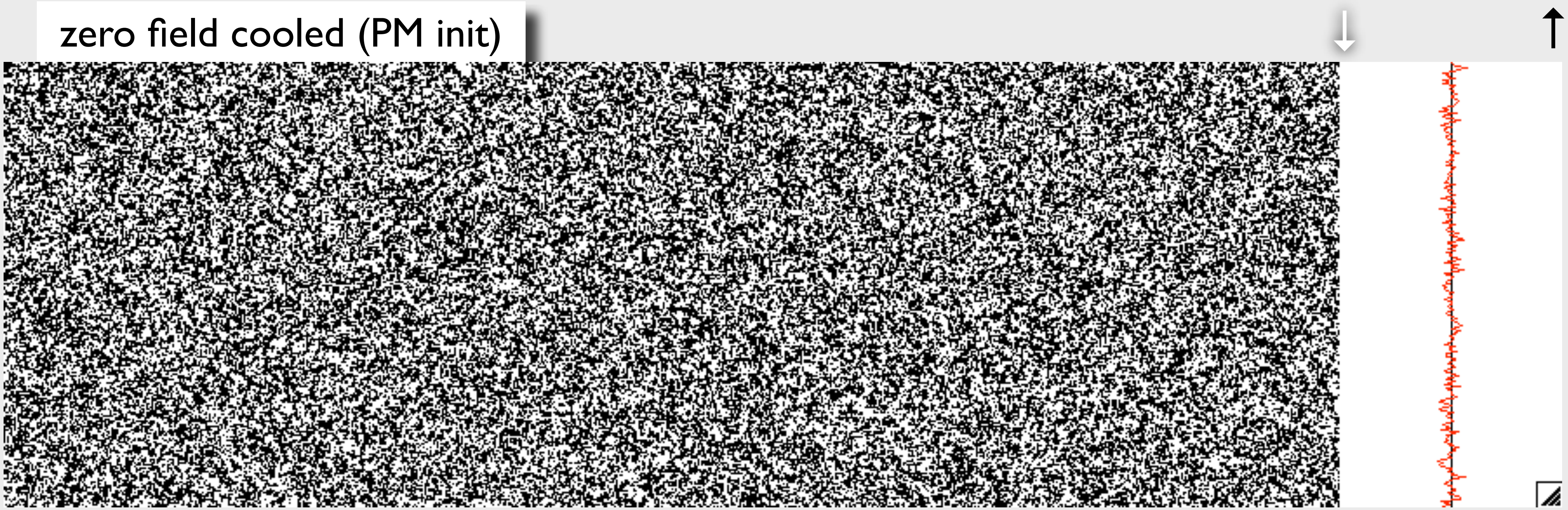
- Example: Quench to surface order state ($v = 16$, $T = 2.4\,J$)

Monte Carlo simulations of surface order

768×256 spins, $v = 16$ spins/MCS, $T = 2.4\,J$, $J_b = J$, 4 MCS/frame, 15 fps

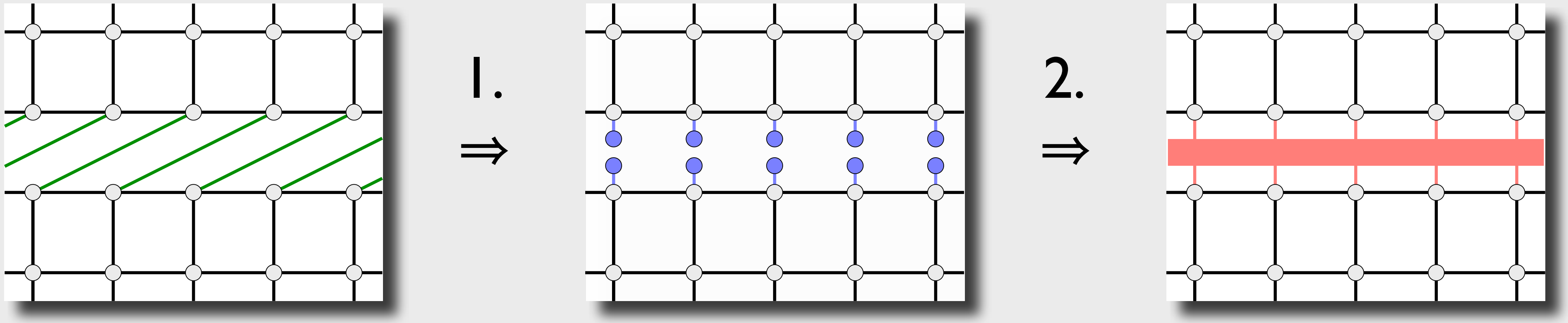
field cooled (FM init)

zero field cooled (PM init)



High Velocities: Exact Solution

High velocities: Exact solution in 3 steps



0. Start with driven system

1. Map boundary couplings K_b to fluctuating fields μ_i with average m_b

2. Replace fluctuating fields μ_i with static field $h_\mathrm{b}(m_\mathrm{b})$

3. Take equilibrium boundary magnetization $m_{\mathrm{b,eq}}(h_\mathrm{b})$ and solve self-consistency condition

$$m_{\mathrm{b,eq}}(h_\mathrm{b}(m_\mathrm{b})) = m_\mathrm{b}$$

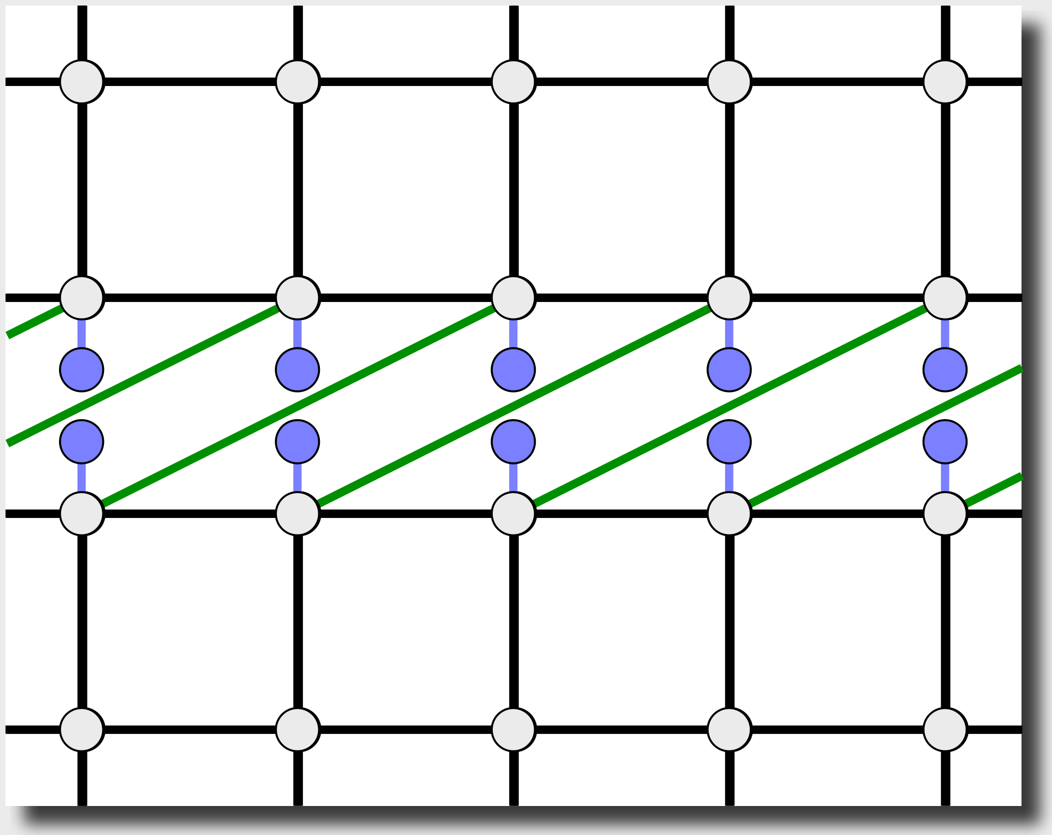
(1.) The limit of high velocities *v*

$$\beta\mathcal{H}(t) = -K\sum_{\langle ij\rangle}\sigma_i\sigma_j - K_\mathrm{b}\sum_{\langle ij\rangle_\mathrm{b}(t)}\sigma_{\bar{ij}}\sigma_j$$

$v \gg 1$: Uncorrelated boundaries

$$\beta\mathcal{H}(m_\mathrm{b}) = -K\sum_{\langle ij\rangle}\sigma_i\sigma_j - K_\mathrm{b}\sum_{i\in\mathcal{B}}\mu_i\sigma_i$$

Condition: $\langle\mu_i\rangle = \langle\sigma_j\rangle_\mathrm{b} = m_\mathrm{b}$

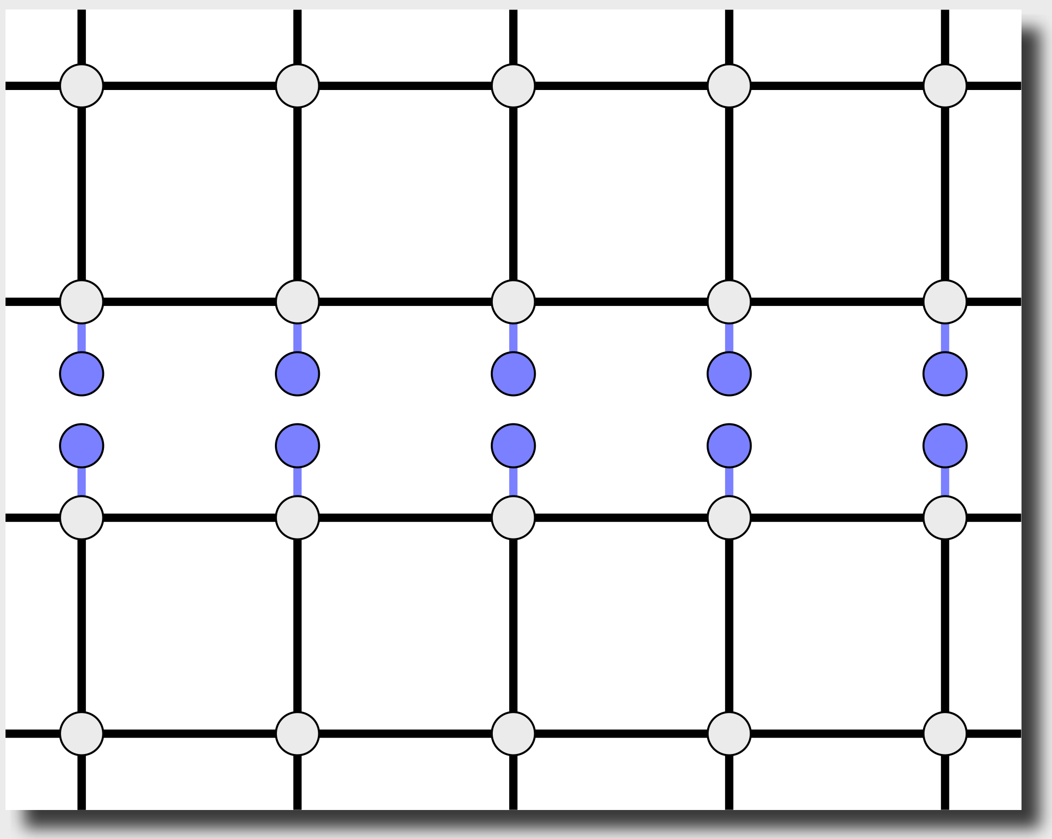


Boundary couplings are replaced by fluctuating fields μ_i with average m_b

(2.) Mapping to equilibrium model

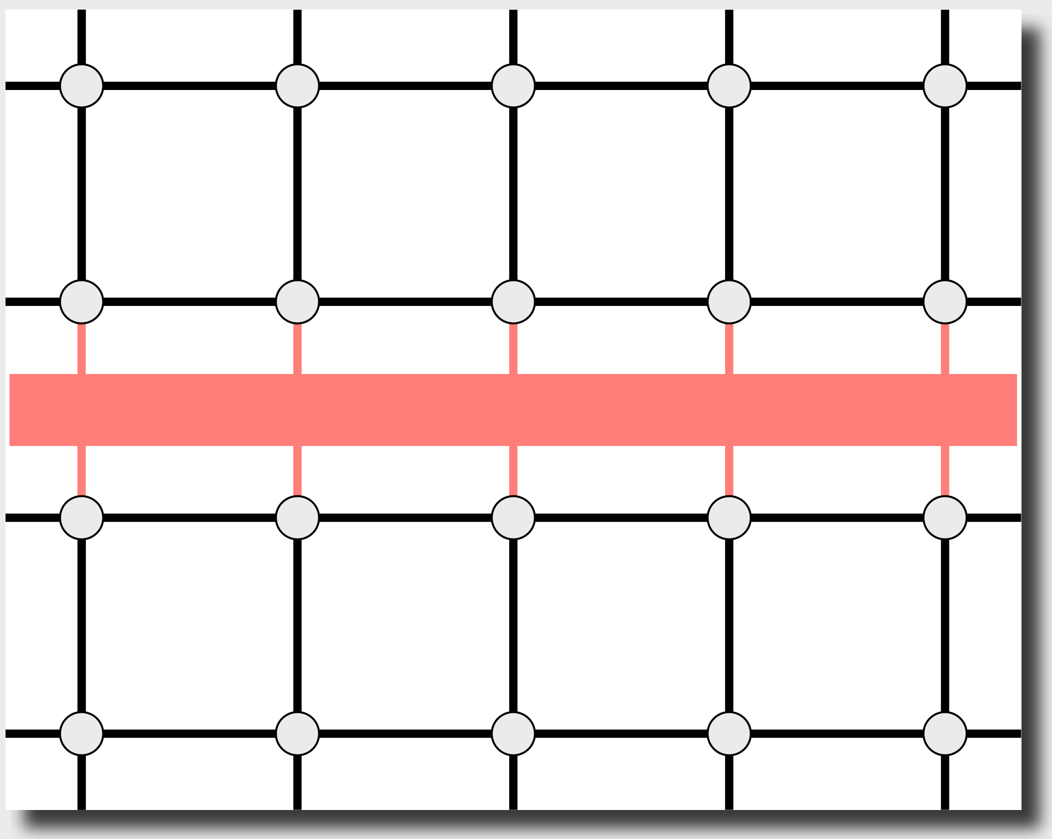
Fluctuating fields

$$\beta\mathcal{H}(m_{\mathrm{b}}) = -K \sum_{\langle ij \rangle} \sigma_i \sigma_j - K_{\mathrm{b}} \sum_{i \in \mathcal{B}} \mu_i \sigma_i$$



Static fields

$$\beta\mathcal{H}_{\mathrm{eq}} = -K \sum_{\langle ij \rangle} \sigma_i \sigma_j - h_{\mathrm{b}} \sum_{i \in \mathcal{B}} \sigma_i$$



(2.) Tracing out some degrees of freedom

Fluctuating fields

$$\beta\mathcal{H}(m_\mathrm{b}) = -K\sum_{\langle ij\rangle}\sigma_i\sigma_j - K_\mathrm{b}\sum_{i\in\mathcal{B}}\mu_i\sigma_i$$

$$\begin{aligned}\mathcal{Z} &= \mathrm{Tr}_{\sigma\mu}\, e^{-\beta\mathcal{H}(m_\mathrm{b})} \\ &= \mathrm{Tr}_\sigma\, e^{-\beta\mathcal{H}_0}\,\mathrm{Tr}_\mu \prod_{i\in\mathcal{B}} e^{K_\mathrm{b}\mu_i\sigma_i} \\ &= \mathrm{Tr}_\sigma\, e^{-\beta\mathcal{H}_0} \prod_{i\in\mathcal{B}}\sum_{\mu=\pm1} e^{K_\mathrm{b}\mu\sigma_i} p(\mu) \\ &= (\cosh K_\mathrm{b})^{N_\mathrm{b}}\,\mathrm{Tr}_\sigma\, e^{-\beta\mathcal{H}_0}\prod_{i\in\mathcal{B}}[1+\sigma_i m_\mathrm{b}\tanh K_\mathrm{b}]\end{aligned}$$

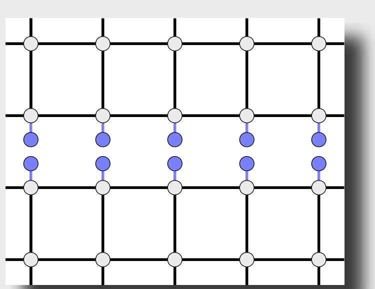
$$\langle\mu_i\rangle = m_\mathrm{b} \quad\Rightarrow\quad p(\mu) = (1+\mu m_\mathrm{b})/2$$

Static fields

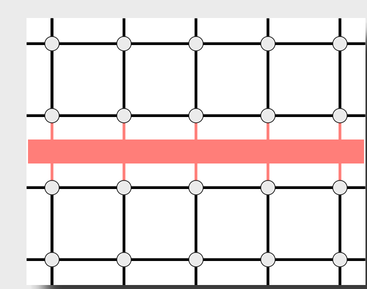
$$\beta\mathcal{H}_\mathrm{eq} = -K\sum_{\langle ij\rangle}\sigma_i\sigma_j - h_\mathrm{b}\sum_{i\in\mathcal{B}}\sigma_i$$

$$\begin{aligned}\mathcal{Z}_\mathrm{eq} &= \mathrm{Tr}_\sigma\, e^{-\beta\mathcal{H}_\mathrm{eq}} \\ &= \mathrm{Tr}_\sigma\, e^{-\beta\mathcal{H}_0}\prod_{i\in\mathcal{B}} e^{h_\mathrm{b}\sigma_i} \\ &= (\cosh h_\mathrm{b})^{N_\mathrm{b}}\,\mathrm{Tr}_\sigma\, e^{-\beta\mathcal{H}_0}\prod_{i\in\mathcal{B}}[1+\sigma_i\tanh h_\mathrm{b}]\end{aligned}$$

$$\left(\begin{aligned} e^{h\sigma} &= \cosh h\sigma + \sinh h\sigma \\ {\scriptstyle(\sigma=\pm1)} &= \cosh h\,[1+\sigma\tanh h]\end{aligned}\right)$$



$$\tanh h_\mathrm{b} \overset{!}{=} m_\mathrm{b}\tanh K_\mathrm{b} \quad\Rightarrow\quad \mathcal{Z} = \left(\frac{\cosh K_\mathrm{b}}{\cosh h_\mathrm{b}}\right)^{N_\mathrm{b}}\mathcal{Z}_\mathrm{eq}$$



(3.) The self-consistency equation

$$h_\mathrm{b} = \mathrm{arctanh}(m_\mathrm{b} \tanh K_\mathrm{b})$$

Equilibrium surface magnetization and zero field susceptibility

$$m_{\mathrm{b,eq}}(K, h_\mathrm{b}) = \frac{\partial \ln \mathcal{Z}_{\mathrm{eq}}}{\partial h_\mathrm{b}} \qquad \chi^{(0)}_{\mathrm{b,eq}}(K) = \left. \frac{\partial^2 \ln \mathcal{Z}_{\mathrm{eq}}}{\partial h_\mathrm{b}^2} \right|_{h_\mathrm{b}=0}$$

$$m_{\mathrm{b,eq}}(K, \mathrm{arctanh}(m_\mathrm{b} \tanh K_\mathrm{b})) \stackrel{!}{=} m_\mathrm{b}$$

$\Leftarrow$ Self-consistency condition

Condition for critical point T_c from expansion around $m_\mathrm{b} = 0$ $\Rightarrow$

$$\chi^{(0)}_{\mathrm{b,eq}}(K_c) \tanh K_{\mathrm{b},c} \stackrel{!}{=} 1$$

$$\tanh h_\mathrm{b} = m_\mathrm{b} \tanh K_\mathrm{b}$$

(3.) The self-consistency equation

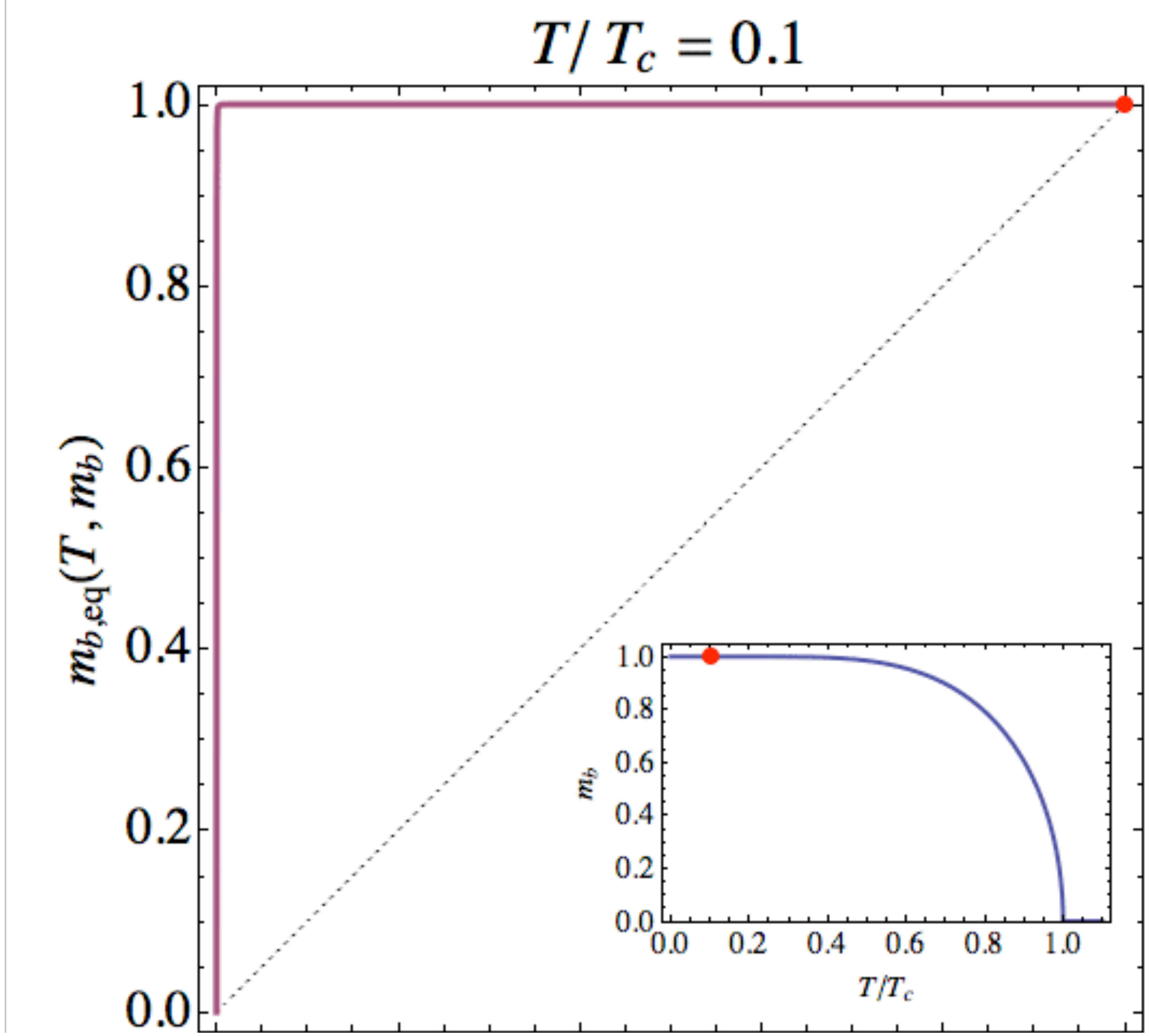
Equilil … tibility

n … $h_{\rm b}\!=\!0$

$m_{\rm b,eq}(K, \text{a}$ … ondition

Condition … 1

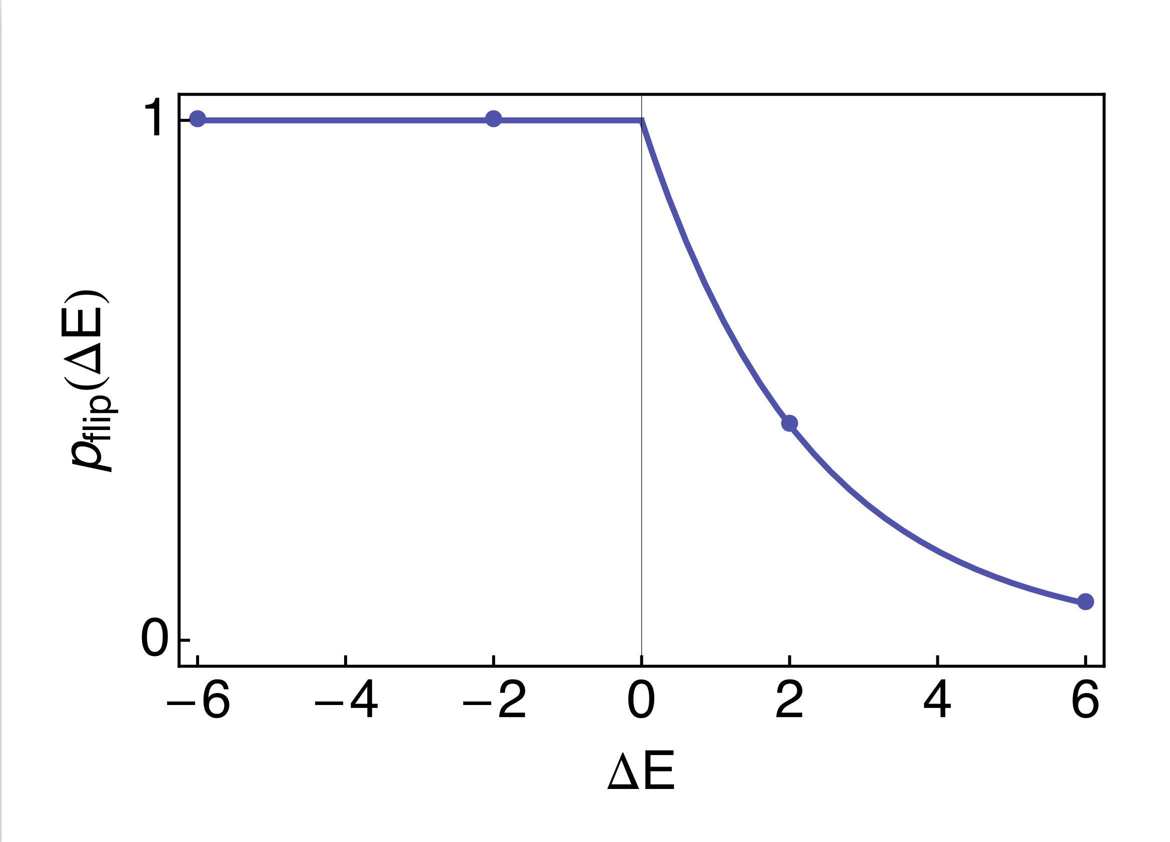
from expa



Mean–field universality class: $\alpha = 0$, $\beta = 1/2$, $\gamma = 1$, $\delta = 3$

An integrable Monte Carlo algorithm

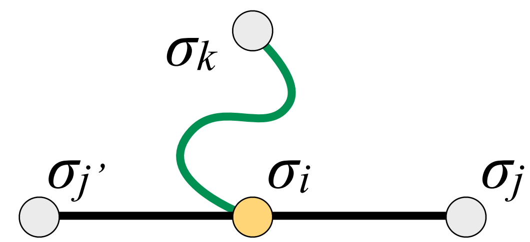
- Critical temperature T_c depends on MC algorithm
- Example:
  $1d$ case with $J_\mathrm{b} = J = 1$
- Acceptance rate $A = \langle p_\mathrm{flip} \rangle$



Metropolis	$p_\mathrm{flip}^\mathrm{MP}(\Delta E) = \min(1, e^{-\beta\,\Delta E})$	$T_\mathrm{c}^\mathrm{MP} = 1.910(2)$	$A_\mathrm{c}^\mathrm{MP} = 0.476(2)$
Heat-Bath	$p_\mathrm{flip}^\mathrm{HB}(\Delta E) = \dfrac{1}{1 + e^{\beta\,\Delta E}}$	$T_\mathrm{c}^\mathrm{HB} = 2.031(2)$	$A_\mathrm{c}^\mathrm{HB} = 0.366(2)$
Exact solution	??	$T_\mathrm{c} = 2.2692...$	$A_\mathrm{c} = 0.24264...$

An integrable Monte Carlo algorithm

- Consider MC update of boundary spin σ_i



$$\Delta E = \underbrace{2J\sigma_i \sum_{\langle j \rangle} \sigma_j}_{\Delta E_1} + \underbrace{2J_\mathrm{b}\sigma_i\sigma_k}_{\Delta E_2}$$

- Usual MC algorithms introduce correlations over boundary: Influence on σ_k depends on σ_j

$$\sigma_i = -\sigma_j \;\rightarrow\; \Delta E_1 = -4J \;\rightarrow\; p_\mathrm{flip} = 1$$
$$\sigma_i = +\sigma_j \;\rightarrow\; \Delta E_1 = +4J \;\rightarrow\; p_\mathrm{flip}(\sigma_k)$$

- Solution: Use algorithm which
  - fulfills detailed balance
  - is multiplicative

$$\frac{p_\mathrm{flip}(\Delta E)}{p_\mathrm{flip}(-\Delta E)} = e^{-\beta \Delta E}$$

$$p_\mathrm{flip}(\Delta E_1 + \Delta E_2) = p_\mathrm{flip}(\Delta E_1)\, p_\mathrm{flip}(\Delta E_2)$$

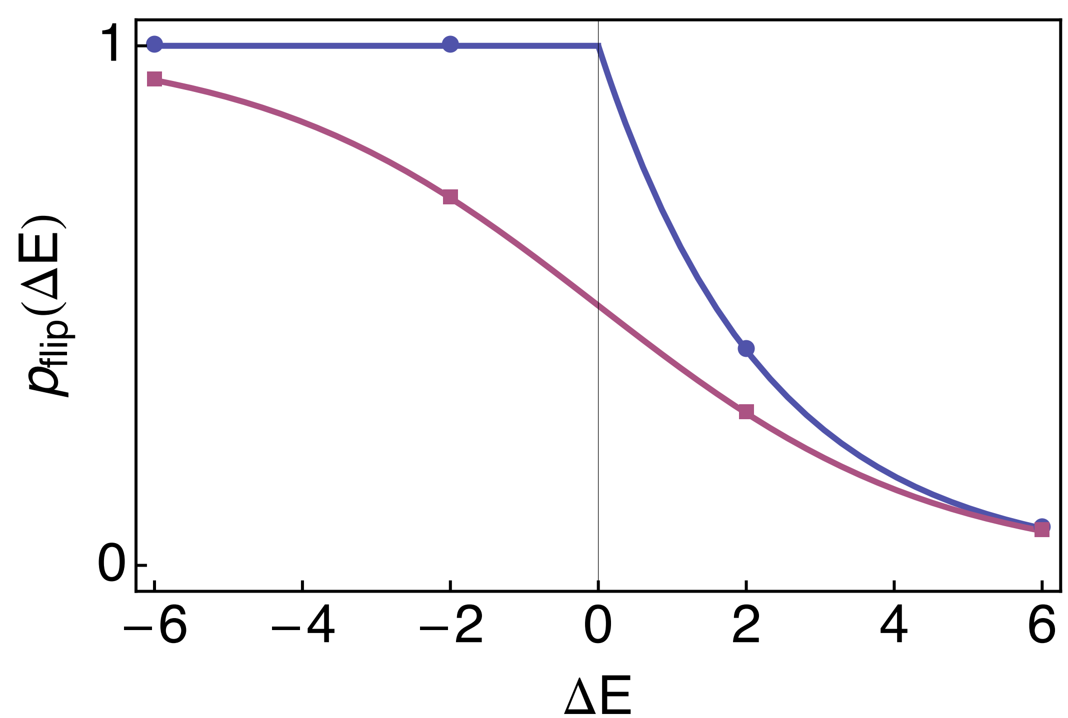
- Result:

$$p^*_\mathrm{flip}(\Delta E) = e^{-\frac{\beta}{2}(\Delta E - \Delta E_\mathrm{min})}$$

An integrable Monte Carlo algorithm

- Critical temperature T_c depends on MC algorithm
- Example: $1d$ case with $J_b = J = 1$

Acceptance rate $A = \langle p_{\mathrm{flip}} \rangle$



Metropolis	$p^{\mathrm{MP}}_{\mathrm{flip}}(\Delta E) = \min(1, e^{-\beta\,\Delta E})$	$T^{\mathrm{MP}}_c = 1.910(2)$	$A^{\mathrm{MP}}_c = 0.476(2)$
Heat-Bath	$p^{\mathrm{HB}}_{\mathrm{flip}}(\Delta E) = \frac{1}{1 + e^{\beta\,\Delta E}}$	$T^{\mathrm{HB}}_c = 2.031(2)$	$A^{\mathrm{HB}}_c = 0.366(2)$
Multiplicative	$p^{*}_{\mathrm{flip}}(\Delta E) = e^{-\frac{\beta}{2}(\Delta E - \Delta E_{\mathrm{min}})}$	$T^{*}_c = 2.269(1)$	$A^{*}_c = 0.242(2)$
Exact solution		$T_c = 2.2692...$	$A_c = 0.24264...$

Application to $1d$ model

- Solution of equilibrium model

$$m_{\mathrm{eq}}(K,h) = \frac{\sinh h}{\sqrt{e^{-4K} + \sinh^2 h}}$$

$$\chi_{\mathrm{eq}}^{(0)}(K) = e^{2K}$$

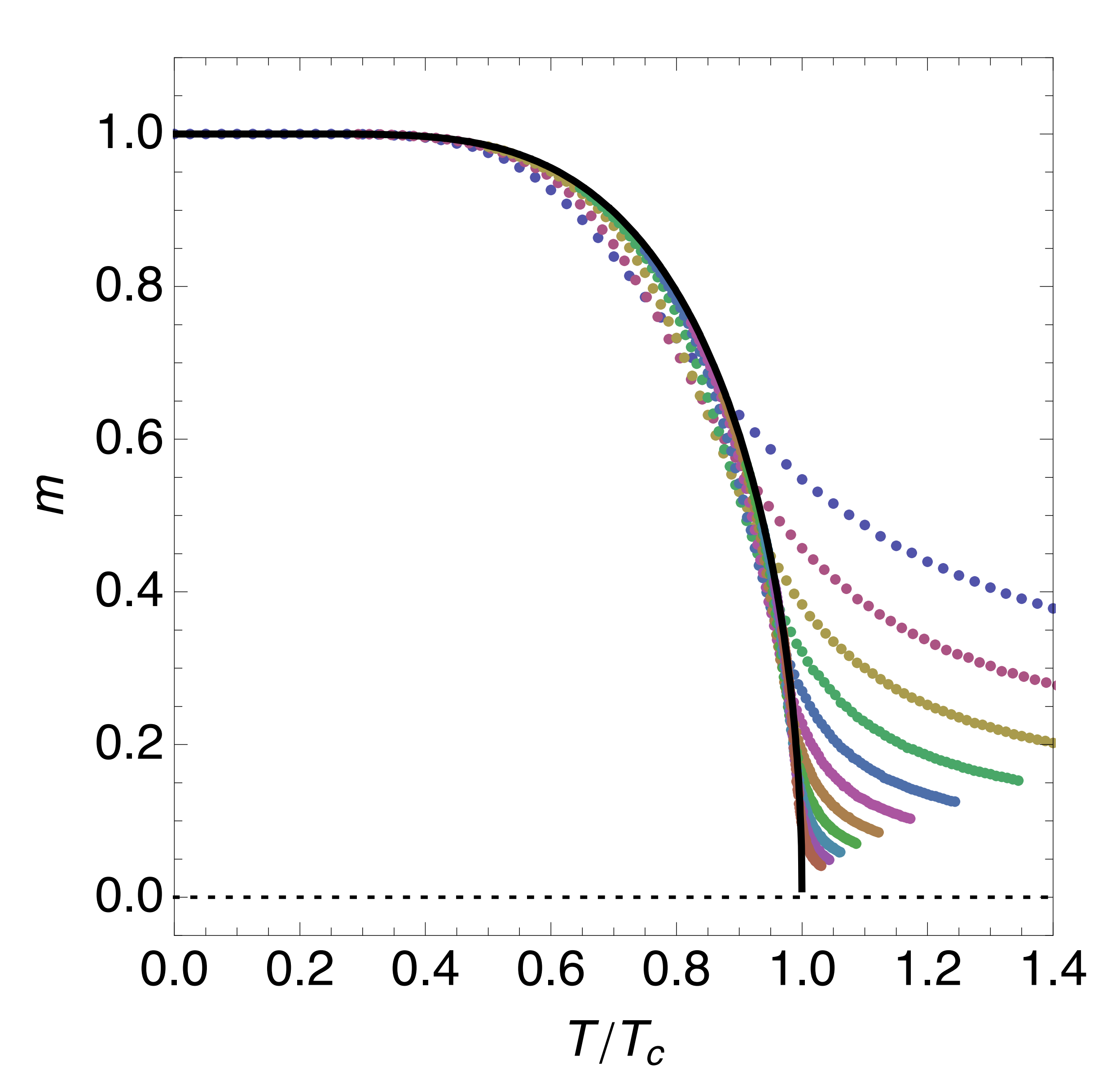
- Critical point at finite T:

$$\chi_{\mathrm{eq}}^{(0)}(K_c)\,\tanh K_{\mathrm{b},c} \stackrel{!}{=} 1$$

$$T_{\mathrm{c}} = 2/\log(\sqrt{2}+1) = 2.26918...$$

- Order parameter m:

$$m(K, K_{\mathrm{b}}) = \sqrt{\frac{\cosh 2K_{\mathrm{b}} - \coth 2K}{\cosh 2K_{\mathrm{b}} - 1}}$$



- **Note**: Identical to surface magnetization of $2d$ Ising model

Other quantities in $1d$: The transfer matrix

- TM of equilibrium $1d$ Ising model: $\mathbf{T}_{\text{eq}} = \begin{pmatrix} e^{K+h} & e^{-K} \\ e^{-K} & e^{K-h} \end{pmatrix}$ $\mathcal{Z}_{\text{eq}} = \operatorname{Tr} \mathbf{T}_{\text{eq}}^{L_\parallel}$
- TM of driven system: $(\sin\psi = m \tanh K_{\text{b}})$

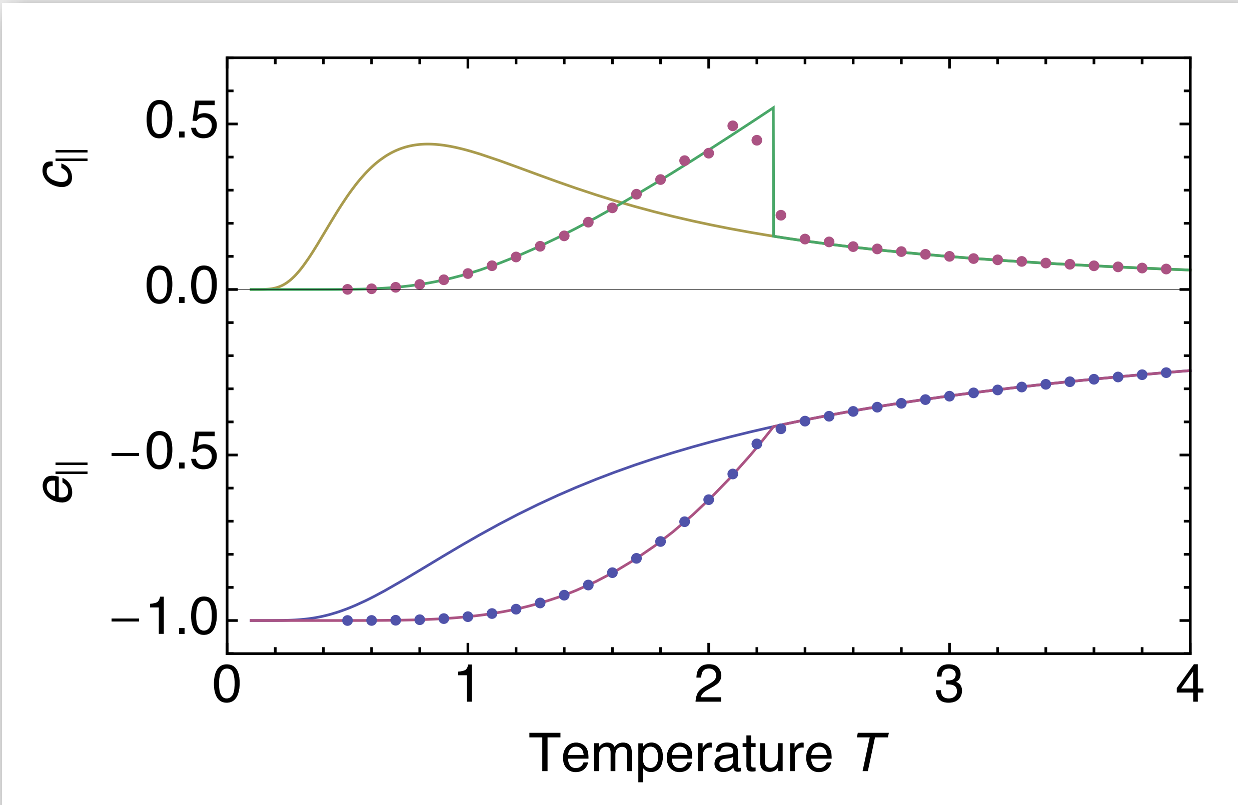
$$\mathbf{T} = \frac{\cosh K_{\text{b}}}{\cosh h} \mathbf{T}^*_{\text{eq}} = \cosh K_{\text{b}} \begin{pmatrix} e^K (1+\sin\psi) & e^{-K}\cos\psi \\ e^{-K}\cos\psi & e^K(1-\sin\psi) \end{pmatrix}$$

- Eigensystem: $\mathbf{T}|t_i\rangle = \lambda_i |t_i\rangle \quad \rightarrow \quad \lambda_{0,1} = \begin{cases} e^{K\pm K_{\text{b}}} & T \le T_{\text{c}} \\ \cosh K_{\text{b}}(e^K \pm e^{-K}) & T \ge T_{\text{c}} \end{cases}, \quad |t_i\rangle = \cdots$
- Physical quantities: $\left[\hat{\mathbf{T}} = \mathbf{T}/\lambda_0,\ \mathbf{M} = \left(\begin{smallmatrix} 1 & 0 \\ 0 & -1 \end{smallmatrix}\right)\right]$

$$m = \langle \sigma_j \rangle = \langle t_0 | \mathbf{M} | t_0 \rangle$$

$$e_\parallel = -J\langle \sigma_j \sigma_{j+1} \rangle = -J \langle t_0 | \mathbf{M}\hat{\mathbf{T}}\mathbf{M} | t_0 \rangle$$

$$\ldots$$



- **Note**: "free energy" of driven system is constant in ordered phase (?)

$$f_< = -\beta^{-1} \log \lambda_0 = -(J + J_{\text{b}})$$

Dynamical quantities in $1d$

- Define configuration probability $\langle \zeta_\ell \, \zeta \, \zeta_r \rangle \langle \zeta_\mu \rangle, \quad \zeta = \uparrow, \downarrow$
- Then, with $\mathbf{P}_\uparrow = \left(\begin{smallmatrix} 1 & 0 \\ 0 & 0 \end{smallmatrix}\right)$, $\mathbf{P}_\downarrow = \left(\begin{smallmatrix} 0 & 0 \\ 0 & 1 \end{smallmatrix}\right)$, e. g.,

$$\langle \uparrow\uparrow\downarrow \rangle \langle \downarrow \rangle = \langle t_0 | \mathbf{P}_\uparrow \hat{\mathbf{T}} \mathbf{P}_\uparrow \hat{\mathbf{T}} \mathbf{P}_\downarrow | t_0 \rangle \langle t_0 | \mathbf{P}_\downarrow | t_0 \rangle$$

- Monte Carlo acceptance rate:

$$A = \sum_{\zeta_\ell, \zeta, \zeta_r, \zeta_\mu} p_{\text{flip}}(\Delta E) \langle \zeta_\ell \, \zeta \, \zeta_r \rangle \langle \zeta_\mu \rangle = \begin{cases} \dfrac{\cosh(K + K_\text{b}) - \sinh(K - K_\text{b})}{4 e^{2(K + K_\text{b})} \sinh K \cosh^2 K \sinh K_\text{b}} & T \leq T_\text{c} \\ e^{-K_\text{b}} \cosh K_\text{b} (1 - \tanh K)^2 & T \geq T_\text{c} \end{cases}$$

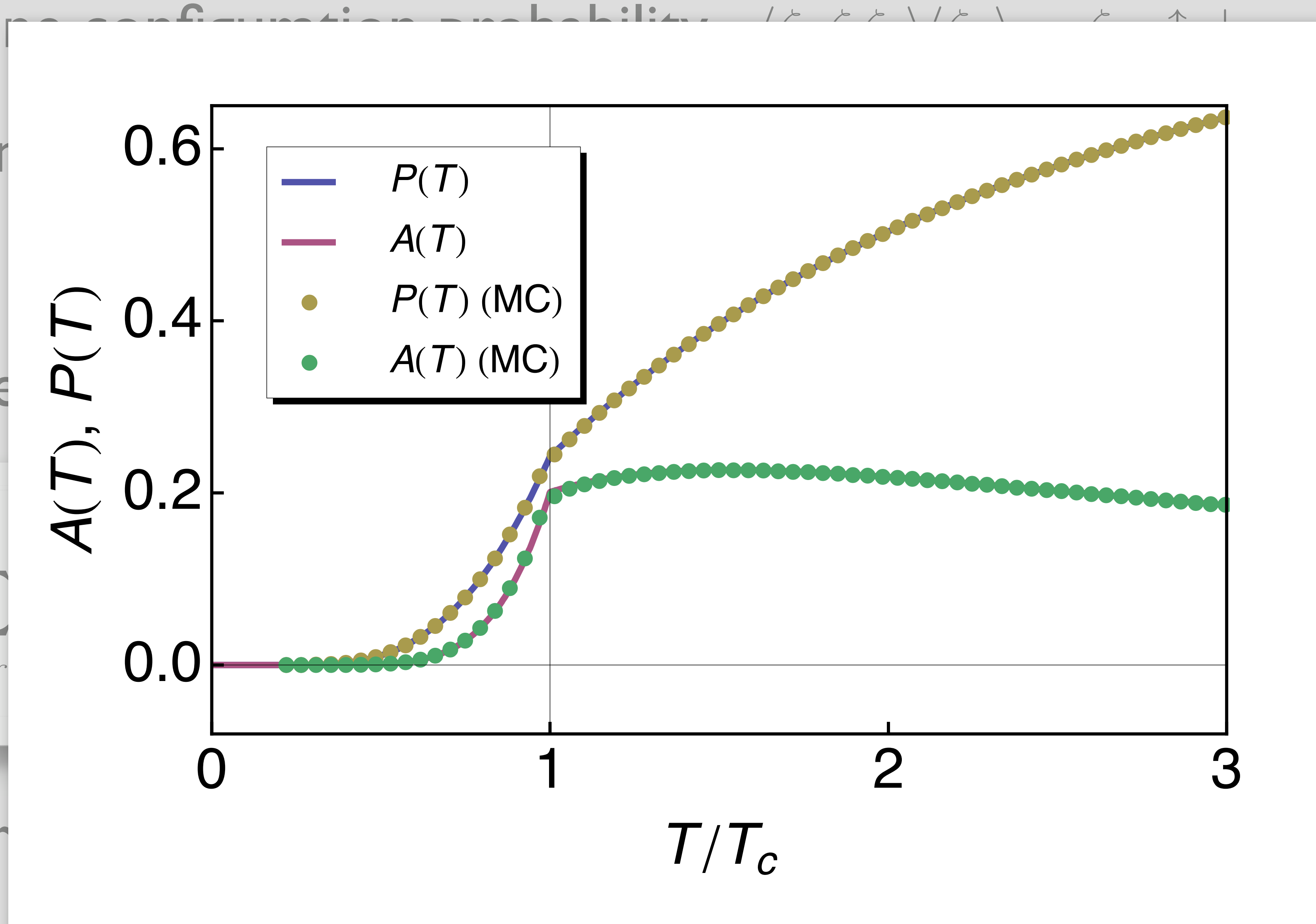
- Energy dissipation rate:

$$\frac{P}{A} = -J_\text{b} \sum_{\zeta = \uparrow, \downarrow} \frac{e^{-2K_\text{b}} \langle \zeta \rangle - \langle \bar{\zeta} \rangle}{e^{-2K_\text{b}} \langle \zeta \rangle + \langle \bar{\zeta} \rangle} = \begin{cases} \dfrac{2 J_\text{b} e^{-4K}}{\tanh K_\text{b}} & T \leq T_\text{c} \\ 2 J_\text{b} \tanh K_\text{b} & T \geq T_\text{c} \end{cases}$$

Dynamical properties

- Defi[…]
- Ther[…]
- Acce[…]

$$A = \sum_{\zeta_\ell,\zeta,\zeta_r} \ldots \qquad T \leq T_\mathrm{c} \qquad T \geq T_\mathrm{c}$$



- Ener[…]

$$\frac{P}{A} = -J_\mathrm{b} \sum_{\zeta=\uparrow,\downarrow} \frac{e^{-2K_\mathrm{b}}\langle\zeta\rangle - \langle\bar{\zeta}\rangle}{e^{-2K_\mathrm{b}}\langle\zeta\rangle + \langle\bar{\zeta}\rangle} = \begin{cases} \dfrac{2J_\mathrm{b}e^{-4K}}{\tanh K_\mathrm{b}} & T \leq T_\mathrm{c} \\ 2J_\mathrm{b}\tanh K_\mathrm{b} & T \geq T_\mathrm{c} \end{cases}$$

Application to $2d$ model

- Surface magnetization of $2d$ Ising model [1]

$$m_{\mathrm{b,eq}}(z,y_{\mathrm{b}}) = y_{\mathrm{b}} + \frac{1-y_{\mathrm{b}}^2}{2\pi} y_{\mathrm{b}} z \int_{-\pi}^{\pi} \frac{4(1+\cos\theta)}{4y_{\mathrm{b}}^2 z(1+\cos\theta) + (1+z^2)(1-2z\cos\theta - z^2) + r(\theta)} \mathrm{d}\theta$$

$$z = \tanh K, \quad y_{\mathrm{b}} = \tanh h_{\mathrm{b}}, \quad r(\theta) = \prod_{\epsilon=\pm 1} \sqrt{(1-\epsilon z)^2 - 2z(1-z^2)\cos\theta + z^2(1+\epsilon z)^2}$$

- Surface susceptibility

$$\chi^{(0)}_{\mathrm{b,eq}}(z) = \left(\frac{1}{z^2} - 1\right)\left[(1+2w-8w^2)\frac{\mathbf{K}(16w^2)}{4\pi w} - \frac{\mathbf{E}(16w^2)}{4\pi w} - \frac{1}{4}\right], \quad w = \frac{z(1-z^2)}{(1+z^2)^2}$$

- Critical point ($J = J_{\mathrm{b}} = 1$)

$$\chi^{(0)}_{\mathrm{b,eq}}(K_c)\ \tanh K_{\mathrm{b},c} \stackrel{!}{=} 1 \quad \Rightarrow \quad T_{\mathrm{c}} = 2.6614725655752...$$

- Boundary magnetization

$$m_{\mathrm{b,eq}}(z, m_{\mathrm{b}} z_{\mathrm{b}})) \stackrel{!}{=} m_{\mathrm{b}} \ \rightarrow \ m_{\mathrm{b}}(z, z_{\mathrm{b}}) = \ldots$$

- Bulk magnetization

$$m_{\mathrm{bulk}} = (1 - \sinh^{-4} 2K)^{1/8}$$

[1] B. M. McCoy & T. T. Wu, The Two-Dimensional Ising Model, Harvard University Press (1973)

Application to $2d$ model

- Sur

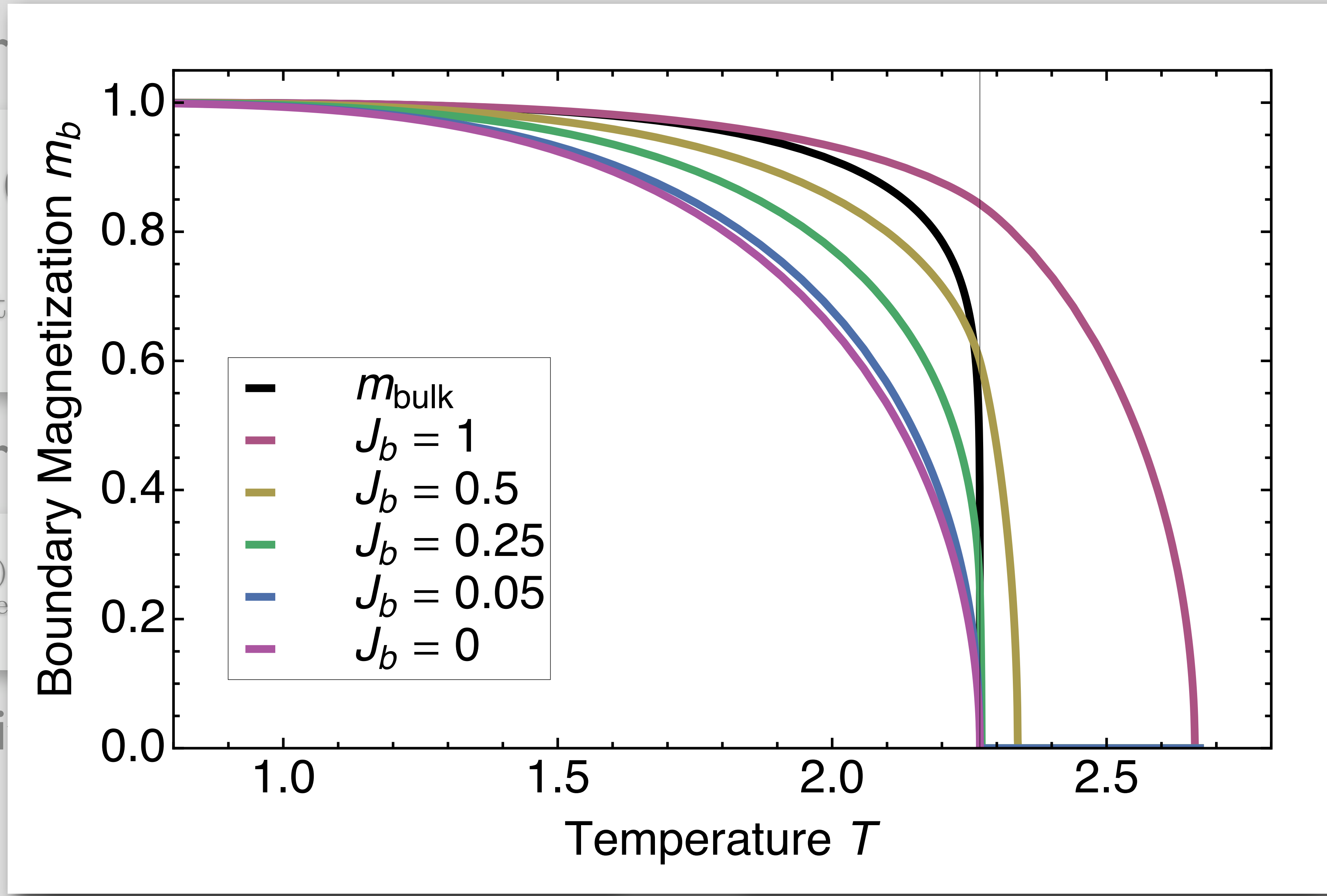
$m_{\mathrm{b,eq}}($ ... $\mathrm{d}\theta$

$z = \mathrm{t}$ ... $\epsilon z)^2$

- Sur

$\chi^{(0)}_{\mathrm{b,e}}$ ... $z^2)$ ... $z^2)^2$

- Cri



- Boundary magnetization

$$m_{\mathrm{b,eq}}(z, m_{\mathrm{b}} z_{\mathrm{b}})) \stackrel{!}{=} m_{\mathrm{b}} \;\; \rightarrow \;\; m_{\mathrm{b}}(z, z_{\mathrm{b}}) = \ldots$$

- Bulk magnetization

$$m_{\mathrm{bulk}} = (1 - \sinh^{-4} 2K)^{1/8}$$

[1] B. M. McCoy & T. T. Wu, The Two-Dimensional Ising Model, Harvard University Press (1973)

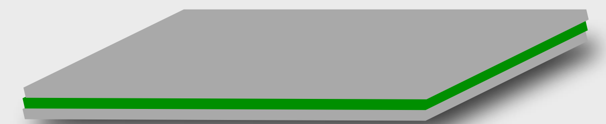
Results for other geometries (sc)

$1d$

$$\chi^{(1d)}(K_c)\tanh K_{\mathrm{b},c} = 1$$

$$T_{\mathrm{c}} = 2/\log(\sqrt{2}+1) = 2.26918\ldots$$

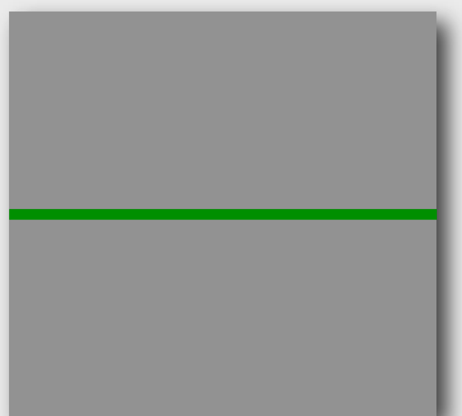
$2d$



$$\chi^{(2d)}(K_c)\tanh K_{\mathrm{b},c} = 1$$

$$T_{\mathrm{c}} = 4.0587824231379800009877750406806(2)$$

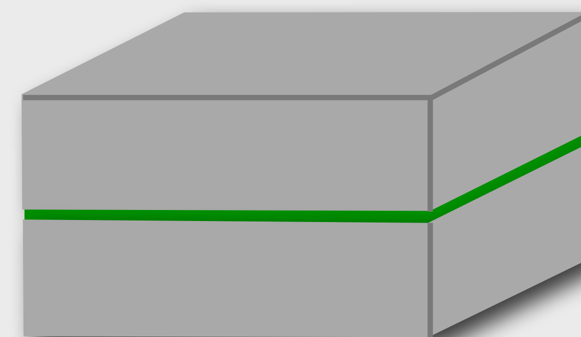
$2d_{\mathrm{b}}$



$$\chi_{\mathrm{b}}^{(2d)}(K_c)\tanh K_{\mathrm{b},c} = 1$$

$$T_{\mathrm{c}} = 2.6614725655752\ldots$$

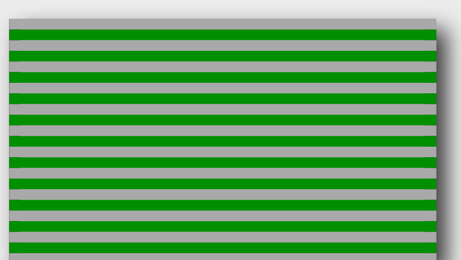
$3d_{\mathrm{b}}$



$$\chi_{\mathrm{b}}^{(3d)}(K_c)\tanh K_{\mathrm{b},c} = 1$$

$$T_{\mathrm{c}} = 8.2(2)$$

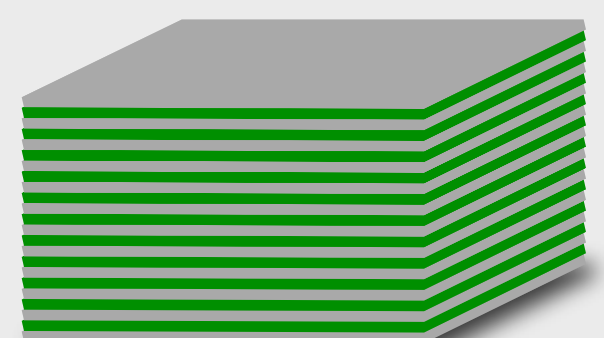
$1{+}1d$



$$2\chi^{(1d)}(K_c)\tanh K_{\mathrm{b},c} = 1$$

$$T_{\mathrm{c}} = 1/\log\left(\frac{1}{2}\sqrt{3+\sqrt{17}}\right) = 3.46591\ldots$$

$2{+}1d$



$$2\chi^{(2d)}(K_c)\tanh K_{\mathrm{b},c} = 1$$

$$T_{\mathrm{c}} = 5.264750414514743550598017203424676392109336009(3)$$

Conclusions & Outlook

- New contribution to friction from spin correlations
- Driven model shows non–equilibrium phase transition
- Exactly solvable for high velocities ($v \to \infty$) in many geometries
- Phase transition in mean–field class for $d > 1$ and $v > 0$

- Cross–over from Ising to mean–field class
- Strong anisotropic phase transition in $d \geq 2$?
- Experimental realization possible?

A.H., Phys. Rev. E (submitted), arXiv:0909.0533