Black Holes in Einstein-Maxwell-Chern-Simons Theory

Alikram Aliev

TUBITAK-BU, Feza Gürsey Institute

Istanbul, TURKEY

Background 1

- **Einstein-Maxwell Theory: The Tangherlini solution for static charged black hole in all higher dimensions.**
- **The Tangherlini metric is also a solution to EMCS theory for any value of the (CS) coefficient.**

$$\mathcal{L} = (R + 12/l^2) \star 1 - \frac{1}{2} \star F \wedge F + \frac{\nu}{3\sqrt{3}} F \wedge F \wedge A \,, \quad (1)$$

- **The construction of exact rotating charged black hole solutions in higher-dimensional EM theory turned out to be a rather complicated problem**

The Kerr-Schild form

- One of the most useful methods for constructing such solutions in four dimensions is based on the use of the Kerr-Schild form for the spacetime metric

$$ds^2 = ds^2_{flat} + H k \otimes k \tag{2}$$

- In 1986, Myers and Perry used the Kerr-Schild form in uncharged case and presented the general AF solution for rotating black holes in all spacetime dimensions
- A generalization to include a cosmological constant was given recently by Gibbons et al (2004).

Background 2

- A new attempt to employ the Kerr-Schild framework for constructing the rotating charged black hole solutions in higher-dimensional EM theory was undertaken in a paper (Aliev, 2006).

- It turned out that the Kerr-Schild approach enables one to obtain the desired rotating charged solutions only in the limit of slow rotation.

- Numerical analysis of the general situation was given by J. Kunz et al (2005).

5D Minimal Gauged SUGRA

- **The addition to the theory of the Chern-Simons term with a particular value of the CS coefficient $\nu = 1$, extends its symmetries, facilitating the search for the exact solutions.**

$$\begin{aligned} \mathcal{L} &= (R + 2\Lambda) \star 1 - \frac{1}{2} \star F \wedge F \\ &+ \frac{1}{3\sqrt{3}} F \wedge F \wedge A \, , \end{aligned} \tag{3}$$

- **The general rotating charged black hole solution in $D = 5$ minimal gauged supergravity was constructed by Chong, Cvetič, Lu and Pope in 2005.**

The Purpose

The purpose of this talk is two-fold:

- **to present a simple Kerr-Schild type framework for rotating charged black holes in $D = 5$ minimal gauged supergravity**
- **to use this framework for constructing new black hole solutions to the Einstein-Maxwell-CS theory, when the CS coefficient $\nu \neq 1$.**

New Kerr-Schild framework

$$ds^2 = ds^2_{flat} + Hk \otimes k \tag{4}$$

Suppose a spacetime metric admits two specific vector fields: a null vector field k and a spacelike vector field ℓ.

$$ds^2 = d\bar{s}^2 + Hk \otimes k + V\left(k \otimes \ell + \ell \otimes k\right)\,, \tag{5}$$

where $d\bar{s}^2$ is the background AdS spacetime, H and V are two scalar functions.

We have

$$k_{a;b}\,k^b = 0\,, \qquad l_{a;b}\,l^b \neq 0\,. \tag{6}$$

New Kerr-Schild framework

$$ds^2 = d\bar{s}^2 + H k \otimes k + V\left(k \otimes \ell + \ell \otimes k\right) \,, \qquad (7)$$

In coordinates $x^\mu = \{t, r, \theta, \varphi, \psi\}$ **we have**

$$k = k_\mu dx^\mu = \left\{ \frac{\Delta_\theta}{\Xi_a \Xi_b} dt \,, 0 \,, 0 \,, -\frac{a \sin^2\theta}{\Xi_a} d\varphi \,, -\frac{b \cos^2\theta}{\Xi_b} d\psi \right\},$$

$$\ell = \ell_\mu dx^\mu = \left\{ \frac{\Delta_\theta}{\Xi_a \Xi_b} \frac{ab}{l^2} dt \,, 0 \,, 0 \,, -\frac{b \sin^2\theta}{\Xi_a} d\varphi \,, -\frac{a \cos^2\theta}{\Xi_b} d\psi \right\}$$

where

$$\Delta_\theta = 1 - \frac{a^2}{l^2} \cos^2\theta - \frac{b^2}{l^2} \sin^2\theta \,, \quad \Xi_a = 1 - \frac{a^2}{l^2} \,, \quad \Xi_b = 1 - \frac{b^2}{l^2} .$$

New Kerr-Schild framework

$$k_\mu k^\mu = 0\,, \quad k_\mu \ell^\mu = 0\,, \quad \ell_\mu \ell^\mu = \frac{a^2\cos^2\theta + b^2\sin^2\theta}{r^2} \qquad (8)$$

The potential one-form for the electromagnetic field is given by

$$A = \frac{\alpha}{\Sigma}\,k\,, \qquad \Sigma = r^2 + a^2\cos^2\theta + b^2\sin^2\theta \qquad (9)$$

$$R_\mu^\nu - 2\left(F_{\mu\lambda}F^{\nu\lambda} - \frac{1}{6}\,\delta_\mu{}^\nu\,F_{\alpha\beta}F^{\alpha\beta}\right) + \frac{4}{l^2}\,\delta_\mu^\nu = 0\,, \qquad (10)$$

$$\nabla_\nu F^{\mu\nu} + \frac{\nu}{2\sqrt{3}\sqrt{-g}}\,\epsilon^{\mu\alpha\beta\rho\tau}F_{\alpha\beta}F_{\rho\tau} = 0\,. \qquad (11)$$

Solution 1

We have a single differential equation for the function V.

$$\frac{\partial V}{\partial r} - \frac{2r}{\Sigma} V + \frac{4Q\nu\, r}{\Sigma^2} = 0\,, \qquad (12)$$

The solution has the form

$$V = \frac{Q\nu}{\Sigma}\,. \qquad (13)$$

for $\nu = 1$, we obtain

$$H = \frac{2M}{\Sigma} - \frac{Q^2}{\Sigma^2}\,, \qquad V = \frac{Q}{\Sigma}\,, \qquad \alpha = \frac{\sqrt{3}}{2} Q\,. \qquad (14)$$

Solution 1

Applying the coordinate transformations

$$
\begin{aligned}
dt &= d\tau - \frac{(r^2+a^2)(r^2+b^2)+abQ}{\Delta_r r^2}\, dr\,, \\
d\varphi &= d\phi + \frac{a}{l^2}\, d\tau - \frac{a(r^2+b^2)(1+r^2/l^2)+Qb}{\Delta_r r^2}\, dr\,, \\
d\psi &= d\chi + \frac{b}{l^2}\, d\tau - \frac{b(r^2+a^2)(1+r^2/l^2)+Qa}{\Delta_r r^2}\, dr\,,
\end{aligned}
\qquad (15)
$$

where

$$
\Delta_r = \frac{\left(r^2+a^2\right)\left(r^2+b^2\right)\left(1+r^2/l^2\right)+2abQ+Q^2}{r^2} - 2M
$$

we obtain the general black hole solution of Chong, Cvetič, Lu and Pope(2005)

Solution 2

At the linear level in a and b, the metric

$$ds^2 = d\bar{s}^2 + H k \otimes k + V\left(k \otimes \ell + \ell \otimes k\right) \,, \qquad (16)$$

is also a solution to the Einstein-Maxwell-CS theory with $\nu \neq 1$.

$$\begin{aligned} dt &= d\tau - \frac{r^2}{\Delta}\, dr \,, \\ d\varphi &= d\phi + \frac{a}{l^2}\, d\tau - \frac{a r^2\left(1 + r^2/l^2\right) + Q b \nu}{\Delta r^2}\, dr \,, \\ d\psi &= d\chi + \frac{b}{l^2}\, d\tau - \frac{b r^2\left(1 + r^2/l^2\right) + Q a \nu}{\Delta r^2}\, dr \,, \end{aligned} \qquad (17)$$

where

$$\Delta = r^2\left(1 + r^2/l^2\right) + Q^2/r^2 - 2M \,, \qquad (18)$$

Solution 2

we obtain the solution in the form

$$ds^2 \;=\; -\frac{\Delta}{r^2}\,d\tau^2 + \frac{r^2}{\Delta}\,dr^2 - 2\left(1 - \frac{\Delta}{r^2}\right) \times \tag{19}$$

$$\left(a\sin^2\theta\,d\phi + b\cos^2\theta\,d\chi\right)d\tau \tag{20}$$

$$-\frac{2Q\nu}{r^2}\left(b\sin^2\theta d\phi + a\cos^2\theta d\chi\right)d\tau + r^2 d\Omega_3^2 \tag{21}$$

in which, the electromagnetic field is described by the potential one-form

$$A = \frac{\sqrt{3}Q}{2r^2}\left(d\tau - a\sin^2\theta\,d\phi - b\cos^2\theta\,d\chi\right)\;. \tag{22}$$

Solution 3

There exists another class of solutions with $\nu \neq 1$ which, unlike the solutions given above, exhibits the singularity (instability) in the regime of slow rotation $a = b$

$$H = \frac{2M}{r^2} - \frac{Q^2}{r^4}\left[1 - 2\ln 2\,(1-\nu^2)\left(1 - \frac{r^2}{a^2}\right)\right], \quad V = \frac{Q\nu}{r^2}\,. \quad (23)$$

We see that for $a \to 0$ the metric components become divergent.

Gyromagnetic ratios

$$g_a = 3\left(1 - \frac{Qb\nu}{2aM + Qb\nu}\right) , \tag{24}$$

$$g_b = 3\left(1 - \frac{Qa\nu}{2bM + Qa\nu}\right) . \tag{25}$$

We see that for $\nu \to 0$, the gyromagnetic ratio tends to its value in the Einstein-Maxwell theory, $g \to 3$.

It also follows that for given parameters of the black hole, the value of the gyromagnetic ratios tend to decrease with the growth (within the linear approximation in rotation parameters) of the CS coefficient.

These results are in qualitative agreement with the numerical analysis of J. Kunz and F. Navarro-Lerida, Phys. Rev. Lett. 96, 081101 (2006).