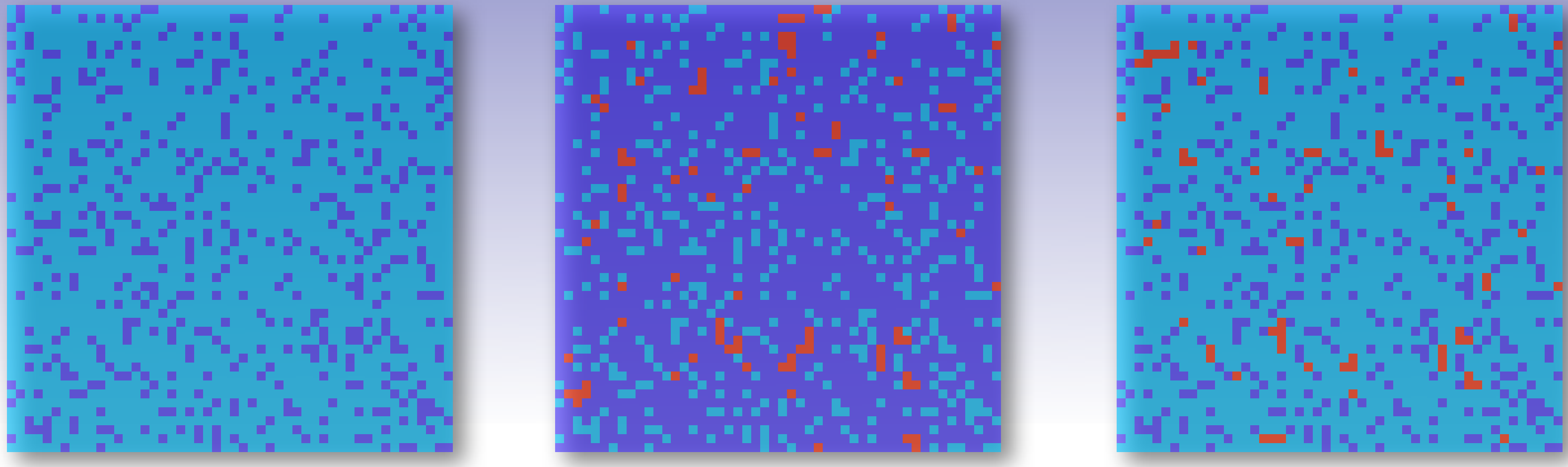
Equilibrium and nonequilibrium properties of spin glasses in a field

ETH Zürich

Equilibrium and nonequilibrium properties of spin glasses in a field

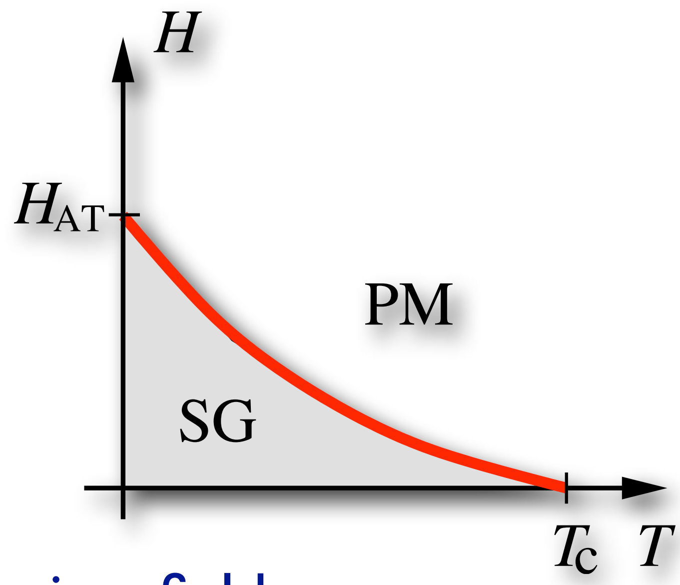


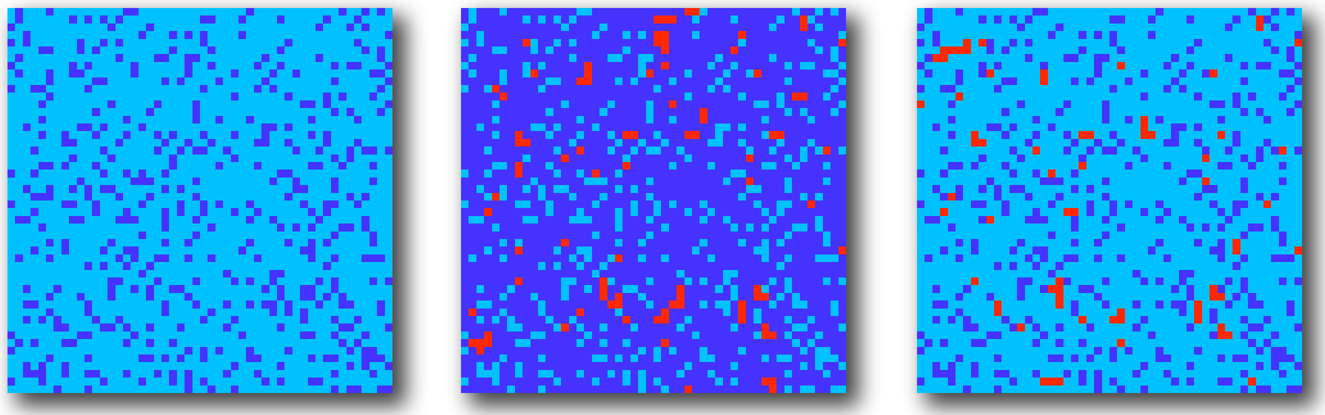
Helmut G. Katzgraber
A. P. Young (UC Santa Cruz)
G. T. Zimányi (UC Davis)

http://katzgraber.org

Outline

- Introduction to spin glasses (disordered magnets)
    - What are spin glasses?
    - Why are they interesting?
- Equilibrium properties of spin glasses in a field
    - Absence of an Almeida-Thouless line below upper critical dimension
- Nonequilibrium properties of spin glasses in a field
    - Return and complementary point memory effects





Introduction to spin glasses

Magnetic systems

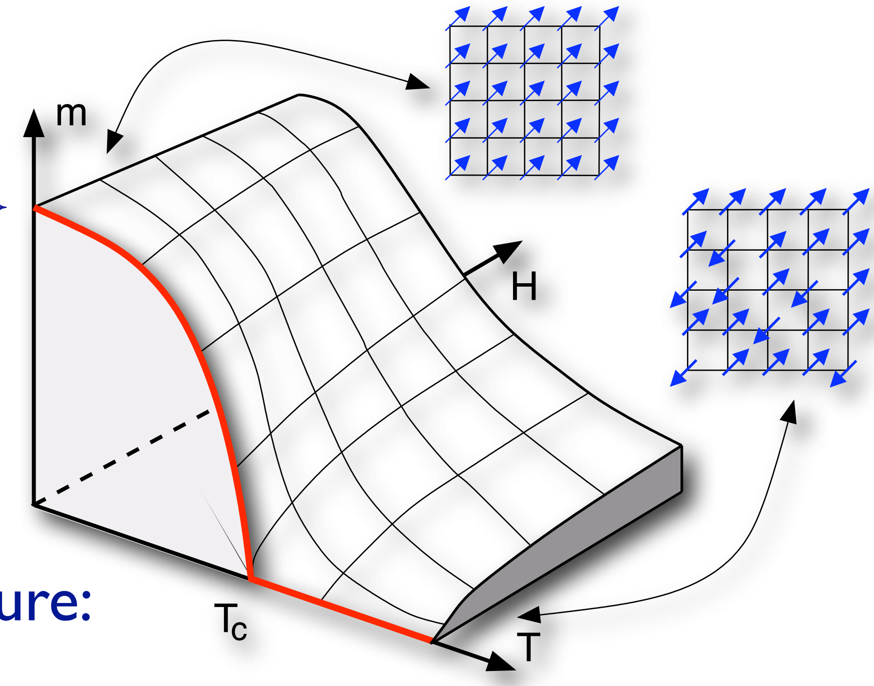
- Prototype model for a magnet:

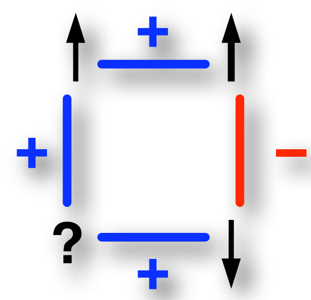
$$\mathcal{H} = -\sum_{\langle i,j \rangle} J_{ij} S_i S_j \qquad S_i \in \{\pm 1\}$$

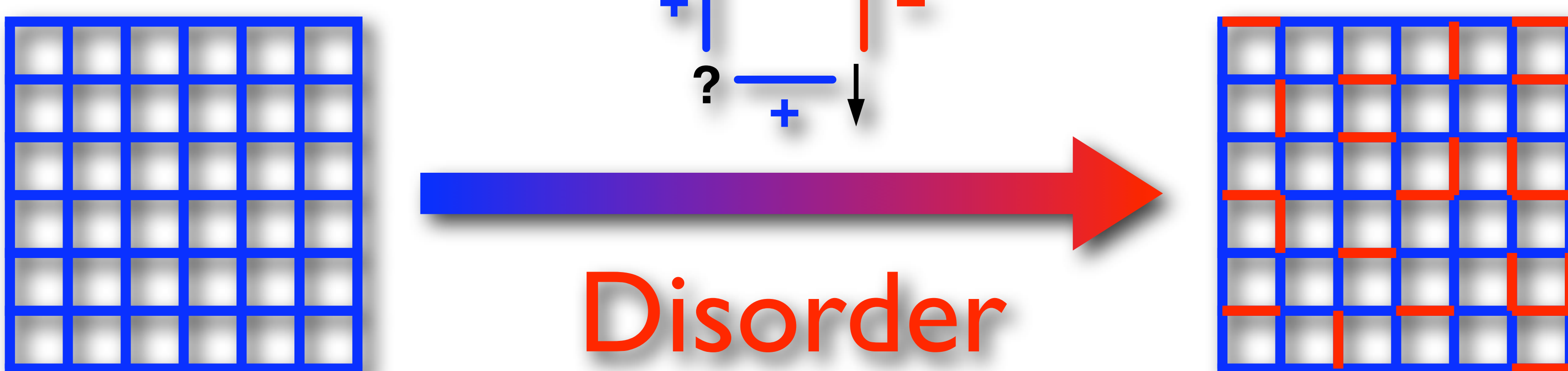
- Order parameter

$$m = \frac{1}{N} \sum_i S_i$$

- Disorder plays an integral role in nature:
  - Properties of materials change.
  - But often neglected.

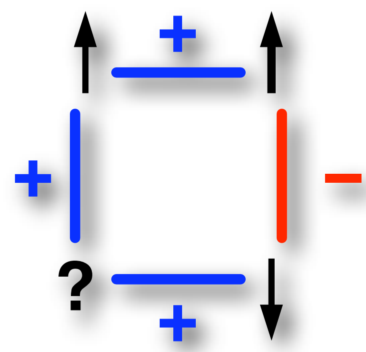


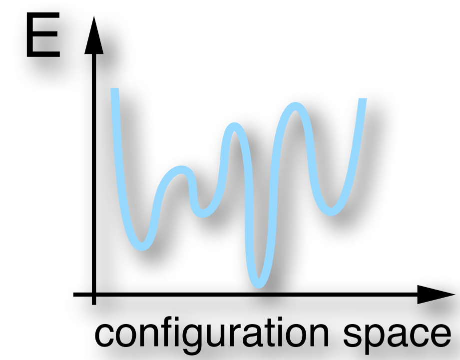




Spin glasses

- Phase transition into a glassy phase with no spatial order
- Complex energy landscape
- Slow dynamics
- Unexpected effects: aging, memory, hysteresis





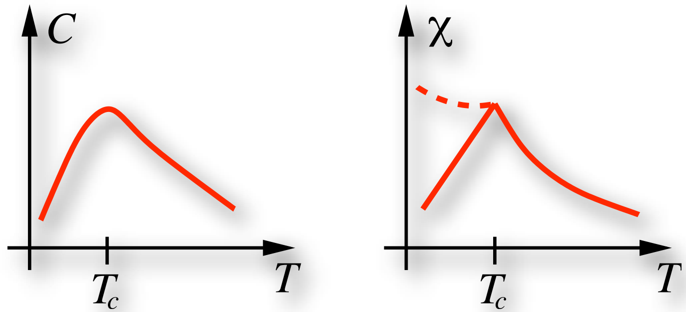
- Problem: Only mean-field model solvable. Solution: Simulations.
- Numerically complex optimization problem, generally NP hard
- Many applications to other fields and problems:
  - Physics: vortex glasses, disordered magnetic media, error correcting codes, structural glasses, ...
  - Computer science and related fields: pattern recognition, combinatorial optimization, economics, ...

Still a lot to be understood!

Brief history

- 1970: Canella & Mydosh see a cusp in $\chi_{\rm SG}$ of Fe/Au alloys. The material has RKKY interactions

$$J_{ij} \sim \frac{\cos(2k_{\rm F}R_{ij})}{R_{ij}^3}$$



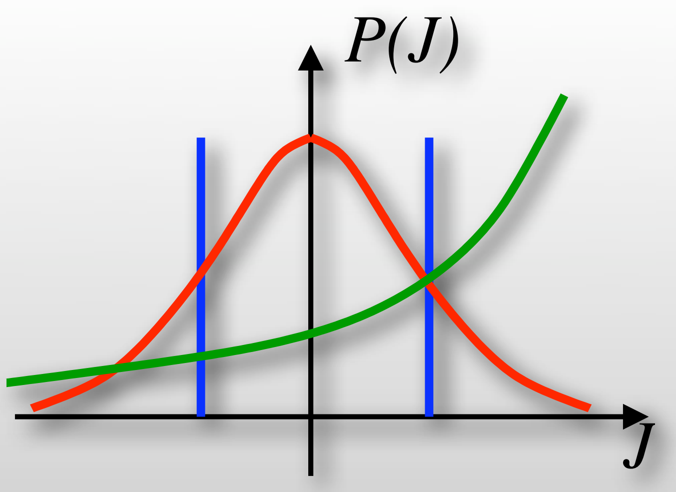
which introduces disorder and frustration, necessary in a spin glass.

- 1975: Introduction of the Edwards-Anderson Ising spin glass model:

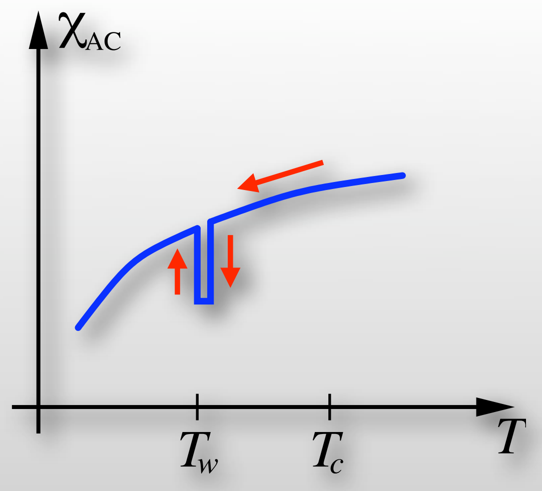
$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j$$

- 1975: The mean-field Sherrington-Kirkpatrick model is introduced.
- 1979: Parisi solution (RSB) of the mean-field model.
- 1986: Fisher & Huse suggest the droplet picture (DP) to describe short-range spin glasses.

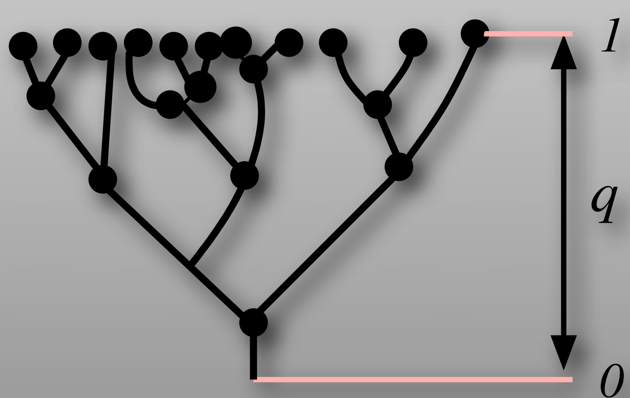
Some open questions...



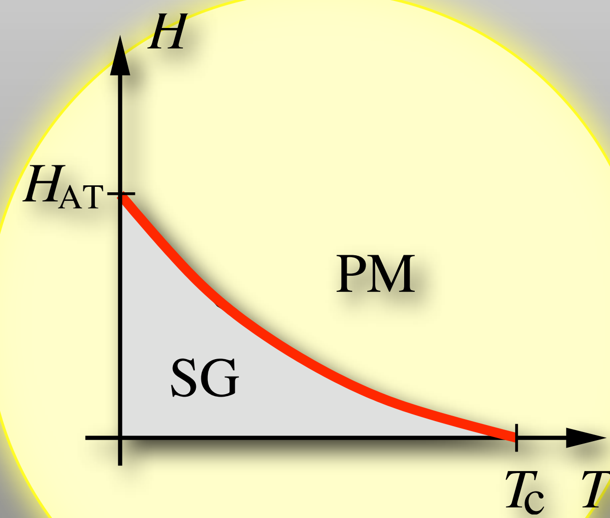
Universality



Memory effect



Ultrametricity

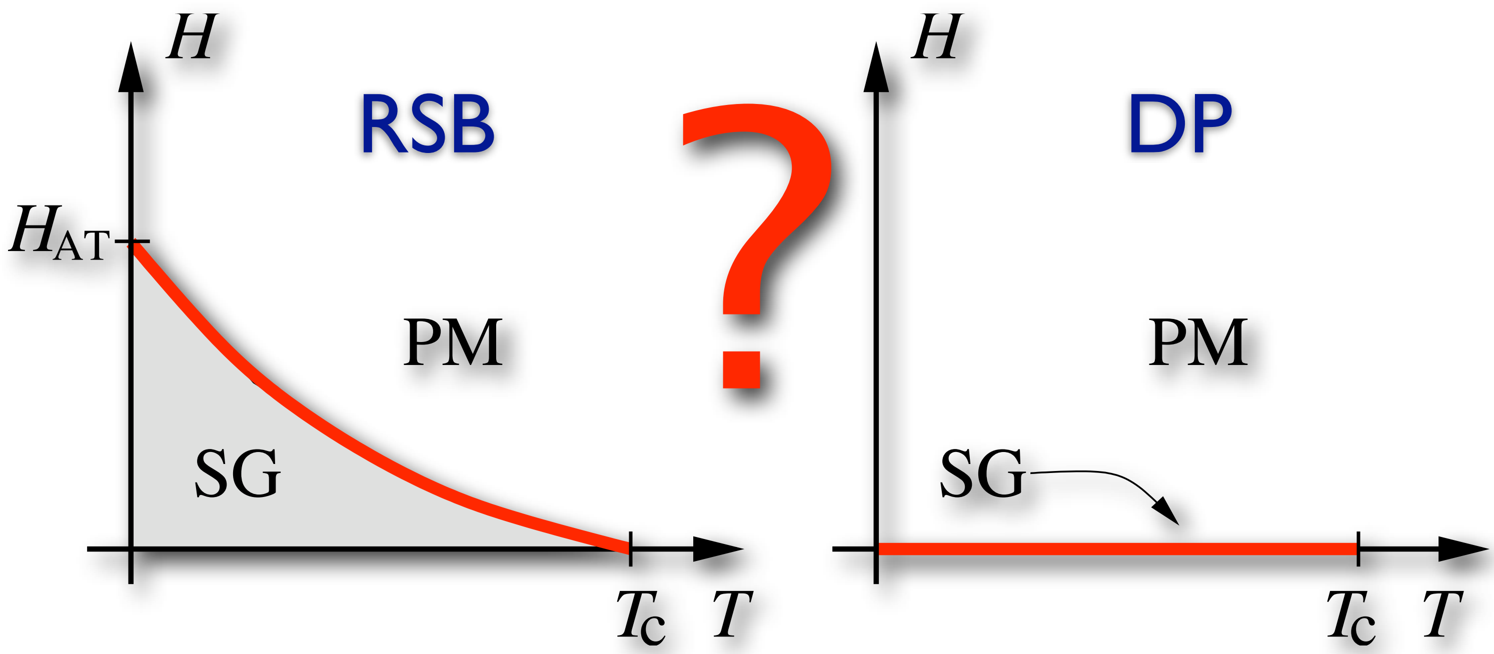


Spin-glass state in a field?

Equilibrium properties in a field

Spin-glass state in a field?

- Two contradicting predictions:
  - Replica Symmetry Breaking: Existence of an instability line [de Almeida & Thouless (78)] for mean-field glasses.
  - Droplet Picture: there is no spin-glass state in a field.



Which of the above pictures is correct?

What has been done?

- Theory: de Almeida & Thouless (78) predict an instability line for the SK model.
- Experiments:
  - Katori & Ito (94): claim existence of an AT line.
  - Mattson et al. (95): no AT line (study divergent relaxation times).
- Simulations:
  - Study of the Binder cumulant [Bhatt & Young (85), Kawashima & Young (96)]: no AT line. Problem: Binder ratio not stable in a field.
  - Out of equilibrium methods [Marinari et al. (98)]: signature of an AT line. Problem: Is the true equilibrium behavior probed?
  - Simulations according to experimental protoco... Hukushima (04)] show no AT line.
  - Zero-T calculations [Houdayer & Ma... find not AT line.

Model

Algorithm

Tool

First approach: 3D EA Ising spin-glass

PRL 93, 207203 (2004)

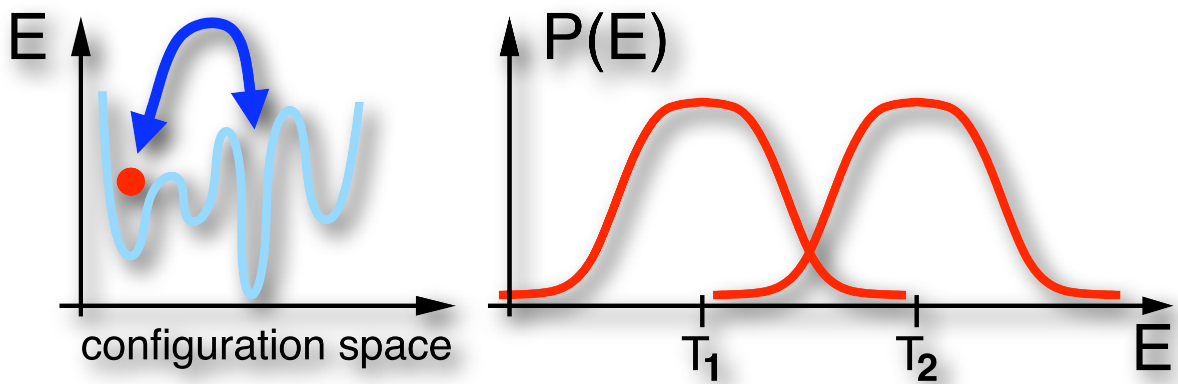
- Edwards-Anderson Ising spin-glass model with random fields:

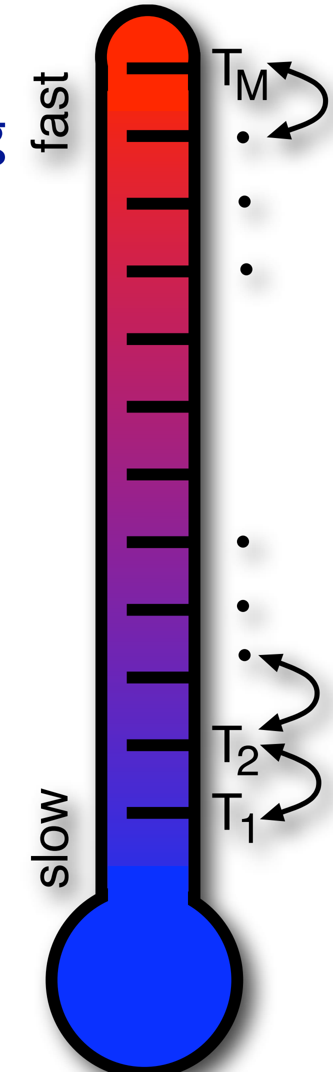
$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - \sum_i h_i S_i \qquad S_i \in \{\pm 1\}$$

- Properties:
  - Sum over nearest neighbors in 3D with Gaussian random bonds.
  - The random fields are Gaussian distributed with zero mean and $[h_i^2]_{\mathrm{av}}^{1/2} = H_{\mathrm{R}}$. This corresponds to a uniform field H_{R}.
  - For zero field $T_{\mathrm{c}} \approx 0.95$.
  - Why do we choose random fields?
    - Equilibration test for the Monte Carlo method
    - Parallel tempering performs slightly better than in a uniform field.

Parallel tempering Monte Carlo

- Simulate M copies of the system at different temperatures with $T_{\max} \gg T_c$.
- Allow swapping of neighboring temperatures: easy crossing barriers!





- Fast equilibration with rough energy landscapes.
- The method obeys detailed balance

$$P(S_{m+1} \leftrightarrow S_m; \beta_{m+1} \leftrightarrow \beta_m) = \begin{cases} e^{-\Delta} & : \quad \Delta > 0 \\ 1 & : \quad \Delta \leq 0 \end{cases}$$

$$\Delta = (\beta_{m+1} - \beta_m)(E_m - E_{m+1})$$

Probing criticality: correlation length

- Ballesteros et al. (00) reintroduce the use of the finite-size correlation length to study phase transitions in spin glasses. Calculation of ξ_L:
  - Wave-vector-dependent connected spin-glass susceptibility:

$$\chi_{\rm SG}(\mathbf{k}) = \frac{1}{N}\sum_{i,j}\left[\left(\langle S_i S_j\rangle_T - \langle S_i\rangle_T\langle S_j\rangle_T\right)^2\right]_{\rm av} e^{i\mathbf{k}\cdot(\mathbf{R}_i - \mathbf{R}_j)}$$

  - Ornstein-Zernicke approximation:

$$[\chi_{\rm SG}(k)/\chi_{\rm SG}(0)]^{-1} = 1 + \xi_L^2 k^2 + \mathcal{O}[(\xi_L k)^4]$$

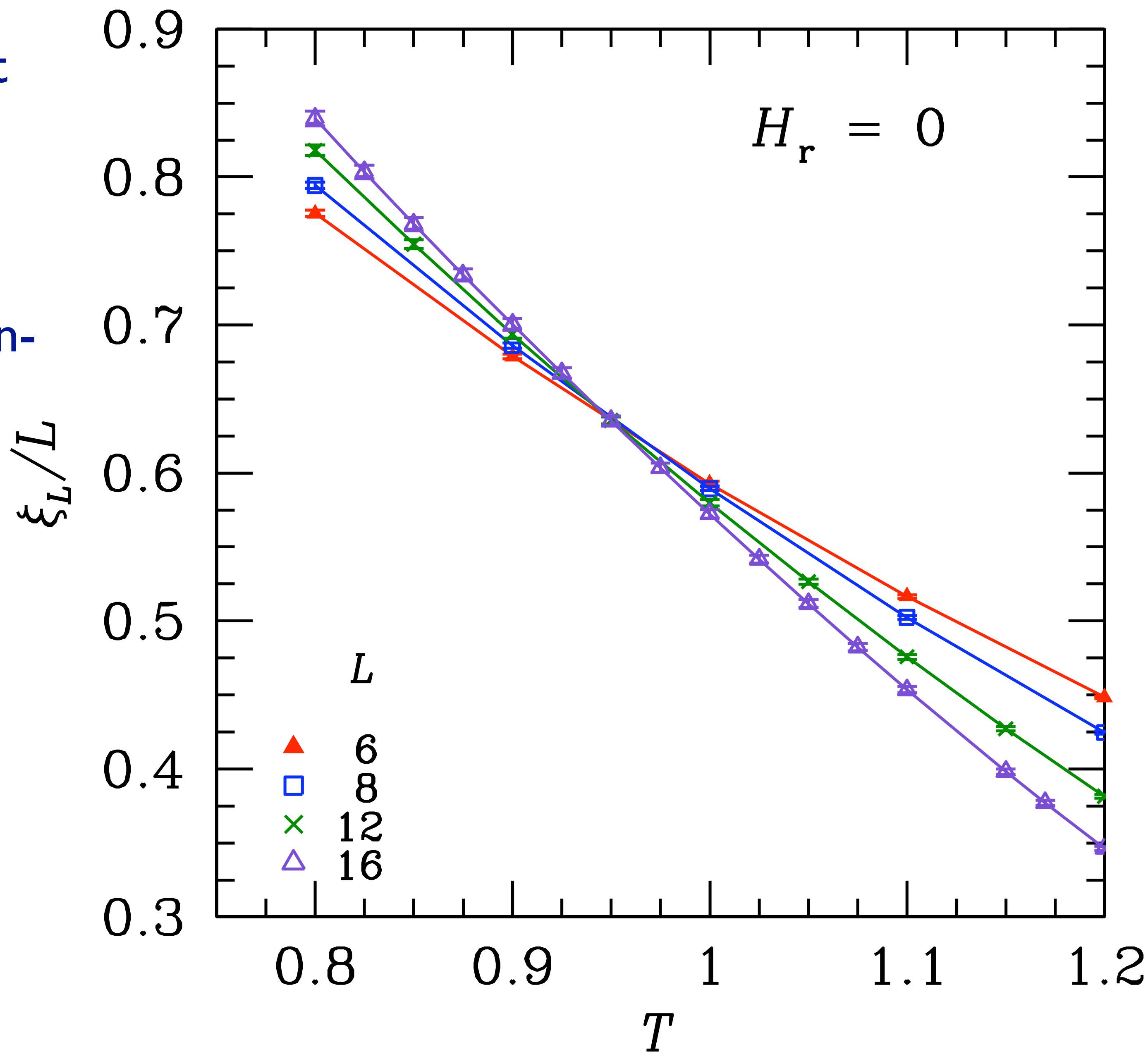
  - Compensate for PBC and finite-size effects and solve for ξ_L

$$\xi_L = \frac{1}{2\sin(k_{\rm min}/2)}\left[\frac{\chi_{SG}(0)}{\chi_{SG}(\mathbf{k}_{\rm min})} - 1\right]^{1/2}$$

  - Finite-size scaling: $$\frac{\xi_L}{L} = \tilde{X}\left(L^{1/\nu}[T - T_{\rm c}(H_{\rm r})]\right)$$

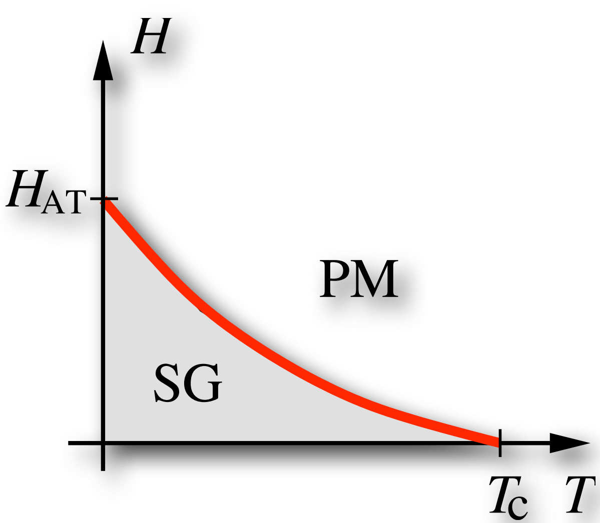
How well does this work for $H = 0$?

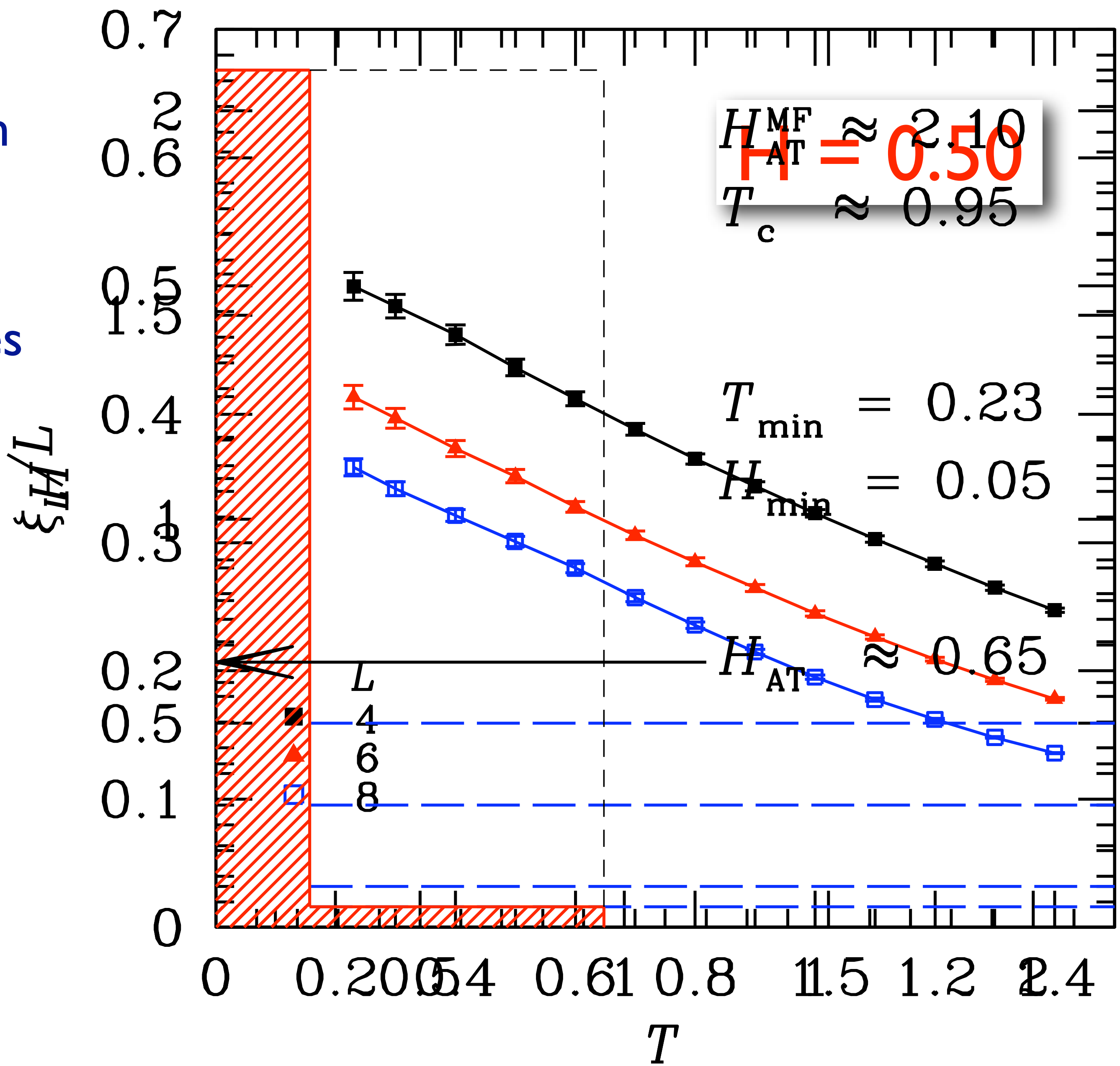
- The data cross at $T_c \approx 0.95$ in agreement with previous results.
- Evidence of a spin-glass state for $T \leq 0.95$.



Finite fields... No transition

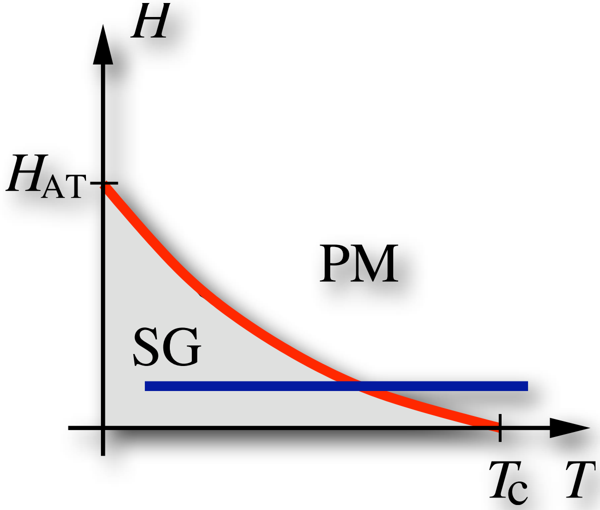
- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.
- Krzakala predicts $H_{AT} \approx 0.65$.

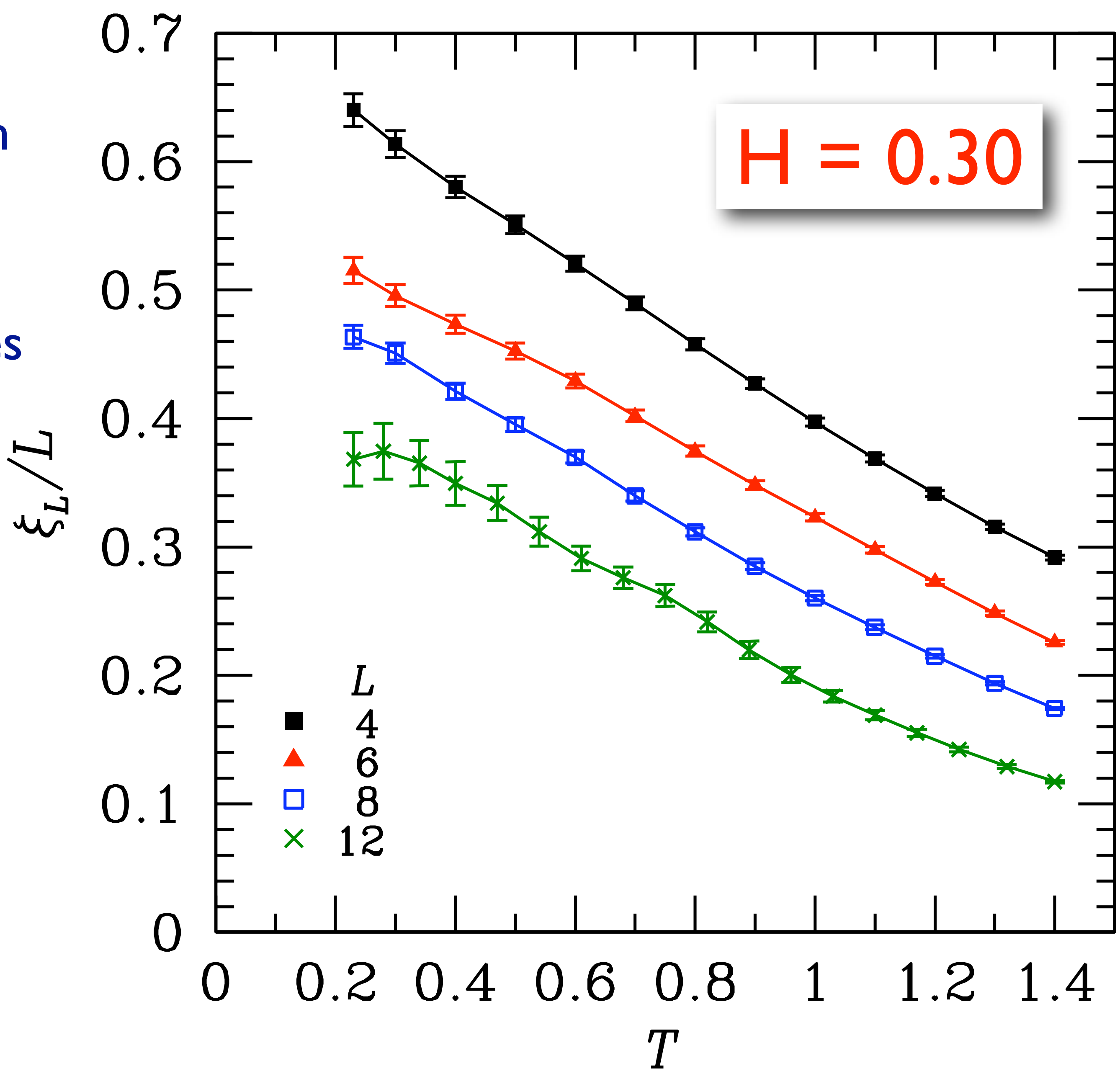




Finite fields... No transition

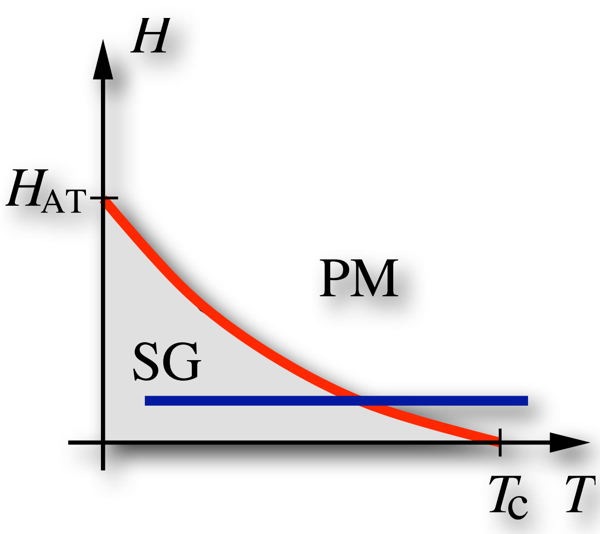
- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.

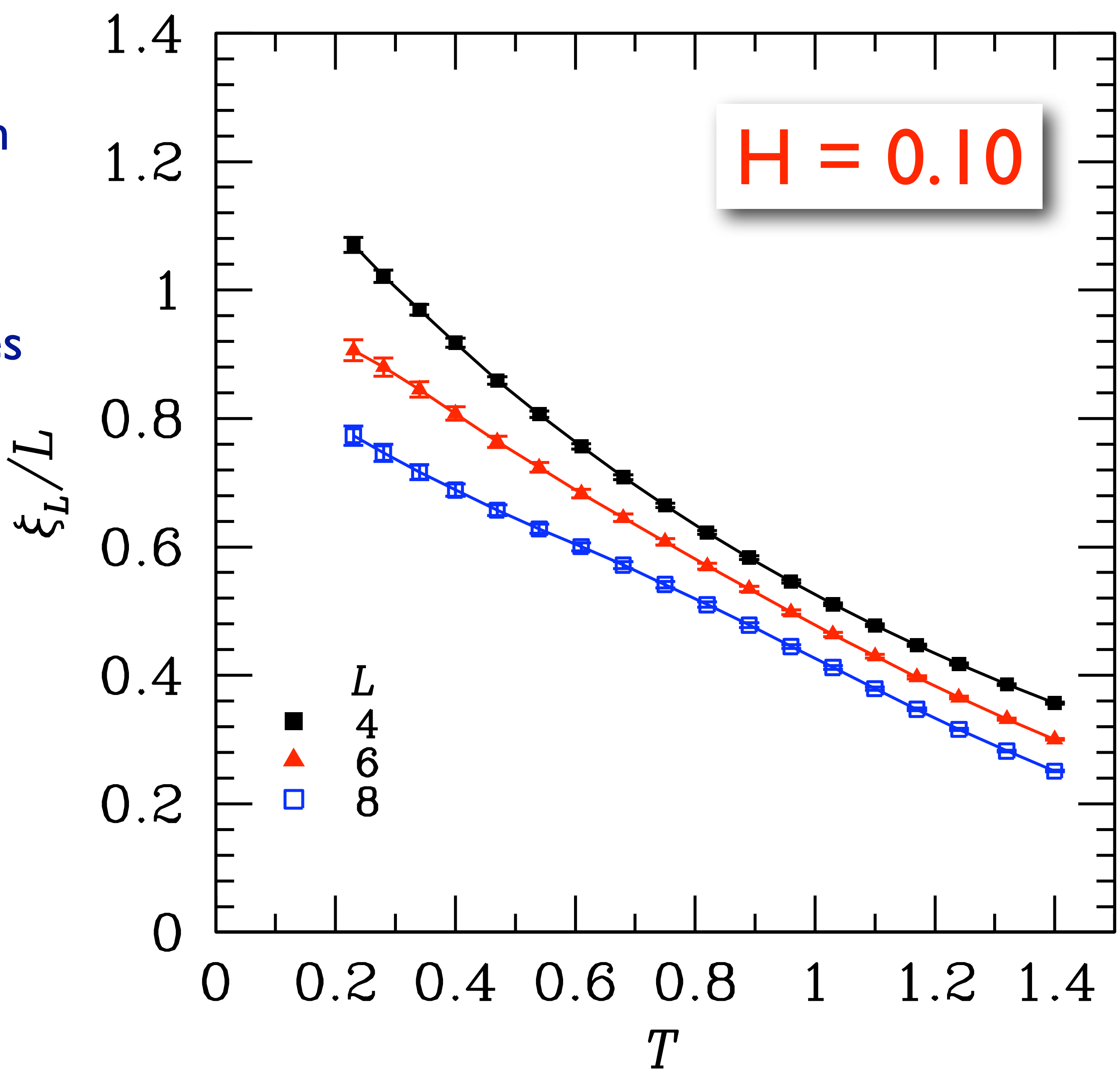




Finite fields... No transition

- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.



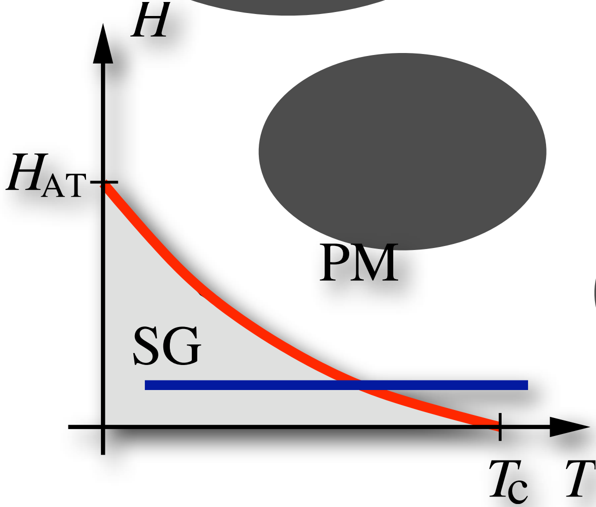


Finite fields... No transition

Problem:
- small systems.

- Using parallel tempering we can scan down to $T = 0.23$.
- We perform slices at different fields.

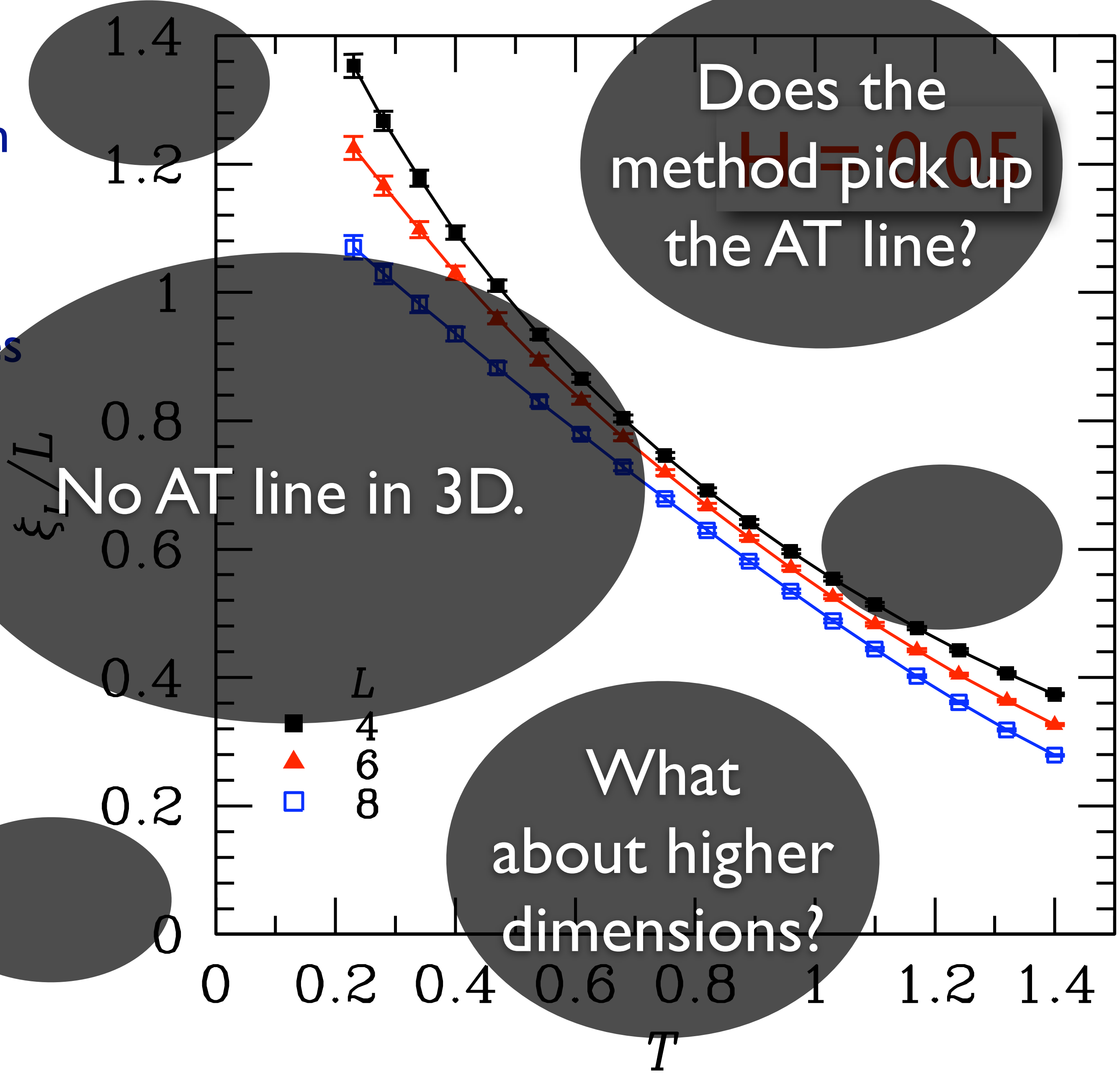
Maybe AT line for d > 6?



No AT line in 3D.

Does the method pick up the AT line?

What about higher dimensions?



Solution: 1D chain

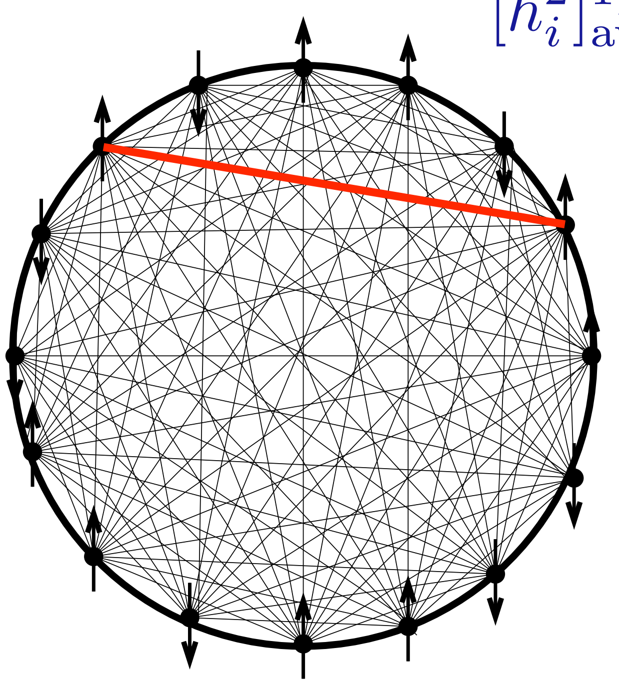
PRB 72, 184416 (2005)

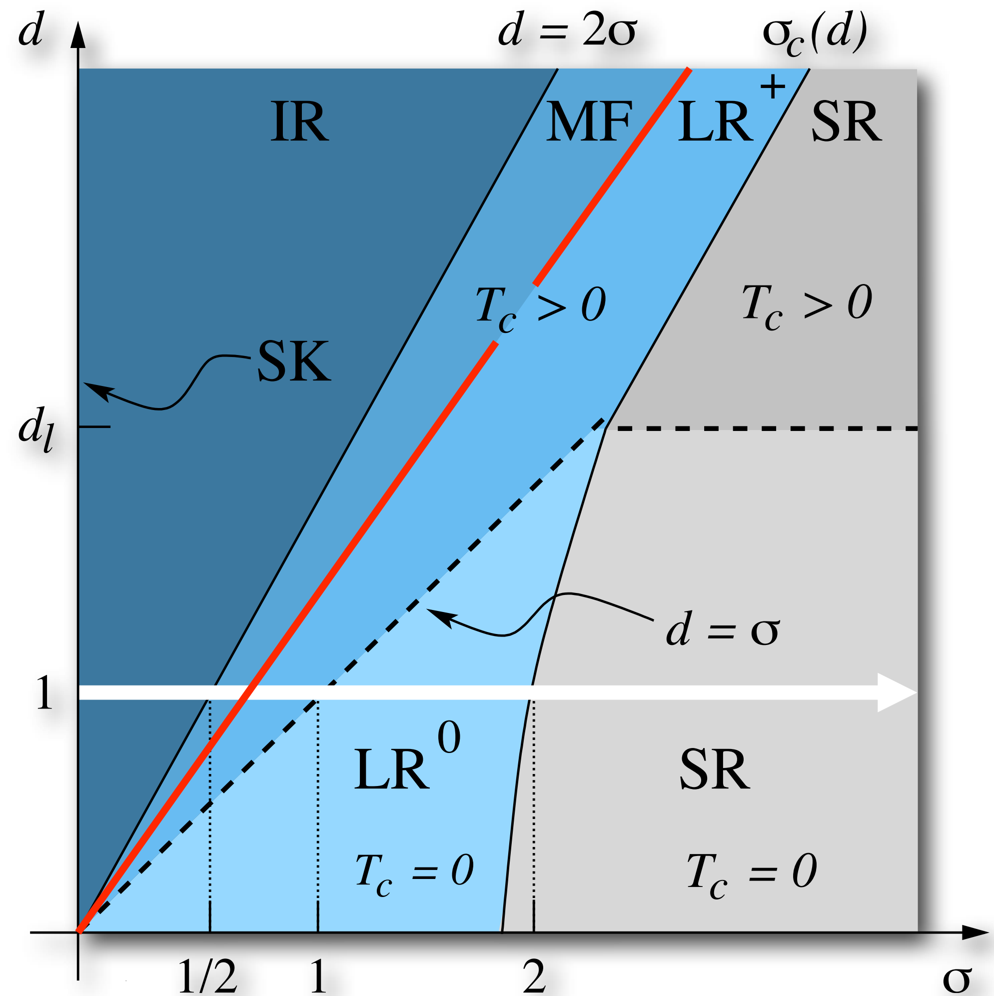
$$\mathcal{H} = -\sum_{ij} J_{ij} S_i S_j - \sum_i h_i S_i$$

- The sum ranges over all spins.
- Gaussian random fields and power-law modulated random bonds (SK model for $\sigma = 0$):

$$[h_i^2]_{\rm av}^{1/2} = H_{\rm r}$$

$$J_{ij} \sim \frac{\epsilon_{ij}}{r_{ij}^{\sigma}}$$



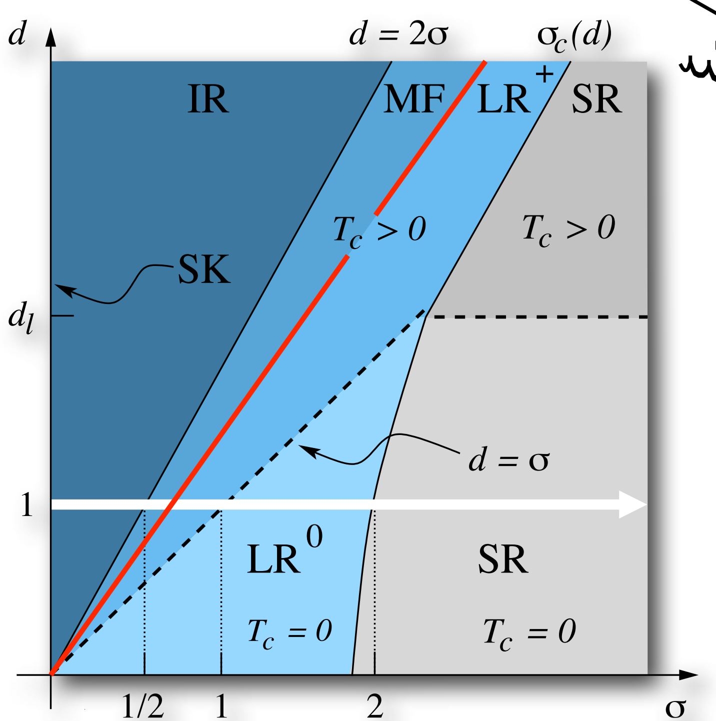


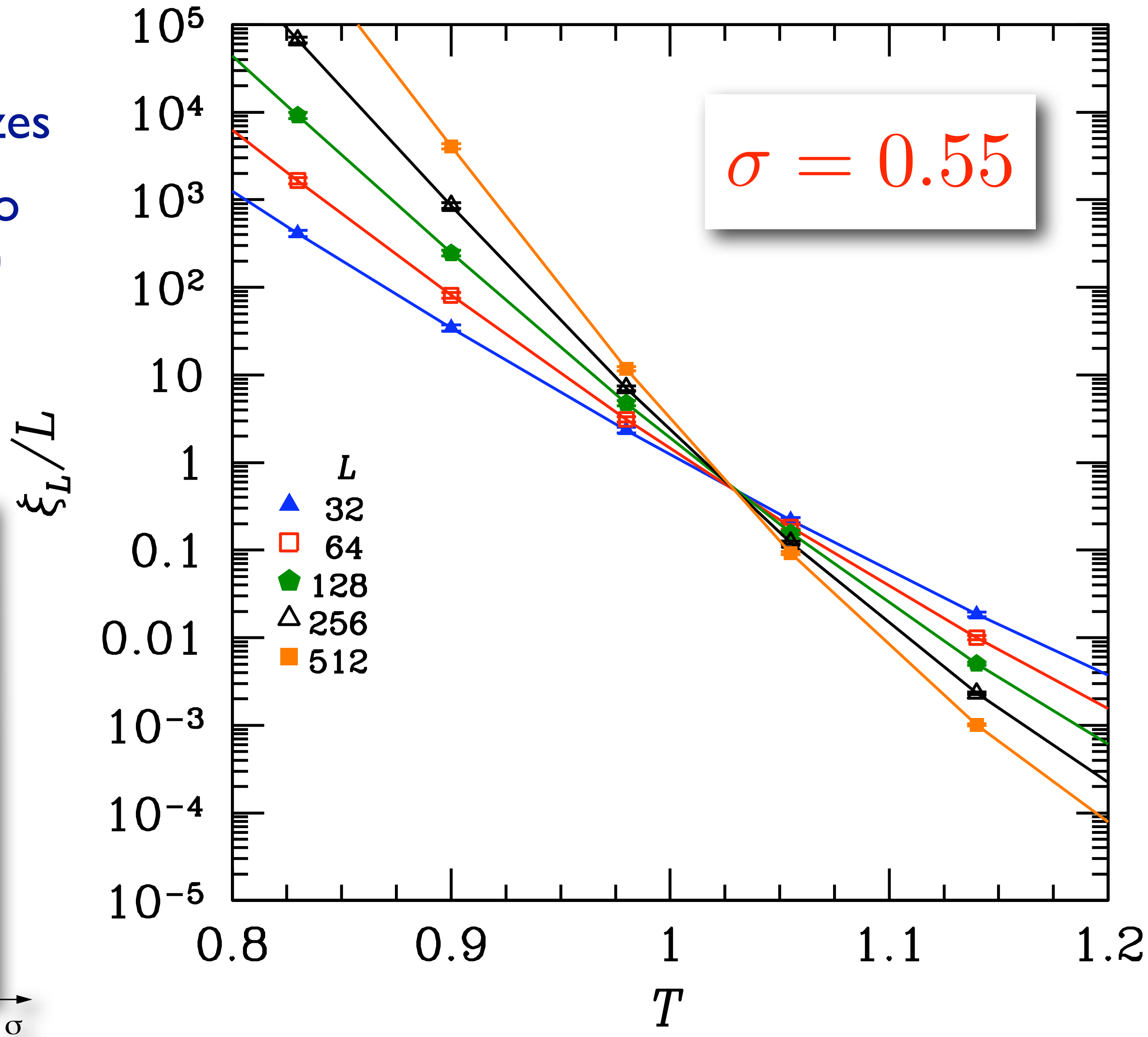
The model allows for a large range of sizes.

Interesting regime: $1/2 < \sigma < 1$

1D chain: zero field

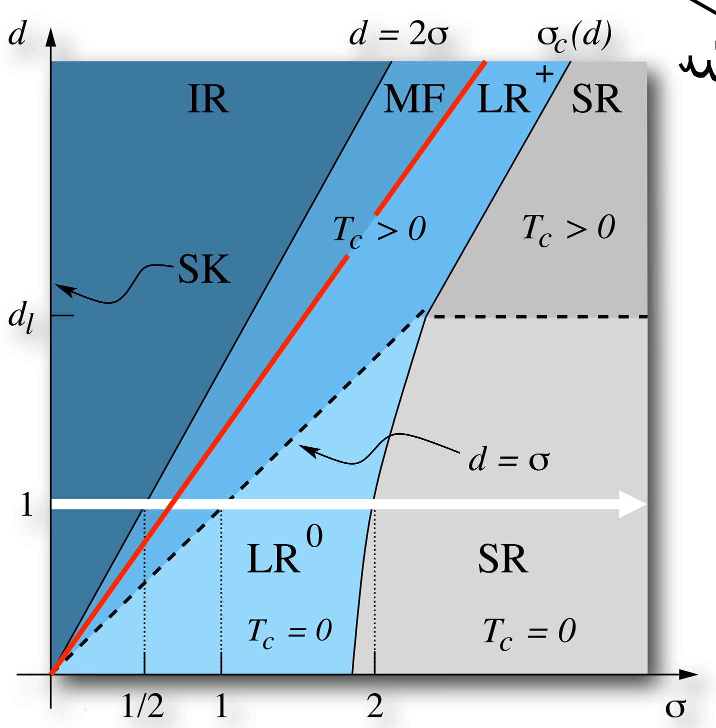
- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 1.00$

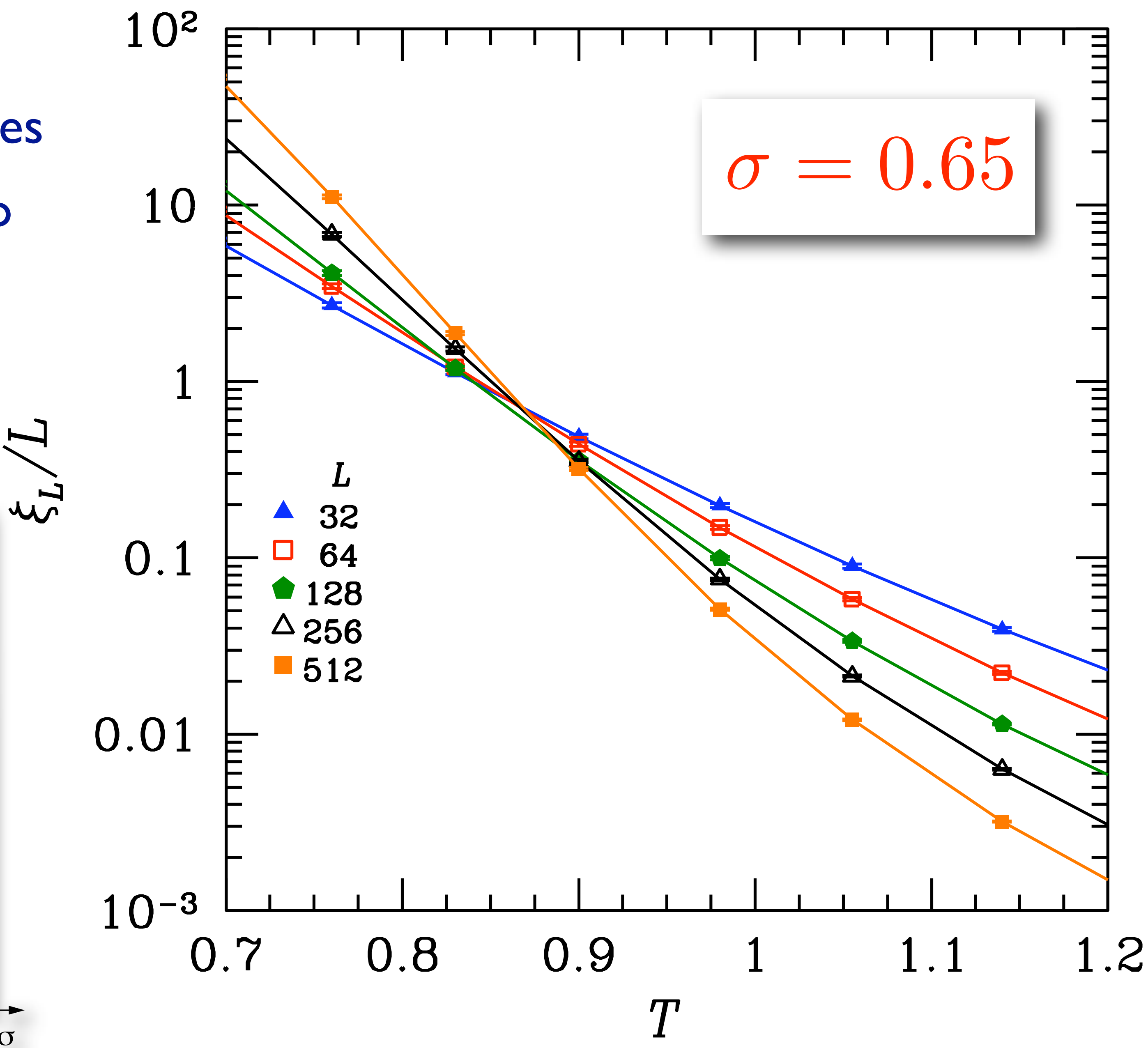




1D chain: zero field

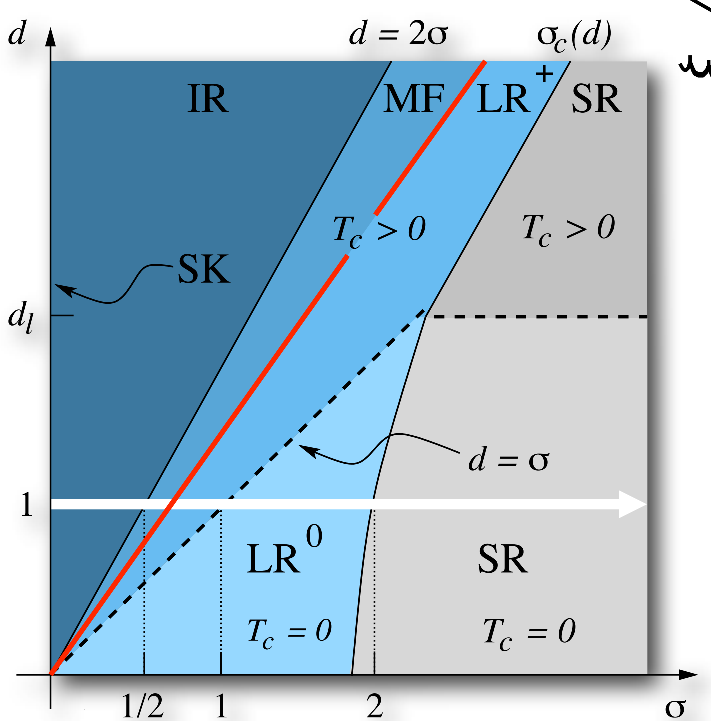
- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 0.85$

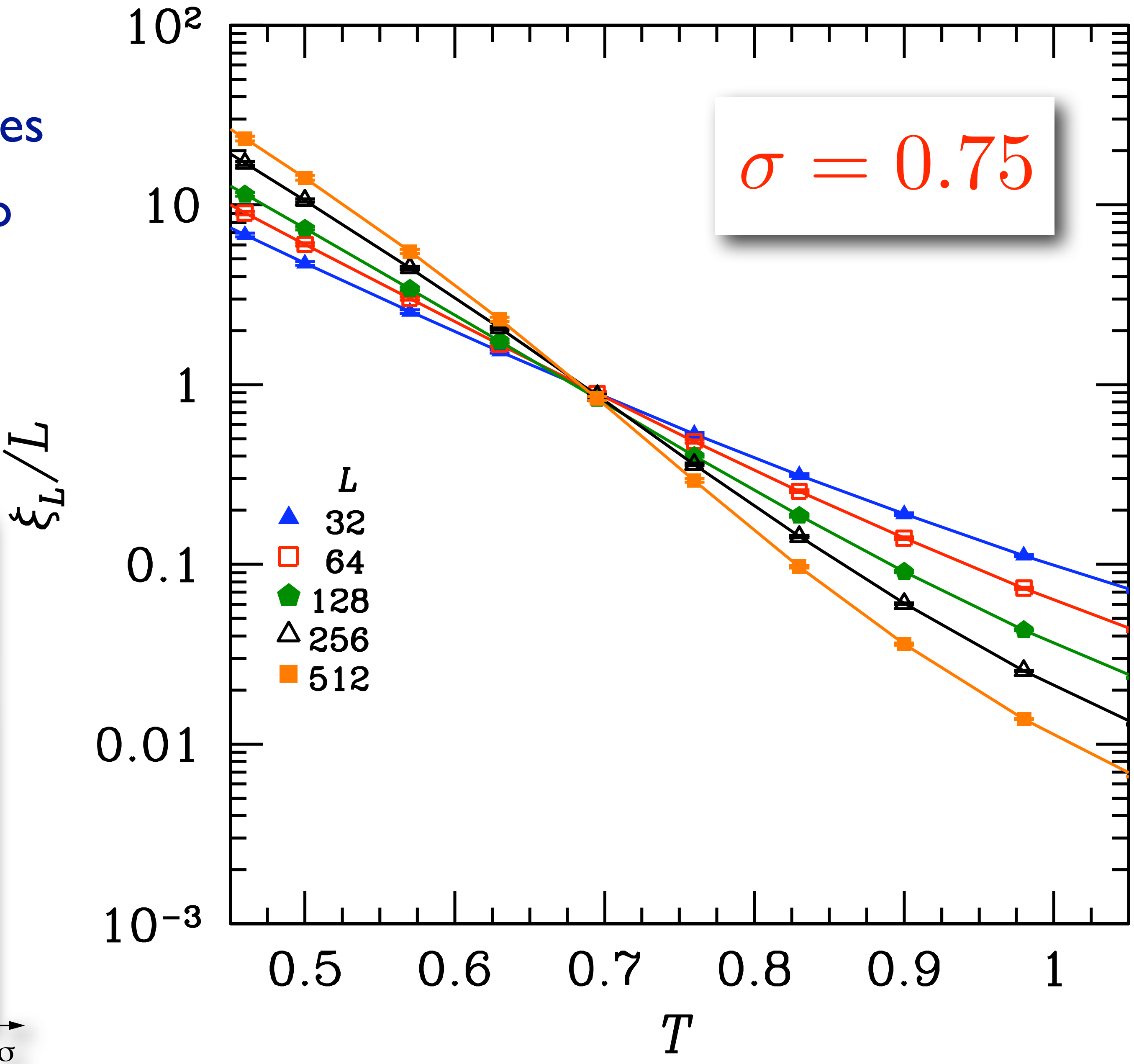




1D chain: zero field

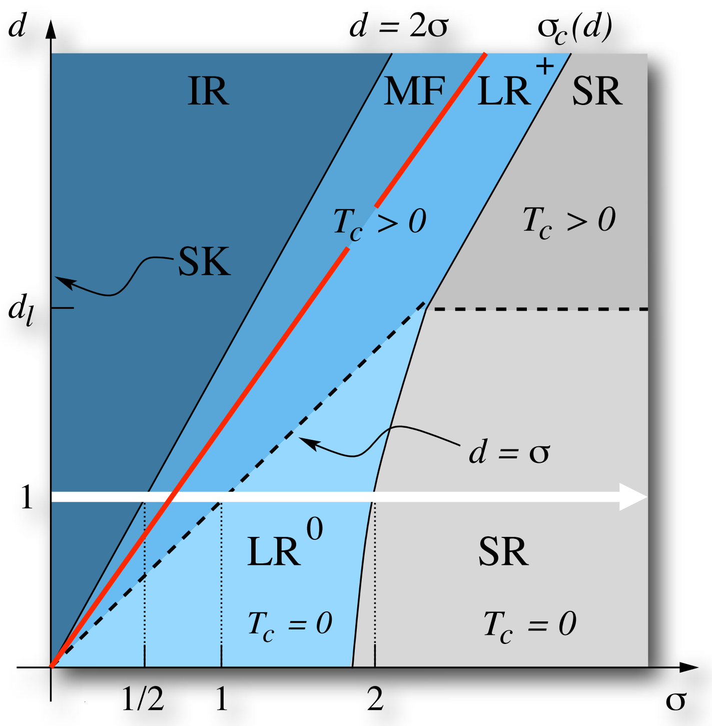
- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 0.68$

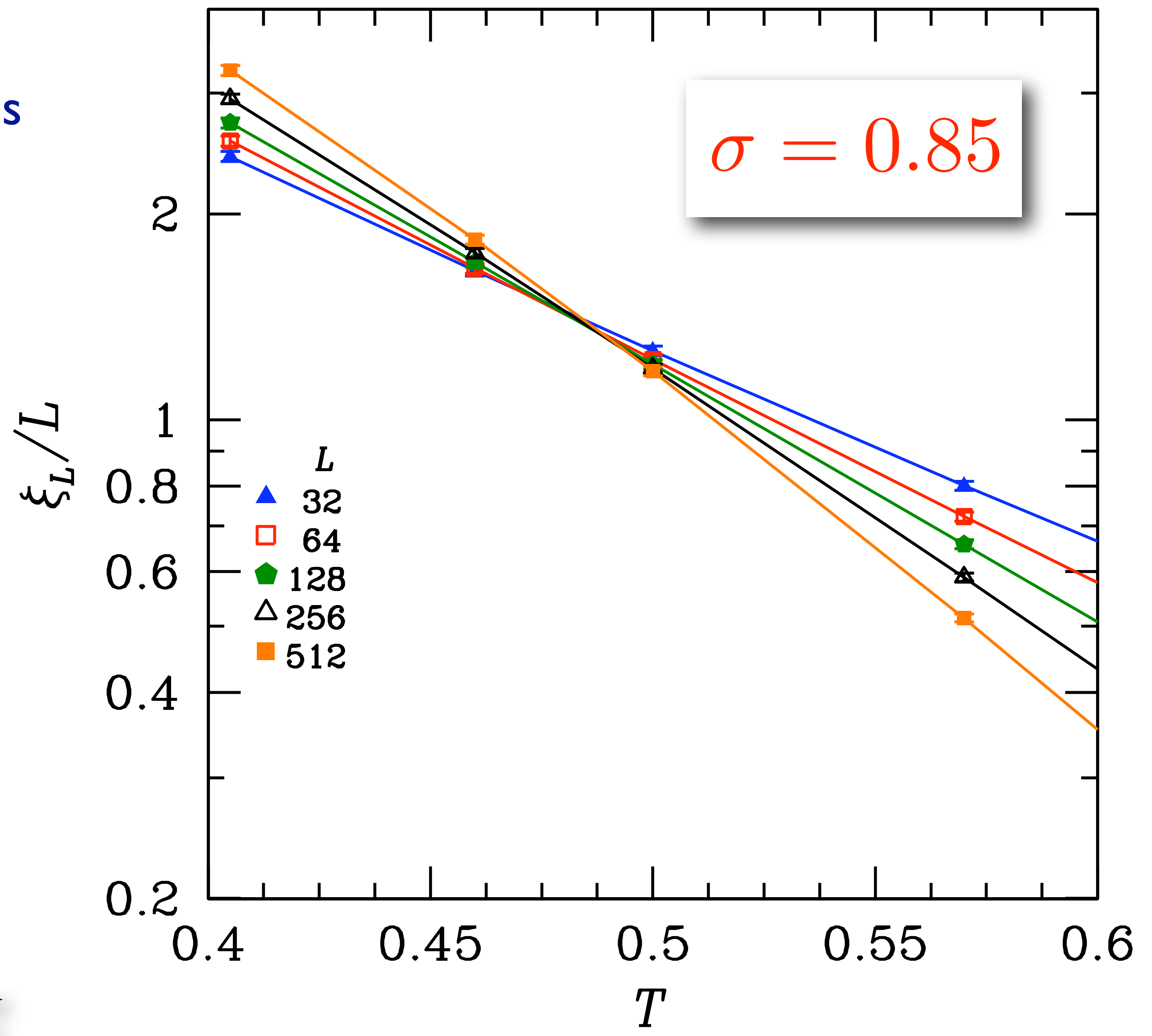




1D chain: zero field

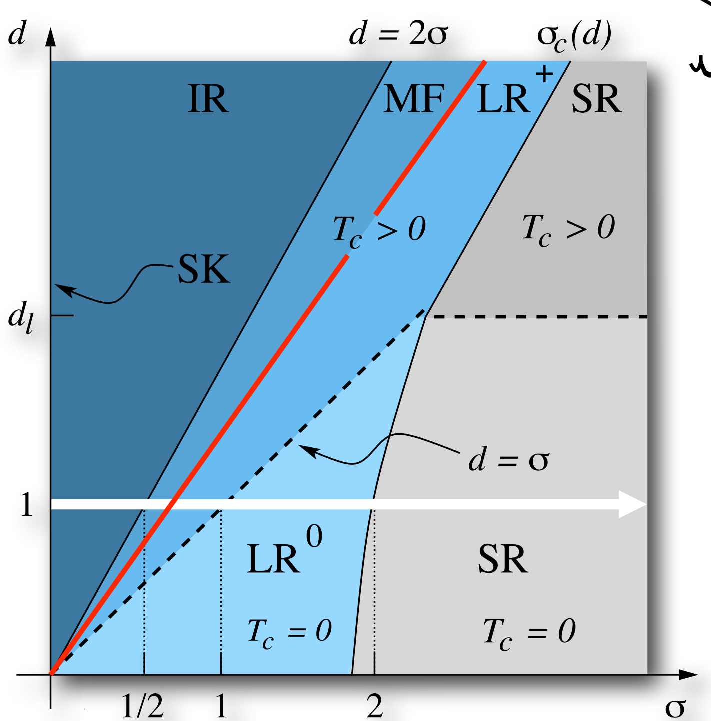
- The data span a large range of sizes
- Transition in zero field for $\sigma < 1.0$
- $T_c \approx 0.48$

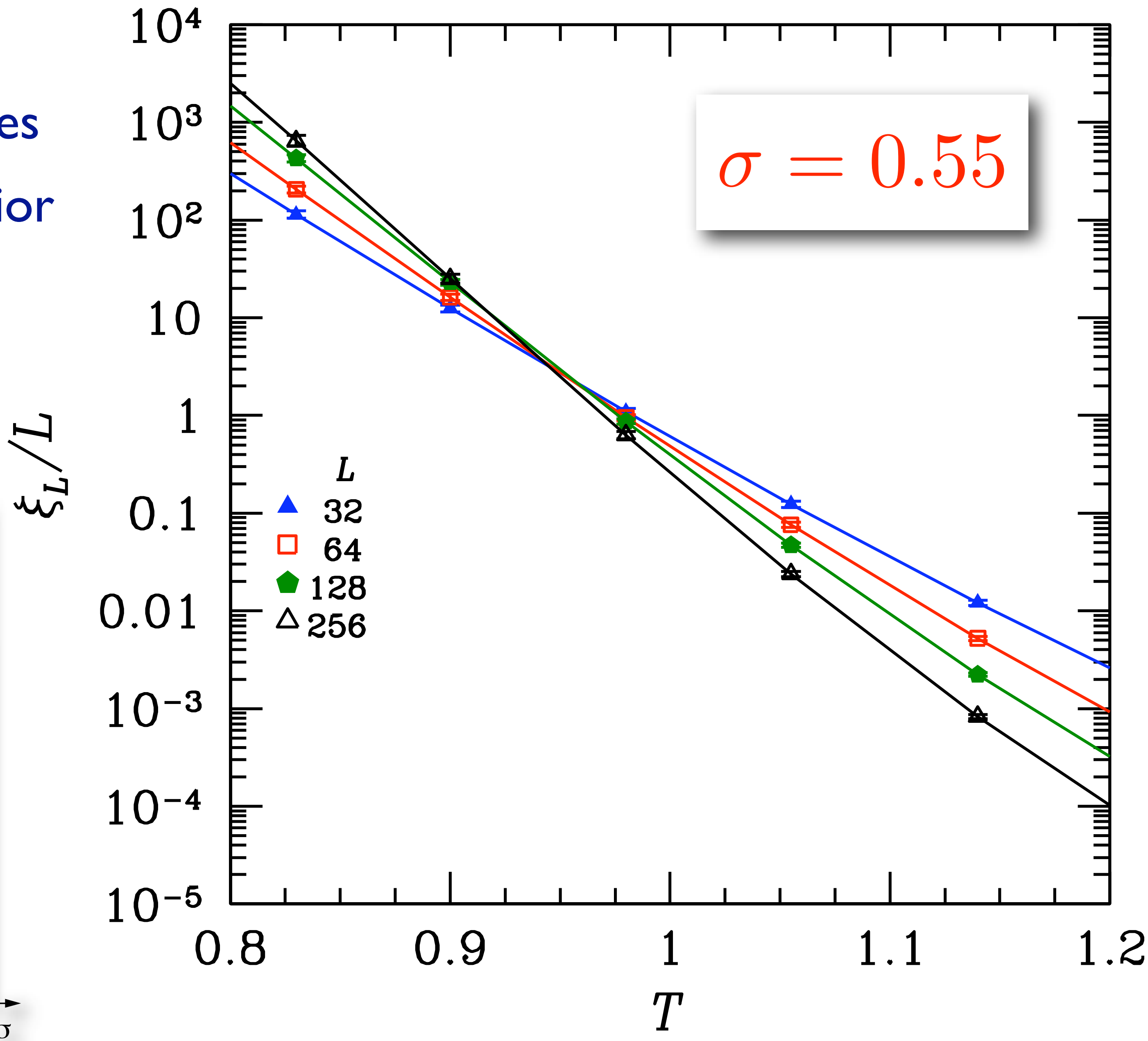




1D chain: finite field (H = 0.10)

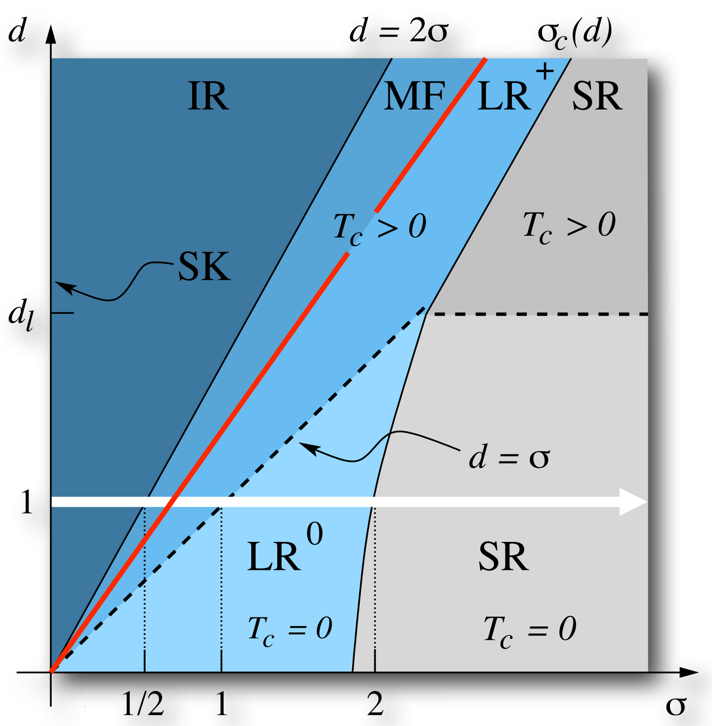
- The data span a large range of sizes
- Mean-field behavior for $\sigma \leq 2/3$
- $T_c \approx 0.92$

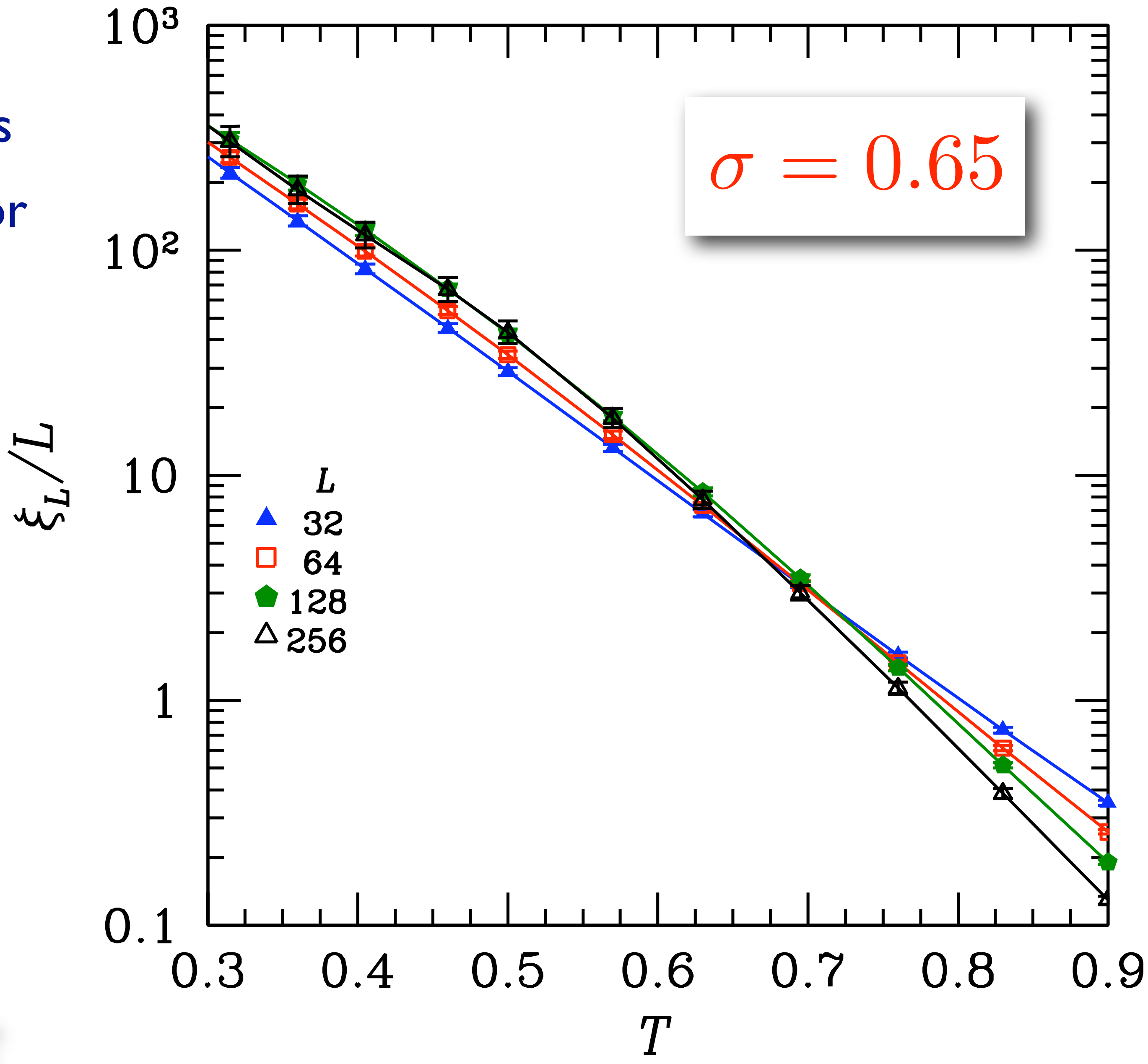




1D chain: finite field (H = 0.10)

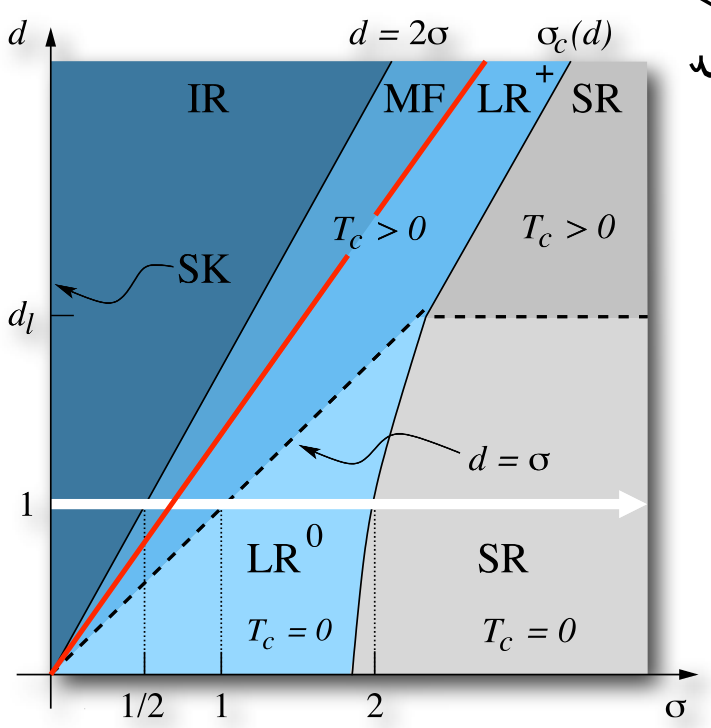
- The data span a large range of sizes
- Mean-field behavior for $\sigma \leq 2/3$
- Crossover regime

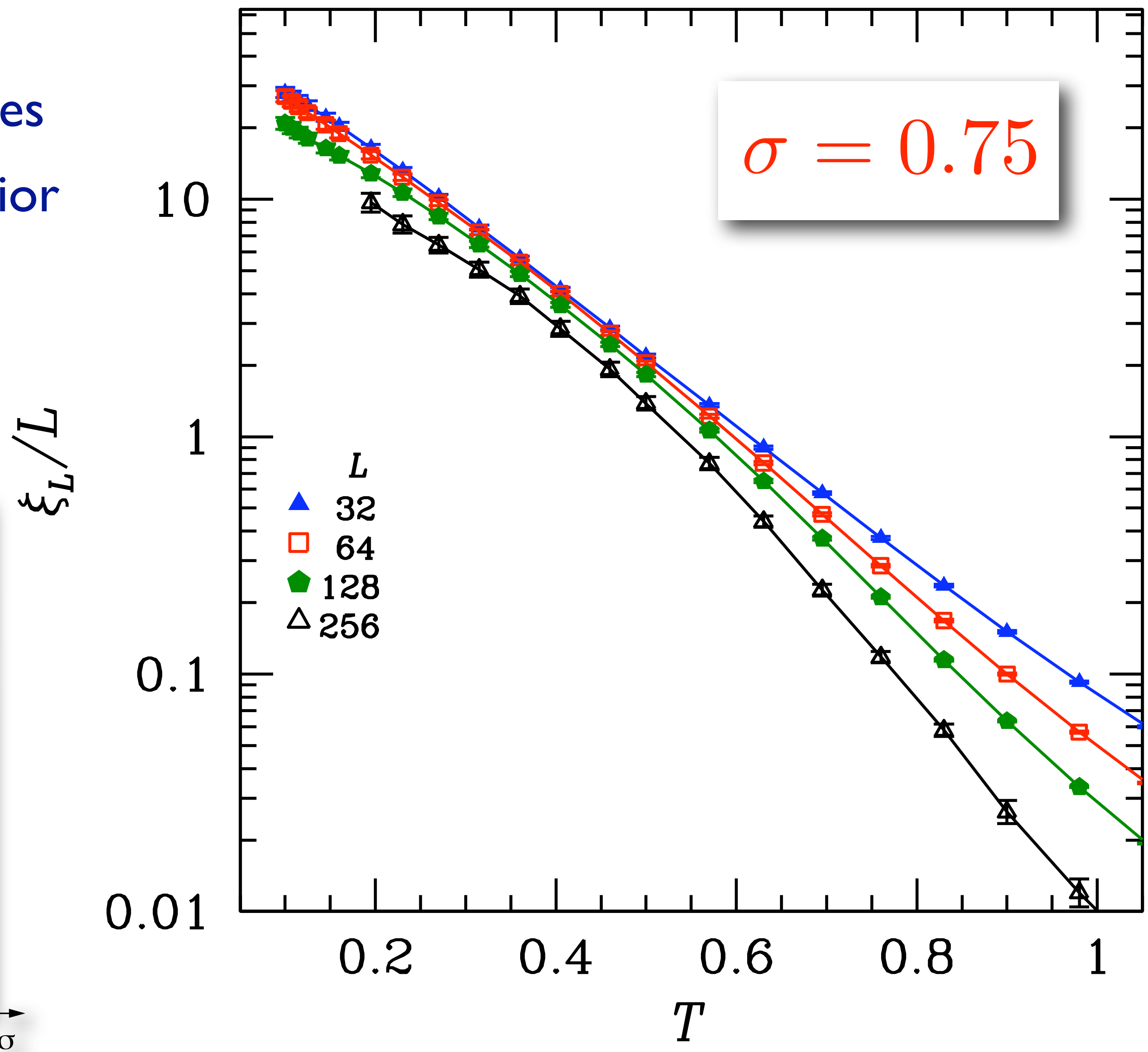




1D chain: finite field (H = 0.10)

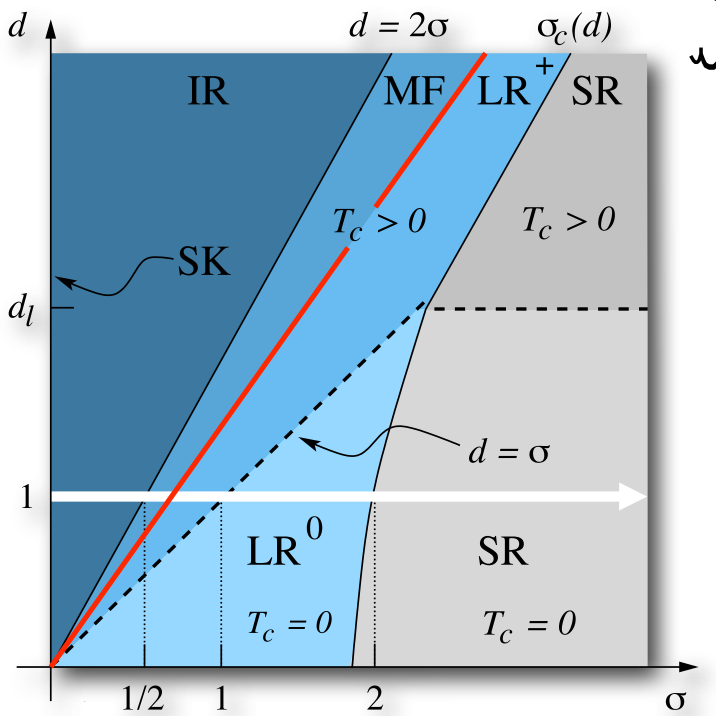
- The data span a large range of sizes
- Mean-field behavior for $\sigma \leq 2/3$
- $T_{\rm c} = 0$

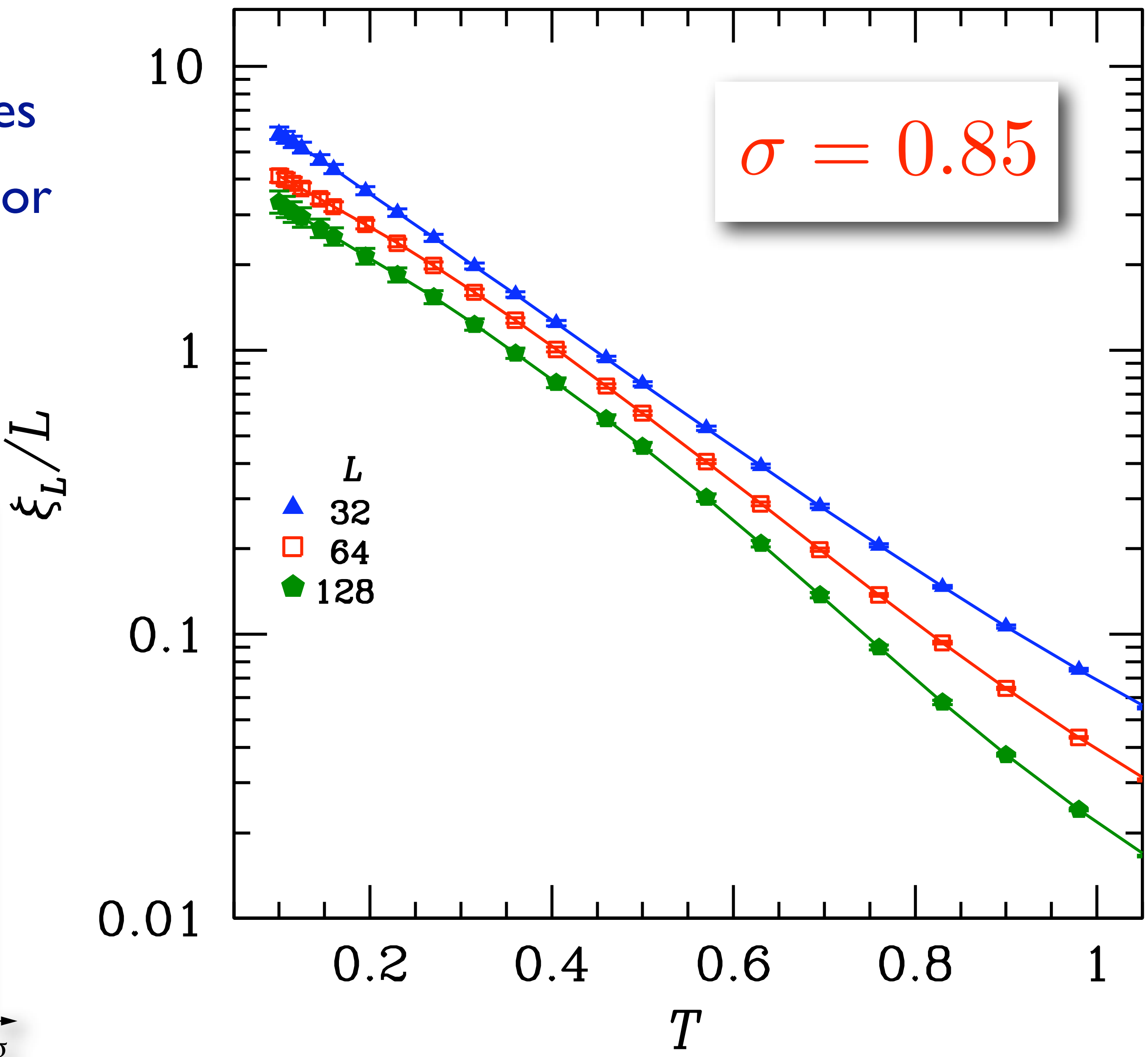




1D chain: finite field (H = 0.10)

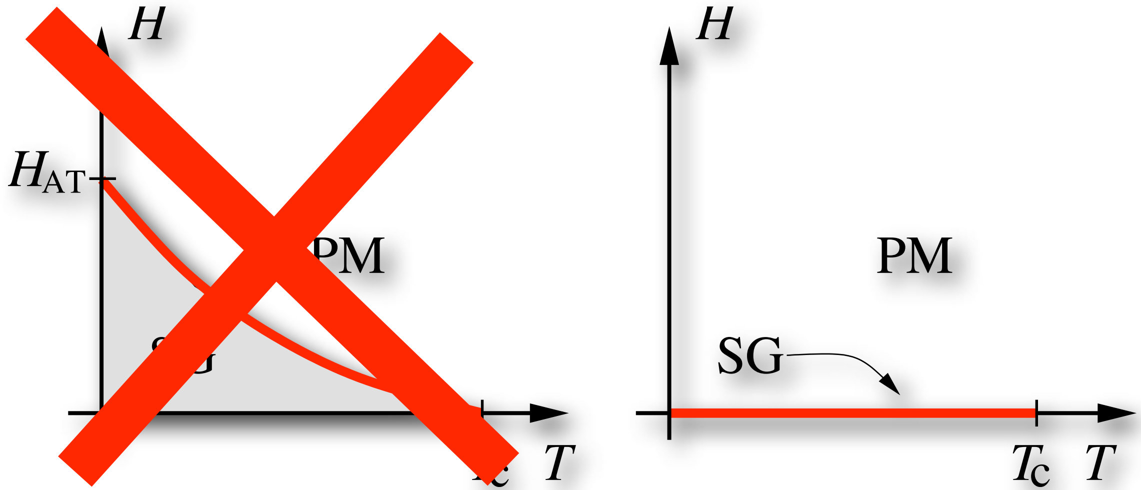
- The data span a large range of sizes
- Mean-field behavior for $\sigma \leq 2/3$
- $T_\mathrm{c} = 0$





What have we learned so far?

- The AT line vanishes when not in the mean-field regime.
- For short-range spin glasses below the upper critical dimension:

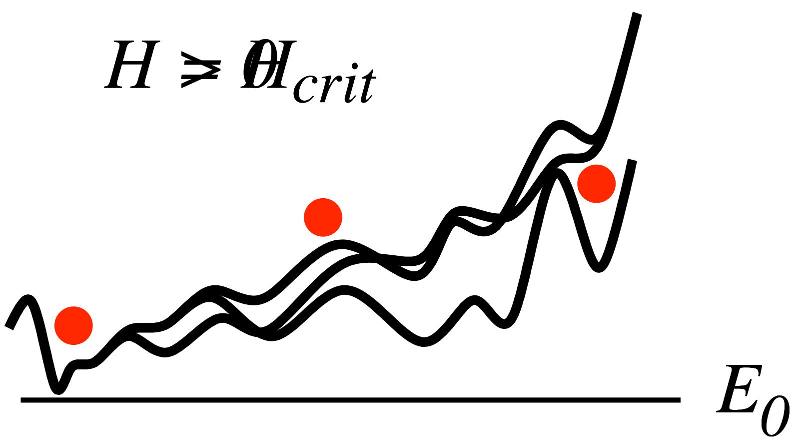


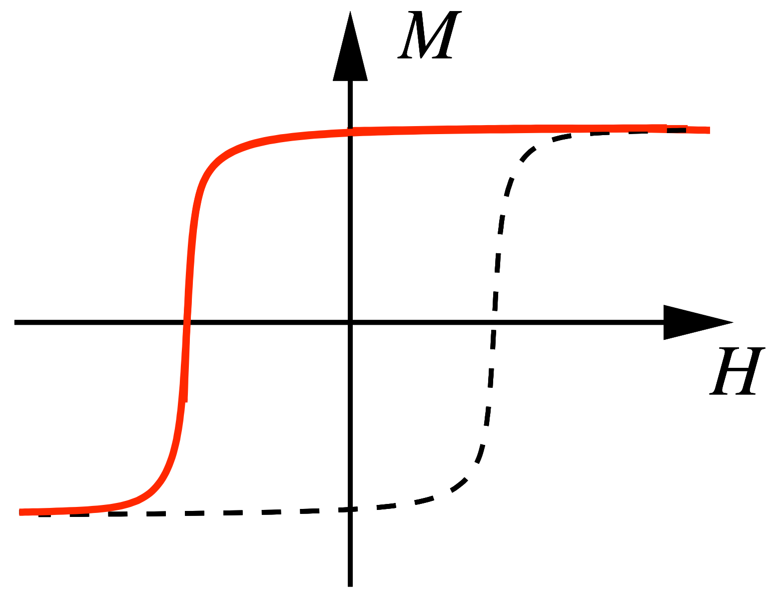
- Related work:
  - Proposal by M.A. Moore (cond-mat/0508087) how RSB might be stable for d < 6 (Temesvari: RSB for d > 8).
  - Proposal by de Dominicis (cond-mat/0509096) of a possible field theory for DP for d < 6.
  - See also: http://jc-cond-mat.bell-labs.com/jc-cond-mat/

Nonequilibrium properties in a field

Hysteresis in disordered spin systems?

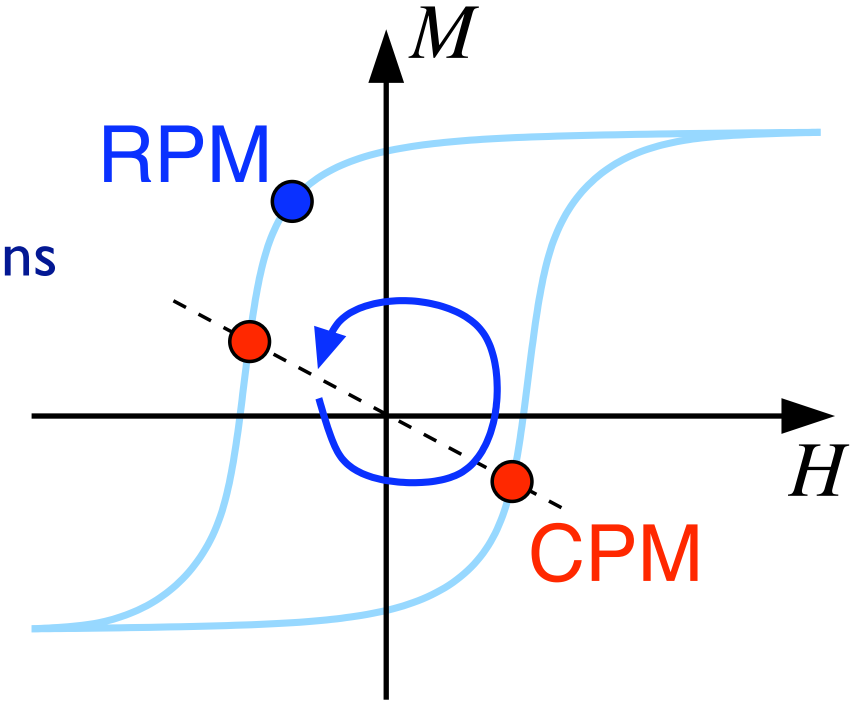
- Due to the randomness the system has a rough energy landscape.
- The rough energy landscape has many metastable states responsible for the hysteresis.





RPM & CPM

cond-mat/0509515 (2005)

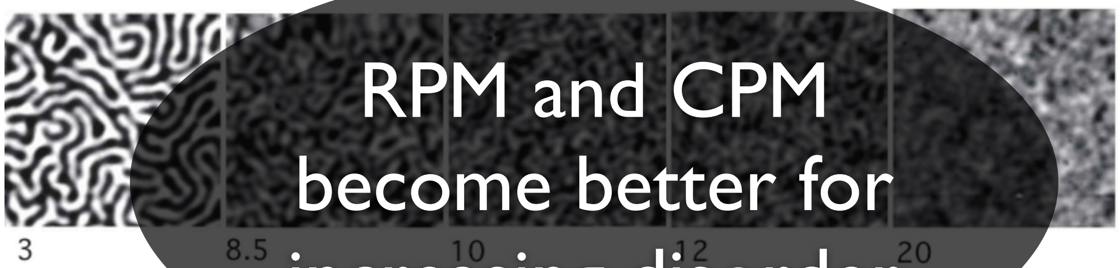


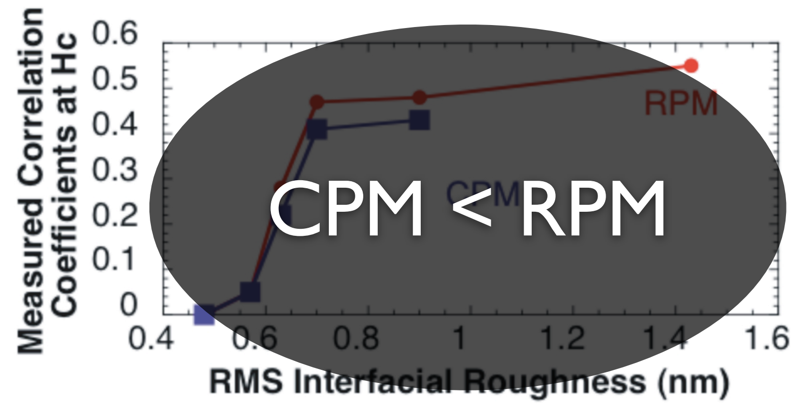
- Definitions:
  - Complementary point memory: Correlations between configurations at $+H^*$ and $-H^*$.
  - Return point memory: Configurations at a given H^* are similar after n loop cycles.
- Example: Barkhausen noise.
- Recent experiments [Pierce et al. (05)] and numerical work [Deutsch & Mai (05), Jagla (05)] suggest the following:
  - RPM and CPM $\longrightarrow$ 0 for decreasing disorder.
  - CPM < RPM < 1 for systems with high disorder.

Can we see these effects in simple models?

Previous results

- Experiments by Pierce et al. (05):





increasing disorder

Measure the effects of disorder on Co/Pt multilayer films using X-ray speckle metrology.

- Simulations by Deutsch & Mai (05) [Jagla (05)] using LLG dynamics:

$$\mathcal{H} = -J\sum_{\langle i,j\rangle}\mathbf{S}_i\mathbf{S}_j - \alpha\sum_i(\mathbf{S}_i\mathbf{n}_i)^2 - w\sum_{i\neq j}\frac{1}{r_{ij}^3}[3(\mathbf{S}_i\cdot\mathbf{e}_{ij})(\mathbf{S}_j\cdot\mathbf{e}_{ij}) - \mathbf{S}_i\mathbf{S}_j] - H\sum_i S_i^z$$

- Results of theory and simulation agree.

Models studied here

- Edwards-Anderson Ising spin glass (EASG):

$$\mathcal{H} = -\sum_{\langle ij \rangle} J_{ij} S_i S_j - H(t) \sum_i S_i \qquad S_i \in \{\pm 1\}$$

  - Gaussian-distributed bonds: $[J_{ij}]_{\rm av} = 0$ and $[J_{ij}^2]_{\rm av}^{1/2} = \sigma_J$
  - Nearest neighbor interactions in two dimensions.
  - From spin reversal symmetry expect: RPM = CPM = 1 for T = 0.

- Random-field Ising model (RFIM):

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j - \sum_i h_i S_i - H(t) \sum_i S_i$$

  - Gaussian-distributed random fields $[h_i]_{\rm av} = 0$ and $[h_i^2]_{\rm av}^{1/2} = \sigma_h$
  - Nearest neighbor interactions in two dimensions.
  - No spin reversal symmetry: CPM < 1
  - Due to the no passing property we expect RPM = 1 for T = 0.

Algorithm

- T = 0
  - Change the external field in small steps. Compute the local fields:

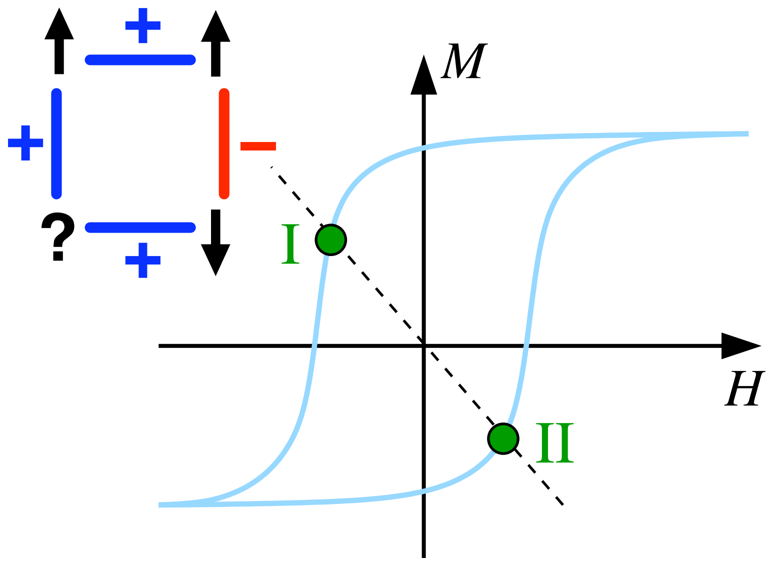
$$h_i = \sum_j J_{ij} S_j + H(t)$$

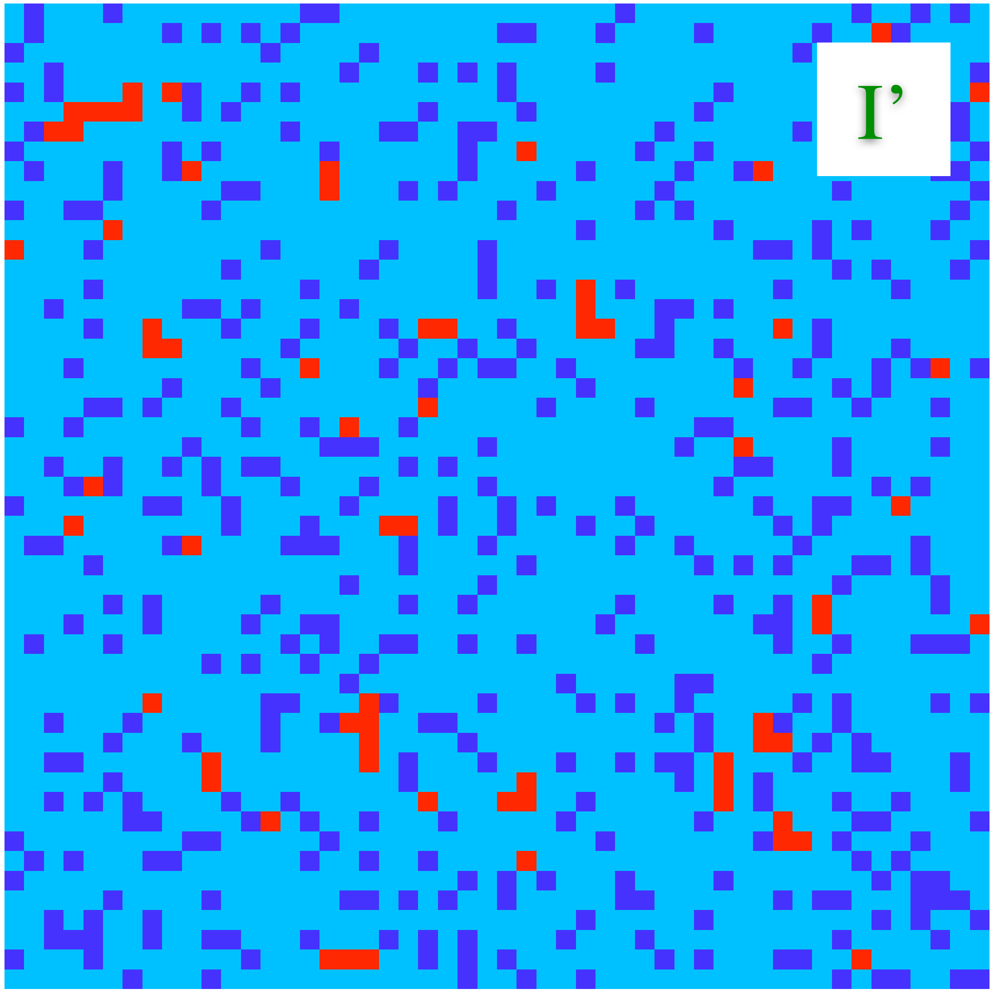
    of each spin S_i . A spin is unstable if $h_i S_i < 0$. Dynamics:
    - Flip a randomly chosen unstable spin.
    - Update the local fields of the neighbors.
    - Iterate until all spins are stable.
- T > 0
  - Change the external field in small steps.
    - For each field step perform a finite-T Mo
    - Iterate until the magnetization is indepe

Average over 500 disorder realizations.

EASG: Qualitative behavior

- Intermediate disorder:
  $$\sigma_J = 1$$
- $T = 0.2$
- Red pixels denote differences between configurations.
- RPM and CPM are not perfect due to frustration.

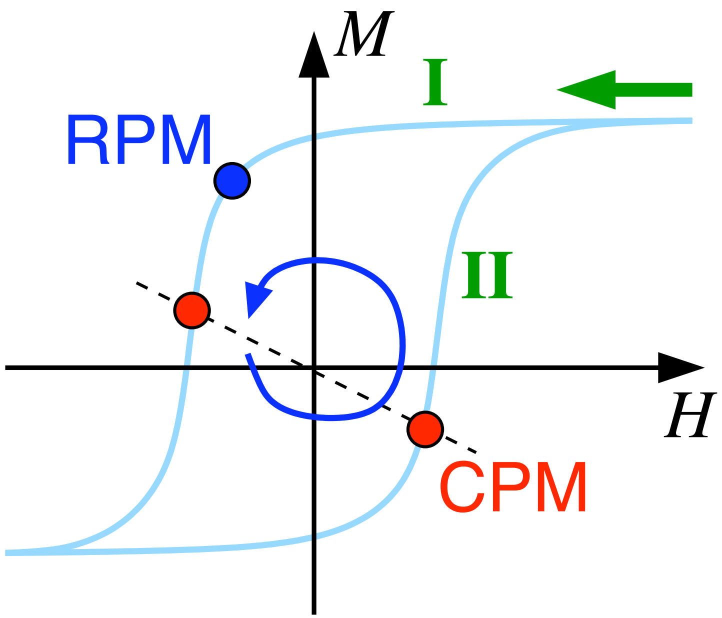




Can we better quantify this?

Overlaps to measure RPM & CPM

- Idea:
  - Study correlations between configurations.
- Start the loop at positive saturation.
- Definition of overlaps: q measures the degree of memory configurations, q' the uniqueness.



- CPM:

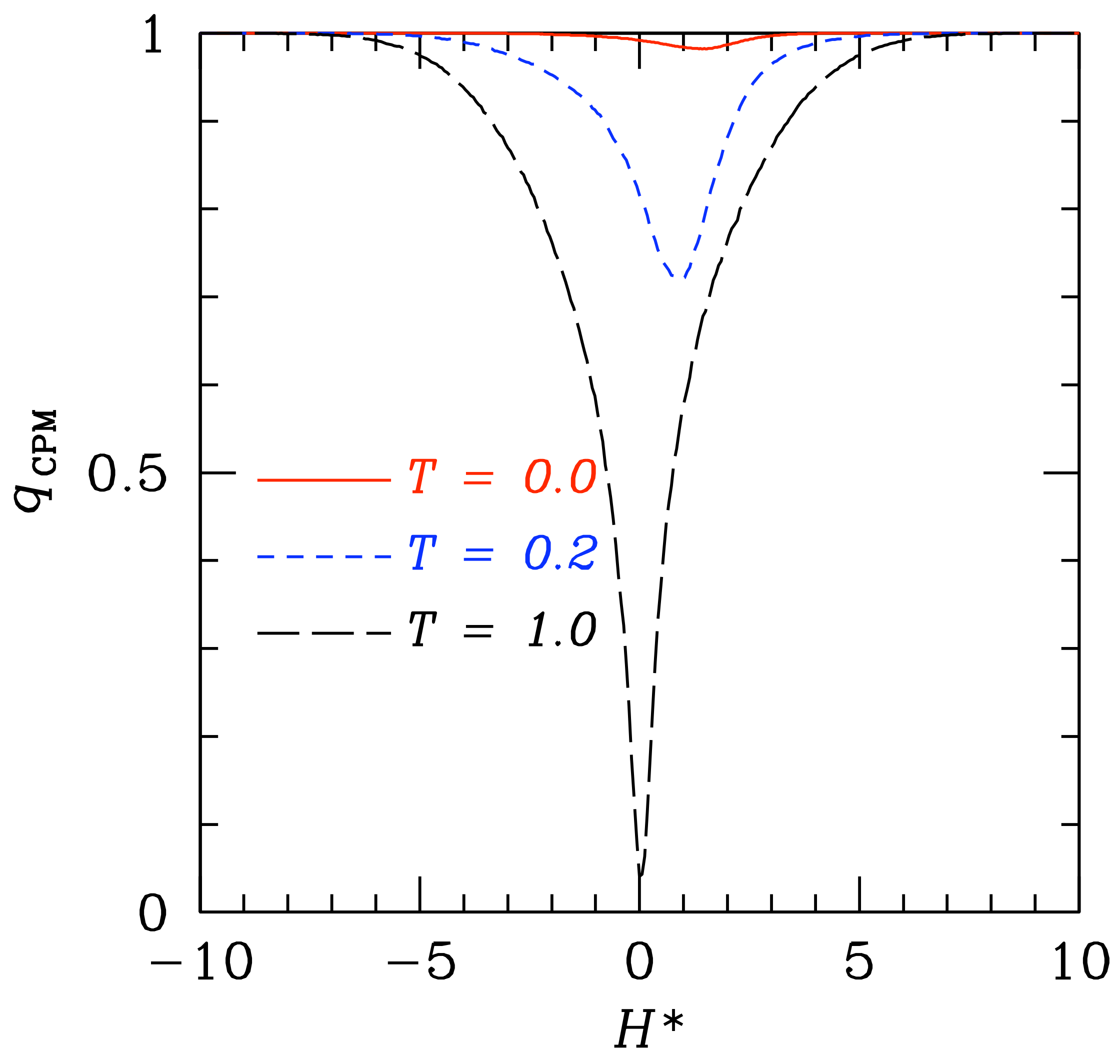
$$q(H^*) = -\frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(-H^*_{\mathrm{II}}) \qquad q'(H^*) = -\frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(H_{\mathrm{II}})$$

- RPM:

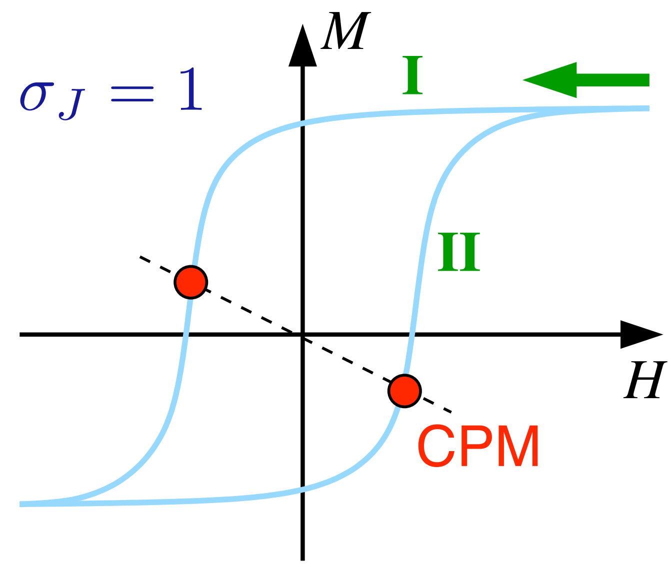
$$q(H^*) = \frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(H^*_{\mathrm{I'}}) \qquad q'(H^*) = \frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(H_{\mathrm{I'}})$$

EASG: Overlap $q(H^*)$

$$q(H^*) = -\frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}}) S_i(-H^*_{\mathrm{II}})$$

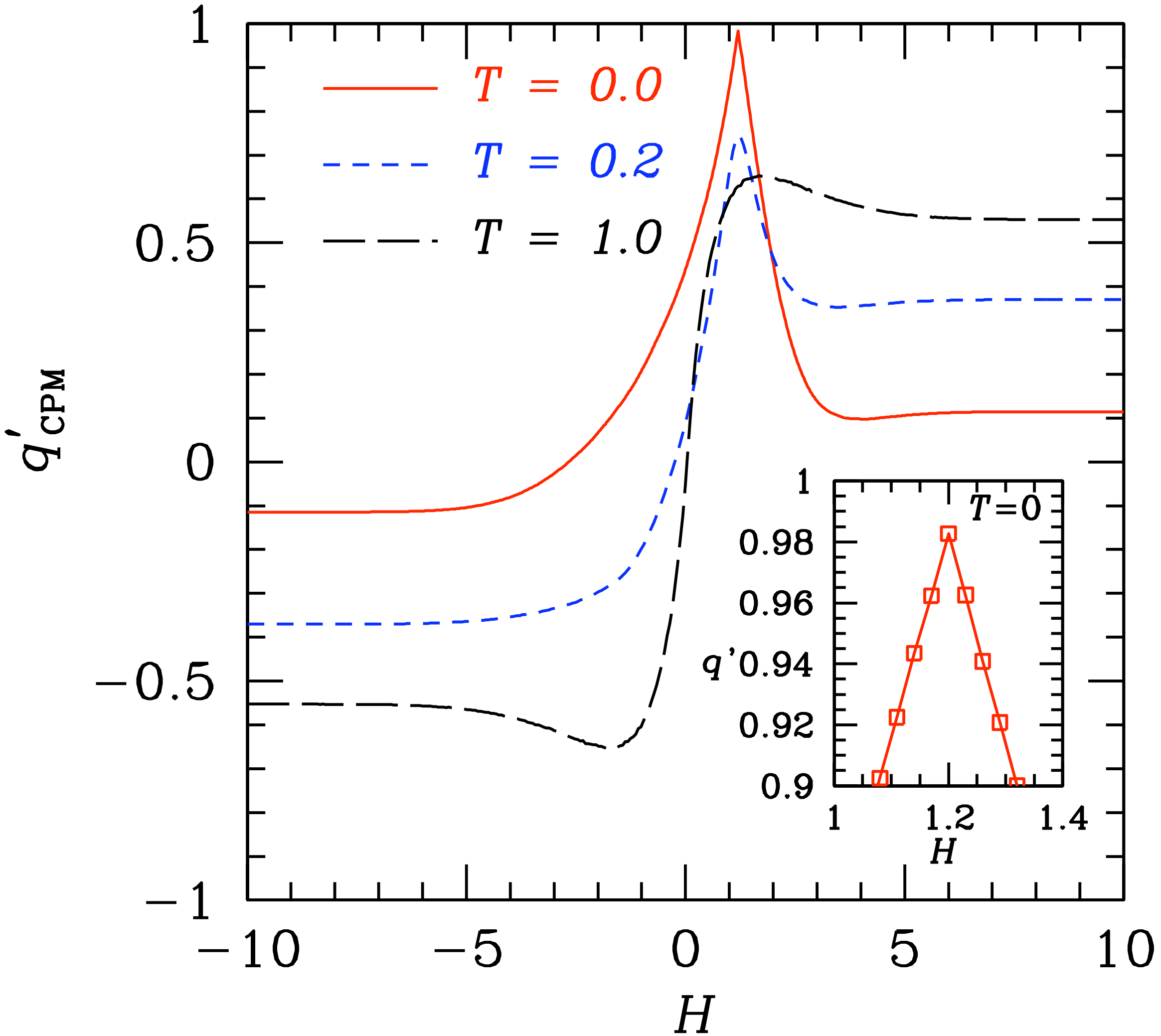


- Data show strong correlations between configurations.
- Memory not perfect even at T = 0.
- Memory decreases with T.

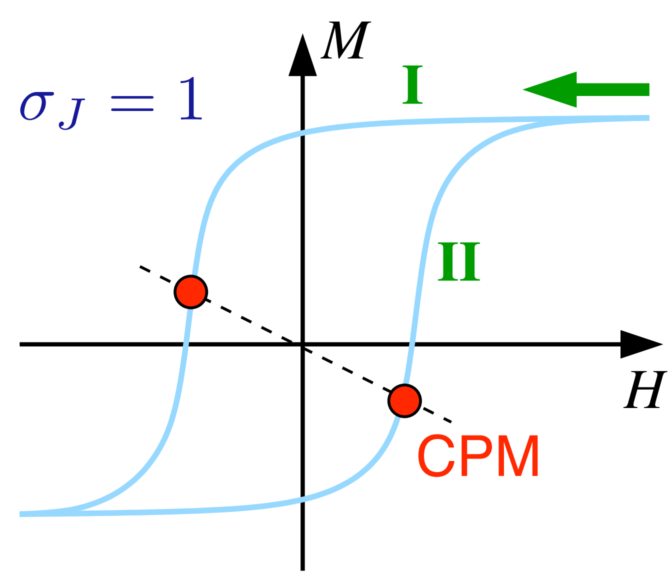


EASG: Overlap $q'(H)$

$$q'(H^*) = -\frac{1}{N}\sum_{i=1}^{N} S_i(H_{\rm I}^*)S_i(H_{\rm II})$$

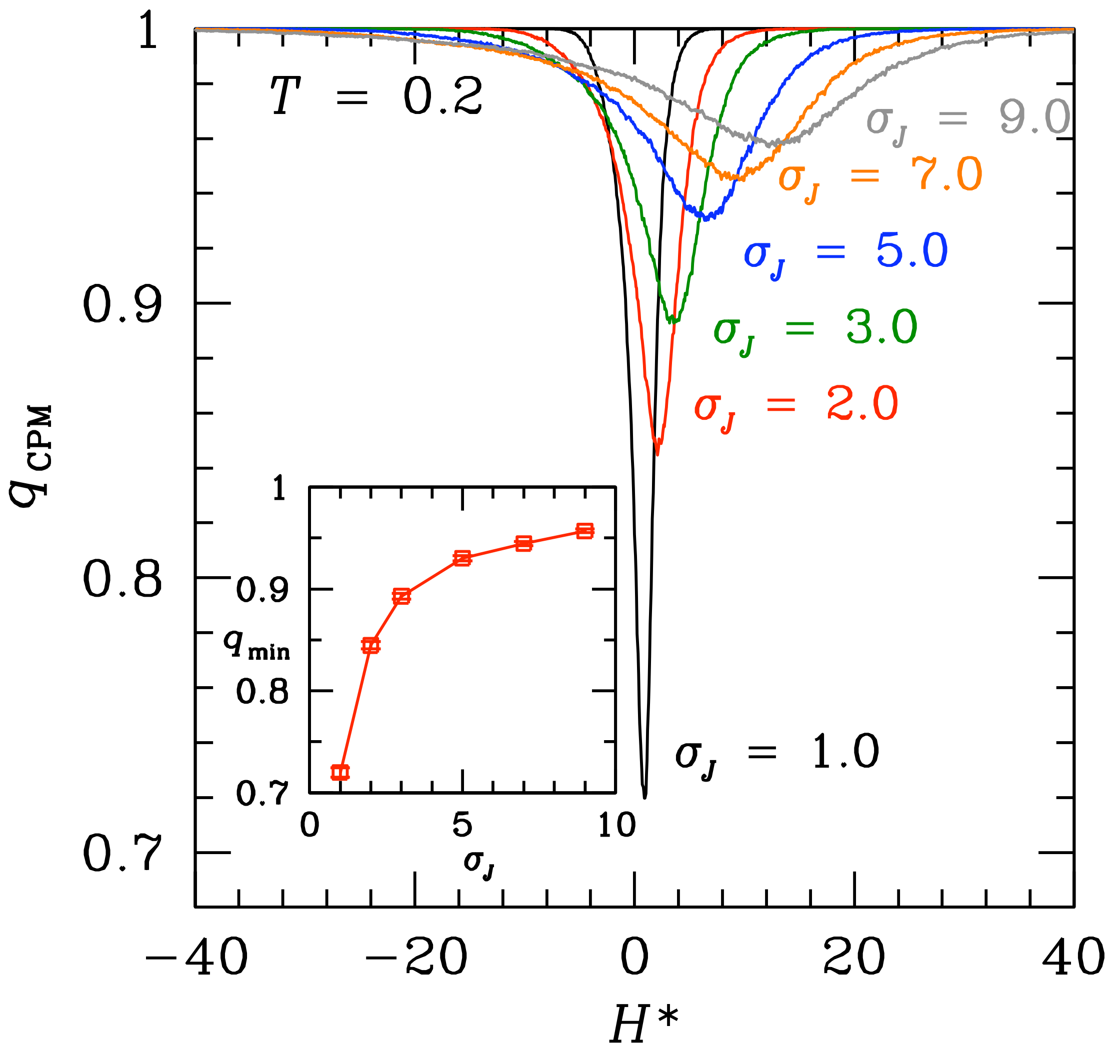


- Correlations unique.
- Memory not perfect even at T = 0.
- Memory decreases with T.
- RPM = CPM.

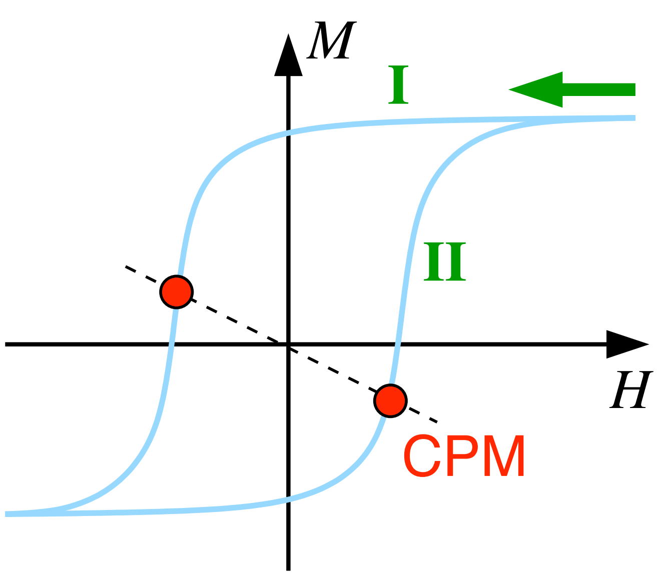


EASG: Overlap $q(H^*)$ disorder scan

$$q(H^*) = -\frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(-H^*_{\mathrm{II}})$$

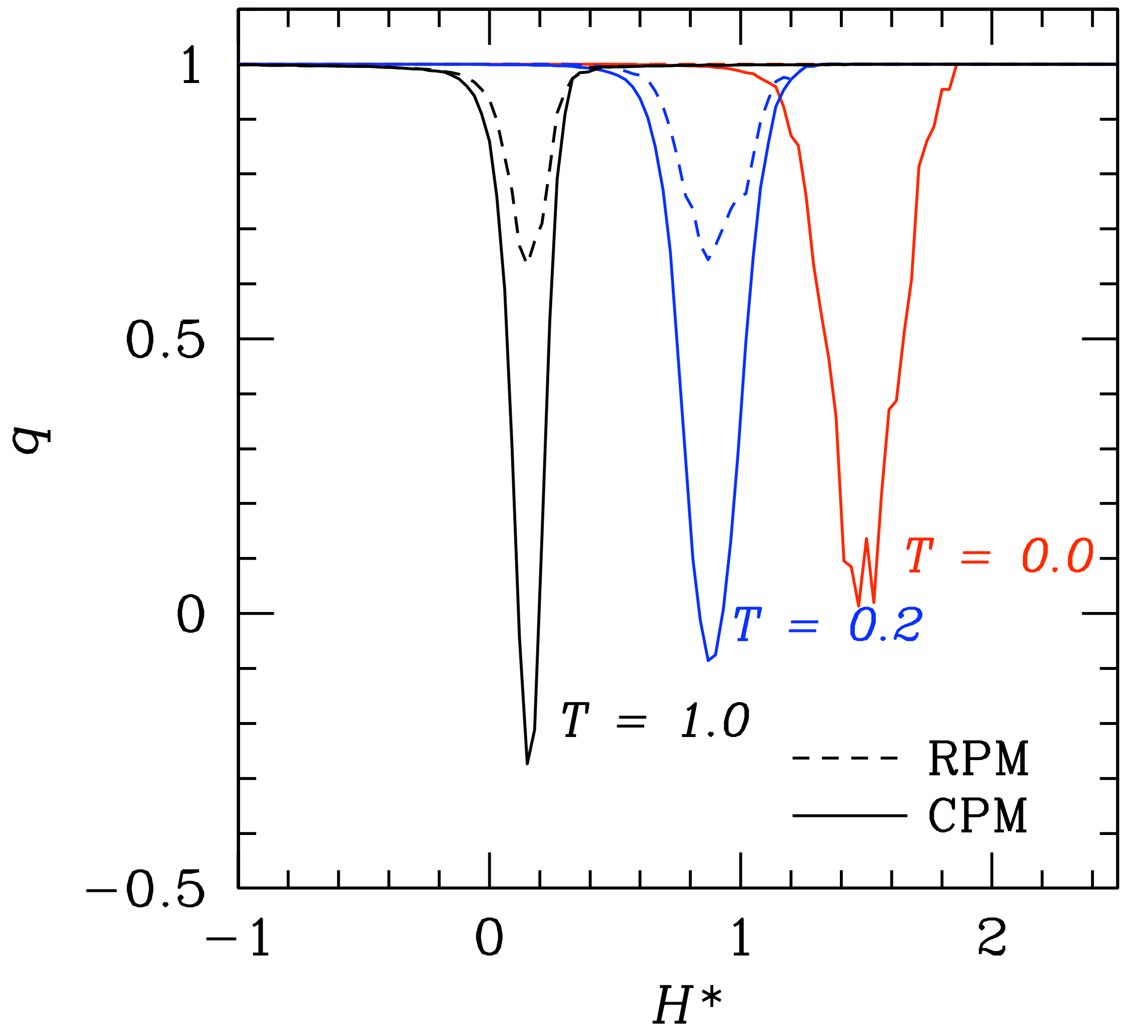


- Data for T = 0.2.
- Memory better for increasing disorder.
- Variable σ_J
- Qualitative agreement with the experiments.

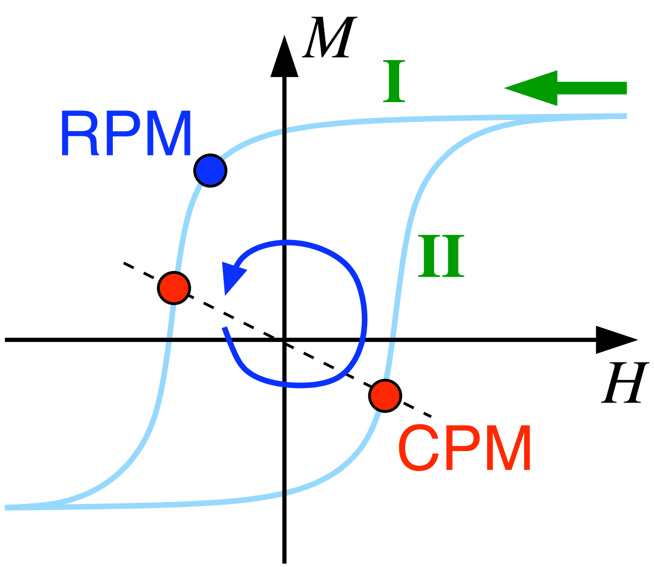


RFIM: Overlap $q(H^*)$

$$q(H^*)_{\mathrm{CPM}} = -\frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(-H^*_{\mathrm{II}}) \qquad q(H^*)_{\mathrm{RPM}} = \frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\mathrm{I}})S_i(H^*_{\mathrm{I'}})$$

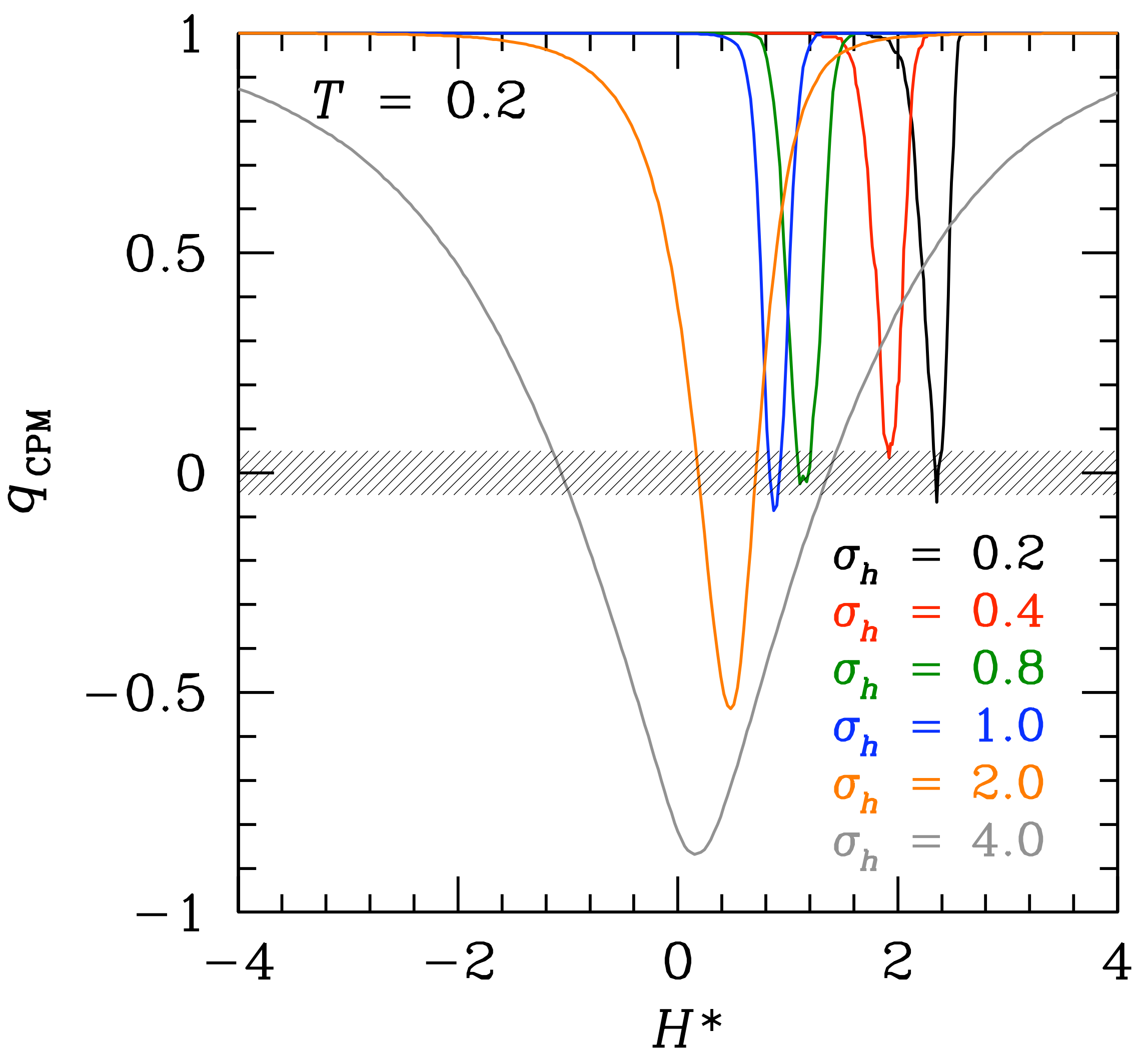


- CPM < RPM.
- RPM = 1 (T = 0).
- CPM < 1 (T = 0).
- $q \to -1$?
- $\sigma_h = 1$

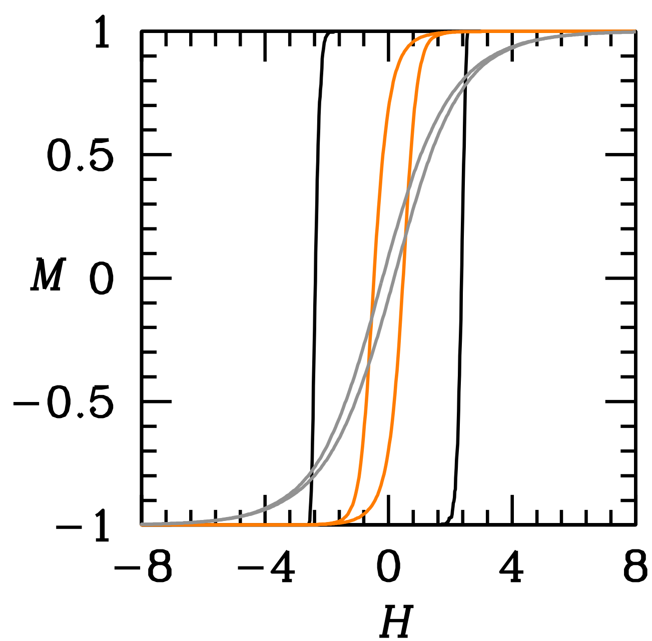


RFIM: Overlap $q(H^*)$ CPM

$$q(H^*) = -\frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\rm I}) S_i(-H^*_{\rm II})$$

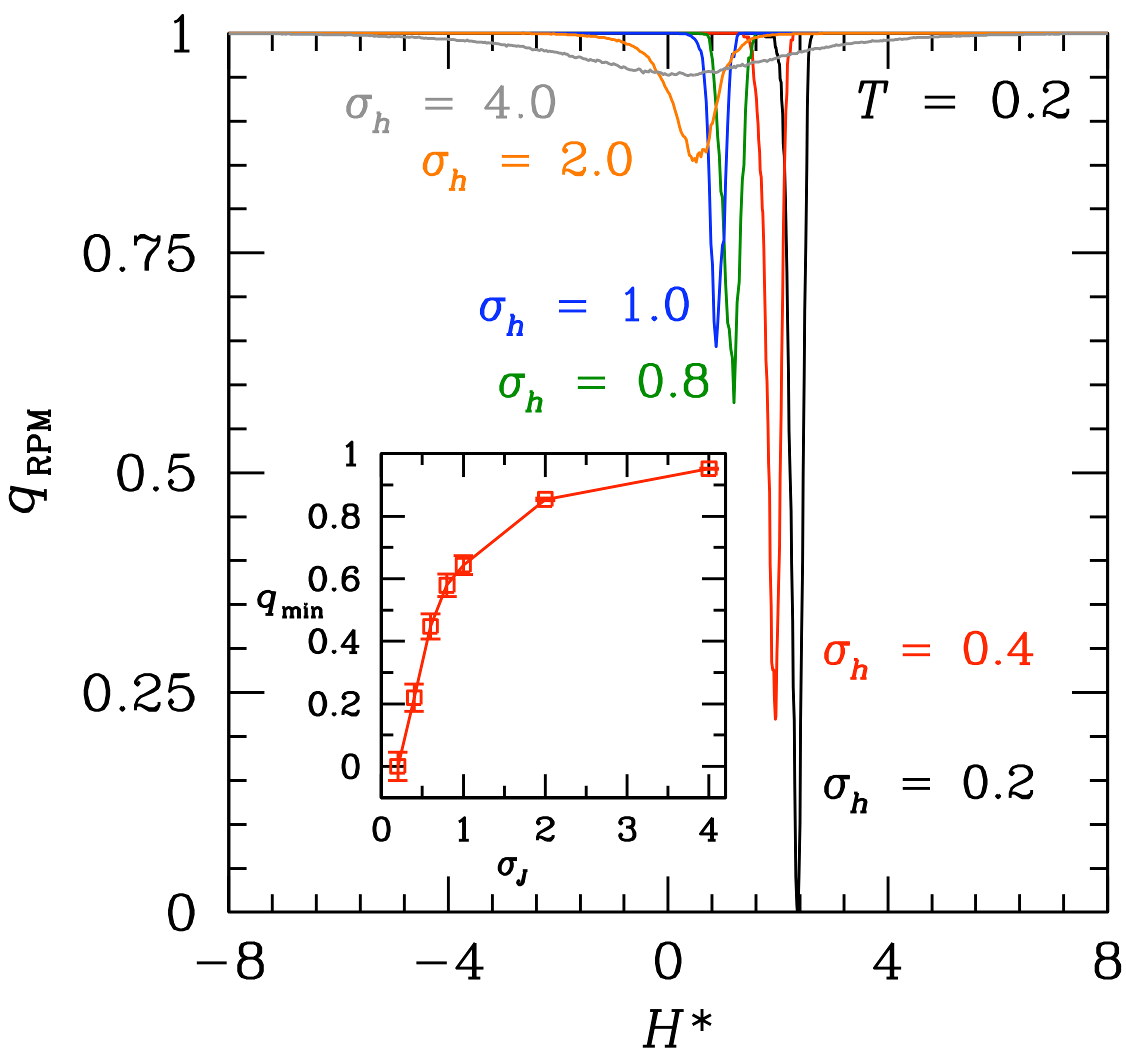


- Variable σ_h
- Data for T = 0.20
- CPM ~ 0
- Anticorrelations for large disorder (loops close).

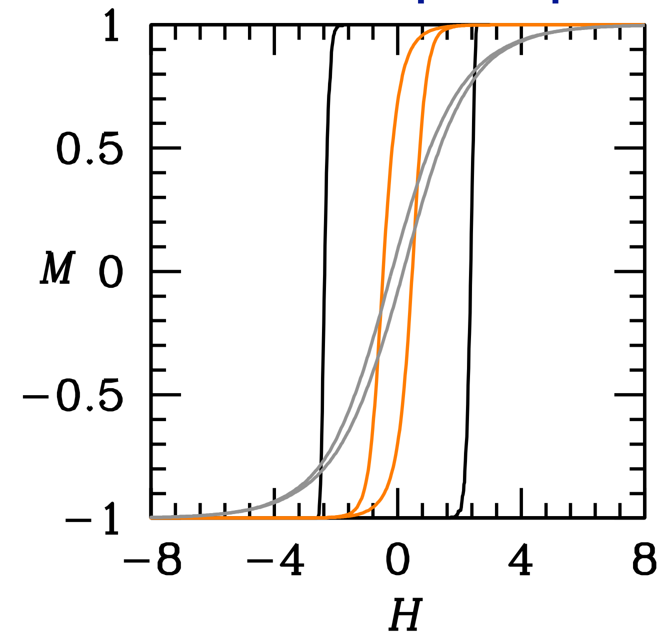


RFIM: Overlap $q(H^*)$ RPM

$$q(H^*) = \frac{1}{N}\sum_{i=1}^{N} S_i(H^*_{\rm I})S_i(H^*_{\rm I'})$$

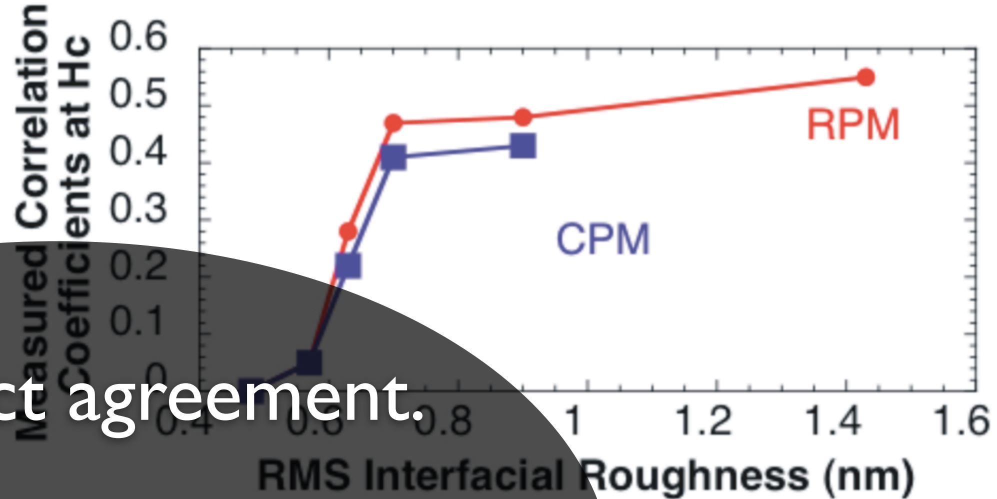


- Variable σ_h
- Data for T = 0.20
- Memory better for higher disorder
- Narrow dips due to sharp loops



Summary & comparison

- Experiments of Pierce et al.:
  - CPM < RPM < 1 for all T
  - No error bars: CPM < RPM?
  - RPM and CPM increase with disorder.
- Edwards-Anderson Ising spin glass:
  - Model has frustration and spin reversal symmetry.
  - RPM = CPM for all T, RPM and CPM < 1 (also T = 0).
  - RPM and CPM increase for increasing disorder.
- Random-field Ising model:
  - No frustration and no spin reversal symmetry.
  - RPM > CPM for all T, RPM = 1 for T = 0, CPM ~ 0 for all T.
  - RPM increases for increasing disorder.

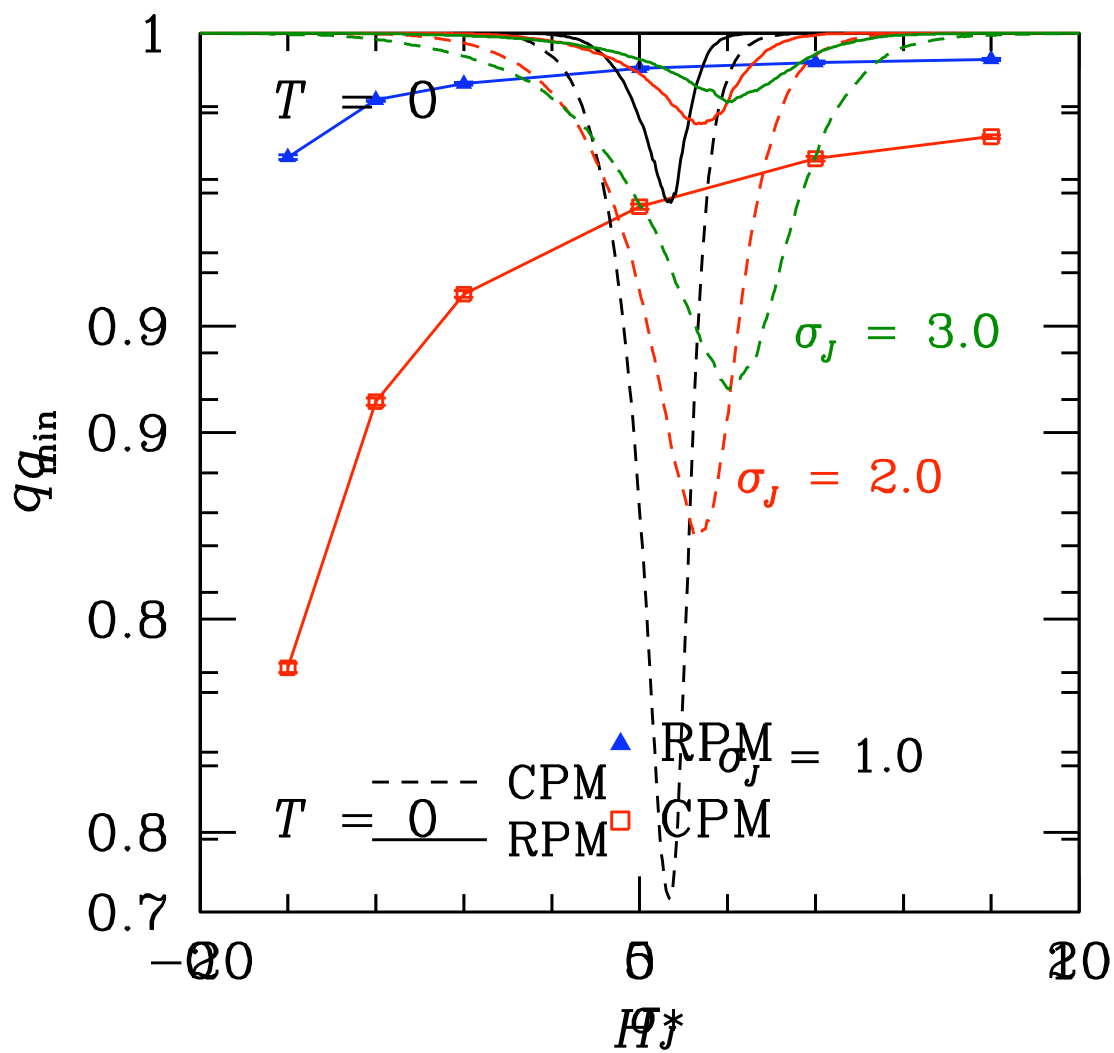


No perfect agreement.

Can we construct a minimal model which reproduces the experiments?

SG+RF: Overlap $q(H^*)$ CPM/RPM

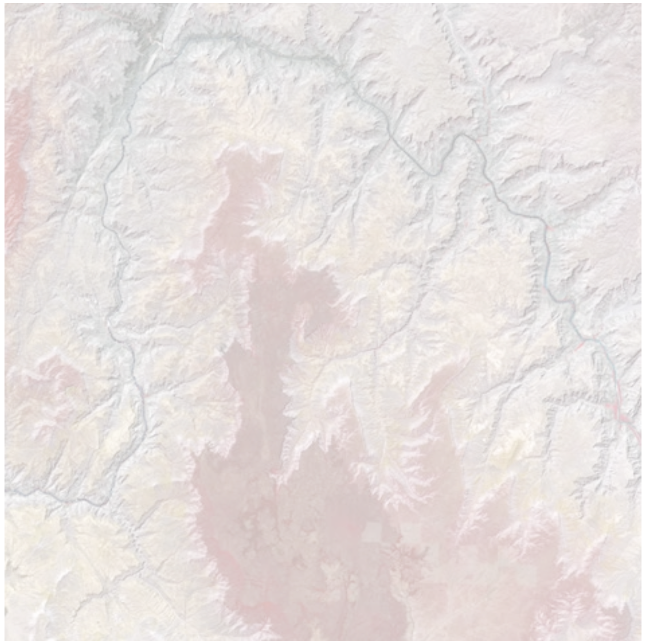
$$\mathcal{H}_{\mathrm{SG+RF}} = -\sum_{\langle i,j\rangle} J_{ij} S_i S_j - \sum_i [H + h_i] S_i$$

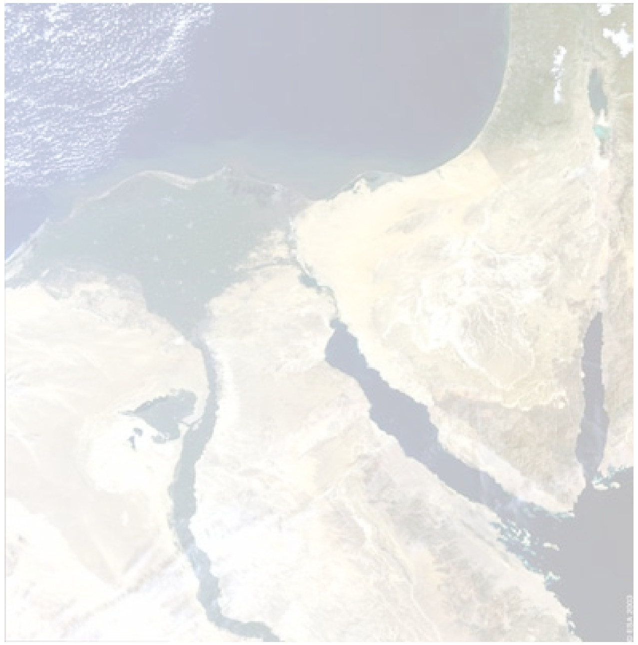


- 5% random fields with $\sigma_h = 1$
- variable σ_J
- CPM < RPM < 1
- Memory increases with increasing disorder
- The random fields break spin-reversal symmetry
- Deutsch: break time-reversal symmetry.

Qualitative explanation

- Why does the memory increase with increasing disorder?
  - Strong disorder → Rough energy landscape.
  - Rough energy landscape → Pinning in configuration space.
  - Pinning in Configuration space → Increased memory.
- Analogy: Colorado River vs Nile Basin.





Concluding remarks

- Equilibrium properties:
  - Simulations on the one-dimensional Ising chain suggest that short-range spin glasses can have an AT line for $d > 6$.
- Nonequilibrium Properties:
  - The random-field Ising model and the EA spin glass show memory effects.
  - The SG+RF model is a minimal model which shows the same behavior as the experiments of Pierce et al.
- Future problems:
  - Probe other characteristics of the mean-field model on short-range systems, such as ultrametricity.
  - Understand the nature of the spin-glass state (RSB favored in zero field, DP favored in a finite field).

Thank you.