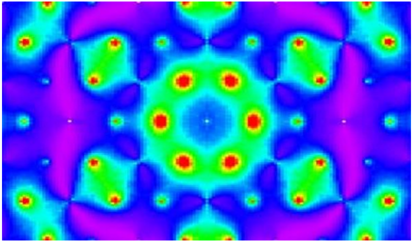
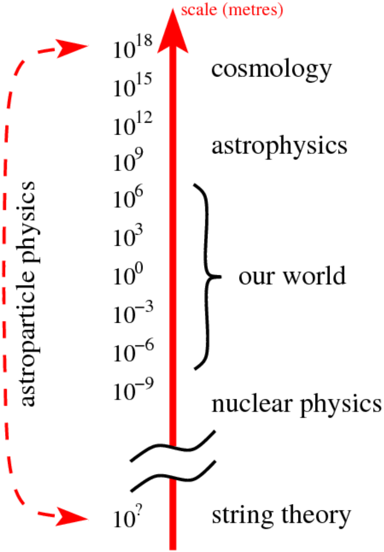
Magnetic monopoles in spin ice

Roderich Moessner



NIST

The physics landscape



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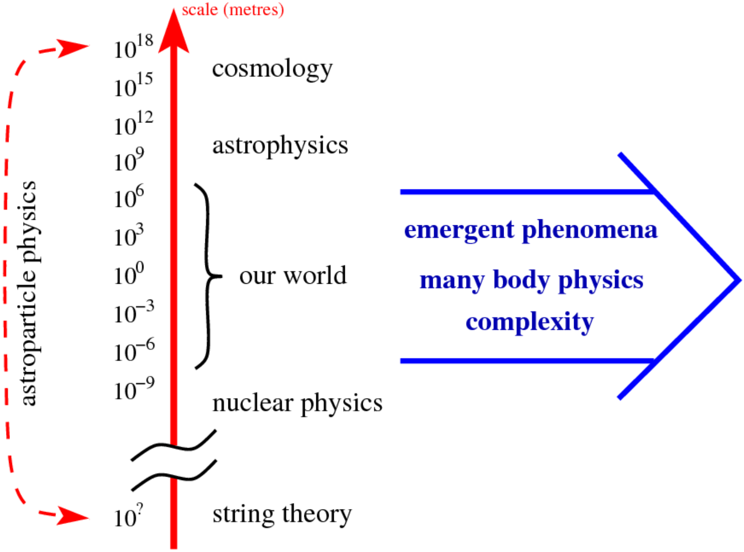
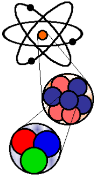


Fundamental questions



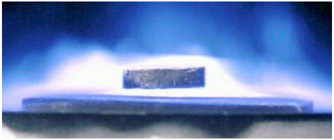
What are *building blocks and interactions* of matter?

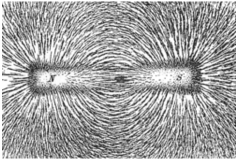
$\Rightarrow$ high energy + particle physics

What is the origin of variety and complexity

$\Rightarrow$ many-body theory:

- understand individual phenomena
  - $\Rightarrow$ 'applications'
- understand variety as such
  - $\Rightarrow$ 'organising principles'





Outline

Spin ice

- history and material
- frustration and degeneracy

Emergent gauge field

- emergence from constraint
- magnetic monopoles and ‘Dirac strings’
- visualisation in experiment

Strings as degrees of freedom

- statistics and Monte Carlo simulations

Cubic RVB liquid

- representation as loop gas
- coexistence of bond criticality and spin order

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Geometrical Frustration in the Ferromagnetic Pyrochlore $Ho_2Ti_2O_7$

M. J. Harris,[1] S. T. Bramwell,[2] D. F. McMorrow,[3] T. Zeiske,[4] and K. W. Godfrey[5]
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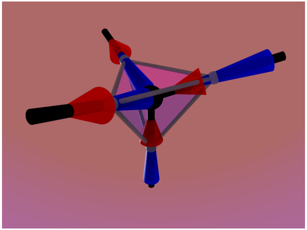
[2]*Department of Chemistry, University College London, 20 Gordon Street, London, WC1H 0AJ, United Kingdom*

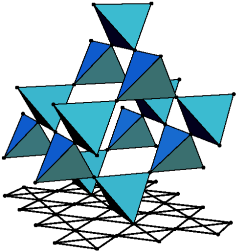
Spin ice compounds Dy/$Ho_2Ti_2O_7$

- local [111] crystal field $\sim$ 200 K

$\Rightarrow$ Ising spins $\sigma = \pm 1$

- large classical spins (15/2 and 8)
- large magnetic moment $|\vec{\mu}| \approx 10\,\mu_B$

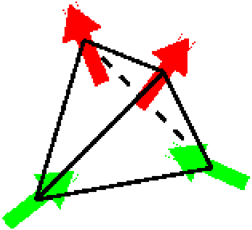


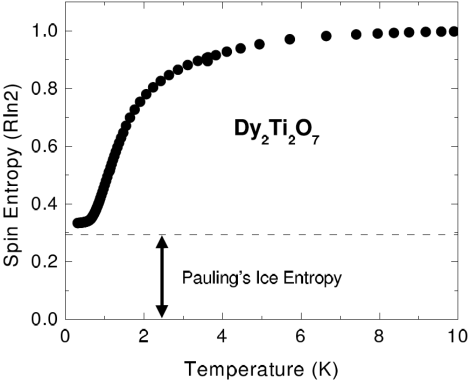


Frustration leads to (classical) degeneracy

(exchange+dipolar) interactions minimised by
2-in, 2-out ice rules ⇒ <u>local constraint</u>

Siddharthan+Shastry 1999, Gingras *et al.* 2000+





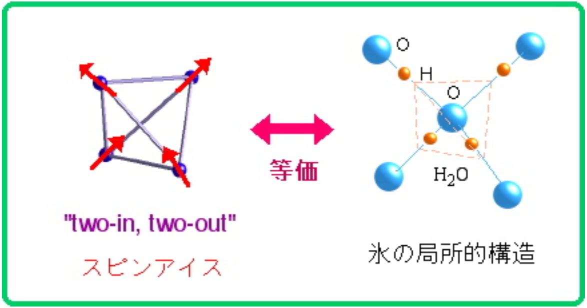
six ground states "per tetrahedron" ⇒ <u>degeneracy</u>
nonzero residual entropy

$$S_p = \ln 2 - \int_{T_0}^{\infty} (C/T) dT$$

Anderson 1956; Ramirez *et al.* 1999

Mapping from ice to spin ice

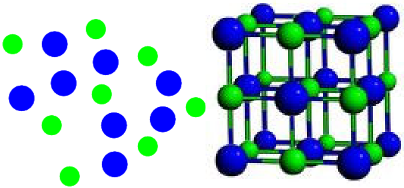
- In ice, water molecules retain their identity
- Hydrogen near oxygen $\leftrightarrow$ spin pointing in



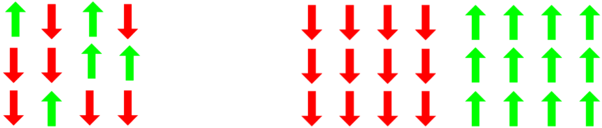
150.69.54.33/takagi/matuhirasan/SpinIce.jpg

Conventional order and disorder

Gas-crystal (e.g. rock salt):



Paramagnet-ferromagnet (e.g. fridge magnet)



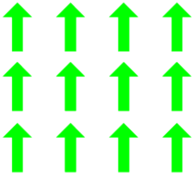
In between: critical points

Anything else???

Is spin ice ordered or not? Henley; Huse et al.; Hermele et al.

No order as in ferromagnet

- extensive degeneracy



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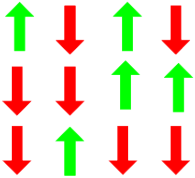
Not disordered like a paramagnet



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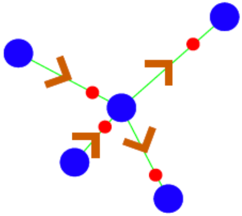

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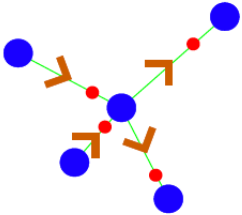
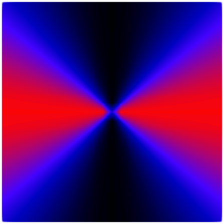


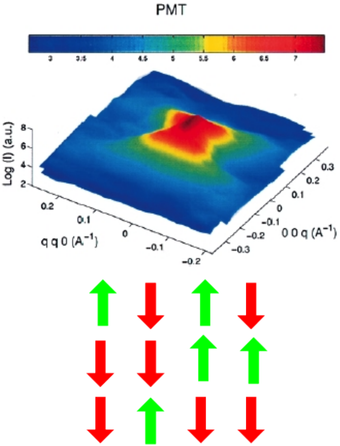
Magnetic moments $\vec{\mu}_i \Leftrightarrow$ (lattice) 'flux'

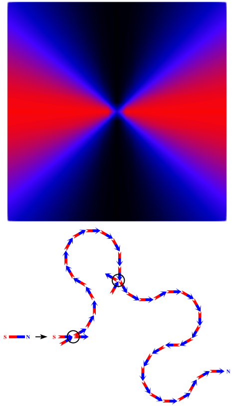
- Ice rules $\Leftrightarrow \nabla \cdot \vec{\mu} = 0 \;\Rightarrow\; \vec{\mu} = \nabla \times \vec{A}$
- Local constraint
  $\Rightarrow$ emergent gauge structure
  $\rightarrow$ algebraic spin correlations
  $\rightarrow$ 'bow-tie' structure factor

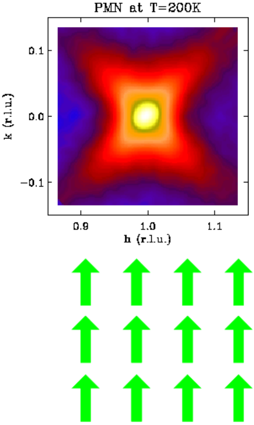


Effective action: $\mathcal{S} = (K/2) \int d^3 r |\nabla \times \vec{A}|^2$

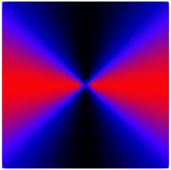
Disorder vs. spin ice vs. order in neutron scattering





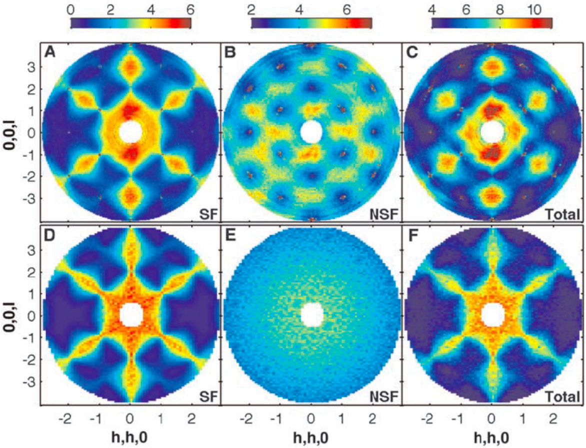


Pinch points in neutron scattering



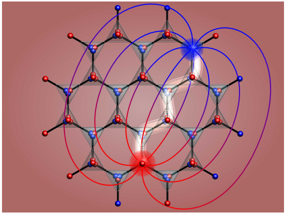
Isakov, RM, Sondhi 2004

Tom Fennell



Fennell+Bramwell *et al.* 2009

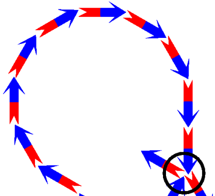
‘Dirac strings’ and emergent magnetic monopoles

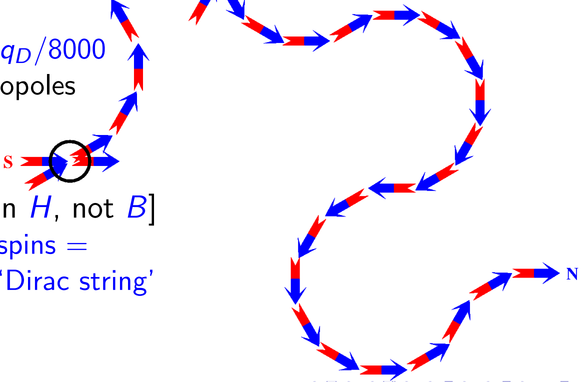


magnetic Coulomb interaction

$$E(r) = -\frac{\mu_0}{4\pi}\frac{q_m^2}{r}$$

- $q_m = 2|\vec{\mu}|/a_d \approx q_D/8000$
- **deconfined** monopoles





[monopoles in H, not B]

flipped spins =

(observable) ‘Dirac string’

‘Dirac strings’ and emergent magnetic monopoles

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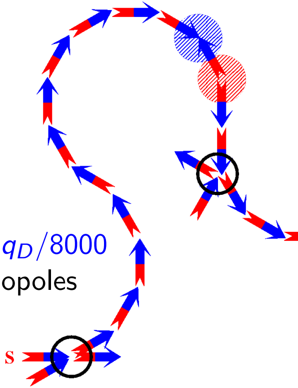

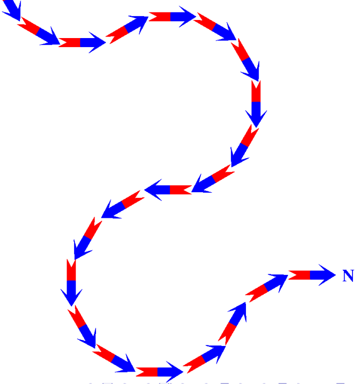

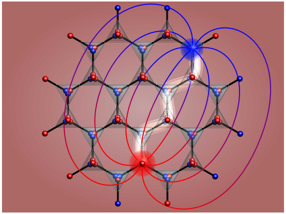

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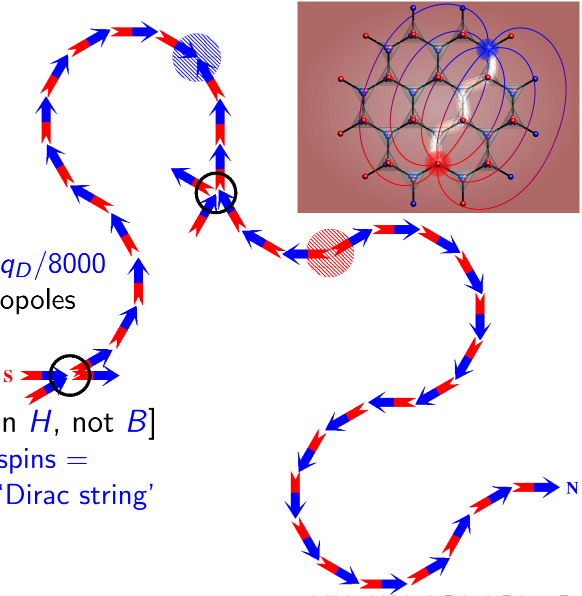

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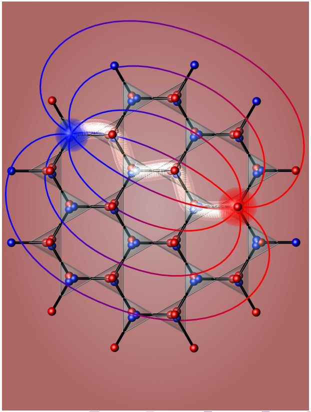
Monopole charge from inverting dipole string

$$V(r) = \frac{|\vec{\mu}|}{a} \int_\Lambda d\vec{r'} \cdot \vec{\nabla} \frac{1}{|r - r'|} = q_m \left(\frac{1}{|r - r_a|} - \frac{1}{|r - r_b|} \right)$$

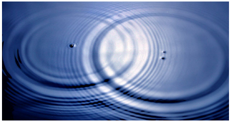
Potential due to a string of dipoles

- same as charges at ends of string
- charge $q_m = |\vec{\mu}|/a =$ moment per unit length of string
- reversing string of dipoles creates (tunable irrational) charges
- fractionalisation/deconfinement



Emergent versus intrinsic gauge charge

Emergence of qualitatively new degrees of freedom is common phenomenon



- low-energy d.o.f. $\neq$ high energy d.o.f.

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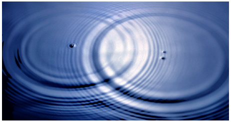
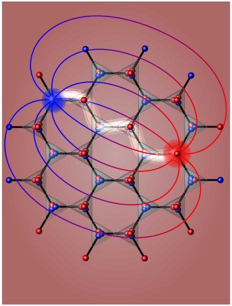


Here: emergent d.o.f. is gauge field

- bow-ties in neutron scattering



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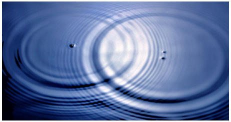
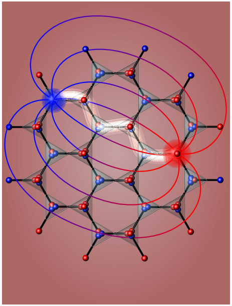
But: we also have high-energy gauge structure

- magnetic dipole moment of spins
- 'intrinsic' magnetic charge of monopole





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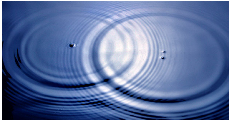
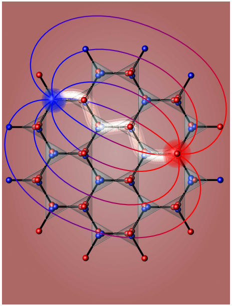
Emergent and intrinsic gauge charges are

- distinct
- (partially) independent





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Here: emergent d.o.f. is gauge field

- bow-ties in neutron scattering

But: we also have high-energy gauge structure

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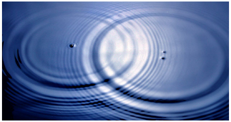
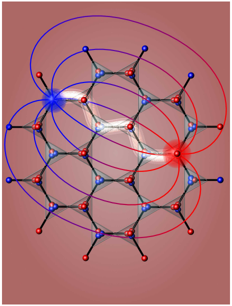
Emergent and intrinsic gauge charges are

- distinct but mathematically identical
- (partially) independent





Dimensional reduction of emergent gauge theory

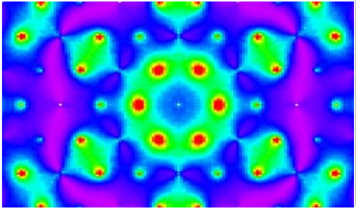
[111] field pins spins in triangular layer

Effective action in $d = 2$ vs. $d = 3$:

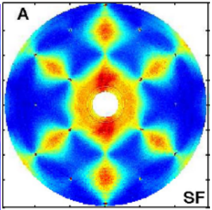
$$3d \; : \; \mathcal{S} = (K/2) \int d^3 r |\nabla \times \vec{A}|^2$$

$$2d \; : \; \mathcal{S} = (K/2) \int d^2 r |\nabla \times h|^2 + \lambda \cos(2\pi h)$$

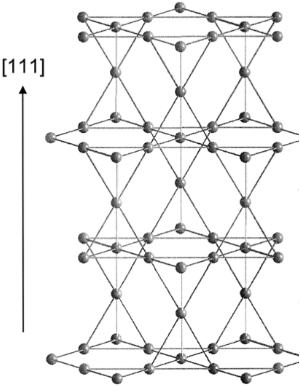
⇒kagome ice



Kadowaki *et al.* 2009



Fennell *et al.* 2009



Additional terms permitted in $2d$ RM+Sondhi 2003

⇒ additional peaks in structure factor

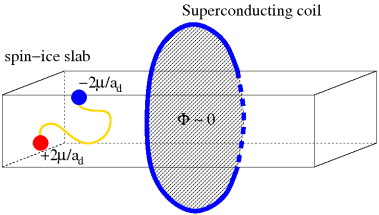
magnetic interaction remains $3d$

Single monopole search: Stanford experiment Cabreras 1982

Monopole passes through superconducting ring

⇒ magnetic flux through ring changes

⇒ e.m.f. induced in the ring ⇒ countercurrent $\propto q_m$ is set up

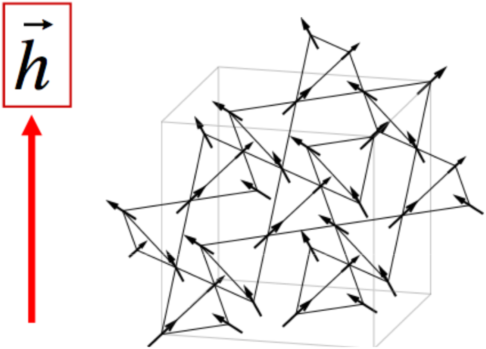


- 'Works' for both fundamental cosmic and spin ice monopoles
- signal-noise ratio a problem

Imagining ‘Dirac strings’

Strings not uniquely defined but

- applying [100] field enforces reference configuration
- motion of monopoles generates strings
- strings execute random walk transverse to field cf. Chalker



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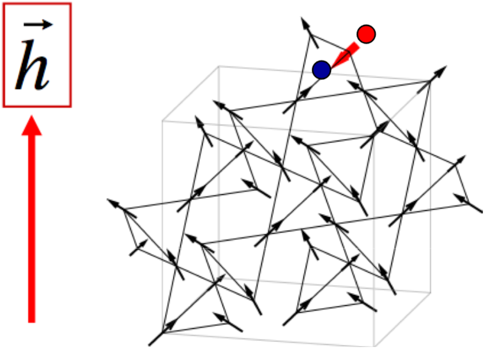

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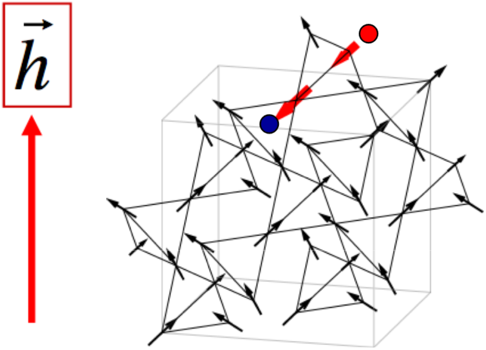

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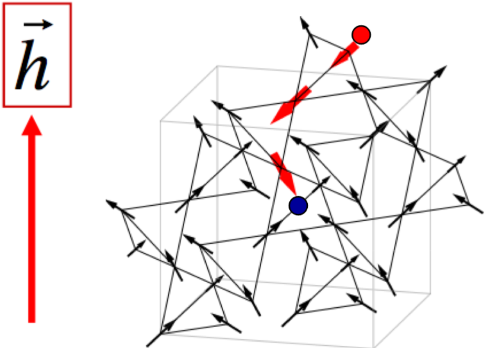

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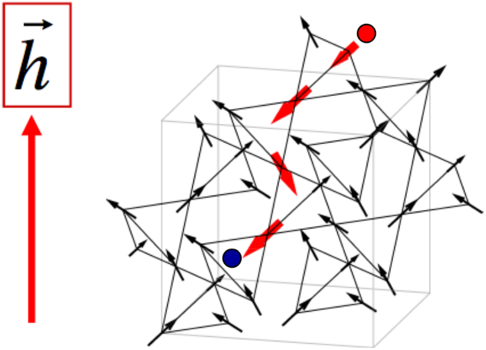

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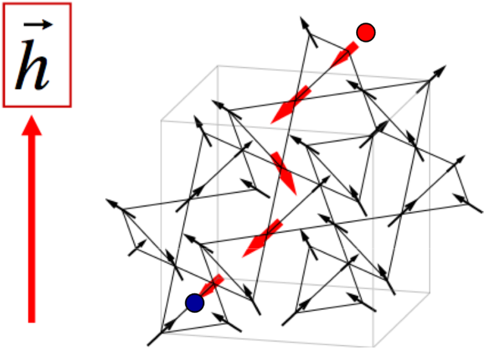

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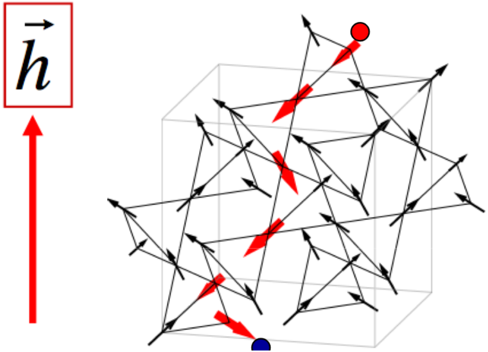

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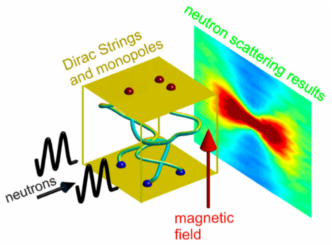
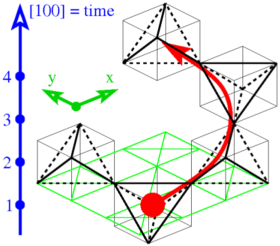
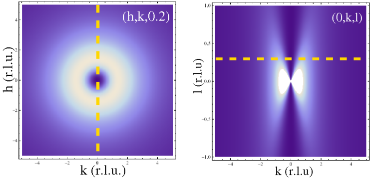


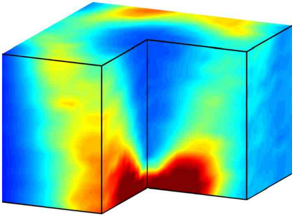
$\Leftrightarrow$



$\Rightarrow$ random walk in 2 dimensions + time



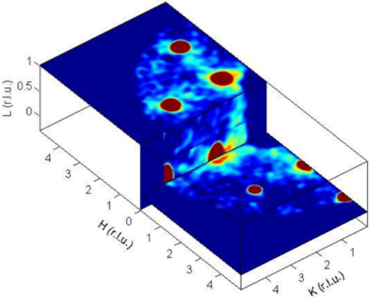
H in the [001] direction



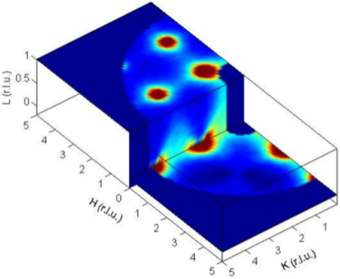
Dirac strings in neutron scattering Morris et al. 2009

Neutrons in fields of order 1T HZB–Tennant group

- compared to random-walk model



Data

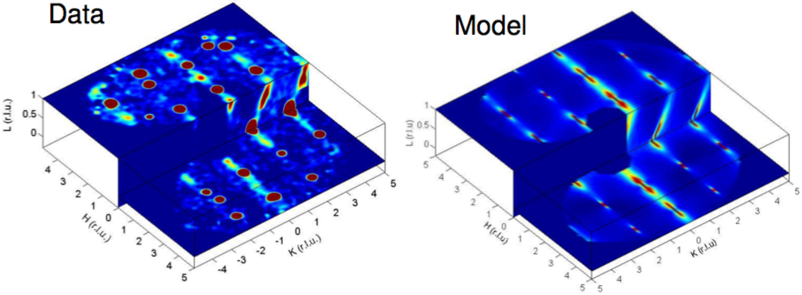


Model

Dirac strings in neutron scattering Morris et al. 2009

Neutrons in fields of order 1T HZB–Tennant group

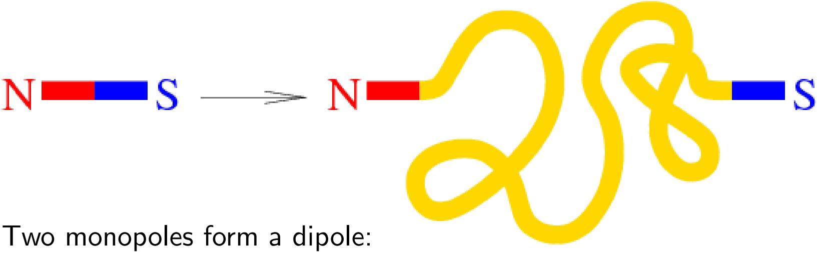
- compared to random-walk model
- tilted field: biased random walk



Intuitive picture for monopoles

Simplest picture does not work: disconnect monopoles

Next best thing: no string tension between monopoles:



Two monopoles form a dipole:

- connected by tensionless 'Dirac string'
- Dirac string <u>is</u> observable

$\Rightarrow$ $q_m \approx q_D/8000$ not in conflict with quantisation of e

Loops and strings/worms in the ice model

Corner-sharing square/tetrahedra

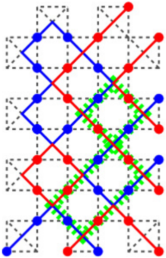
- Ising spins as basic d.o.f.

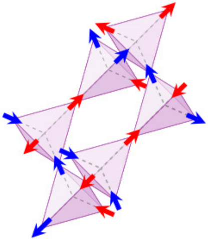
Each square/tetrahedral unit

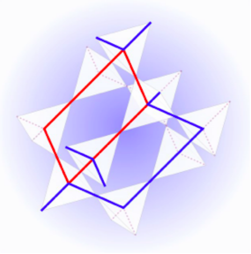
- two up/two down spins
- realises six-vertex model

Two red and two blue sites each

- strings = alternating red/blue
  - emergent gauge flux = spins
- adjacent red (blue) spins form red (blue) loops
  - fully-packed two-color loop model Kondev+Henley







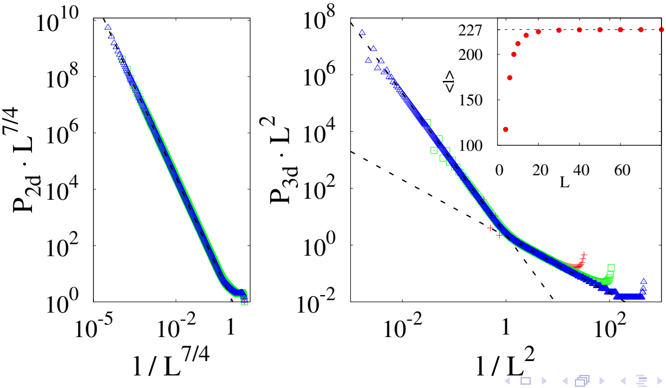
Statistics of strings in spin ice Jacobsen 90s; Jaubert, Haque, RM 2011

Algebraic length distribution, finite average length (24 vs. 227)

- 2d Kondev vs. 3d are different: two populations in 3d cf. random walk

Different effective descriptions

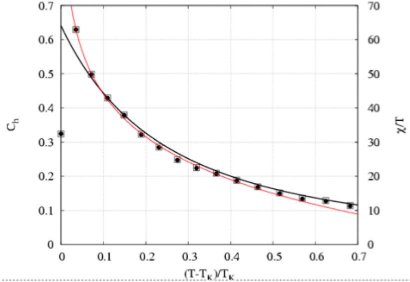
- 2d critical percolation; 3d Brownian motion
  - topological phase!



Use for numerical simulations Newman+Barkema; Gingras et al; Isakov et al; . . .

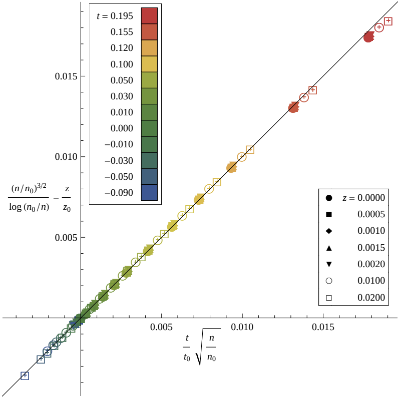
Algorithm flips worms – weighted by length of worm

- in $d = 3$, each MC move flips finite fraction of sample
- can simulate unconventional phase transition very accurately
  - log-corrections at upper critical dim. of Kasteleyn transition



$$\frac{t}{t_0}\left(\frac{n}{n_0}\right)^{1/2} = \frac{1}{\ln(n_0/n)}\left(\frac{n}{n_0}\right)^{3/2} - \frac{z}{z_0}$$

Powell, unpub (2012)



Néel and dipolar correlations in RVB

Albuquerque, Alet, Damle, R.M.

Resonating valence bond wavefunctions

- parent of superconducting state? PWA
- singlet-dominated phase

Encodes magnetic correlations

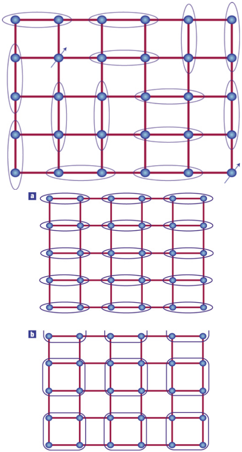
- on square lattice, long(short)-range RVB have (no) Néel order Liang etal

Nature of bond (energy) correlations?

- proximity to valence-bond solid in 2D
- what happens on 3D cubic lattice?

Consider RVB wave function of n.n. dimer coverings, $|c\rangle$ Rokhsar+Kivelson

$$|\Psi\rangle = N_c^{-1/2} \sum_c |c\rangle$$



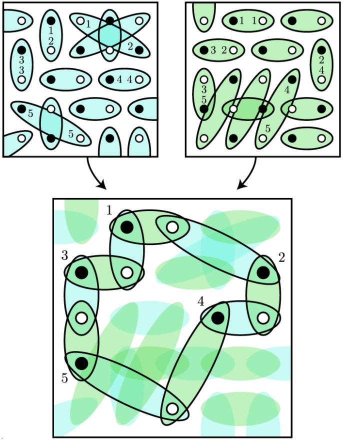
Sachdev

Correlations from RVB wavefunctions Sutherland; Beach, Sandvik

$\langle S_i \cdot S_j \rangle = N_c^{-1} \sum_{c,d} \langle d | S_i \cdot S_j | c \rangle$

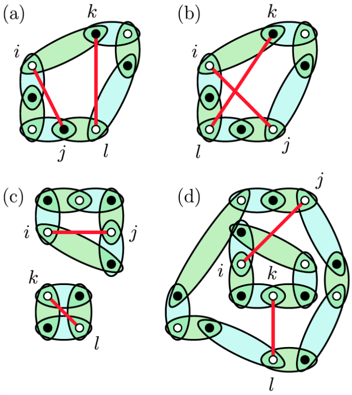
- contribution if i,j on same loop

$\Rightarrow$ properties of loop soup?



Bond correlators

- contributions more complex



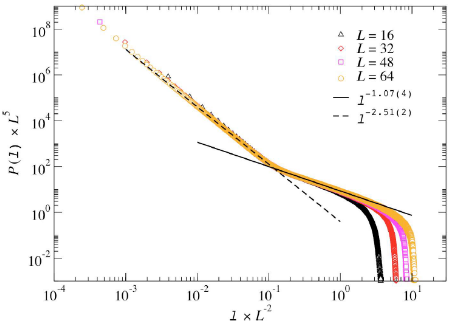
Results for cubic n.n. RVB

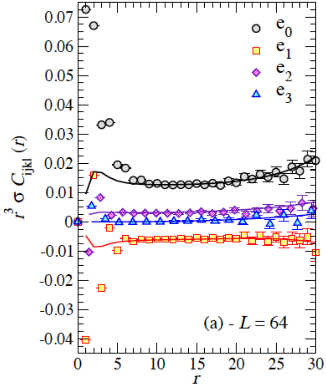
Loop soup has two populations: long loops give rise to Neel order

- Bond correlators have algebraic dipolar form
  - different power law from conventional Néel state

Field theory: two emergent gauge fields

- Néel order can disappear independently





Collective behaviour: magnetic Coulomb liquid

Debye-Hückel theory for low temperatures CMS 2008

- sparse charges without strings
- screening of Coulomb interaction

'Magnetolyte' chemistry + 'magnetricity' Bramwell et al. 2009

- Wien effect: nonequilibrium response to changing field
- transient magnetic currents in response to field steps

[111] magnetic field = chemical potential CMS 2008

- liquid gas transition
- dimensional reduction to 2d

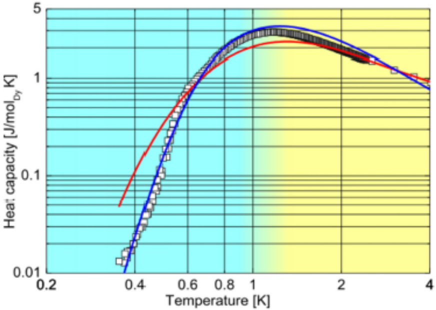
Specific heat of magnetic Coulomb liquid

- Debye-Hückel theory of monopole gas (blue)

  (no free parameters!)

- Bethe lattice calculation (red)

  (tuning J_{eff} to fit the data)



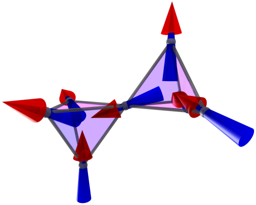
expt by Grigera/Tennant groups 2009

Interacting Coulomb liquid

point-like charged excitations + magnetic Coulomb interaction

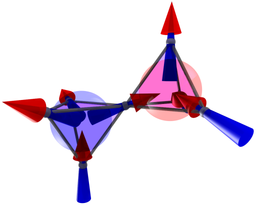
(i) interaction strength $\Gamma \propto (q_m^2/\langle r \rangle)/T \sim \exp[-\Delta/T]/T$
vanishes at high and low T

(ii) [111] magnetic field acts as chemical potential
$\Rightarrow$ can tune $\langle r \rangle$ and T separately



$\vec{B}$

$\Uparrow$



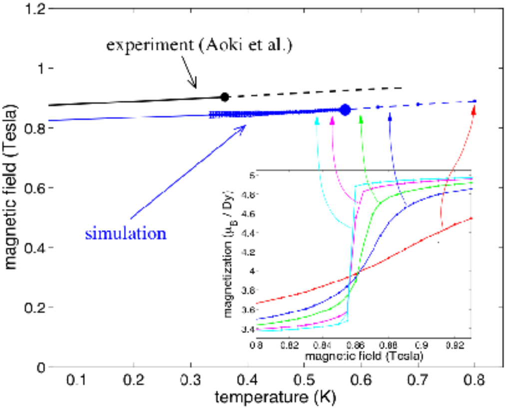
Liquid-gas transition in a [111] field CMS 2008

- first-order transition with critical endpoint — Fisher *et al.*

- observed experimentally
  Sakakibara+Maeno

  ” *unprecedented*
  in localized
  spin systems”

- confirmed numerically



The Wien effect in a 'magnetolyte'

Bramwell et al. 2009

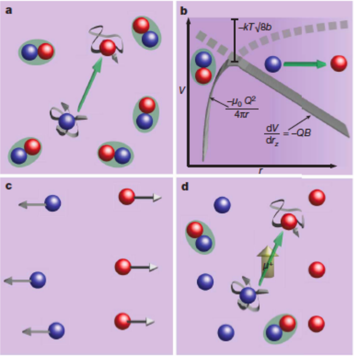
Double equilibrium: vacuum $\leftrightarrow$ bound monopoles $\leftrightarrow$ free monopoles

- applied magnetic field alters bound $\leftrightarrow$ free reaction constant Onsager

$$\frac{K(B)}{K(0)} \simeq 1 + \frac{\mu_0 Q^3 B}{8\pi k_B^2 T^2}$$

- buffering: vacuum $\leftrightarrow$ bound equilibrium unchanged

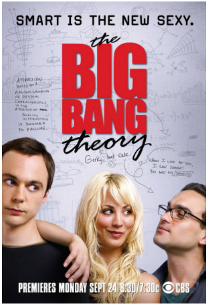
$\Rightarrow$ free charges increase in field in universal fashion

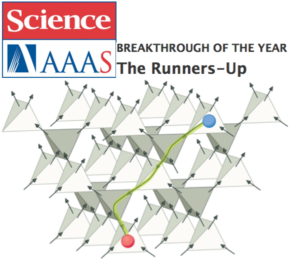


Expt: magnetic fluctuations/dynamics

Sean Giblin

‘Outreach’





FAZ—Welt—Sächsische Zeitung—. . .

Spektrum der Wissenschaft

Physics Today

Physics World

Physik Journal

. . .

Collaborators

Coulomb phase:
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S. Parameswaran
S. Sondhi

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S. Piatecki
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3D RVB:
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F. Alet
K. Damle

String expt–HMI:
S. Grigera
B. Klemke
J. Morris
A. Tennant

Discussions:
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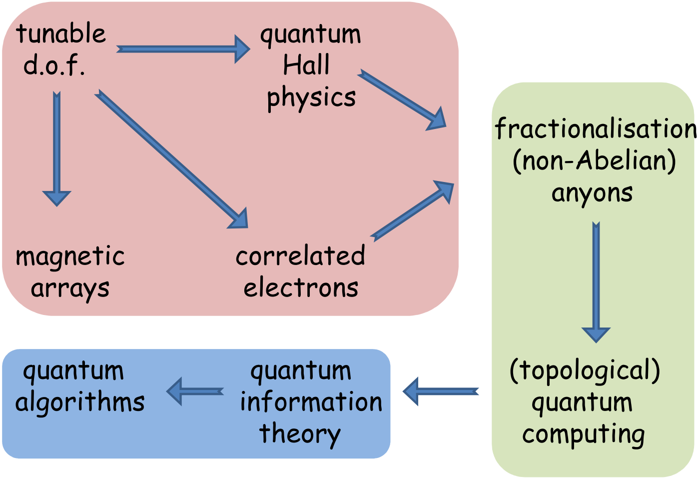
P. Fulde
P. McClarty
A. Nahum
F. Pollmann
A. Sen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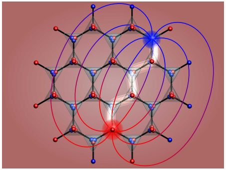
Gauge fields and strings in spin ice

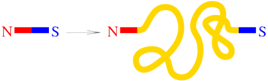
Emergent gauge field, fractionalisation

- topological physics in $d = 3$
- deconfined magnetic monopoles



Neutron scattering

- emergent gauge field: pinch points
- dimensional reduction in a field



'Dirac string': emergent gauge flux

- tensionless; MC simulations; ...

Loops in RVB physics

- long-range magnetic order independent of dipolar bond order