Power Grids as Network Dynamical Systems
Theoretical Challenges & Applications

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Network Dynamics
Max Planck Institute for Dynamics & Self-Organization

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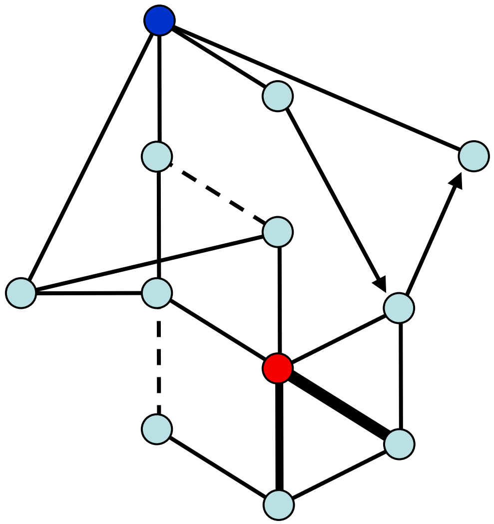
Georg August University, Göttingen

Networks are everywhere - most are dynamic

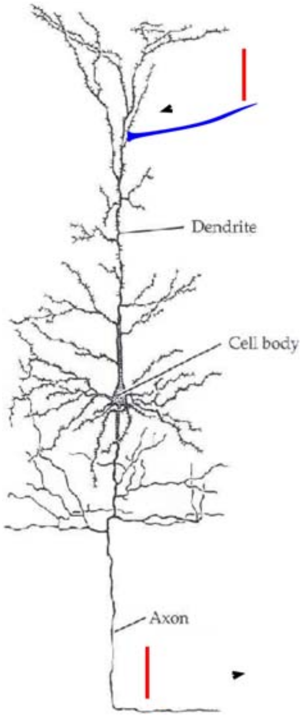


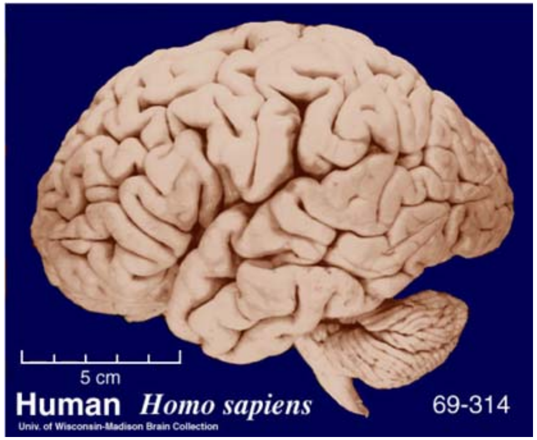
Nodes interacting with other **nodes**

Social Network:	**Person A** is friend with **person B**
Neural Circuit:	**Neuron A** sends signal to **neuron B**
Gene Network:	**Protein A** activates **gene B**
Traffic Network:	**Airport A** offers flights to **airport B**
Computer Network:	**Server A** interacts with **host B**
Economic Network:	**Stock A** rises/falls with **stock B**
Power Grid:	**Power plant** influences **consumer B**
…	**…**

All essential for everyday life!

Self-organized dynamics in networks





<u>Biological Networks</u>

$(10^{-3} - 10^{10}s;\ 10^{-5} - 10^{-1}m)$

- Neurons & neural circuits
- Horizontal gene transfer & evolutionary networks
- information transfer in biological networks

<u>Networks of physical & artificial units</u>

$(10^{-2} - 10^{10}s;\ 10^{-9} - 10^{6}m)$

- Network growth & disordered media
- Modern power grids (mind the renewables!)
- Autonomous robots & network control

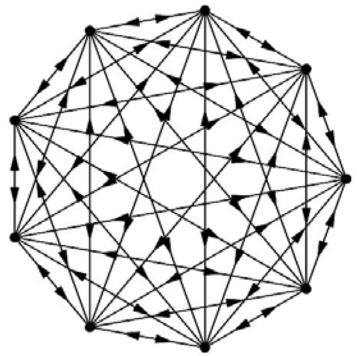
Mathematical challenges for theory

Simultaneous occurrence of:

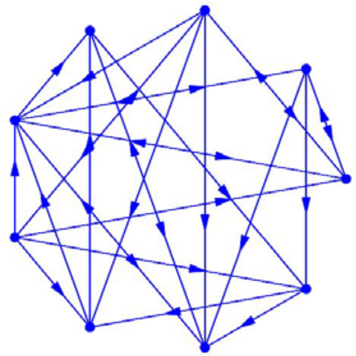
- **Nonlinearity**
- **High dimensionality**
- Complicated Network **Connectivity**
- Interaction **Delays**
- Strong **Heterogeneities**
- **Stochasticity**

<u>common approach:</u>

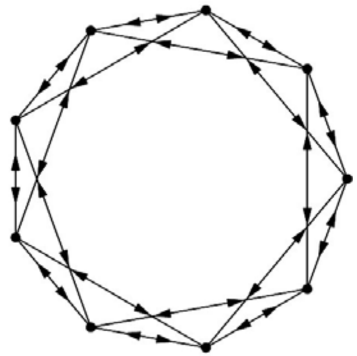
Mean Field Theories, Statistical Description, e.g. averaging over network



all–to–all
(regular)



general
(irregular)

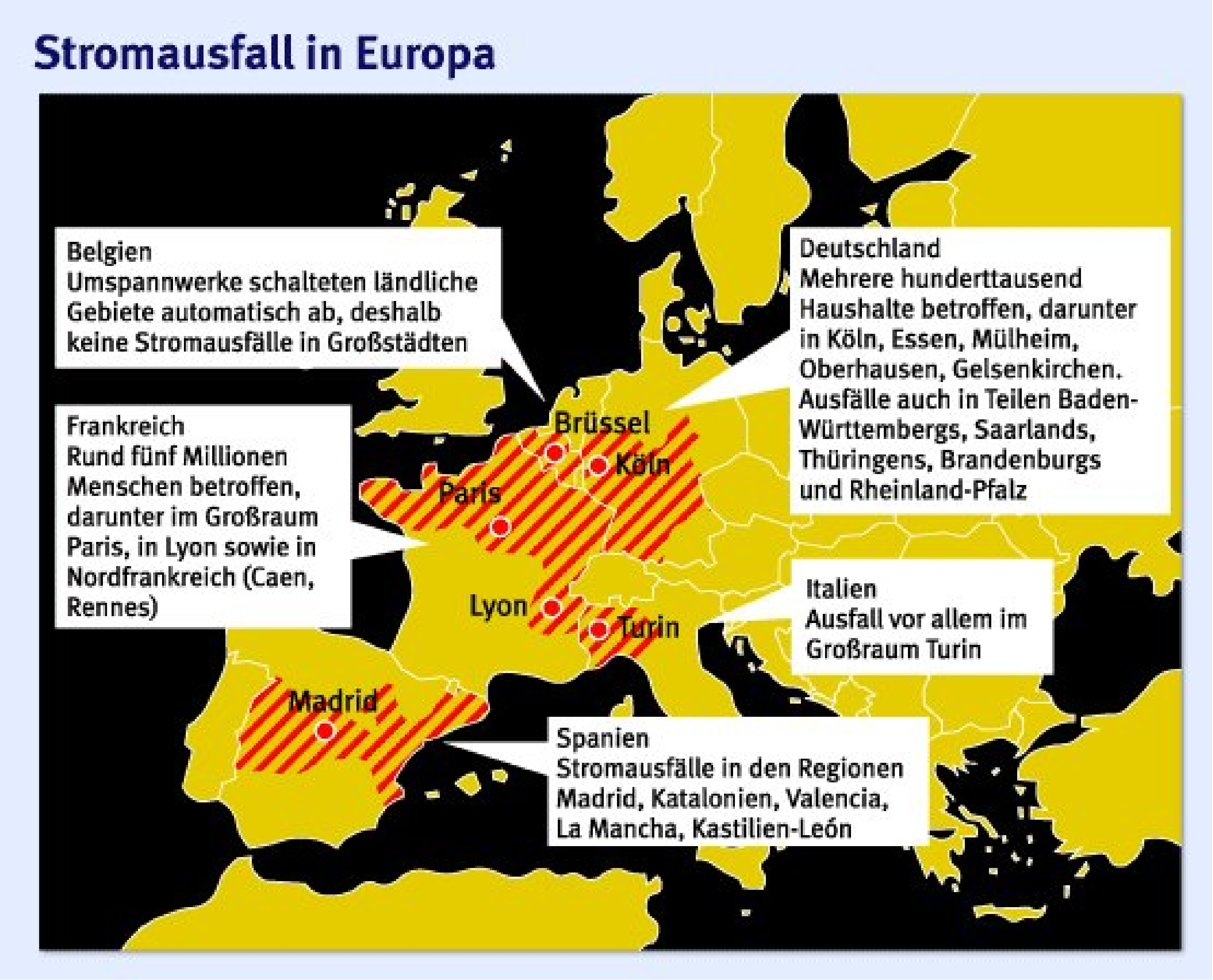


local
(regular)

Mind the specifics (links, events, realizations, …) !

Distributed collective grid dynamics …

Local change ➔ **Nonlocal** impact



Frankf. Allg. Zeitung, Nov 5,2006

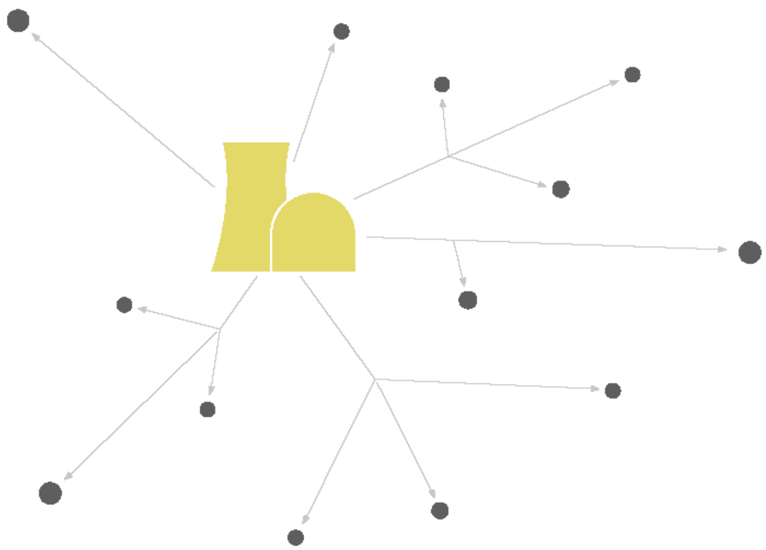
… far from fully understood

>>**"In the past, these operations were often performed with no problems",**
E.O.N. officials declared in great surprise<< *(Softpedia)*

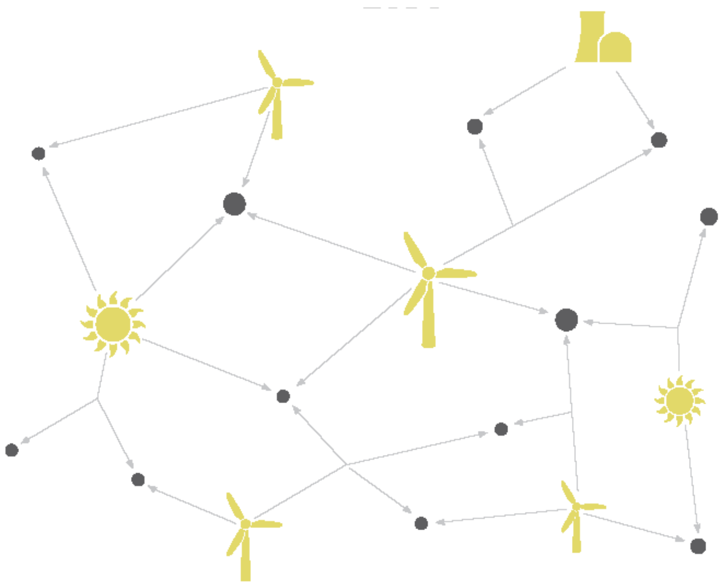
„**We need more interconnections**“
says A.Merlin of RTE, France's power-grid operator *(Bloomberg)*

Which factors determine the collective dynamics of power grids?

How does the grid self-organize dynamically?



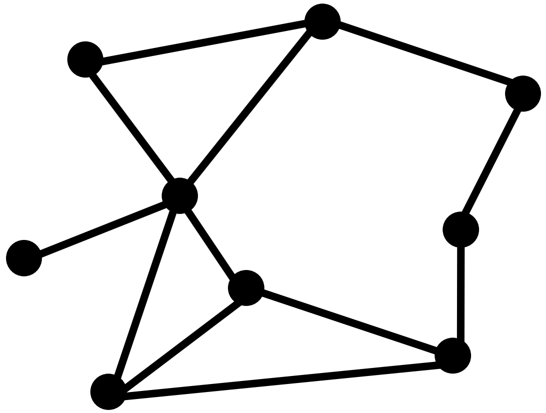
2000
- largely centralized
- dominated by large sources
- almost central control



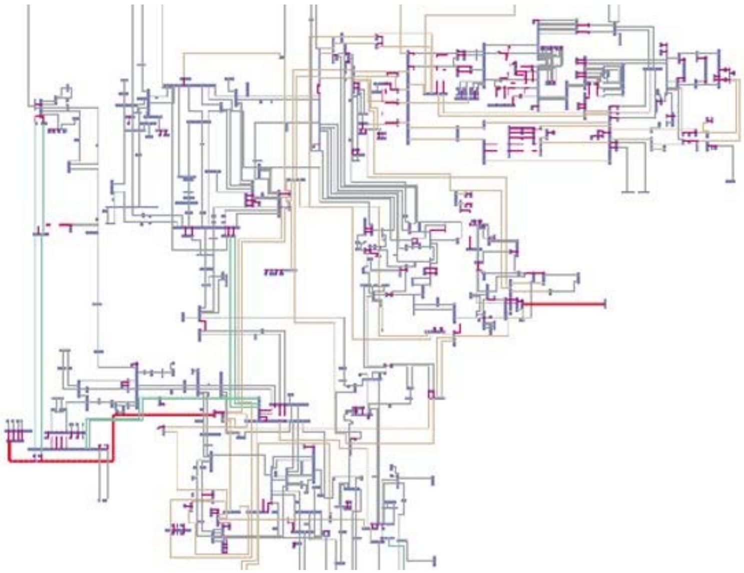
2050
- more distributed
- smaller sources
- less controllable

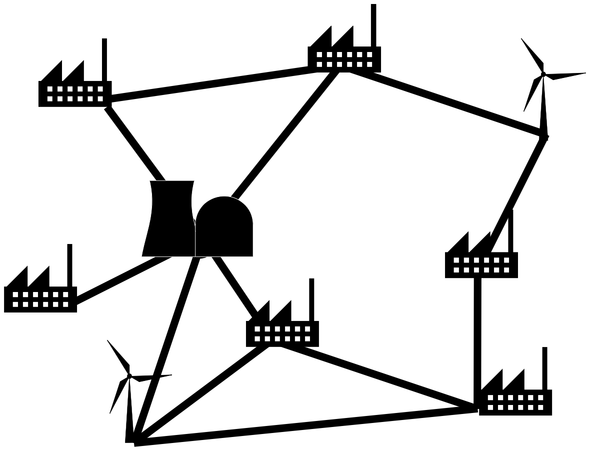
Dynamic models between abstract and detailed

Abstract: homogeneous, statistic, quasi-static, large-scale



Detailed: heterogeneous, component-level, dynamic, small-scale

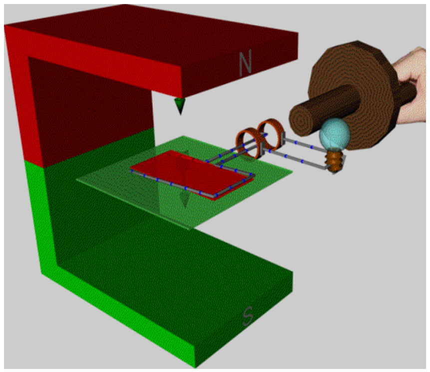


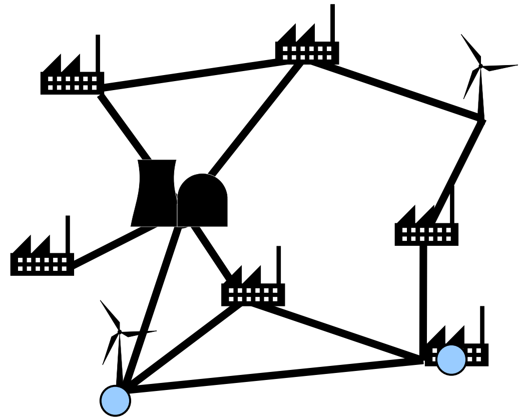


Oscillator model:

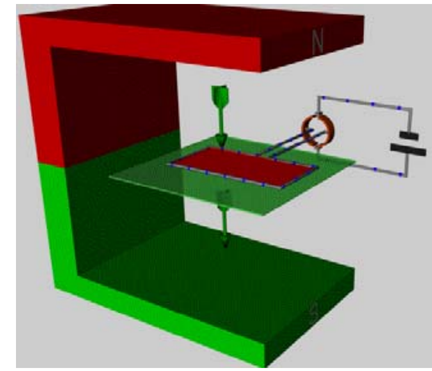
- heterogeneous
- coarse-grained
- dynamic
- large-scale

Coarse, dynamic oscillator model of power grids



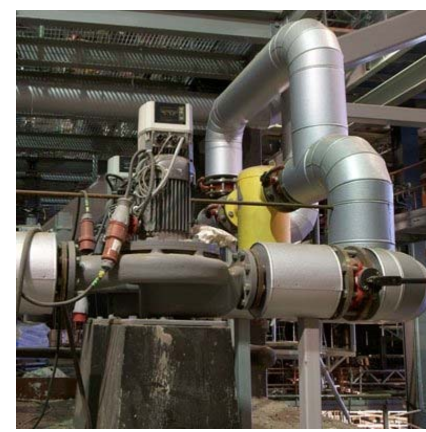


$\theta_A(t)$ $\theta_B(t)$



Generator

Motor



$\theta(t)$	Phase at time t
P	input/output power
K	Transmission capacity

Images: physik3d.de, Wikipedia

Dynamic oscillator model

$$\frac{\mathrm{d}^2\phi_j}{\mathrm{d}t^2} = P_j - \alpha\frac{\mathrm{d}\phi_j}{\mathrm{d}t} + \sum_{i=1}^{N} K_{ij}\sin(\phi_i - \phi_j)$$

$\phi_j(t)$ - phase deviation from base Ωt, $\Omega = 2\pi 50\,\mathrm{Hz}$

α - effective dissipation

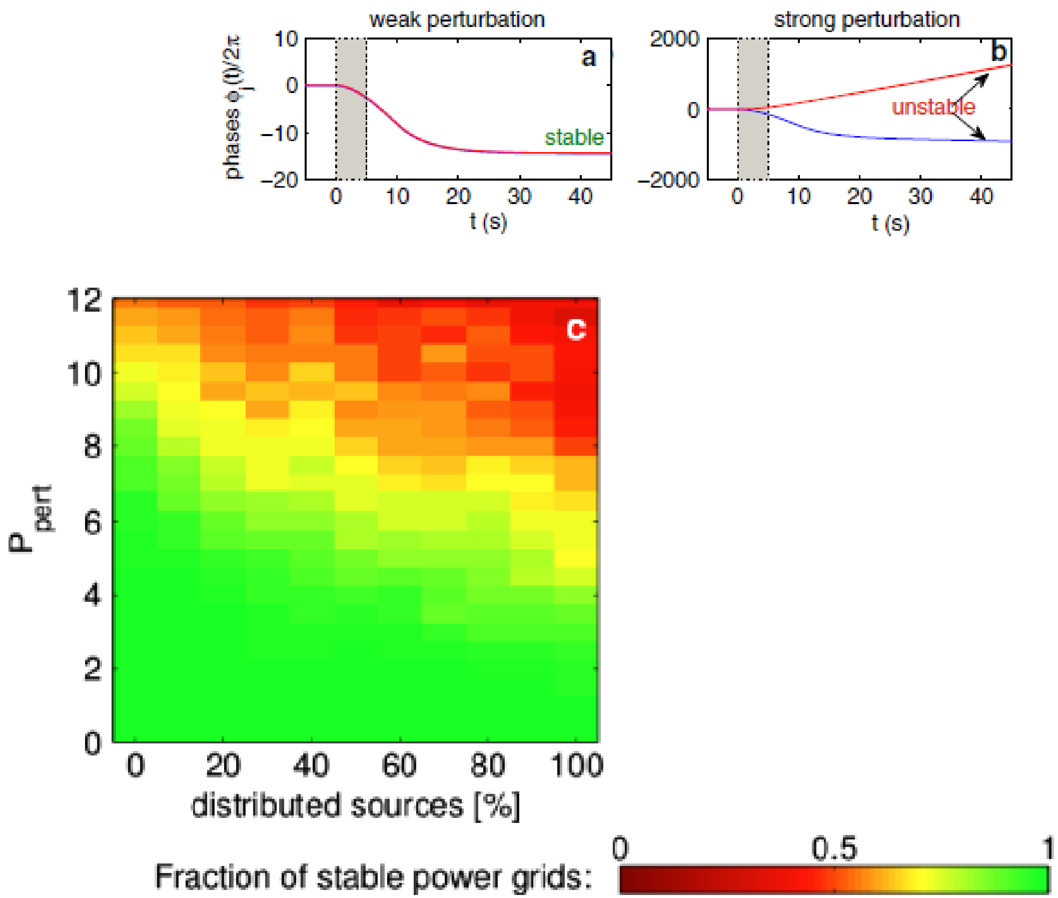
P_j - power consumed (< 0) or generated (> 0)

K_{ij} - Transmission capacity of line

DEs derived from physics of synchr. machines in limit $|\dot{\phi}_j| \ll \Omega$

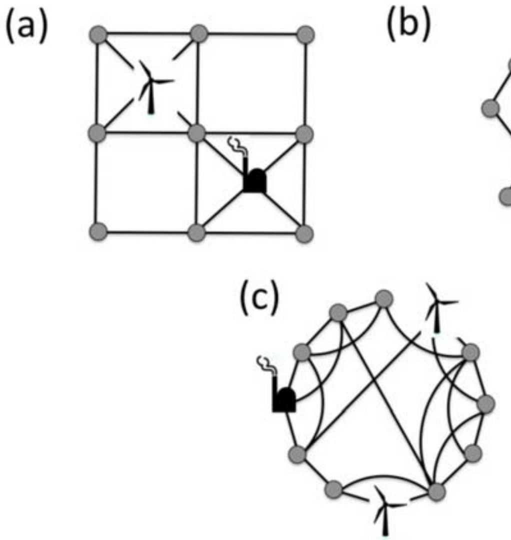
Filatrella et al., *Eur. Phys. J. B*, 61:485 (2008)

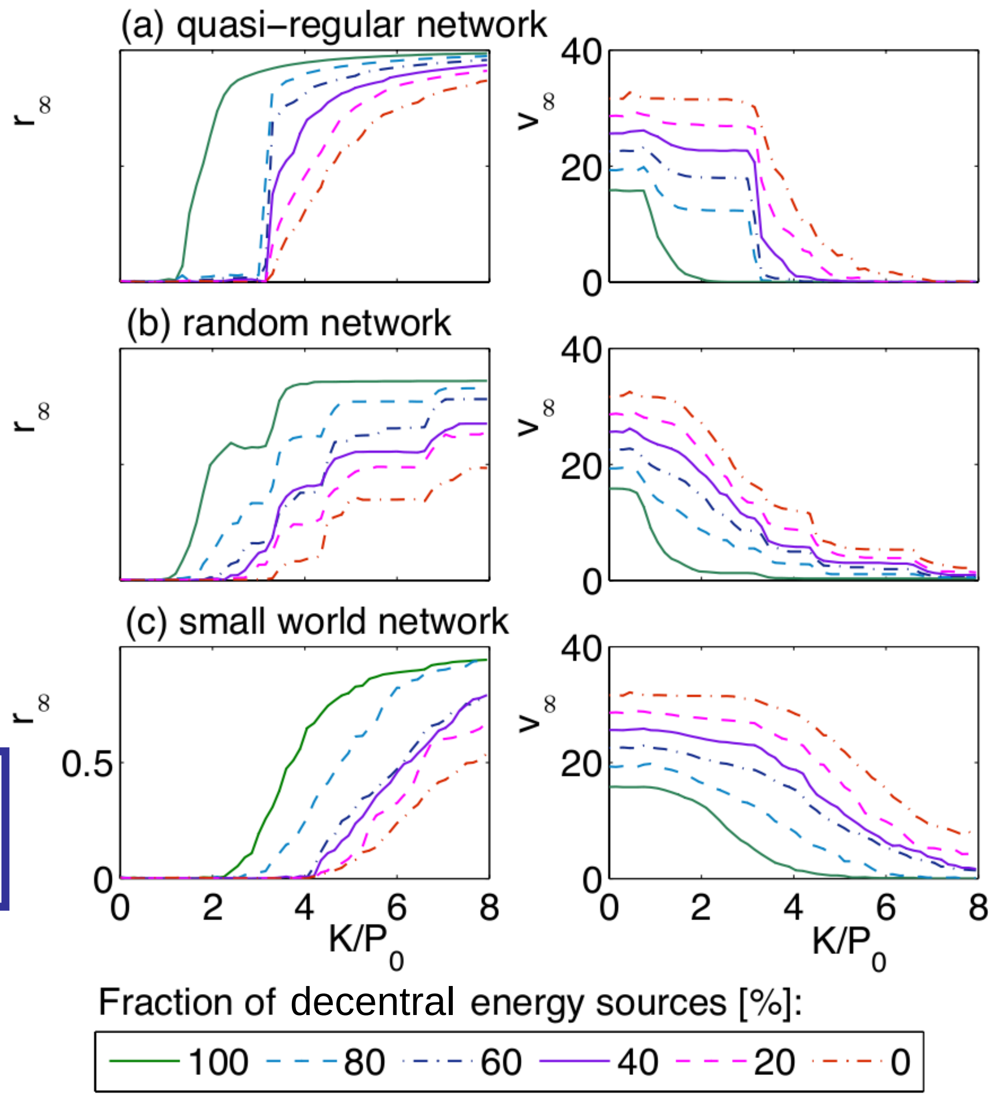
Part I: Decentralization (slightly) decreases stability



Robustness to dynamical perturbations of same order of magnitude

Decentralizing **increases structural robustness**

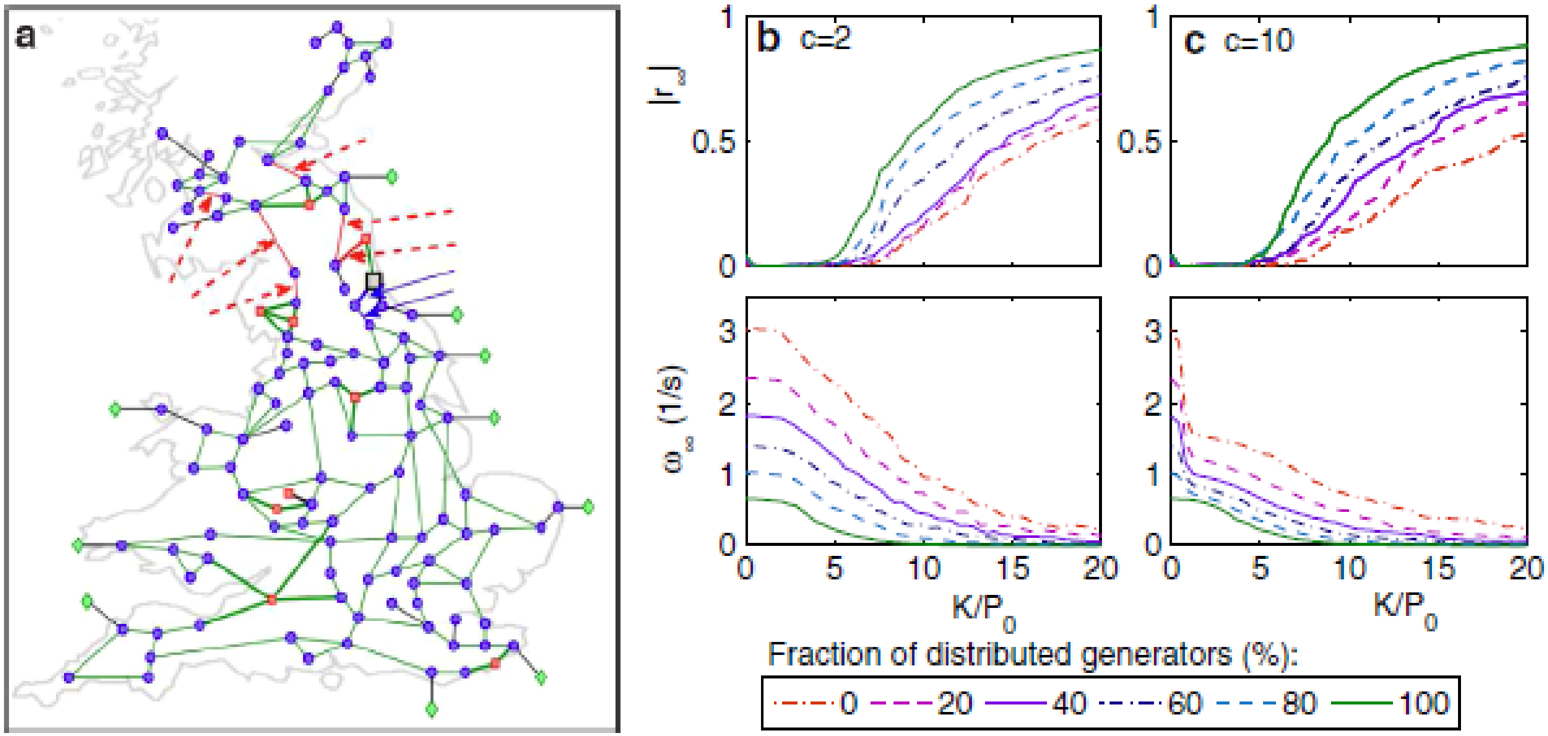




More & smaller sources
➔ larger stationary stability

Rohden et al., *Chaos*, **24**:013123 (2014);

… for model topologies … and for British grid



Dezentralization decreases synchronization threshold

M. Rohden et al., *Phys. Rev. Lett.* 109:064101 (2012)

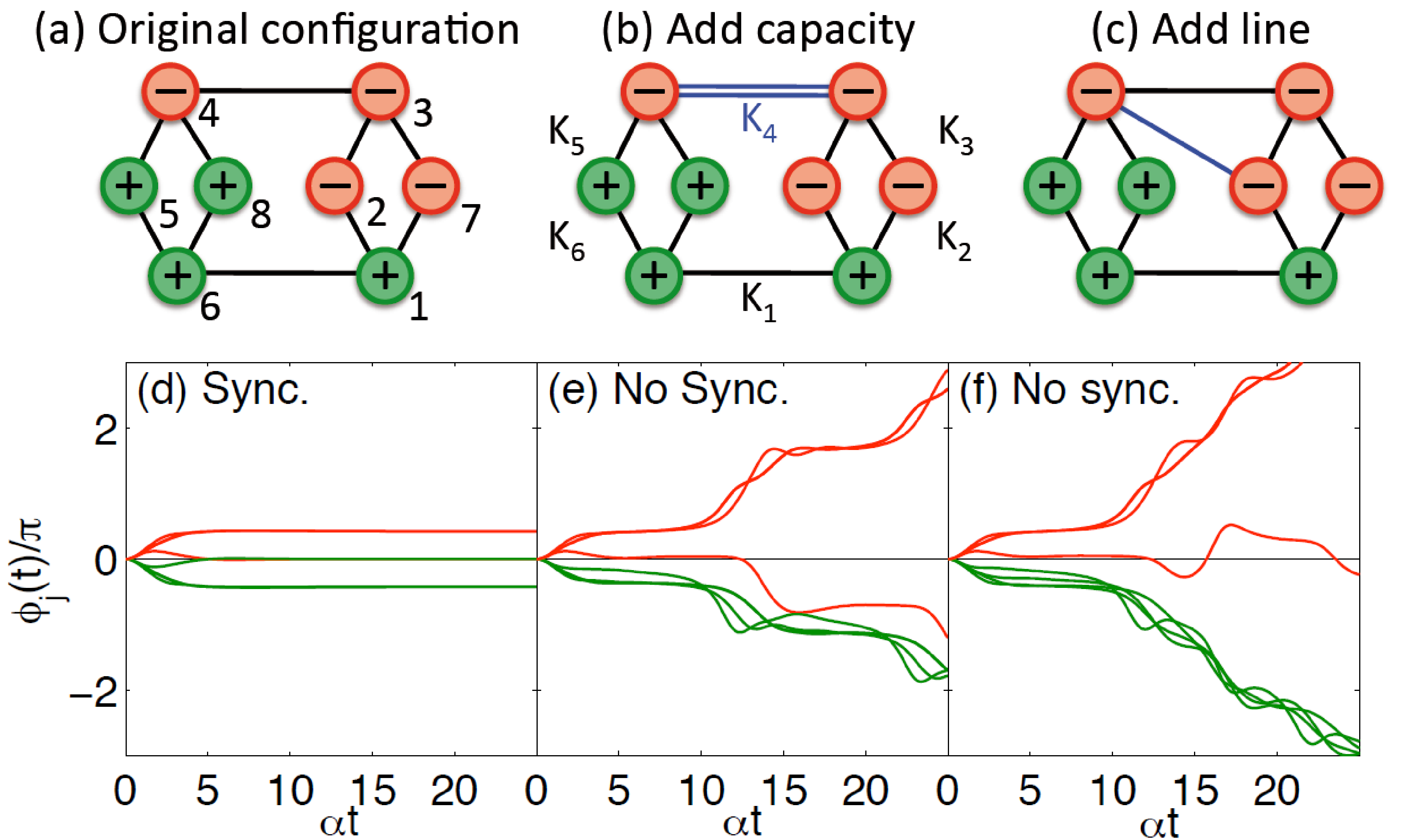
Counteracting effects of decentralization

Dezentralization

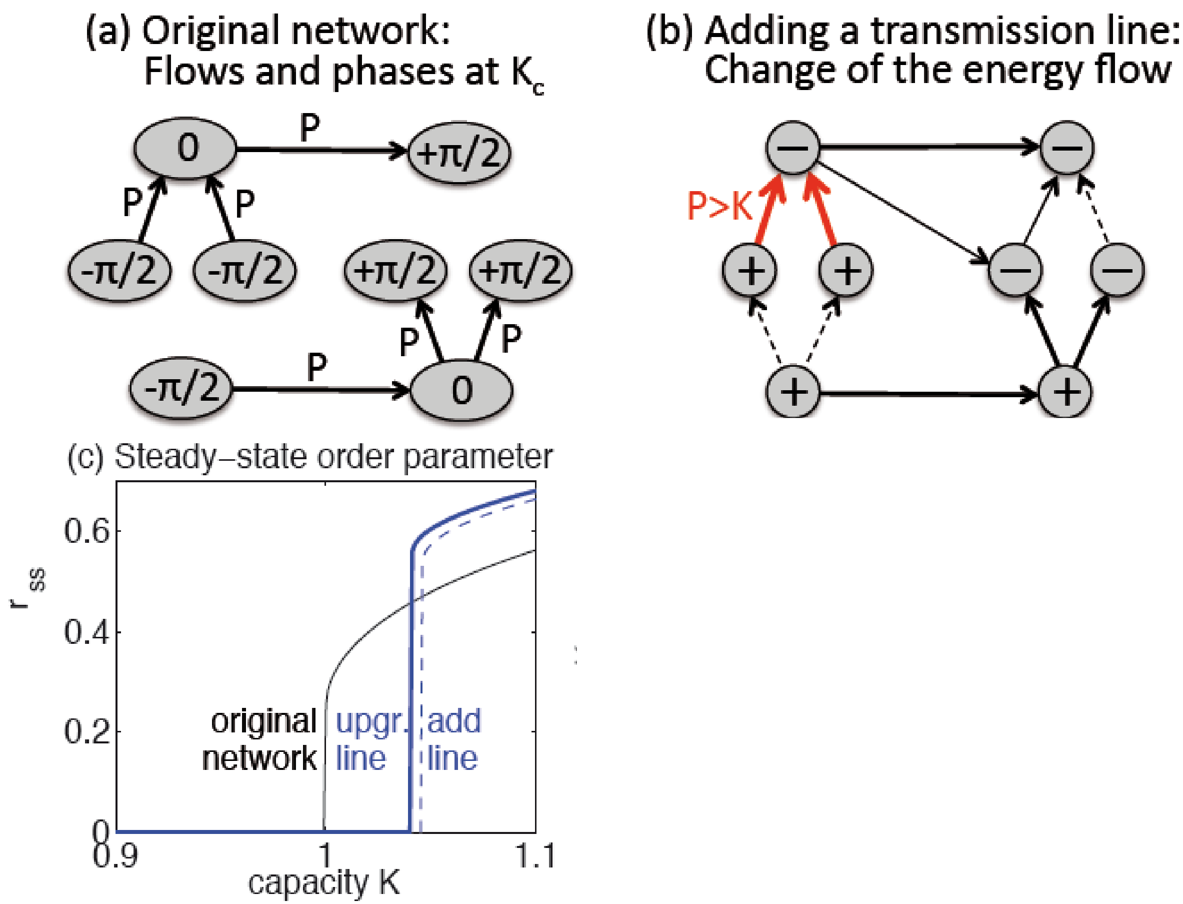
- **slightly deminishes** dynamic **stability** against instantaneous perturbations

- decreases synchronization threshold = **increases structural robustness**

Part II: Braess' paradox: adding lines may cause failures



Mechanism of Braess paradox: geometric frustration



$$\sum_{(i,j)\in\mathcal{C}}(\phi_i-\phi_j)=2\pi n,\ \ n\in\mathbb{Z}$$

„geometric frustration“ → Additional capacity „breaks the balance“

Geometric Frustration in Flow Networks

Flow balance
at every node *j*

$$P_j + \sum_{l=1}^{N} K_{j,l} S_{j,l} = 0$$

hyper-plane
(N=|V| eqns
for M=|E| variables)

No overload
at every edge *(j,l)*

$$|S_{j,l}| \leq 1$$

hyper-polygon
(M inequalities)

Load relate to
phase differences

$$\Delta^{+}_{j,l} = \arcsin(S_{j,l})$$

$$\Delta^{-}_{j,l} = \pi - \arcsin(S_{j,l})$$

2^M discrete points

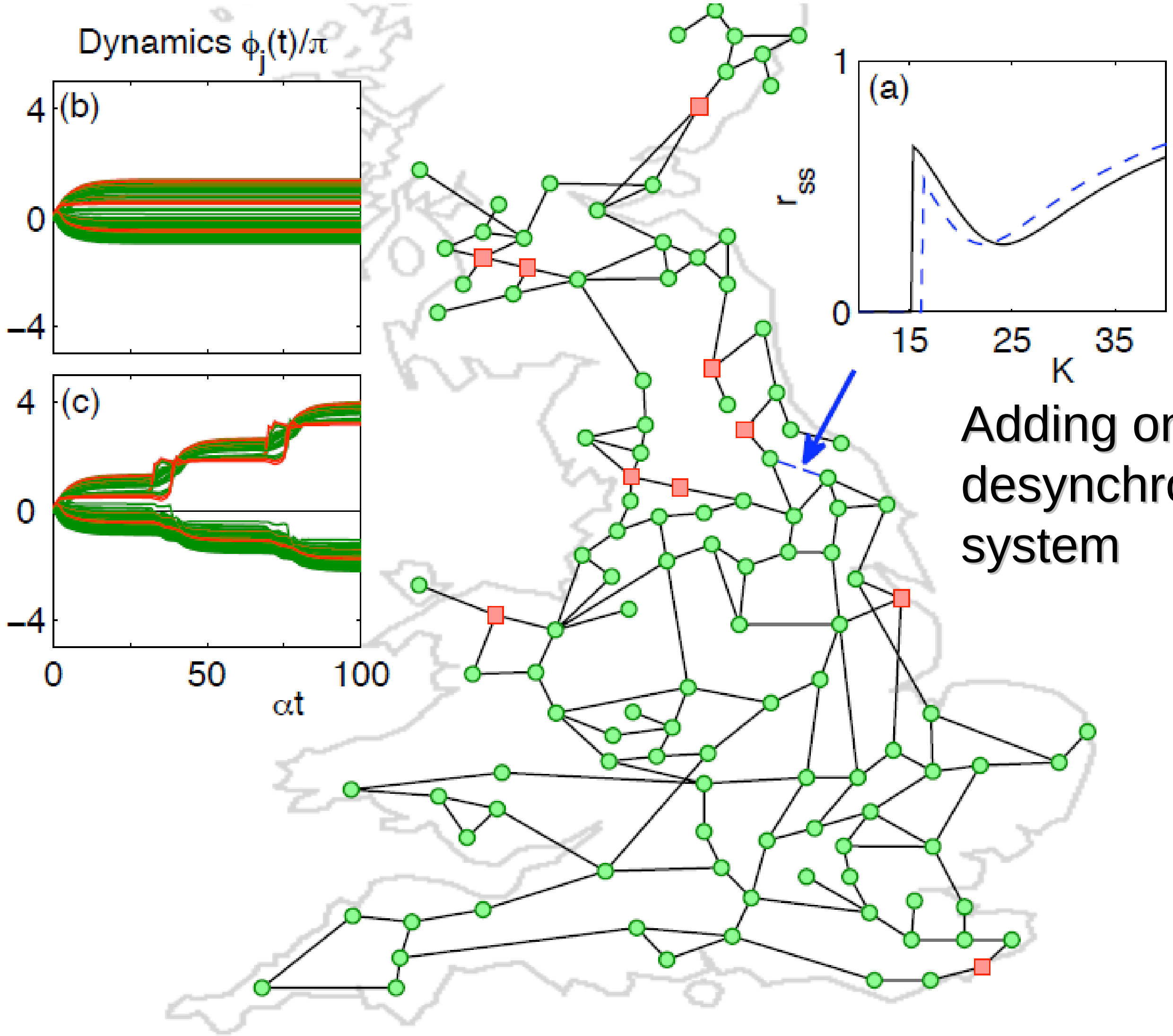
Geom. constraints
(cycles in networks)

$$\sum_{\text{cycle}} \Delta_{j,\ell} = 0 \pmod{2\pi}$$

excludes certain combinations

In addition: stability conditions

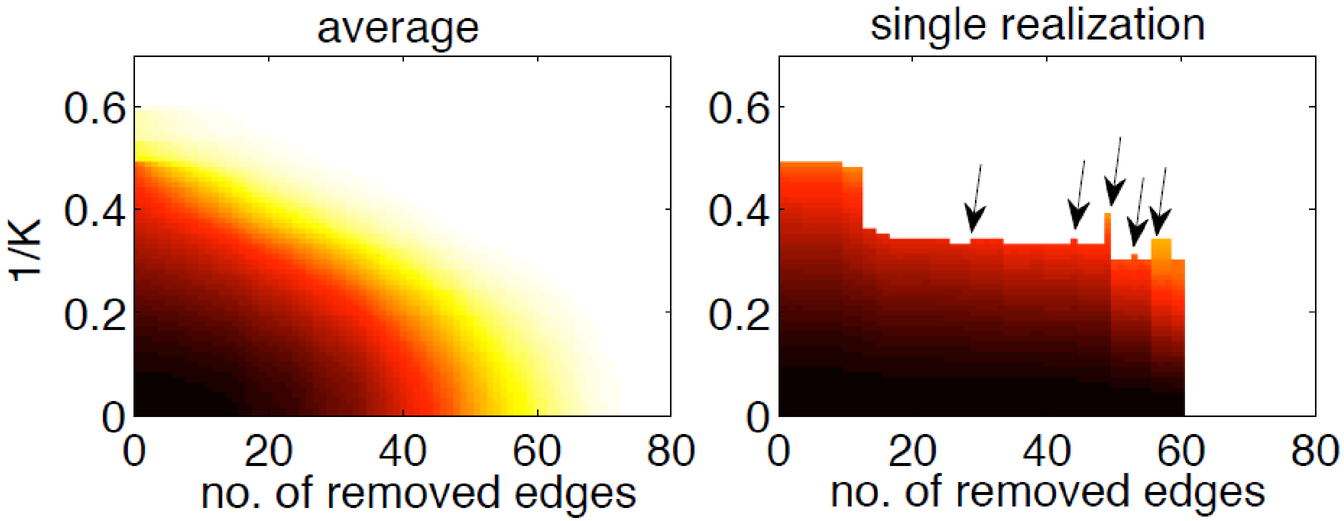
Braess' paradox on real topology



Adding one line desynchronizes system

Braess‘ paradox prevails …

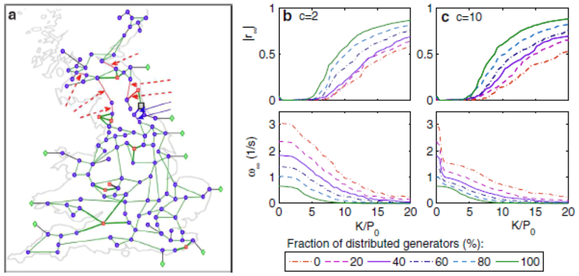
e.g., small world networks:



… but not seen in „mean field“

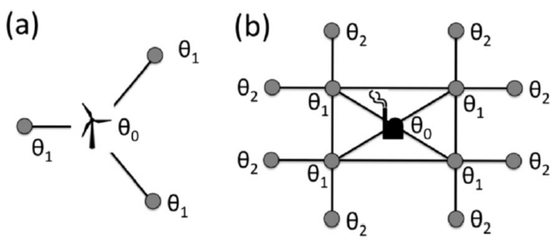
Dirk Witthaut & MT, *New J. Phys.* 14:083036 (2012b)

Overview of Recent Advances …



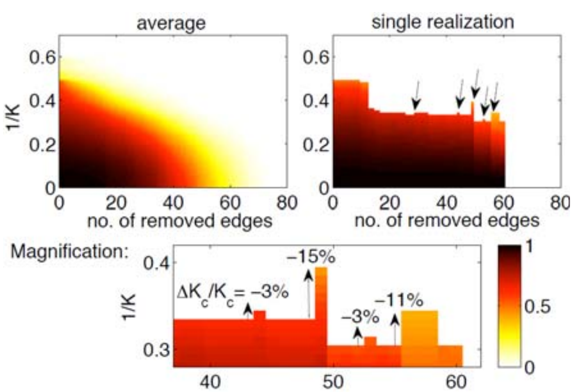
<u>Counter-acting impact of **decentralization:**</u>

Phys. Rev. Lett. (2012) – Editorial Suggestion



<u>**Motif**-guided stability analysis</u>

Chaos (2014);



<u>**Braess‘ paradox**</u>

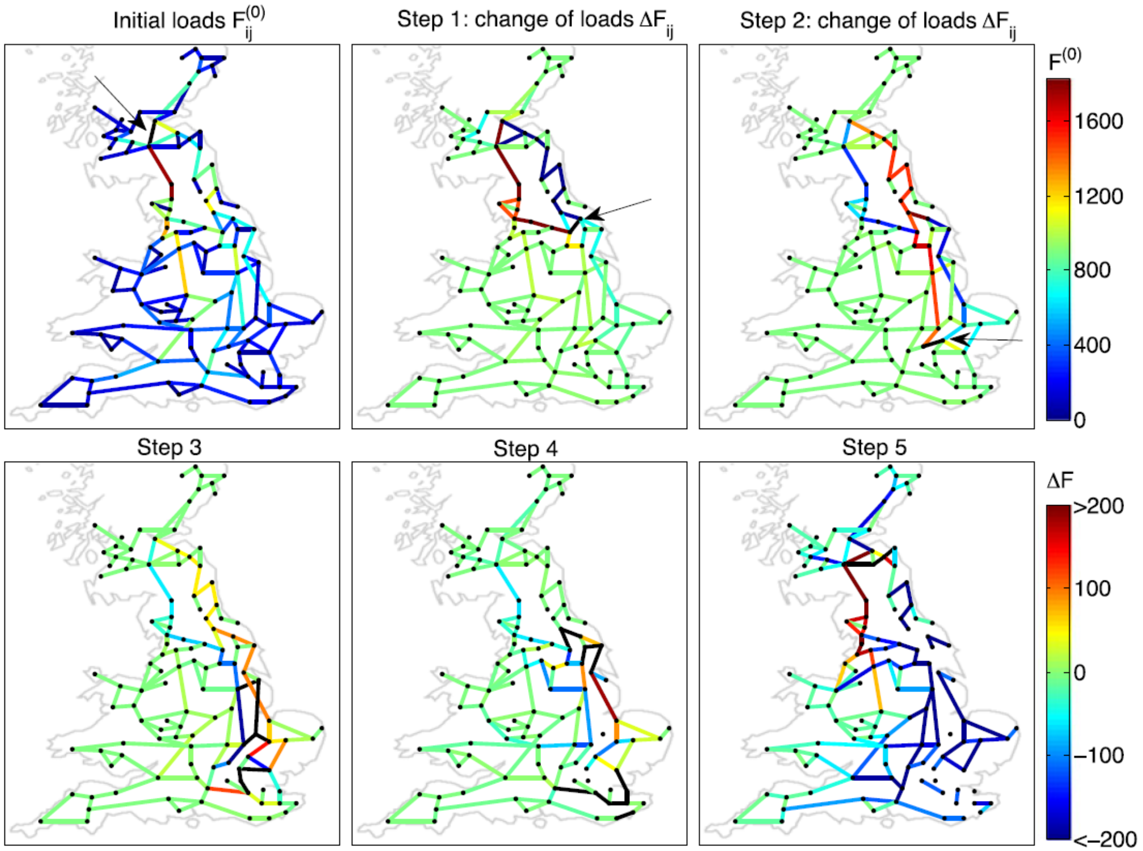
In flow networks with relevant **phase relations**

New. J. Phys. (2012);

Current Directions I -- nonlocal failure propagation

Multiple feedbacks collectively induce nonlocal failures

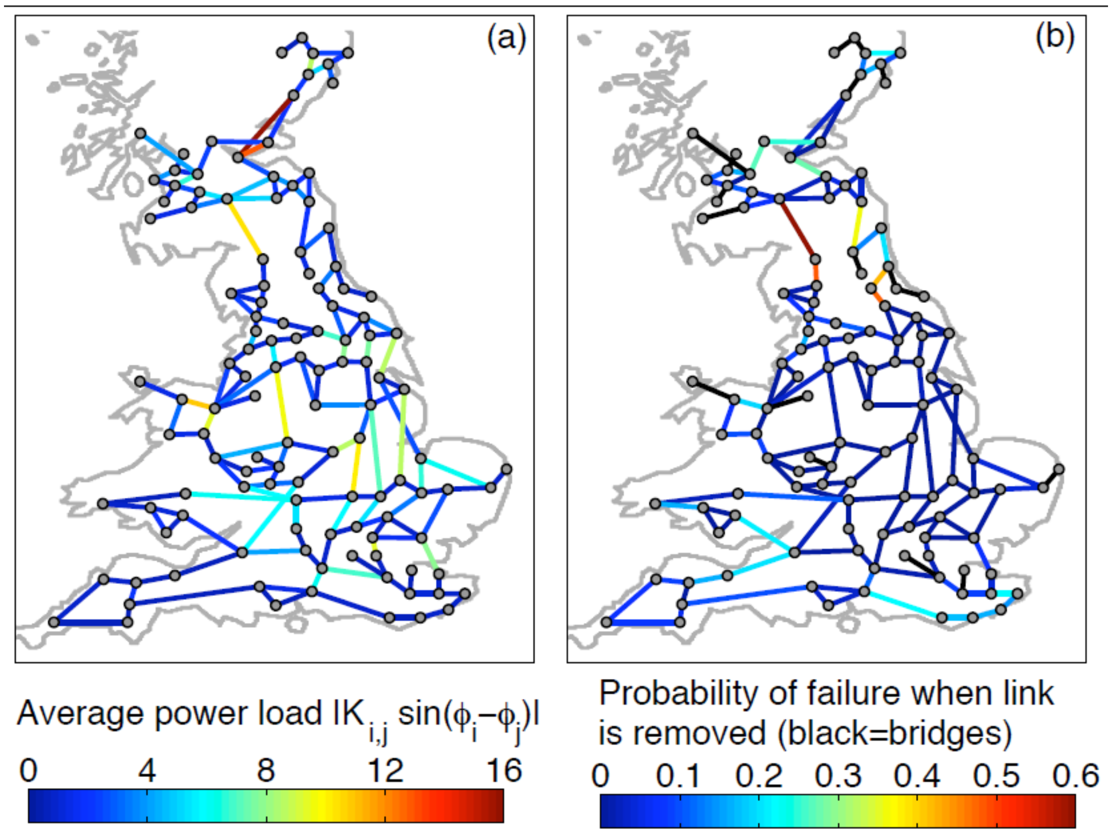
Eur. Phys. J. B (2013)



Mechanisms?

Current Directions II – predicting critical edges prior to outage

Network redundancies complement loads



Optimal Prediction?

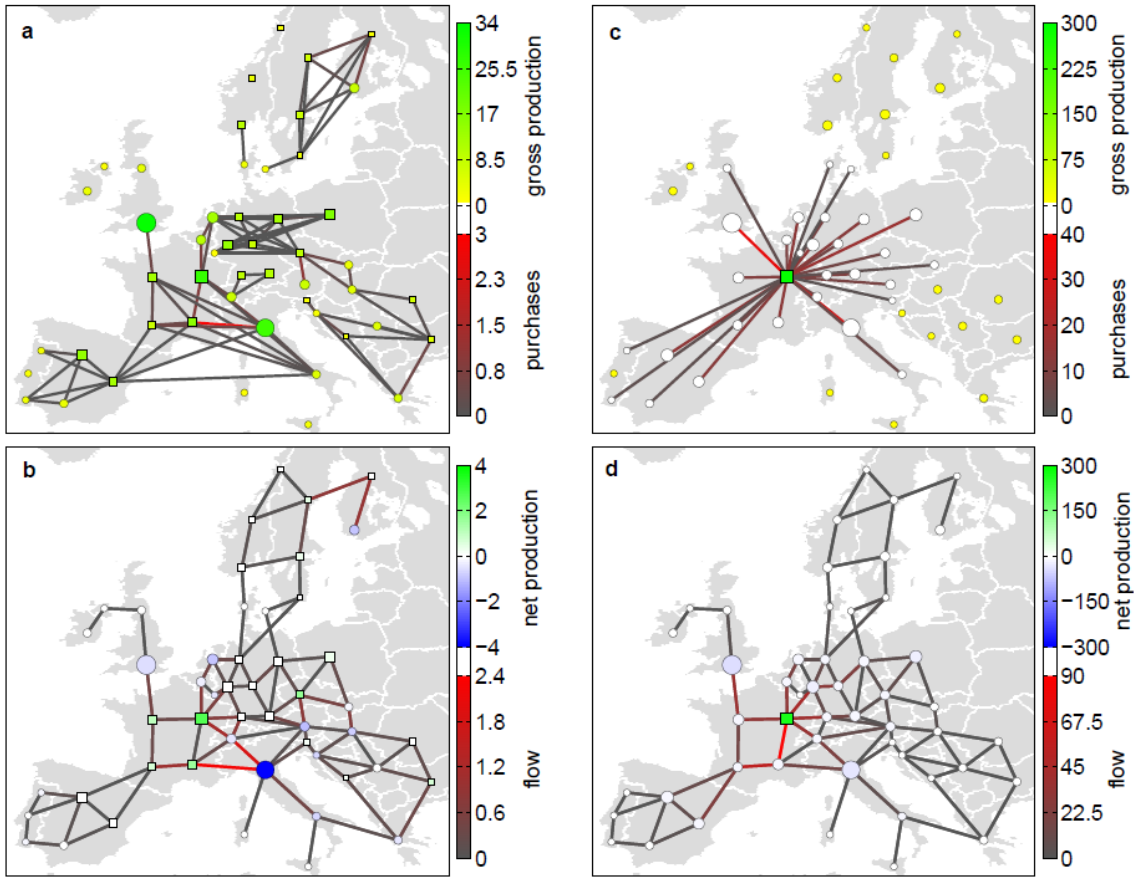
Current Directions III -- coupling market networks to dynamics

Decreasing returns
Low transmission price

Increasing returns
Low transmission price

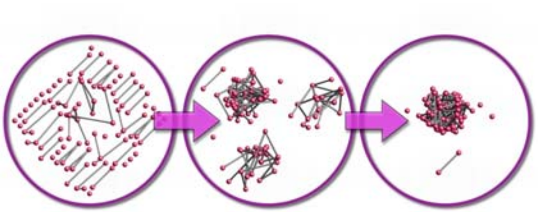
Trading Network
(who buys where
& how much)

Flow Network
(power distribution
over the grid)



How to set up economic incentives?

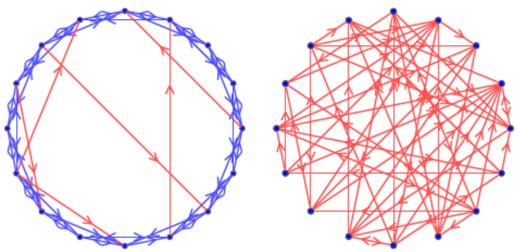
Recent Progress – **Fundamentals**



<u>Network Growth</u>

single link matters!

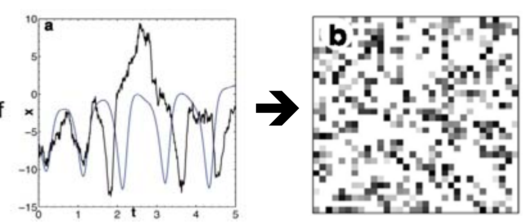
Nature Phys. (2011);



<u>Small World Networks</u>

structure: Watts & Strogatz, *Nature* (1998), >21000 citations

now relaxation **dynamics** *Phys. Rev. Lett.* (2012a)

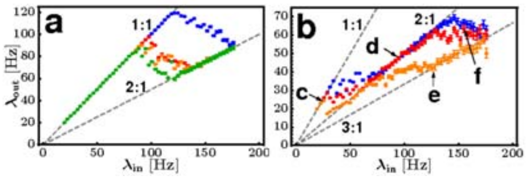


<u>Network Inverse Problem:</u>

inference and **design**

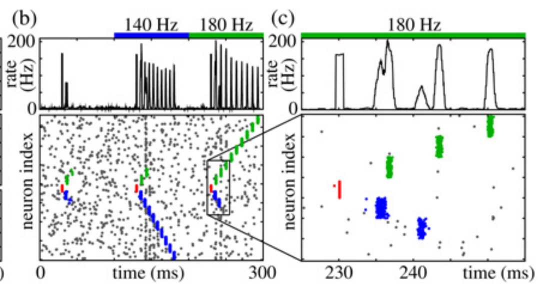
Front. Comput. Neurosci. (2011); *New. J. Phys.* (2011);

Recent Progress – **Neurophysics**



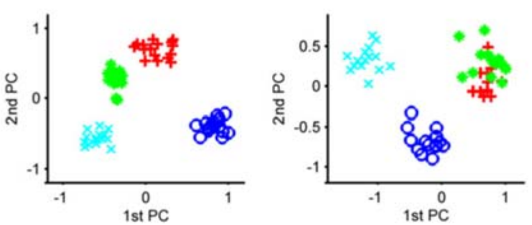
<u>Non-monotonic</u> <u>spike sequence processing in neurons</u>

BMC Neurosci. (2013); *PLoS Comput. Biol.* (under review, 2014)



<u>Non-additive coupling</u> <u>& plasticity in networks</u>

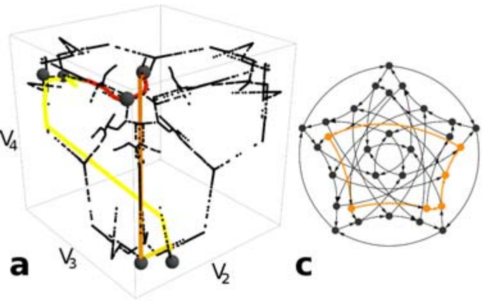
PLoS Comput Biol. (2012, 2013); *Phys. Rev. X* (2012)



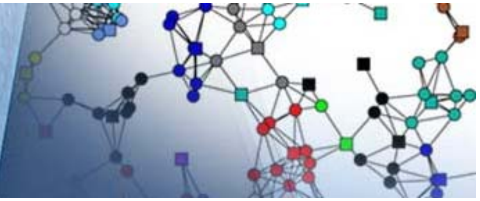
<u>Combinatorial Processing</u> <u>exploiting heterogeneities</u>

joint PhD students with A. Fiala (Uni-Bio); *PLoS ONE* (2011);

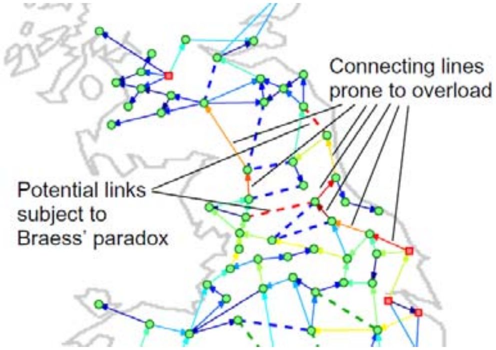
Recent Progress – **Dynamics in Engineering & Computation**



Intelligent Dynamical Systems: **heteroclinic computing**
Phys. Rev. Lett. (2012b)



Control for Communication Networks & Robotics
➔**self-organize versatile functions**
Nature Phys. (2010); **patent** (2013); *New J. Phys.* (2012a);



Dynamically Smart Power Grids
Phys. Rev. Lett. (2012c), *New J. Phys.* (2012b);
EPJ B (2013); *Chaos* (2014); EPJ-ST, under review (2014)

Funding & Cooperation

<u>Public</u>

- Max Planck Society
- University of Göttingen
- IMPRS Physics of Biological and Complex Systems
- GGNB - Graduate School for Neurosciences & Molecular Biosciences
- DFG – German Science Foundation
- BMBF – German Ministry for Education and Research

<u>Industry</u>

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Network Dynamics

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Raoul-Martin Memmesheimer **Nijmegen**

Srinivas Gorur Shandilya **Yale**

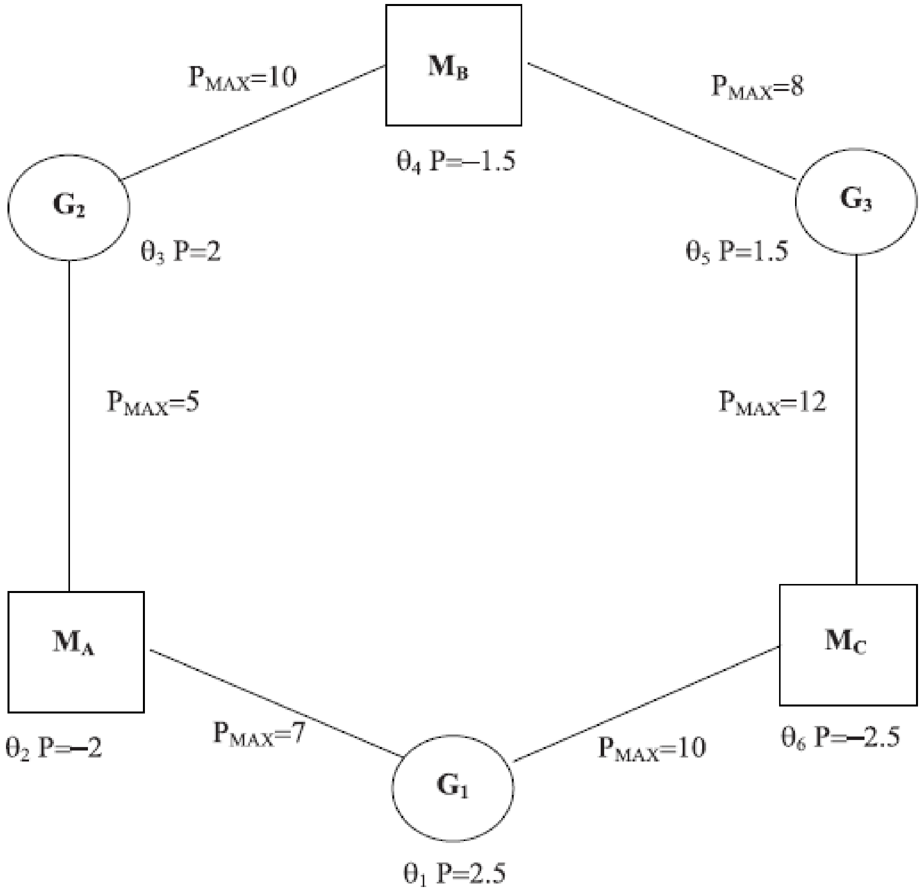
Moritz Matthiae **Aarhus**

YOU all for your attention !

Questions & Comments Welcome!

Previous studies on this model

Approximation to grid of Zealand (Denmark)



- simple topology
- specific parameters

- simulations of impact of temporary perturbations

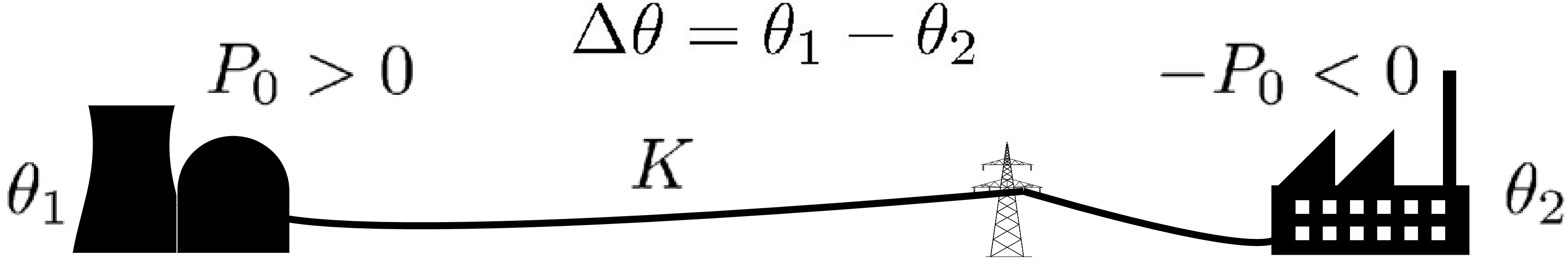
Filatrella et al., *Eur. Phys. J. B*, 61:485 (2008)

Part 0: Analysis of stationary dynamics (first N=2)

$$\ddot{\theta}_1 + \dot{\theta}_1 - K\sin(-\Delta\theta) = P_0$$

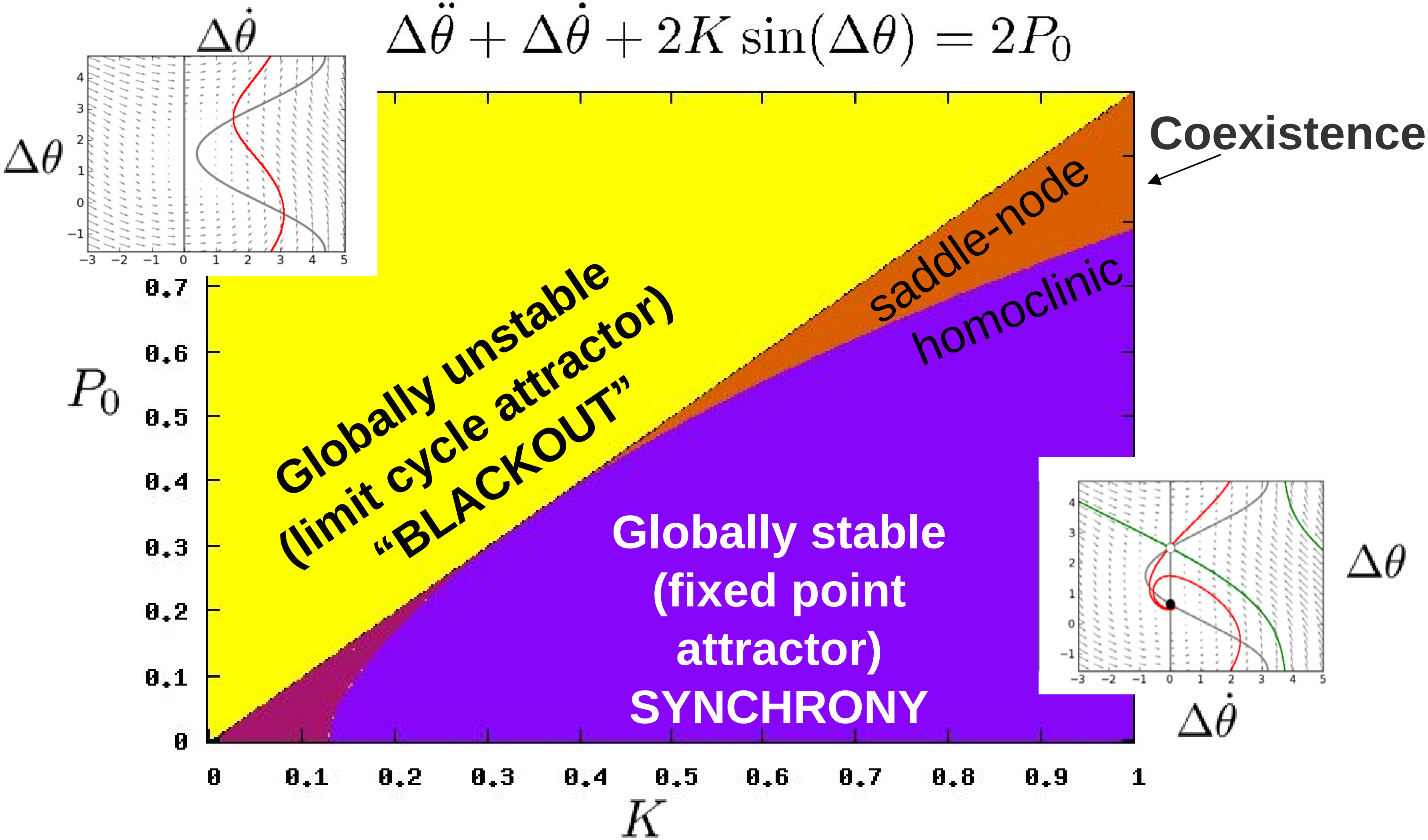
$$-\quad \ddot{\theta}_2 + \dot{\theta}_2 - K\sin(\Delta\theta) = -P_0$$

$$\Delta\ddot{\theta} + \Delta\dot{\theta} + 2K\sin(\Delta\theta) = 2P_0$$



Coexistence of blackout and stable operation

$$\Delta\ddot{\theta} + \Delta\dot{\theta} + 2K\sin(\Delta\theta) = 2P_0$$



Rohden et al., *Chaos*, **24**:013123 (2014);
Manik, Witthaut et al., in prep. (2014)