New static black holes with nonspherical horizon topology

(work done together with B. Kleihaus and J. Kunz)

Eugen Radu

Institut für Physik, Universität Oldenburg, Germany

- Motivation
- The solutions
- Further remarks

based on:

B. Kleihaus, J. Kunz and E. Radu:

- $d \geq 5$ *"static black holes with $S^2 \times S^{d-4}$ event horizon topology"*, e-Print: arXiv:0904.2723

  (Phys. Lett. B: 301, 2009)

Motivation:

–gravity in higher dimensions – a lot of interest in the last decade

–the $d = 5$ back ring solution (Emparan and Reall 2001)

$d > 5$ *solutions with nonspherical topology of the horizon?*

- the phase structure becomes increasingly intricate and diverse
- no closed form useful solutions (apart from the Myers-Perry black holes)
- approximate solutions: the method of asymptotic expansions (Emparan et. al.); however, limited valability...

numerical (-nonperturbative) approach?

our proposal:

–metric ansatz ($d \geq 5$):

$$ds^2 = -f_0(r,z)dt^2 + f_1(r,z)(dr^2 + dz^2) + f_2(r,z)d\psi^2 + f_3(r,z)d\Omega^2_{d-4}$$

("generalized" Weyl coordinates+rod structure)

–the Einstein equations:

$$\nabla^2 f_0 - \frac{1}{2f_0}(\nabla f_0)^2 + \frac{1}{2f_2}(\nabla f_0)\cdot(\nabla f_2) + \frac{(d-4)}{2f_3}(\nabla f_0)\cdot(\nabla f_3) = 0,$$

$$\nabla^2 f_1 + \ldots = 0,$$

$$\nabla^2 f_2 + \ldots = 0,$$

$$\nabla^2 f_3 + \ldots = 0.$$

no obvious structure except for $d = 5$

Known solutions

the $d=5$ static black ring:

the metric functions are known in closed form (Emparan and Reall 2001):

$$f_0 = \frac{R_2+\xi_2}{R_1+\xi_1},$$
$$f_1 = \frac{(R_1+\xi_1+R_2-\xi_2)((1-c)R_1+(1+c)R_2+2cR_3)}{8(1+c)R_1R_2R_3},$$
$$f_2 = \frac{(R_2-\xi_2)(R_3+\xi_3)}{R_1-\xi_1}, \quad f_3 = R_3-\xi_3 \ ,$$

where

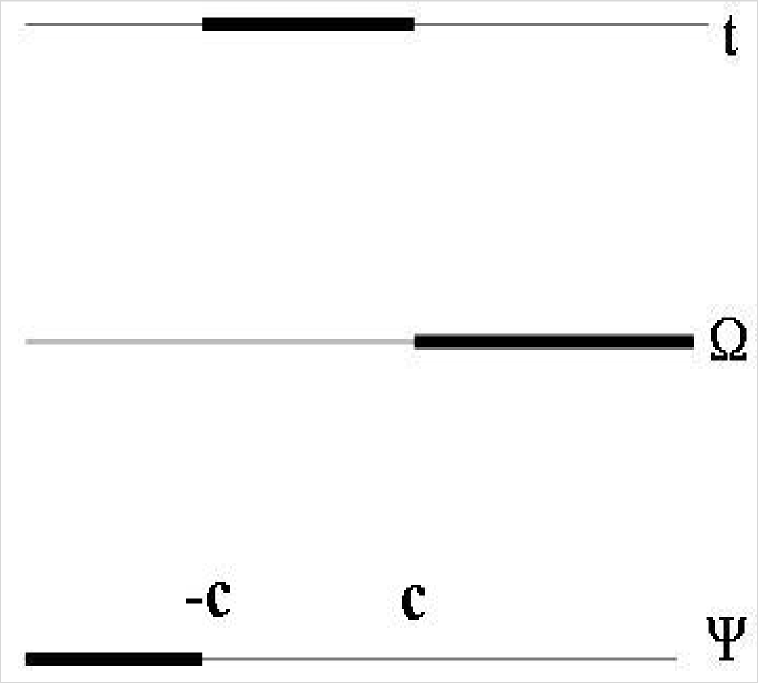
$$\xi_i = z - z_i, \ \ R_i = \sqrt{r^2+\xi_i^2} \ \text{ and } \ z_1 = -a, \quad z_2 = a, \quad z_3 = b,$$

a and b positive constants, $c = a/b < 1$.

$$M^{(5)}, \quad A_H^{(5)}, \quad T^{(5)}: \ \ functions\ of\ (a,b),$$

conical singularity!

$d \geq 5$: the Schwarzschild-Tangerlini solution: S^{d-2} topology of the event horizon



$$f_0 = \left(\frac{v^{(d-3)/2} - 1}{v^{(d-3)/2} + 1}\right)^2, \quad f_1 = c\frac{(v^{(d-3)/2} + 1)^{4/(d-3)}}{4v(z^2\frac{(v^2-1)^2}{(v^2+1)^2} + r^2\frac{(v^2+1)^2}{(v^2-1)^2})},$$

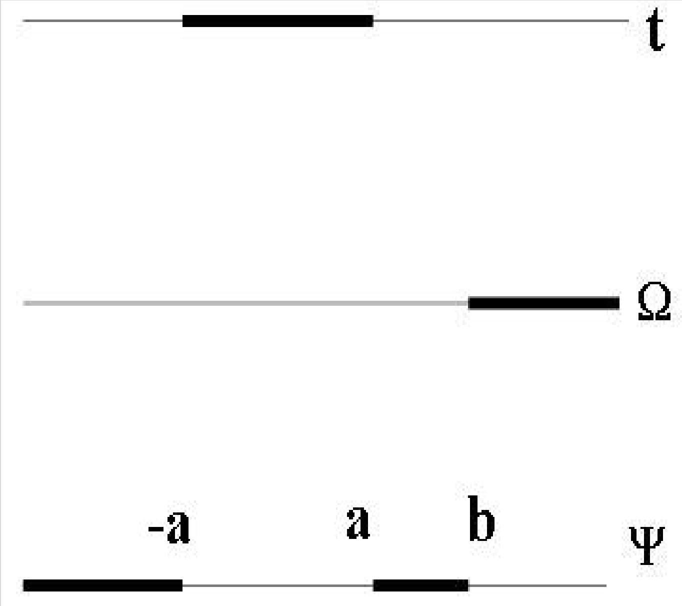
$$f_2 = \ldots, \quad f_3 = \ldots, \quad \text{with} \quad v = v(r,z)$$

complicated expressions (see arXiv:0904.2723)

New solutions:

Black holes with $S^2 \times S^{d-4}$ topology of the event horizon

–rod structure:



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d=5: *static black ring*!

numerical solutions only!

- computation done with the new metric functions F_i
- f_i^0 –the static black ring solution: impose the rod structure

$$f_i = f_i^0 F_i$$

- boundary conditions:

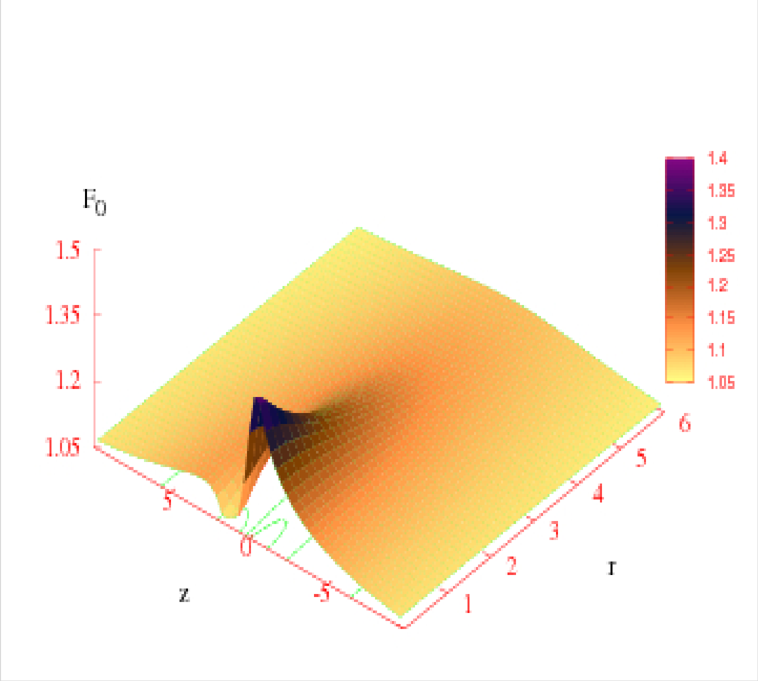
$$\partial_r F_i|_{r=0} = 0, \qquad \text{for} \quad -\infty < z \leq b,$$
$$\partial_r F_0|_{r=0} = 0, \quad \partial_r F_1|_{r=0} = 0, \quad \partial_r F_2|_{r=0} = 0, \quad F_1|_{r=0} = F_3|_{r=0}$$

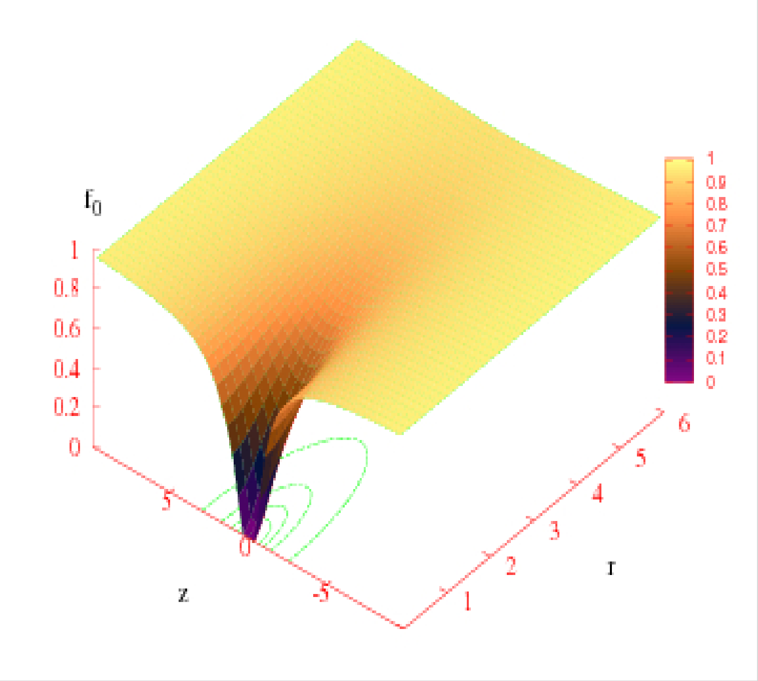
and $F_i = 1$ as $r \to \infty$ or $z \to \pm\infty$.

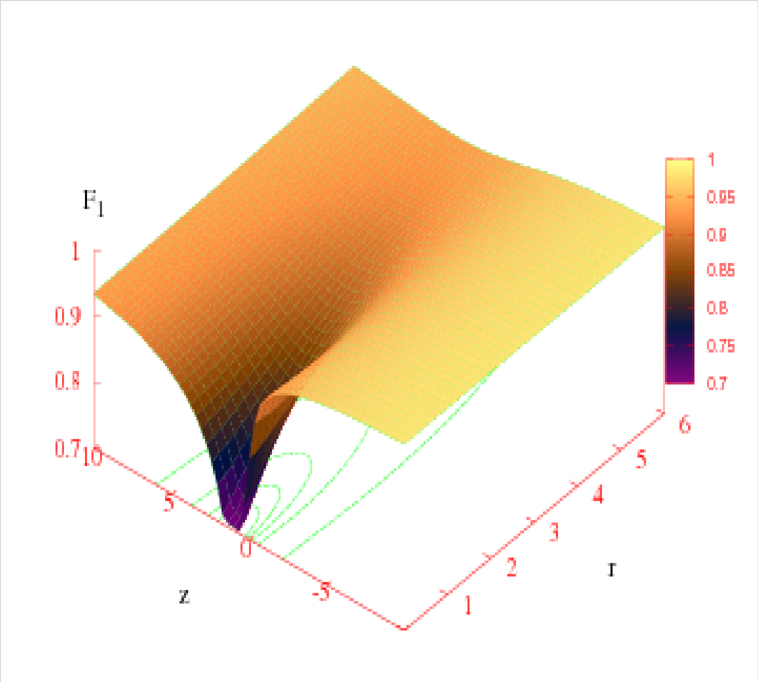
- solve the elliptic partial differential equations for F_i

main idea : increase the value of d (dimension of spacetime)

- $d = 6, 7$ solutions with good accuracy
- $d > 7$?







F1
1
0.9
0.8
0.7
10
5
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-5
z
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2
3
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6
r
1
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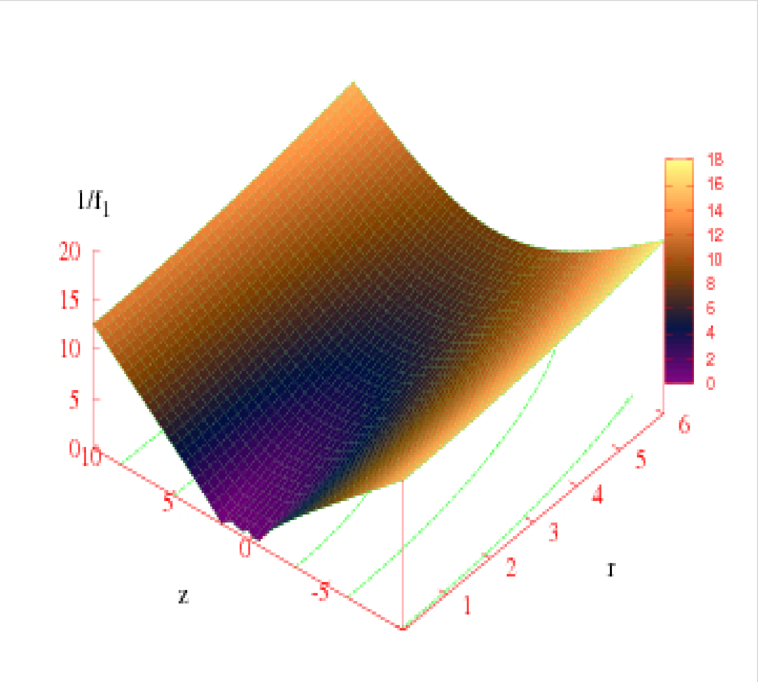
0.9
0.85
0.8
0.75
0.7



1/f1
20
15
10
5
0
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z
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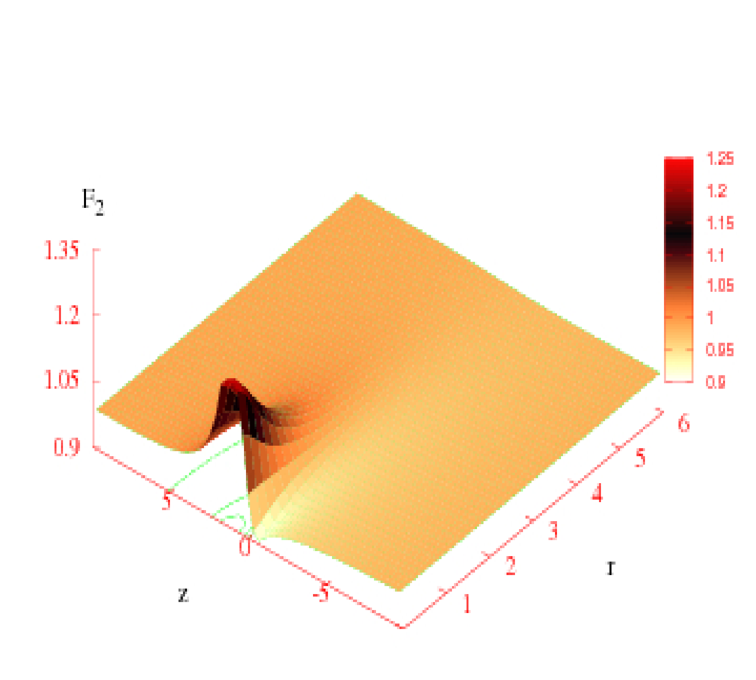
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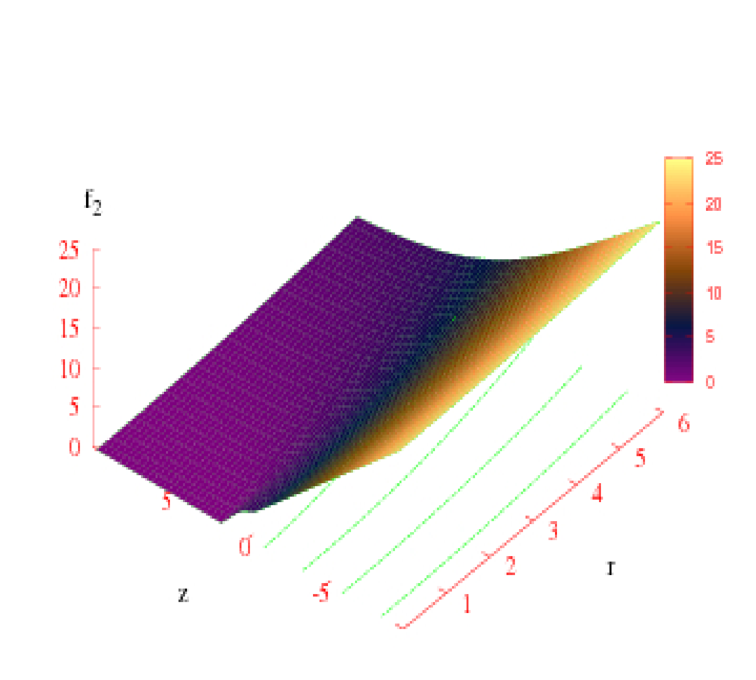
1.2
1.05
0.9
1.25
1.2
1.15
1.1
1.05
1
0.95
0.9
5
0
-5
z
1
2
3
4
5
6
r


f2
25
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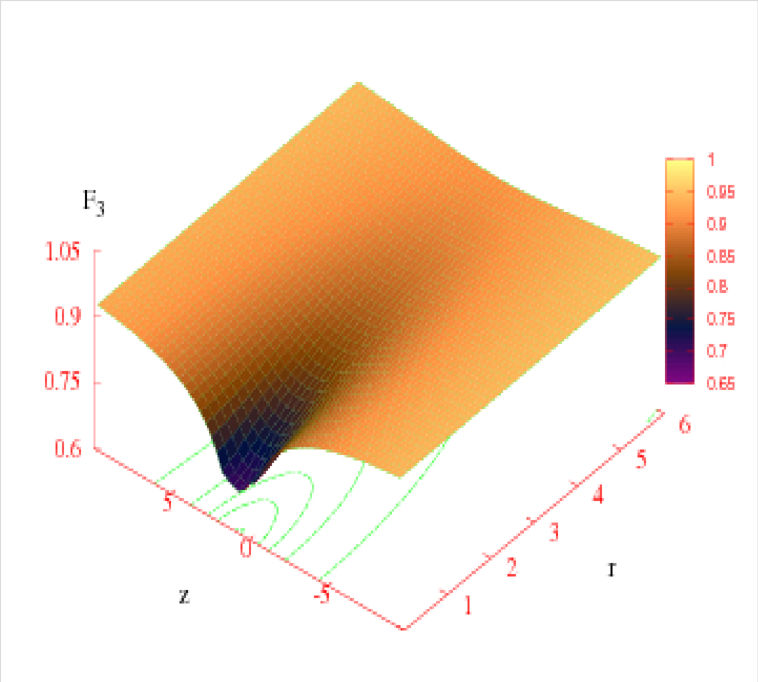
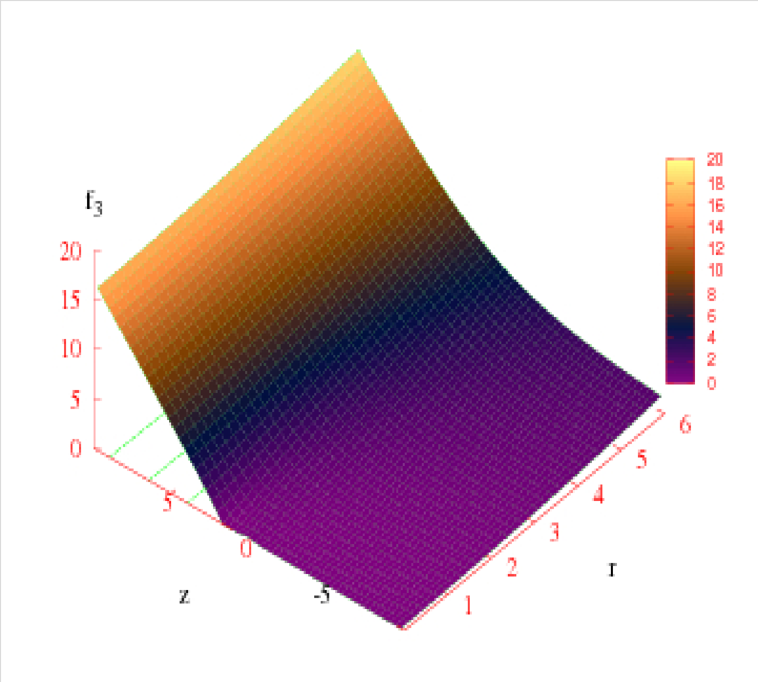


F3
1.05
0.9
0.75
0.6
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
z
5
0
-5
r
1
2
3
4
5
6



f3
20
15
10
5
0
20
18
16
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12
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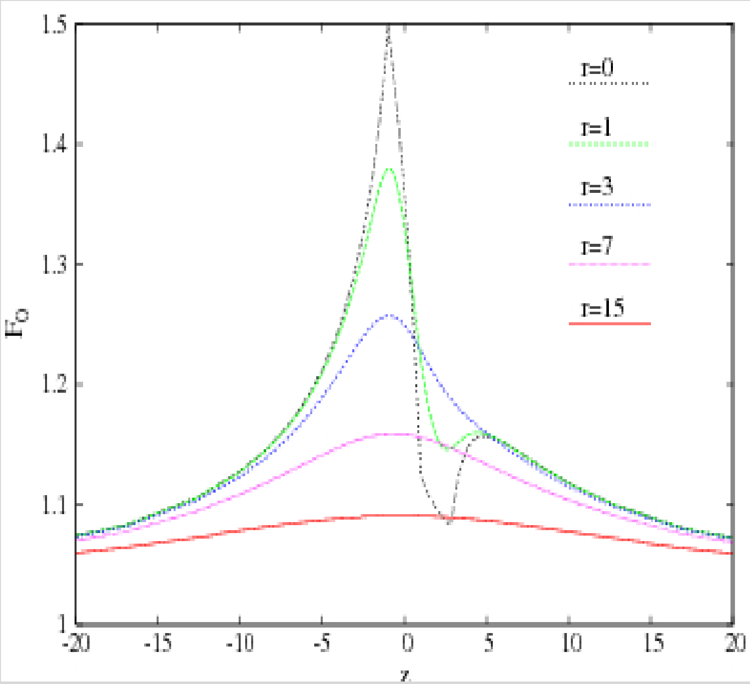
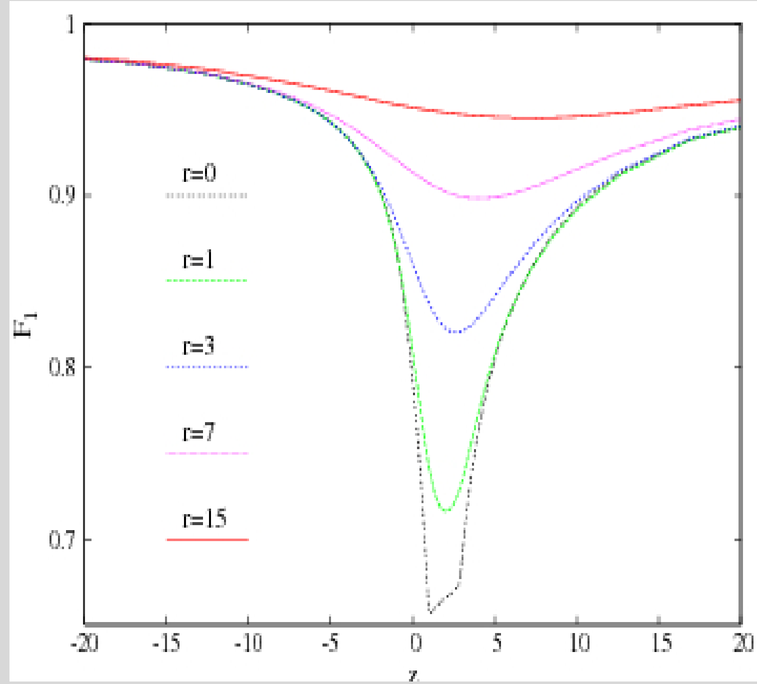
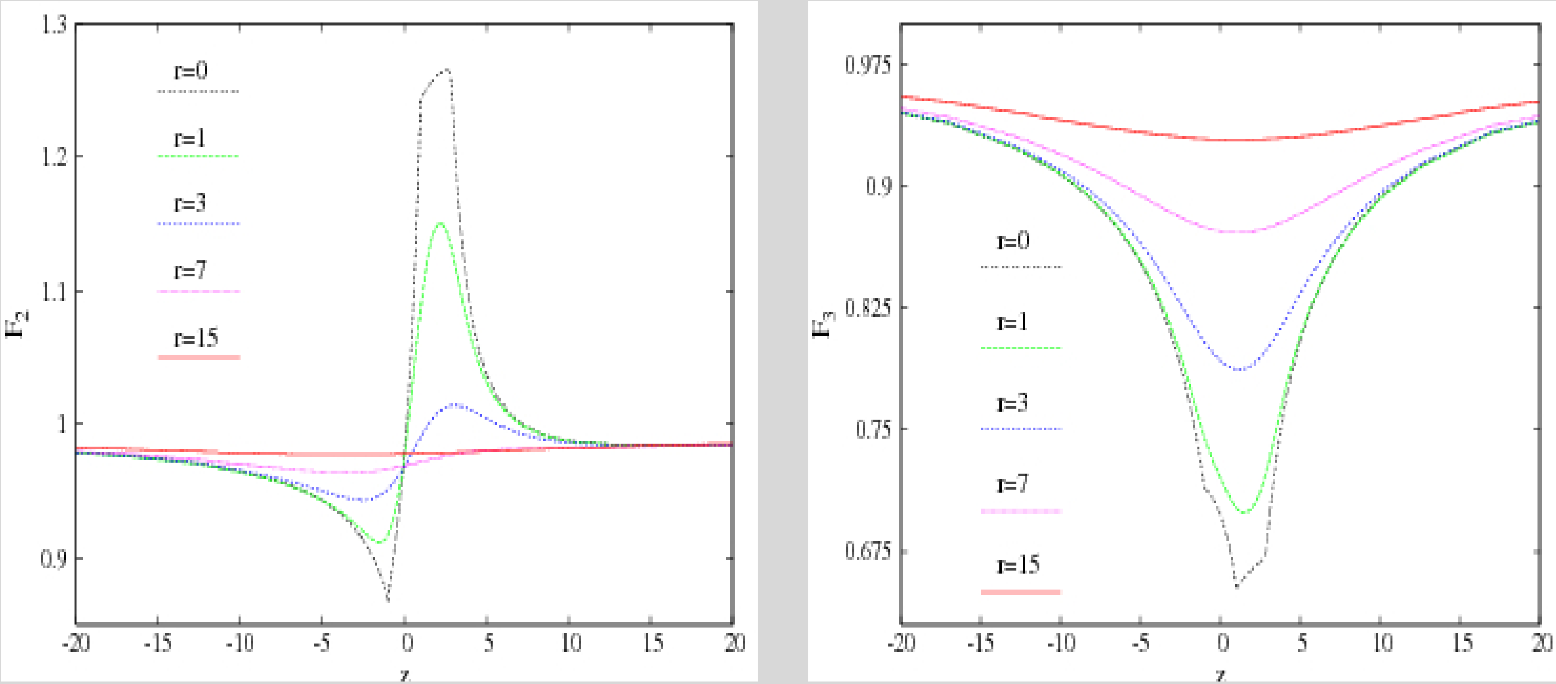
-5
r
1
2
3
4
5
6



1.5
1.4
1.3
1.2
1.1
1
F0
-20
-15
-10
-5
0
5
10
15
20
z
r=0
r=1
r=3
r=7
r=15



1
0.9
0.8
0.7
F1
-20
-15
-10
-5
0
5
10
15
20
z
r=0
r=1
r=3
r=7
r=15



1.3
1.2
1.1
1
0.9
F2
r=0
r=1
r=3
r=7
r=15
-20
-15
-10
-5
0
5
10
15
20
z
0.975
0.9
0.825
0.75
0.675
F3
r=0
r=1
r=3
r=7
r=15
-20
-15
-10
-5
0
5
10
15
20
z

properties of the solutions:

The horizon metric:

$$d\sigma^2 = f_1(0,z)dz^2 + f_2(0,z)d\psi^2 + f_3(0,z)d\Omega^2_{d-4},$$

The event horizon area:

$$\begin{aligned} A_H &= \Delta\psi V_{d-4} \int_{-a}^{a} dz \sqrt{f_1 f_2 f_3^{d-4}} \\ &= 2\Delta\psi V_{d-4} \frac{2^{(d-4)/2} a}{\sqrt{a+b}} \int_{-a}^{a} dz \ (b-z)^{(d-5)/2} \sqrt{F_1 F_2 F_3^{d-4}} \end{aligned}$$

The Hawking temperature:

$$T = \frac{1}{4\pi a}\sqrt{\frac{a+b}{2}}\sqrt{\frac{F_0}{F_1}} \ ,$$

Mass: (from the asymptotics of f_0)

$$f_0 \sim 1 - \frac{16\pi G M}{(d-2)V_{d-2}(r^2+z^2)^{(d-3)/4}} + \ldots \ .$$

the Smarr law: **(test of numerics!)**

$$M = \frac{d-2}{4G(d-3)} T A_H \ .$$

However, all solutions have a conical singularity!

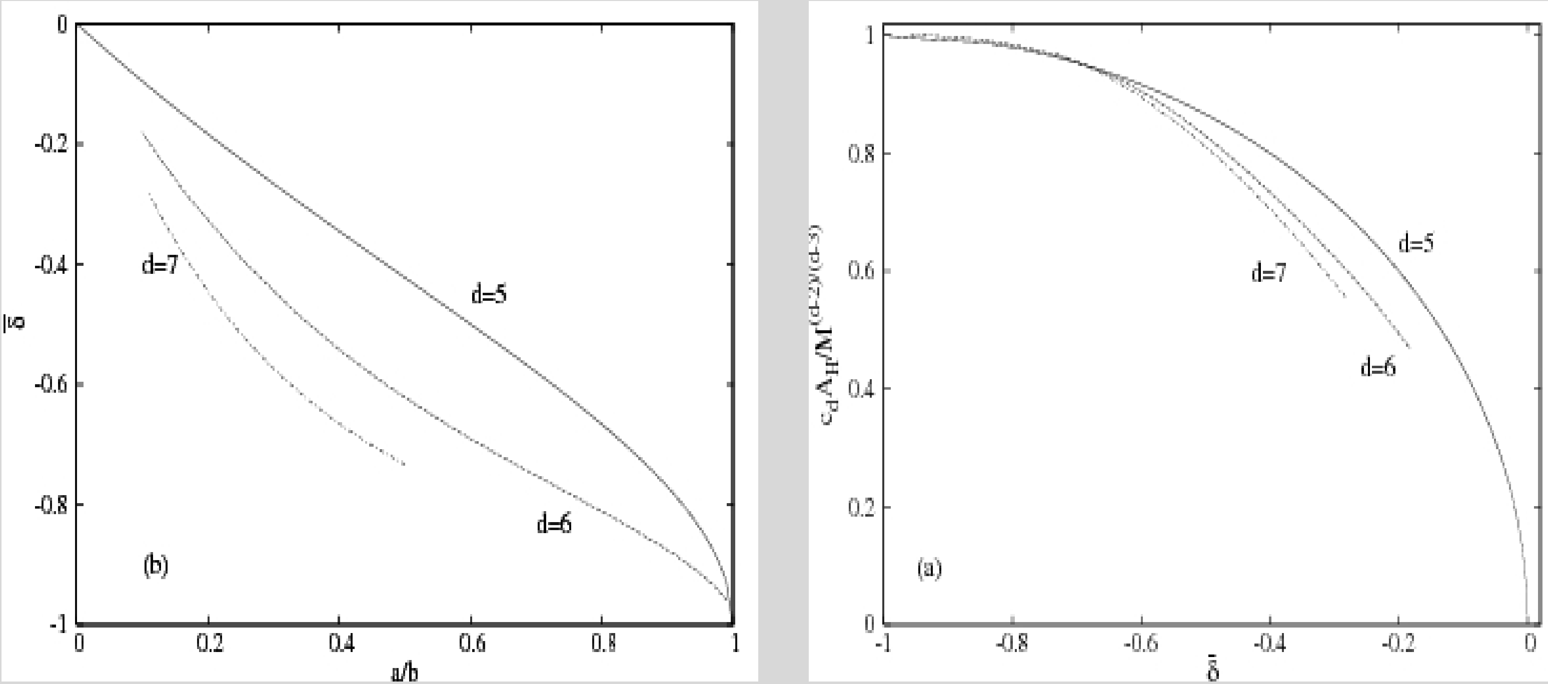
(the same pathology of the $d = 5$ static black ring)

–the value of the conical excess of ψ for $a < z < b$ is

$$\delta = 2\pi\left(1 - \sqrt{\frac{b+a}{b-a}}\sqrt{\frac{F_2}{F_1}}\right).$$

–the 'relative angular excess' ($-1 \leq \bar{\delta} \leq 0$):

$$\bar{\delta} = \frac{\delta/(2\pi)}{1-\delta/(2\pi)},$$



(left): The relative angular excess $\bar{\delta}$ is shown as a function of the ratio between the two length scales a/b.

(right): The scale free ratio $A_H/M^{(d-2)/(d-3)}$ is shown as a function of the relative angular excess $\bar{\delta}$. The value of the parameter c_d there is $c_d = (d-2)/(16\pi)^{(d-2)/(d-3)} V_{d-2}^{1/(d-3)}$ and has been choosen such that the point $(1,-1)$ on the plot corresponds to the Schwarzschild-Tangerlini black hole.

Further remarks/Conclusions

- Q: Charged static solutions?
- A: Yes! *(however, the same pathology...)*

– electrically carged solutions generated from vacuum via an Harrison transformation

$$I = \frac{1}{16\pi G}\int d^d x\sqrt{-g}\left(R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{4}e^{-2\bar{a}\phi}F^2\right)$$

– a second Harrison transformation $\Longrightarrow$ regular solutions in a magnetic Melvin Universe (details in e-Print: arXiv:0904.2723)

situation similar to the $d = 5$ static black rings...

- Q: rotating solutions? $d > 5$ black rings? (within this approach)
- A: very likely. (however, difficult numerical problem (3D eqs., unclear metric ansatz etc) – not impossible...)

- Q: asymptotically (A)dS solutions with this method? (e.g. $d = 5$ AdS black rings)
- A: unlikely

generalizations (currently under study):

- Schwarzschild-Tangerlini + $S^2 \times S^{d-4}$ solution
- new solutions with bubbles