

# A FETCH DEPENDENT MODEL OF SEA SURFACE ROUGHNESS FOR OFFSHORE WIND POWER UTILISATION

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**ABSTRACT:** The sea surface roughness  $z_0$  is usually determined from friction velocity  $u_*$  with the Charnock relation as  $z_0 = z_{ch} u_*^2 / g$ , where  $g$  is the gravitational acceleration and  $z_{ch}$  an empirical parameter, which was meant to be a constant, but turned out to be site specific for sites with coastal influence. Several attempts to improve this relation aim on finding a power law between a non-dimensional sea surface roughness and a non-dimensional group describing the influence of the wave field. The Rødsand field measurement was used to test several proposed relations.

A significant correlation with sea surface roughness was found only if wave age,  $c_p/u_*$ , was used as non-dimensional group for the wave field influence, where  $c_p$  is the velocity of the peak wave component. However, it was also shown that this scaling suffers from the general problems of self-correlation and wind speed dependence. In a test application only a small improvement of the predicted 10 m wind was obtained using this wave age dependent relation. An empirical relation for the dependence of wave age on fetch was used to derive a fetch dependent relation for the sea surface roughness. The 10 m wind speed predicted with this relation was only insignificantly better than with the Charnock equation itself.

**Keywords:** Off-shore, Resources, Roughness, Coastal Sea Areas, Waves, Rødsand

## 1 INTRODUCTION

Large offshore wind farms are being built in several countries in Europe. The economic viability of such projects depends on the favourable wind conditions of offshore sites, since the higher energy yield has to compensate the additional installation and maintenance cost. A reliable prediction of the wind resource is therefore crucial. The sea surface roughness plays a key role here, since it is the main reason for the high wind speeds offshore. However, this sea surface roughness depends on the wave field present, which in turn depends on wind speed, upstream fetch, water depth, etc. For open sea conditions the Charnock relation (Charnock, 1955) has been applied successfully to determine the sea surface roughness:

$$z_0 \approx z_{ch} \frac{u_*^2}{g} \quad (1)$$

$z_{ch}$  is an empirical parameter, which was meant to be a constant, but turned out to be site specific for sites with coastal influence. In these near coastal areas it has been found that quantities other than the friction velocity (or wind speed) also play an important role. The reason is that the sea state is not only determined by the wind speed, but also significantly by upstream fetch.

Numerous attempts have been made to find an empirical relation for the sea surface roughness with an improved description of the wave field. The aim has been to find a power law relation between non-dimensional scaling groups  $Y$  describing the sea surface roughness and  $X$  describing the wave field, where  $A$  and  $B$  are empirical constants:

$$Y \approx AX^B \quad (2)$$

For sea surface roughness group  $Y$  usually either the Charnock parameter  $z_{ch} = z_0 g / u_*^2$  (where  $z_0$  is sea surface roughness,  $g$  gravitational acceleration and  $u_*$  friction velocity) or the ratio of roughness  $z_0$  and significant wave height  $H_s$  are used. The wave field group  $X$  is most commonly parameterised by the ratio of peak wave velocity  $c_p$  and friction velocity  $u_*$  (or alternatively the 10m wind speed  $u_{10}$ ), the so called wave age  $c_p/u_*$  or  $c_p/u_{10}$  (Smith et al., 1992), (Monbaliu, 1994), (Johnson et al., 1998). Alternatively, the wave steepness has been proposed, which is the ratio of significant wave height  $H_s$  and peak wave length  $L_p$ , (Taylor and Yelland, 2001).

A generally accepted relation has not emerged yet and in

this paper we try to evaluate different proposed relations

with measurement data from the Rødsand site (see section

2), which can be seen as a typical site for an offshore

wind farm.

In section 3 the correlations between the non-dimensional groups of the different proposed scaling groups are investigated. Some general problems of the proposed type of power law relations are discussed in section 4. All relations are then tested in their capability to predict the wind speed at 10 m height from measured friction velocity and wave quantities in section 5. A description of the sea surface roughness as function of wave age is combined with an empirical parameterisation of wave age depending on fetch and wind speed in section 6. This leads to a description of the sea surface roughness with easily available quantities. Finally, conclusions are given in the last section.



Figure 1: Rødsand measurement site

## 2 THE RØDSAND MEASUREMENT

A 50 m high meteorological measurement mast has been established at Rødsand in October 1996. It is situated about 11 km south of the coast of Lolland (see Figure 1). Simultaneous wind and wave measurements were performed from 1998 to 2001. For a description of the measurement and the quality control of the data see (Lange et al., 2001). For this analysis data from 1998 to 2000 have been used. The Monin-Obukov-length  $L$  was determined by different methods and records were selected by requiring the different values of  $L$  to be consistent. Additionally, extreme situations ( $0 > L > -20$ m and  $0 < L < 50$ m) and a single peaked wave spectra (wave spectrum bandwidth  $< 0.25$ ) This excluded about 60% of the available data. Wind speeds were corrected for flow distortion of the mast and booms with a method described in (Højstrup, 1999). Records with wind speed sensors in the shade of the mast were excluded. The final data set consisted of about 3500 half-hourly records.

## 3 COMPARISON OF POWER LAW RELATIONS

The non-dimensional groups of the proposed power law relations have been calculated from the measurement data of the Rødsand field measurement. For each combination of non-dimensional groups a linear fit has been made and the factor and power of a power law relationship has been determined as well as the coefficient of determination  $r^2$ . The results are shown in Table 1.

The correlation is clearly highest if wave age,  $u_*/c_p$ , is used as wave scaling group. It can also be seen that the choice of the wave scaling group seems to have a larger influence than the scaling group for sea roughness.

Table 1: Results of a power law fit to the scaling groups

relation		fitted parameters		
sea roughness	wave influence	factor A	power B	$r^2$
$z_{ch}$	$u_*/c_p$	95.5	3.06	0.19
$z_{ch}$	$u_{10n}/c_p$	3.24 E-2	-2.48	0.07
$z_{ch}$	$H_s/L_p$	1.66 E-8	-3.96	0.06
$z_0/H_s$	$u_*/c_p$	87.1	4.50	0.33

$z_0/H_s$	$u_{10n}/c_p$	2.57 E-4	-1.16	0.01
$z_0/H_s$	$H_s/L_p$	4.57 E-8	-2.39	0.02

## 4 PROBLEMS OF POWER LAW RELATIONS

### 4.1 Self-correlation

Some of the relations investigated above contain common variables in both the sea surface roughness and the wave influence scaling group, especially since the sea surface roughness  $z_0$  can not be measured independently, but has to be derived from measured data by assuming a log-linear wind profile:

$$z_0 \stackrel{?}{=} \frac{z}{\exp \left( \frac{u_{z,n}}{u_*} \right)} \quad (1)$$

Here  $\stackrel{?}{=}$  is the von Karman constant  $(\kappa = 0.41)$ ,  $u_{10n}$  the neutral wind speed at 10 m height and  $u_*$  the friction velocity. The neutral wind speed is calculated from  $u_{z,n} = u_*/(\kappa \ln(z/L))$ , where  $\ln(z/L)$  is the Businger/Dyer flux-profile relationship for momentum. Common variables lead to self-correlation effects, which influence the correlation found between the groups. This effect has been investigated by a numerical experiment. A random 'data' set has been produced, where the values of the quantities were random, but follow the probability distributions of the measured quantities. In this way they are independent of each other and therefore not correlated. The random data have been analysed in the same way as the measured data. Any correlation found with such a random 'data' set is the effect of self-correlation.

Figure 2 shows a comparison between the Rødsand data and the random 'data' for the Charnock parameter versus inverse wave age. The data have been bin-averaged with respect to inverse wave age. It can be seen that the correlations are clearly different, e.g. the correlation found in the measured data is not entirely due to self-correlation. However, it can also be seen that the curvatures for high inverse wave ages are similar, which might be seen as an influence of self-correlation.

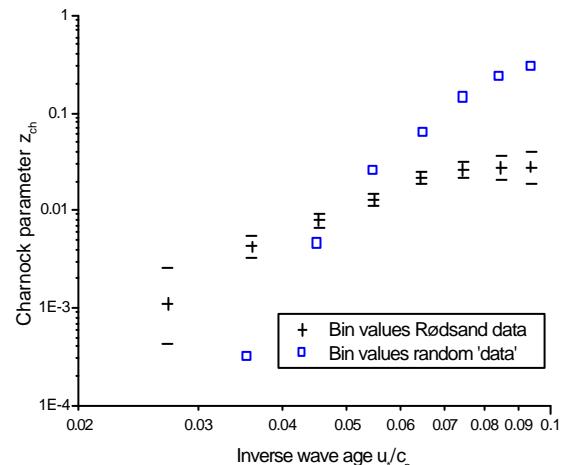


Figure 2: Comparison of Charnock parameter versus inverse wave age, bin-averaged with respect to inverse

wave age for Rødsand measurement data and random 'data'

#### 4.2 Influence of wind speed distribution

For ideal scaling parameters the remaining scatter in the data around the line given by the power law should be independent of other physical quantities. This can be tested by stratifying the data into different classes with respect to one quantity, e.g. the 10 m wind speed, and comparing the correlation between the scaling groups for the different classes. This is shown for wind speed classes and the Charnock parameter versus inverse wave age relation in Figure 3. The data have been bin-averaged with respect to inverse wave age. For the line with crosses all data have been used, the other lines are for data stratified according to wind speed. For an ideal scaling all points should collapse onto one line. This is clearly not the case. It can be seen that the steepness of the relations of the single wind speed classes is larger than that of the total data set, where the bin-averaging has been made irrespective of wind speed.

This means that the wind speed probability distribution indeed has an influence on the result of the analysis. A narrow wind speed distribution of only 2m/s width would e.g. result in a much larger steepness as can be seen from Figure 3.

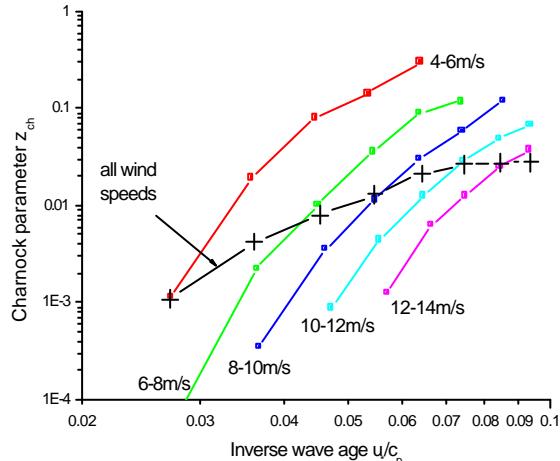


Figure 3: Comparison of Charnock parameter versus inverse wave age, bin-averaged with respect to inverse wave age; for the line with crosses all data have been used, the other lines are for data stratified according to wind speed

#### 4.3 Determination of empirical constants

In Figure 4 the parameters for the relation between Charnock parameter and inverse wave age found from the Rødsand data are compared with values taken from literature. Given the problems discussed in the previous paragraphs it is not astonishing, that the relations differ. The largest difference is found with the relation given by (Toba et al., 1990). The relations from (Smith et al.,

1992), (Monbaliu, 1994) and (Johnson et al., 1998) do not differ much from each other, but are somewhat less steep than the one fitted to the Rødsand data.

(Toba et al., 1990) used a data set including laboratory data with a much wider range of inverse wave ages from 0.04 to 5. Already in the Rødsand data set a curvature is visible in the high end of inverse wave ages, which stems from self-correlation and indicates that a power law relation is only an approximation for a limited range of inverse wave ages. This might be one reason for the different parameter values found.

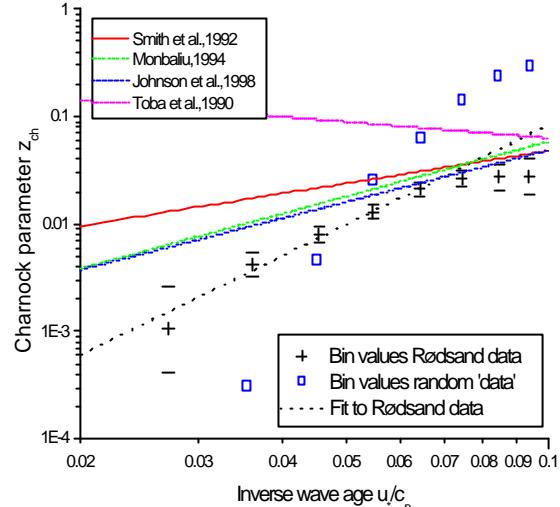


Figure 4: Comparison of Charnock parameter versus inverse wave age, bin-averaged with respect to inverse wave age for Rødsand measurement data with relations from the literature

Another potential problem is the influence of the wind speed distribution of the data set on the correlation. As it is shown in section 4.2 the wind speed distribution of the data set influences the values of the empirical constants found from a power law fit. This might further contribute to differences between the different data sets.

#### 5 TEST APPLICATION

To test the prediction capabilities of the different power law relations they were used to model a time series of neutral wind speed at 10 m height from the measured quantities needed in the relations. The empirical parameters of the power laws were determined by fits to the data (see Table 1). The root mean square error of modelled to measured neutral wind speed at 10 m height was calculated. It is given in Table 2.

Compared to the Charnock relation the power law relation parameterising the Charnock parameter with inverse wave age leads to a small decrease in rms-error. The other two scaling groups (wave age with wind speed and wave steepness) fail to improve the prediction made with the Charnock relation alone.

Table 2: Root mean square error of modelled to measured neutral wind speed at 10 m height for a power law fit of different scaling groups

	Charnock [m/s]	$u_*/c_p$ [m/s]	$u_{10n}/c_p$ [m/s]	$H_s/L_p$ [m/s]
$z_{ch}$	1.19	1.15	1.30	1.29
$z_0/H_s$		1.20	1.57	1.59

## 6 FETCH DEPENDENT ROUGHNESS

The wave parameters used in the above relations are usually not available for wind power studies. Therefore a relation is needed to determine these from an easily available parameter like fetch.

The influence of fetch on wave parameters has been determined by field experiments with winds blowing approximately perpendicular to a straight coastline. To use these relations an effective fetch for a given directions  $\theta$  has been defined as the integral over all direction from  $-90^\circ$  to  $+90^\circ$  weighted by a cosine squared term, normalised and divided by the fetch which would result from a straight coastline.

$$x_{eff} = \frac{\int_{-90^\circ}^{+90^\circ} \cos^2 \theta d\theta}{4\pi} \quad (2)$$

(Kahma and Calkoen, 1992) found the following empirical relation between the dimensionless peak frequency and the dimensionless fetch:

$$\frac{u_*}{g} = C \left( \frac{x}{u_*} \right)^D \quad (3)$$

Here  $\omega_p$  is the peak frequency,  $x$  the fetch in metres and  $u_*$  the friction velocity. The constants were found to be  $C=3.08$  and  $D=-0.27$ .

Using the deep water approximation of the dispersion relation for water waves the left hand side of eq. (2) can be identified as the inverse wave age  $u_*/c_p$ .

This relation can therefore be used to eliminate the wave parameters from the inverse wave age relations. Using the above relation of Charnock parameter and inverse wave age the following equation can be given for the sea surface roughness:

$$z_0 = \frac{u_*^2}{g} A_l C^{B_l} \left( \frac{g}{u_*^2} x_{eff} \right)^{B_l D} \quad (4)$$

with  $A_l=95.5$  and  $B_l=3.06$ . This relation has again been tested in its capability to model a time series of neutral wind speed at 10m height. Now only the friction velocity and the fetch are needed as input parameters. The result is an rms-error of 1.18 m/s, which is about the same as for the Charnock relation alone.

A direct fit of a power law between Charnock parameter and non-dimensional effective fetch

$$z_0 = \frac{u_*^2}{g} A \left( \frac{g}{u_*^2} x_{eff} \right)^B \quad ?$$

yields with  $A'=141$  and  $B'=-0.62$  a rms-error of 1.16 m/s. Here only a direct fit to the measurement data has been used. This can therefore be seen as the limit of this method to use any empirical relation for sea surface roughness from non-dimensional fetch.

## 7 CONCLUSIONS

Different power law relations have been investigated in their capability to improve the description of sea surface roughness compared to the Charnock relation by introducing an effect of the wave field. The dependence of sea surface roughness on different wave field descriptions proposed in the literature was evaluated with the Rødsand data set. A significant correlation was only found for the wave age,  $c_p/u_*$ , not for the wave age with wind speed,  $c_p/u_{10n}$ , or the wave steepness,  $H_s/L_p$ .

A trend of increasing Charnock parameter with inverse wave age was found for the Rødsand data. However, the dependency seems to deviate from the simple power law assumed, especially for high inverse wave ages. It was shown that self correlation effects are a possible explanation for this behaviour. Additionally, it was found that the relation is wind speed dependent and actual wind speed distributions of a data set influence the dependency found.

The different relations were tested in their capability to predict the neutral wind speed at 10 m height from measured quantities. The wave age dependent Charnock parameter was the only relation with results slightly better than the Charnock relation itself.

An empirical relation relating the wave age to fetch was used to derive a fetch dependent relation for the sea surface roughness. The 10 m wind speed predicted with this relation was only insignificantly better than the Charnock equation itself.

It can be concluded that only small improvements compared to the Charnock equation could be obtained by the introduction of wave field dependent parameters in a power law relation.

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