Formation of Higher Dim Topological Black Holes

Filipe C. Mena

Minho, Portugal

with J. Natario and P. Tod

in *gr-qc arXiv:0906.3216*

Paris, July 2009

Motivation and Previous Results

- **Why study collapse to topological black holes?**
  - Stability of gravitational collapse
  - Only a few exact models for non-spherical collapse
  - 'Landscape' of vacua states in string theory

- **Previous results in 4D with spherical Λ models**
  - FLRW, Lemaitre-Tolman, Szekeres matched to Kottler

    (Balbinot et al., Nakao, Markovic & Shapiro, Debnath et al.)

- **Previous in 4D with planar and hyperbolic Λ models**
  - $k < 0$ FLRW matched to Kottler (Smith & Mann, 1997)
  - $k = 0$ FLRW matched to radiating Vaidya (Lemos, 1998)
  - Inhomogeneous spaces matched to Kottler (Mena, Natario, Tod, 08)

Objectives and Matching

- **Objectives**
  - Construct models of inhomogeneous and anisotropic collapse to topological BHs in higher dim
  - Study global structure

- **Matching Conditions**

$$q_{ij}^{+} = q_{ij}^{-}, \qquad H_{ij}^{+} = H_{ij}^{-}$$

  - Equality of 1st and 2nd fund. forms at matching surface
  - If matching fluids with vacuum then p vanishes at boundary
  - If dust fluid then boundary ruled by geodesics

Kottler spacetime

$$ds_+^2 = -VdT^2 + V^{-1}dr^2 + r^2(d\theta^2 + \Sigma^2(\theta)d\varphi^2)$$

$$V = b - \frac{2m}{r} - \frac{\Lambda}{3}r^2$$

with $\Sigma(\theta) = \theta, \sin\theta, \sinh\theta$ (planar, spherical, hyperbolic) for $b = 0, 1, -1$.

- If $\Lambda < 0$ and $m > 0$, V has unique zero and solution describes BHs with corresponding symmetry
- If $b = 0$ or -1 is possible to make identifications of 2-metric of constant T and r to obtain toroidal and higher genus BHs
- $\mathscr{I}$ will have the same topology (times $\mathbb{R}$).

Kottler spacetime

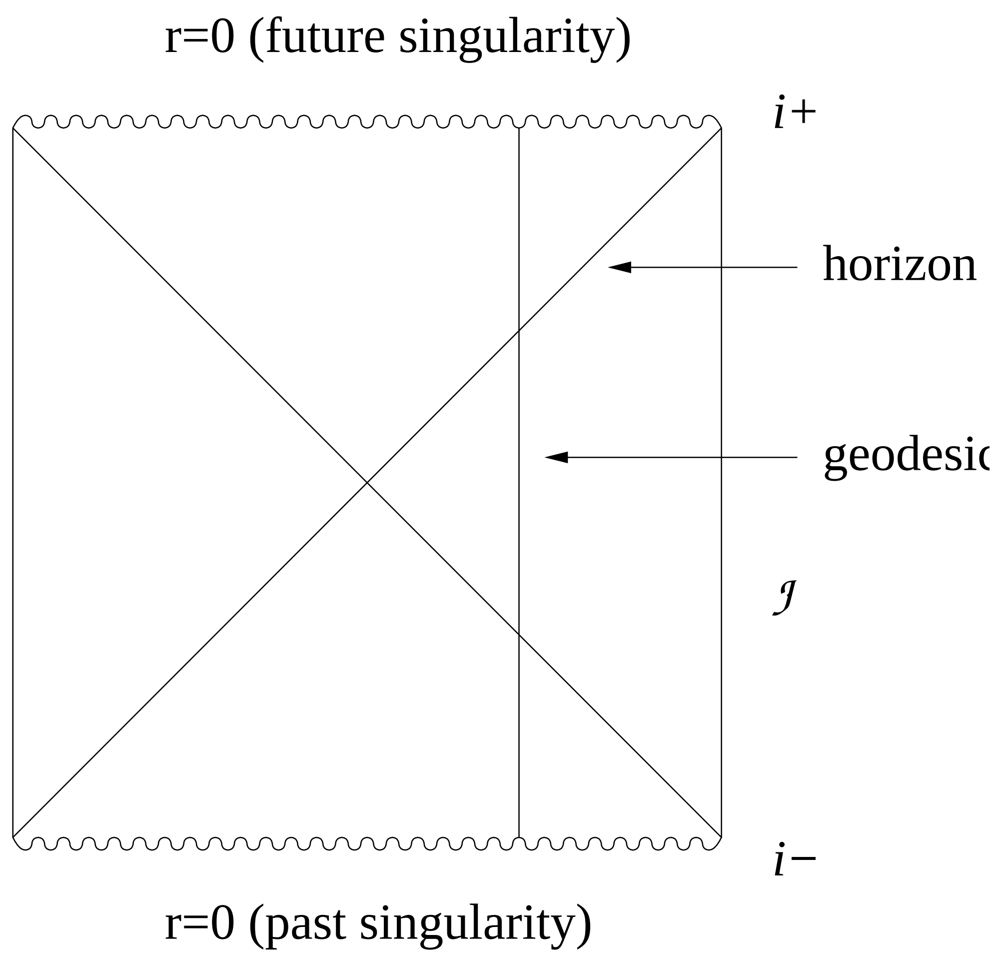

Figure 1: Penrose diagram for the Kottler solution with $\Lambda < 0$

A family of higher-dim black holes

Einstein equations

$$R_{ab} - \frac{1}{2} R g_{ab} + \frac{n\Lambda}{2} g_{ab} = \kappa T_{ab},$$

$(N, d\sigma^2)$ is n-dim Riemannian Einstein manifold with $R = n\lambda$

(M, ds^2) is $(n+2)$-dim Lorentzian manifold $M = \mathbb{R} \times I \times N$

$$ds^2 = -V(r)dt^2 + \frac{1}{V(r)} dr^2 + r^2 d\sigma^2$$

is a solution of the vacuum Einstein equations with Λ for

$$V(r) = \frac{\lambda}{n-1} - \frac{2m}{r^{n-1}} - \frac{\Lambda r^2}{n+1}$$

Properties

Proposition 1: The metrics are conformally-compactifiable and weakly-asymptotically-simple

- If $\Lambda = 0$, $\lambda > 0$ and $m > 0$, V has a single zero, corresponding to an event horizon.

- If $\Lambda > 0$, $m > 0$ and large enough positive λ, $V(r)$ is positive between two zeroes

- If $\Lambda < 0$ and $m > 0$, V again has a single zero, corresponding to an event horizon

Interior Spacetimes

The $(n+2)$-dim dust Λ-Friedman-like metric

$$ds^2 = -dt^2 + R^2(t)(dr^2 + f^2(r)d\sigma^2)$$

with

$$\frac{\dot{R}^2}{R^2} + \frac{k}{nR^2} = \frac{2\kappa\mu}{n(n+1)} + \frac{\Lambda}{n+1}.$$

The $(n+2)$-dim dust Λ-Lemaître-Tolman-Bondi-like metric

$$ds^2 = -dt^2 + A(t,r)^2 dr^2 + B(t,r)^2 d\sigma^2$$

with

$$\dot{B}^2 B^{n-1} + \left(\frac{\lambda}{n-1} - \frac{1}{(1+w(r))^2} - \frac{\Lambda}{n+1}B^2\right)B^{n-1} = \frac{2\kappa M(r)}{n}$$

The Matching

- **Matching FLRW-like to Kottler-like**

  across time-like boundary $\rho = \rho_{\mathbf{0}}$ ruled by geodesics

$$\begin{aligned} r &= R(\tau) f(\rho_0) \\ f'(\rho_0) &> 0 \end{aligned}$$

  together with

$$m = \frac{\kappa \mu_0 f(\rho_0)^{n+1}}{n(n+1)}$$

  This includes planar and hyperbolic cases

- **Matching LTB-like to Kottler-like**

  similar conditions

Global structure

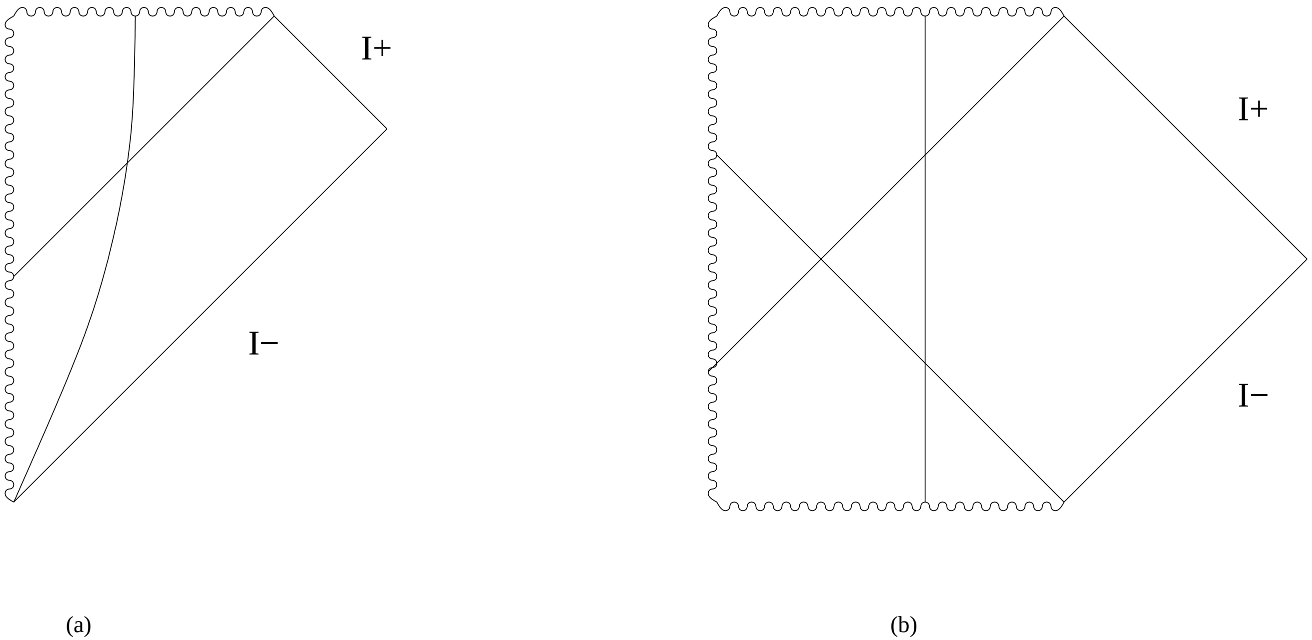

Figure 2: Penrose diagram for $\Lambda = 0$ and (a) $k \leq 0$; (b) $k > 0$, showing the matching surfaces and the horizons.

Global Structure

Proposition 2: If $\Lambda = 0$ and $(N, d\sigma^2)$ is not an n-sphere then the naked singularity at $\rho = 0$ can be hidden if $k > 0$ and $n \geq 4$.

Proposition 3: For $\Lambda < 0$ the FLRW-Kottler spacetime satisfies:

- If $\lambda > 0$ and $(N, d\sigma^2)$ is not an n-sphere then the naked singularity at $\rho = 0$ can always be hidden

- If $\lambda = 0$ then the cusp singularity is not locally naked

- If $\lambda < 0$ then no causal curve can cross the wormhole from $\mathscr{I}$

Spacetime Topology

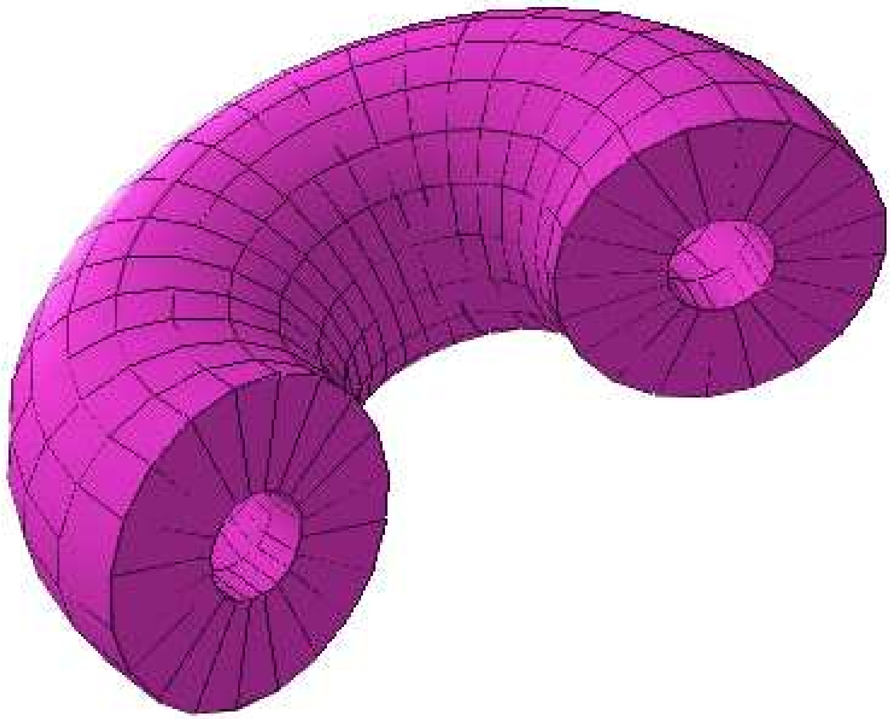

Figure 3: Horizon is a surface interior to a solid torus

Radiating Collapse: the exterior

Bizoń-Chmaj-Schmidt metric

$$ds^{2+} = -Ae^{-2\delta}dt^2 + \frac{1}{A}dr^2 + \frac{r^2}{4}e^{2B}(\sigma_1^2 + \sigma_2^2) + \frac{r^2}{4}e^{-4B}\sigma_3^2$$

where A, δ and B are functions of t and r. The one-forms σ_i are

$$\begin{aligned} \sigma_1 &= \cos\psi d\theta + \sin\theta \sin\psi d\phi \\ \sigma_2 &= \sin\psi d\theta - \sin\theta \cos\psi d\phi \\ \sigma_3 &= d\psi + \cos\theta d\phi \end{aligned}$$

where θ, ψ, ϕ are Euler angles on S^3.

The Schwarzschild limit is obtained for $B = 0$

For $B \neq 0$ pure gravitational waves with radial symmetry

Radiating Collapse: the exterior

The $(4+1)$-dimensional vacuum EFEs give

$$\begin{aligned}
\frac{\partial A}{\partial r} &= -\frac{2A}{r} + \frac{8e^{-2B} - 2e^{-8B}}{3r} - 2r\left(\frac{e^{2\delta}}{A}\left(\frac{\partial B}{\partial t}\right)^2 + A\left(\frac{\partial B}{\partial r}\right)^2\right) \\
\frac{\partial A}{\partial t} &= -4rA\frac{\partial B}{\partial t}\frac{\partial B}{\partial r} \\
\frac{\partial \delta}{\partial r} &= -2r\left(\frac{e^{2\delta}}{A^2}\left(\frac{\partial B}{\partial t}\right)^2 + \left(\frac{\partial B}{\partial r}\right)^2\right)
\end{aligned}$$

together with the quasi-linear wave equation for B

$$\frac{\partial}{\partial t}\left(\frac{e^{\delta}r^3}{A}\frac{\partial B}{\partial t}\right) - \frac{\partial}{\partial r}\left(\frac{Ar^3}{e^{\delta}}\frac{\partial B}{\partial r}\right) + \frac{4r(e^{-2B} - e^{-8B})}{3e^{\delta}} = 0$$

Radiating Collapse: the interior

Eguchi-Hanson metric: self-dual to the Euclidean EFEs

$$h_{EH} = \left(1 - \frac{a^4}{\rho^4}\right)^{-1} d\rho^2 + \frac{\rho^2}{4}(\sigma_1^2 + \sigma_2^2) + \frac{\rho^2}{4}\left(1 - \frac{a^4}{\rho^4}\right)\sigma_3^2$$

with a constant.

The FLRW-like metric built on this is

$$ds^{2-} = -dt^2 + R^2(t) h_{EH}$$

with the Einstein equations for a dust source reducing to

$$\mu R^4 = \mu_0, \qquad \dot{R}^2 = \frac{\kappa \mu_0}{6R^2}$$

The Matching

Theorem: Proved local existence of the radiating exterior. The initial boundary data can be chosen close to Schwarzschild.

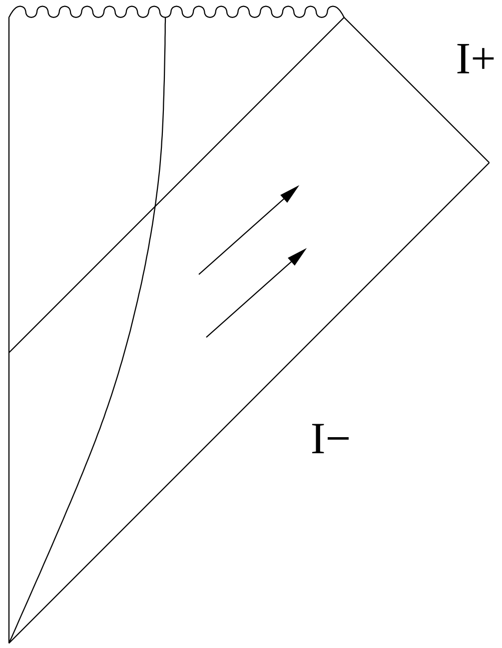

Figure 4: Penrose diagram for $\Lambda = 0$ and (a) $k \leq 0$; (b) $k > 0$, showing the matching surfaces and the horizons.

Conclusions and Questions

- We have obtained large classes of models of gravitational collapse to static BHs in higher dim

- Naked singularities? Wormholes?
- Cosmic Censorship Conjecture in Higher Dim?
- Global existence of radiating collapsing solution?
- Model of collapse to a rotating BH?