Higher-dimensional numerical relativity: Formulation and code tests

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arXiv:0907.2760 [gr-qc]

July 17, 2009: MG12 @ Paris

Contents

- Introduction
- BSSN formulation
- Cartoon method
- Codes
- Summary

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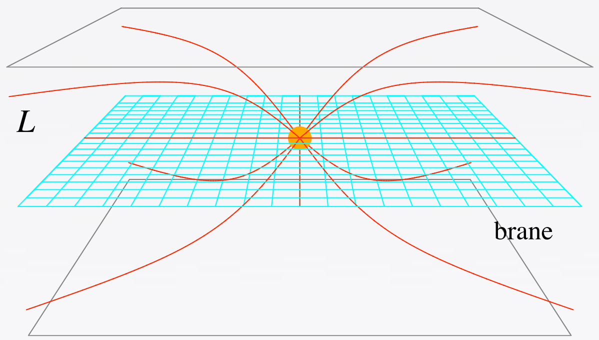
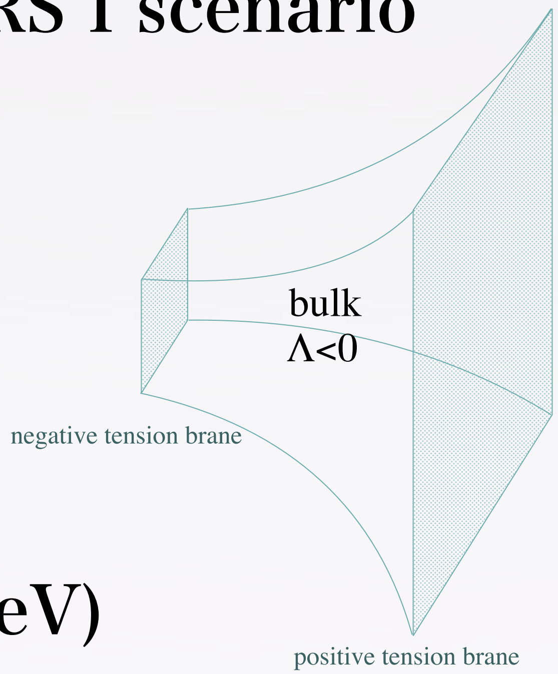
Why higher dimensions?

- BH production at the LHC
  - ADD scenario
  - RS I scenario





Planck Energy = O(TeV)

- Theoretical motivation

- In 4D, long-term evolutions of BHs have become possible

  Pretorius (2005) and other various groups.

- In 5D, the temporal evolution of Gregory-Laflamme instability was studied.

  Choptuik, Lehner, Olabarrieta, Petryk, Pretorius and Villegas, PRD68, 044001 (2003).

We develop codes of numerical relativity in higher dimensions.

Target physics

- Black hole formation in high-energy particle collisions

  with Shibata

- Stability of Kerr black holes

  with Shibata

- Evolution of BHs in Randall-Sundrum II scenarios

  with Tanaka, Tanahashi and Shibata

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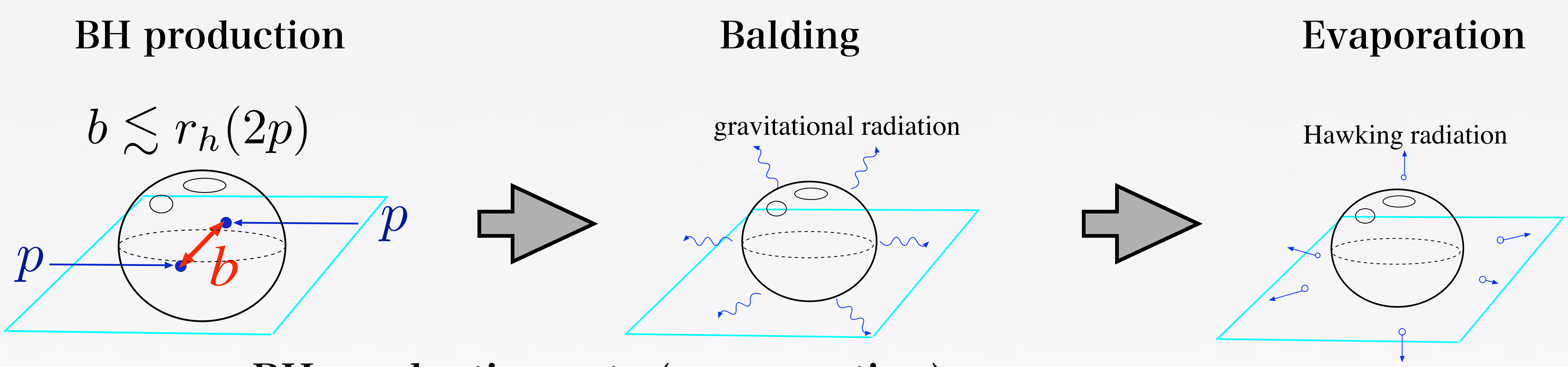
BH formation in high-energy particle collisions

- Expected BH phenomena at the LHC.

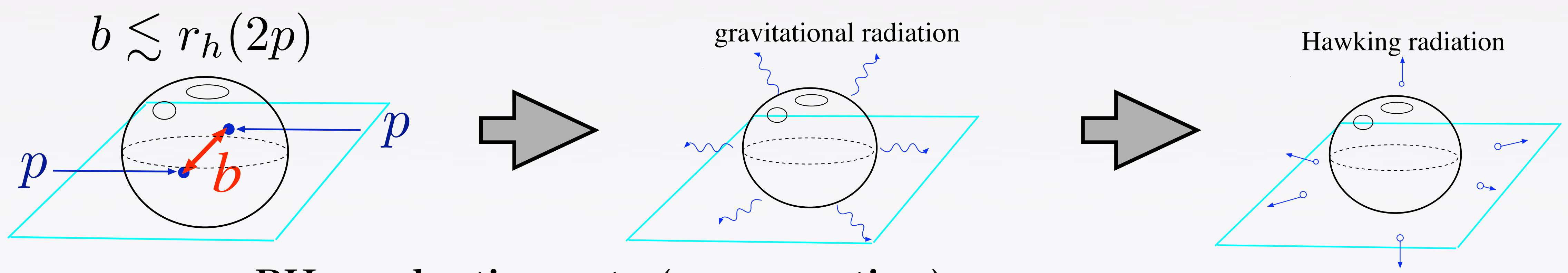


- BH production rate (cross section)
- M and J after the balding (radiation efficiency)

BH formation in high-energy particle collisions

- Expected BH phenomena at the LHC.

BH production   Balding   Evaporation



  - BH production rate (cross section)
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- AH studies in high-energy particle system (instant of collision).

Yoshino and Nambu, PRD 67, 024009 (2003).
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D	4	5	6	7	8	9	10	11
$\sigma_{\mathrm{AH}}/\pi[r_h(2p)]^2$	0.76	1.54	2.15	2.52	2.77	2.95	3.09	3.20

BH formation in high-energy particle collisions

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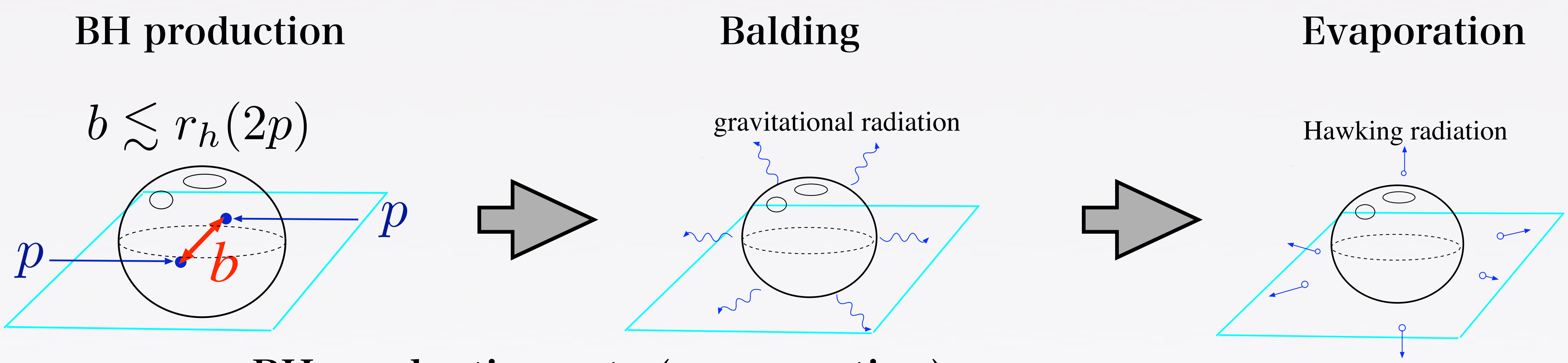

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- Numerical simulations of BH high-velocity collision in 4D.

Sperhake, Cardoso, Pretorius, Berti and Gonzalez, PRL101, 161101 (2008).
Shibata, Okawa and Yamamoto, PRD78, 101501(R) (2008).
Sperhake, Cardoso, Pretorius, Hinderer and Yunes, arXiv:0907.1252.

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Stability of Kerr BHs

- Separation of variables of fields in a high-D Kerr spacetime has been done for scalar fields and Dirac fields.

  e.g., Alberta separatists

- However, separation of variables in metric perturbations has been only partially done.

  Ohta and Yasui, arXiv:0812.1623 [hep-th].

  Murata and Soda, PTP 120, 561 (2008) [arXiv:0803.1371 [hep-th]].

- The multi-dimensional analysis shows that a Kerr BH is unstable for large Kerr parameter at least for $D \geq 7$.

  Emparan and Myers, JHEP 0309, 025 (2003).

  Dias, Figueras, Monterio, Santos and Emparan, arXiv:0907.2248 [hep-th].

- The numerical relativity may be able to give the answer for onset of instability and subsequent evolution.

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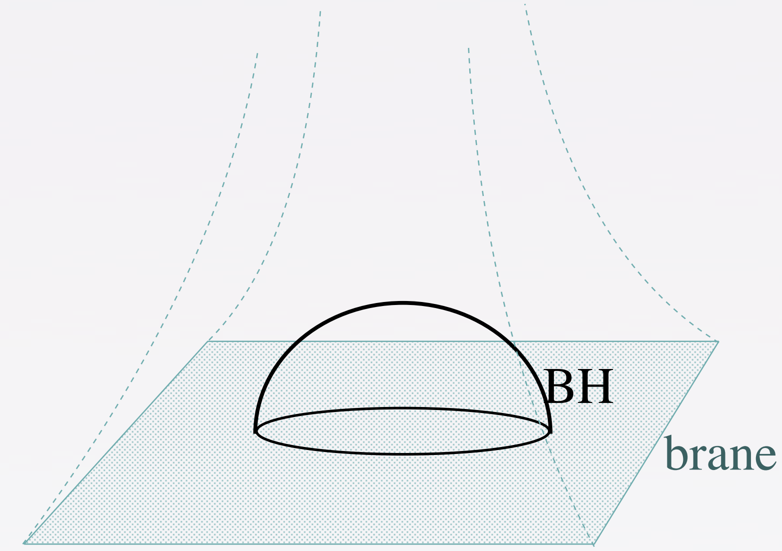
Evolution of BHs in RS II braneworld scenarios



- The exact solution of the braneworld BH exists in 4D case.
- However, no one has discovered an exact solution in 5D case.
- AdS/CFT correspondence may imply the non-existence of such solutions.

Tanaka, Prog. Theor. Phys. Suppl. 148, 307-316 (2002).
Emparan, Fabbri and Kaloper, JHEP08, 043 (2002).

A 4D BH with quantum fields ⟷ A 5D classical BH

↑ ↑

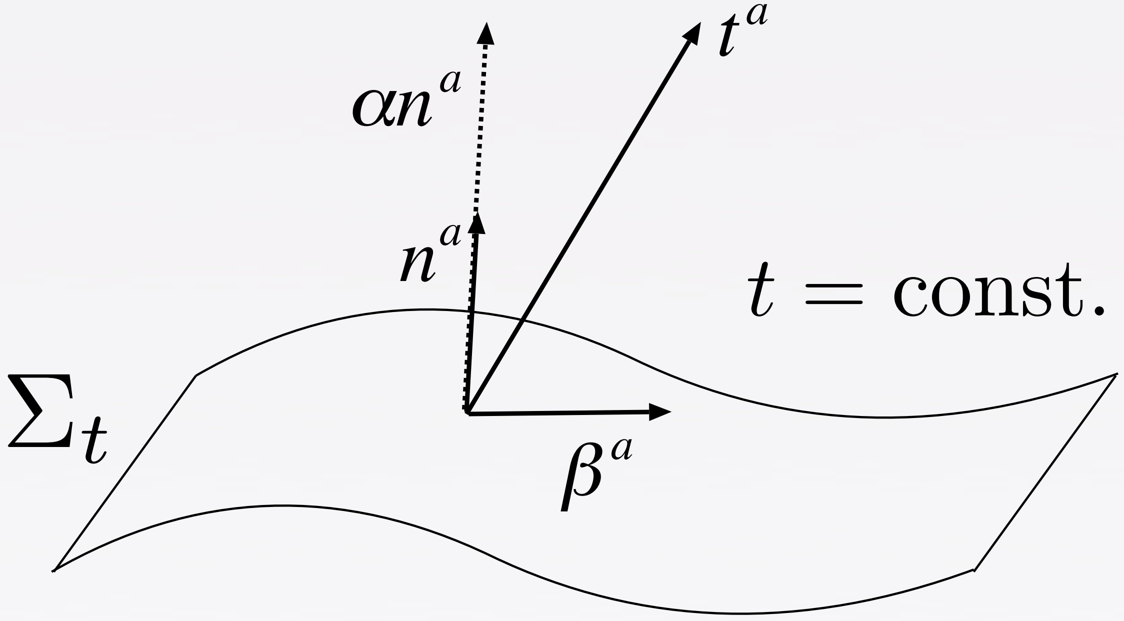
evaporates by Hawking rad. ⟷ escapes into the bulk (?)

- The recent numerical calculation by myself also supports the non-existence of a static BH solution.

Yoshino, JHEP 0901, 268 (2009) [arXiv:0812.0465].

Contents

ADM formulation



$$(\gamma_{ij},\ K_{ij})$$

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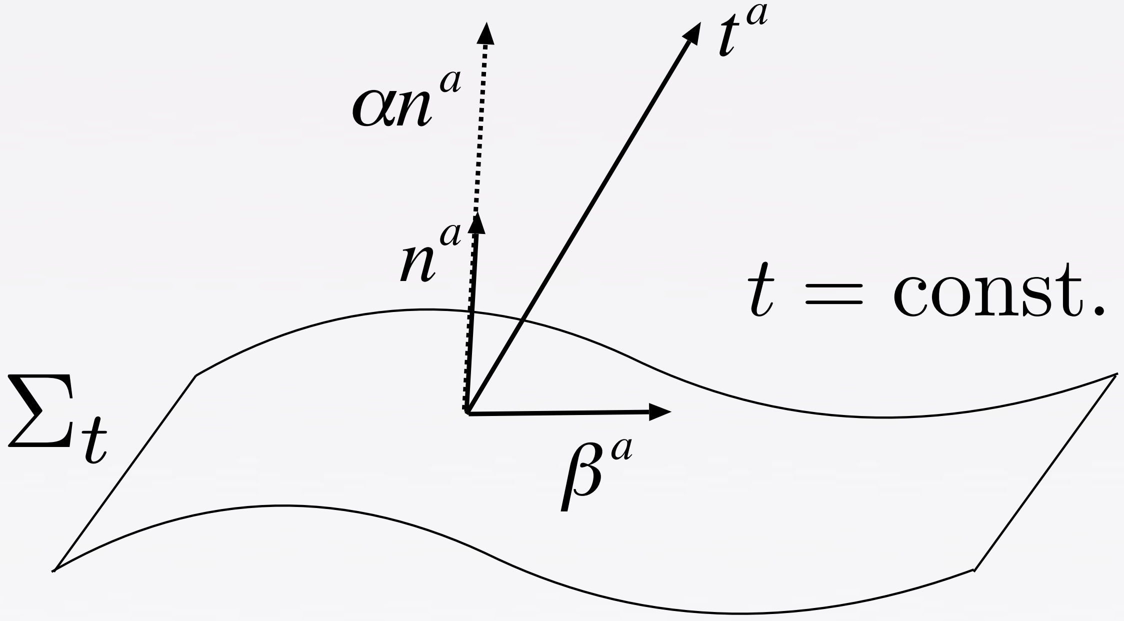

$$(\gamma_{ij},\ K_{ij})$$

- $^{(D-1)}R + K^2 - K_{ij}K^{ij} = 16\pi\rho$
- $D_j K^j_i - D_i K = 8\pi j_i$

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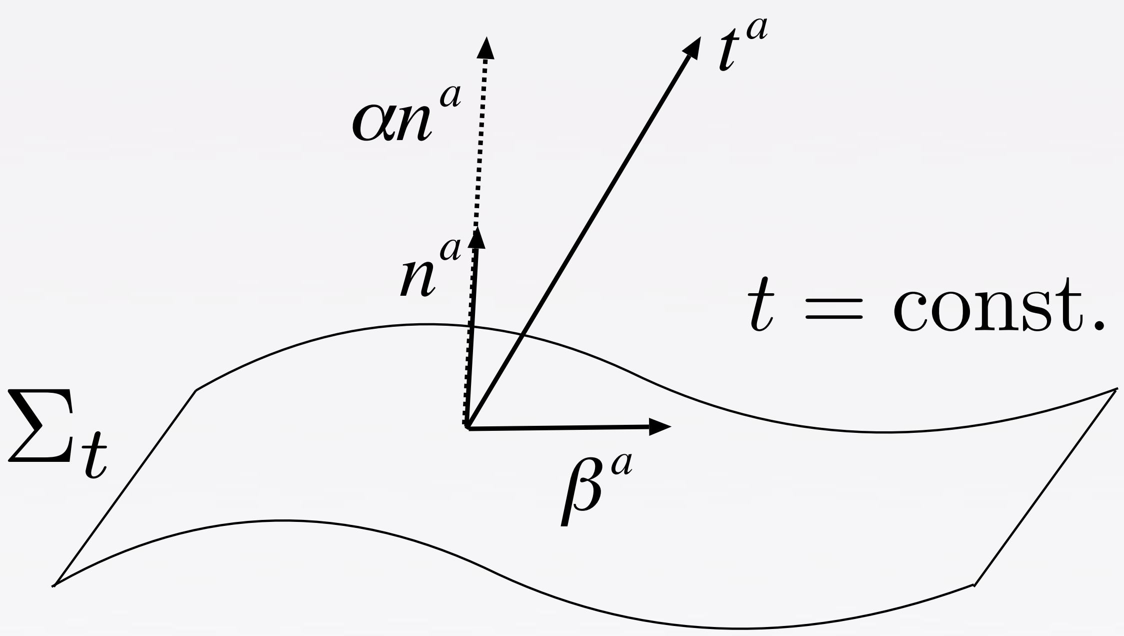

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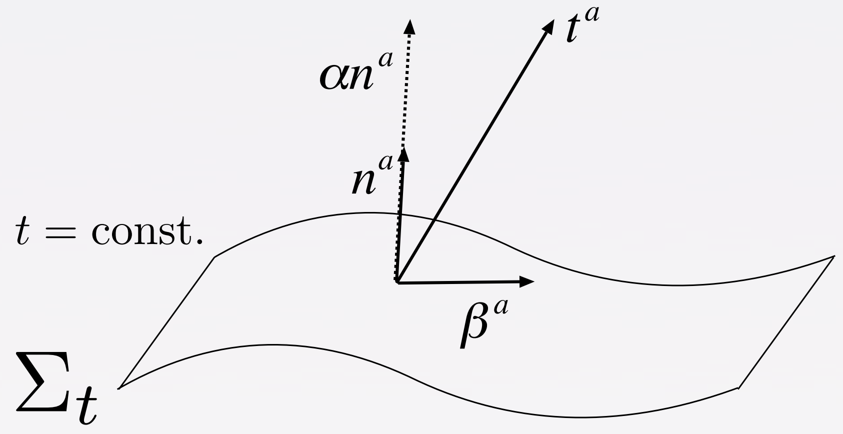
- $$\partial_t K_{ij} = -D_i D_j \alpha + \alpha\left({}^{(D-1)}R_{ij} - 2K_{ik}K^k{}_j + K_{ik}K\right)$$
$$-8\pi G_D \alpha\left[S_{ij} + \frac{1}{D-2}\gamma_{ij}\left(\rho - S^k{}_k\right)\right] + \beta^k D_k K_{ij} + D_i\beta^k K_{kj} + D_j\beta^k K_{ki}$$

BSSN formulation



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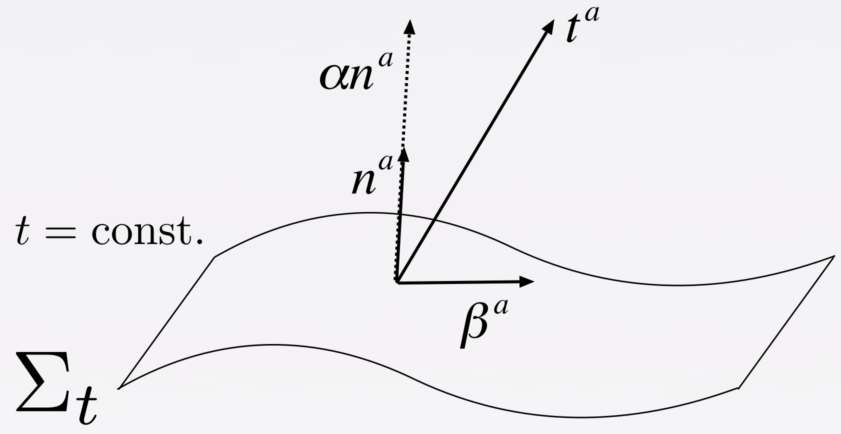

- $\bar{\gamma}_{ij} = \chi \gamma_{ij}$
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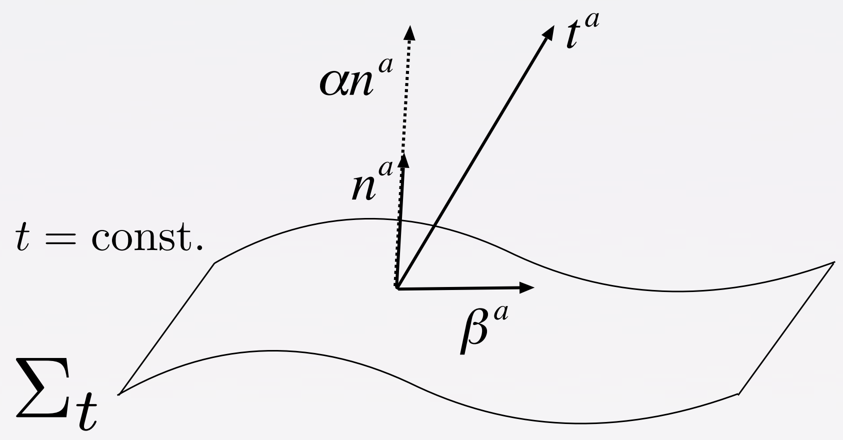

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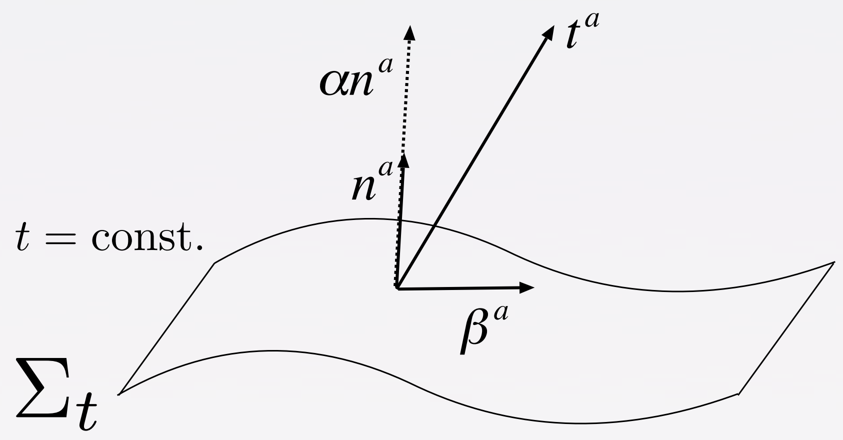

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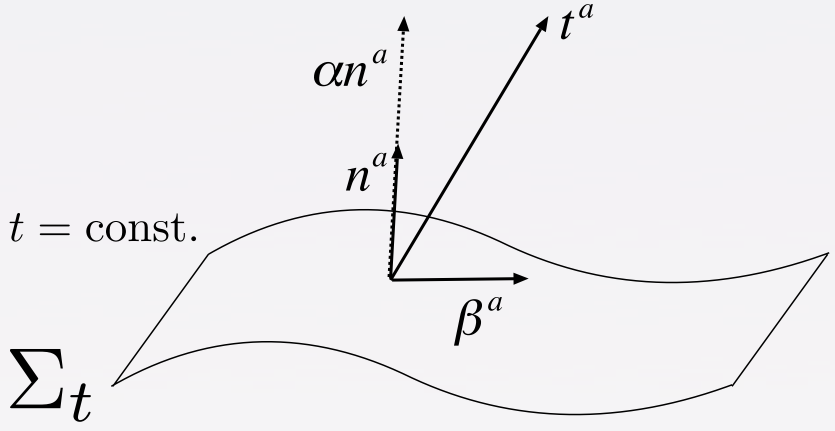

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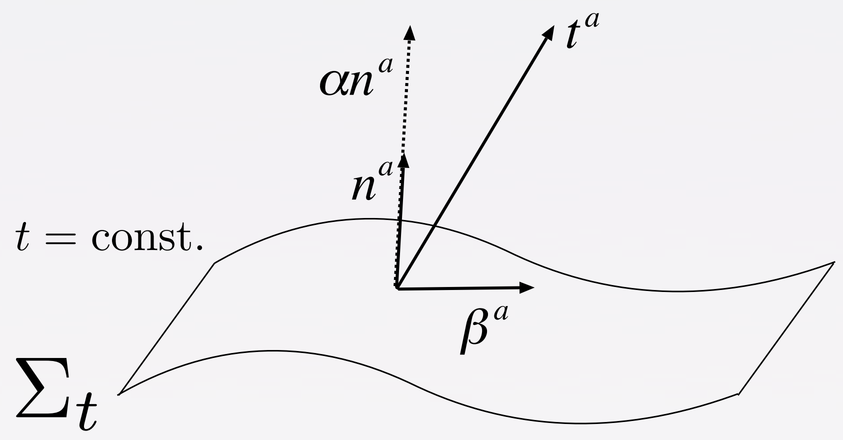

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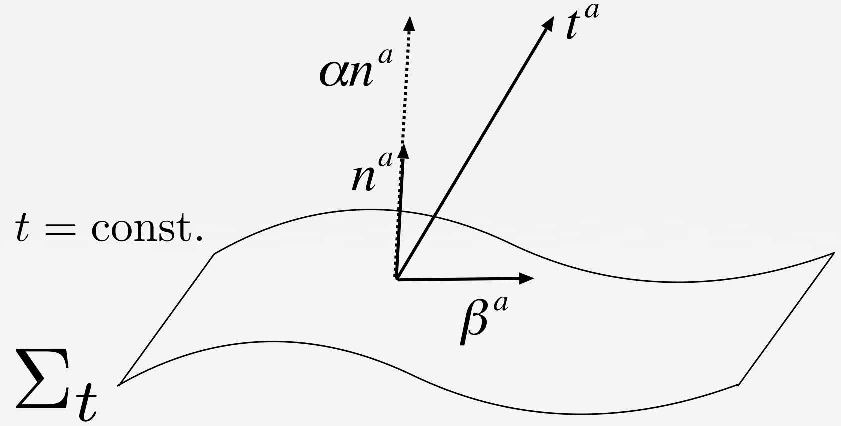

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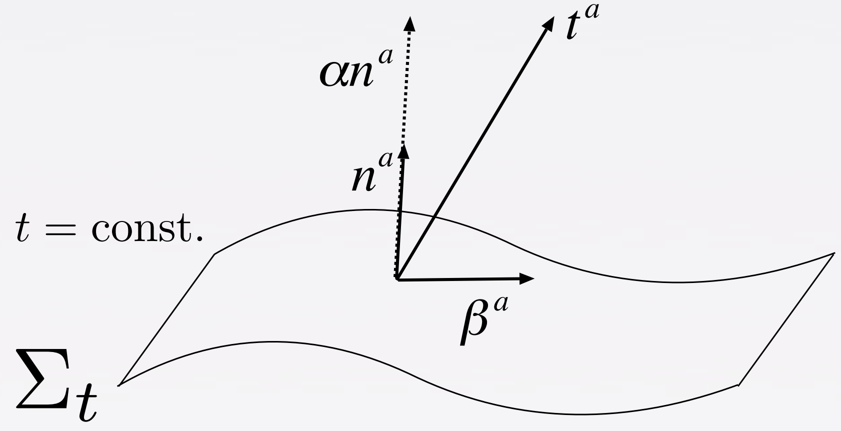

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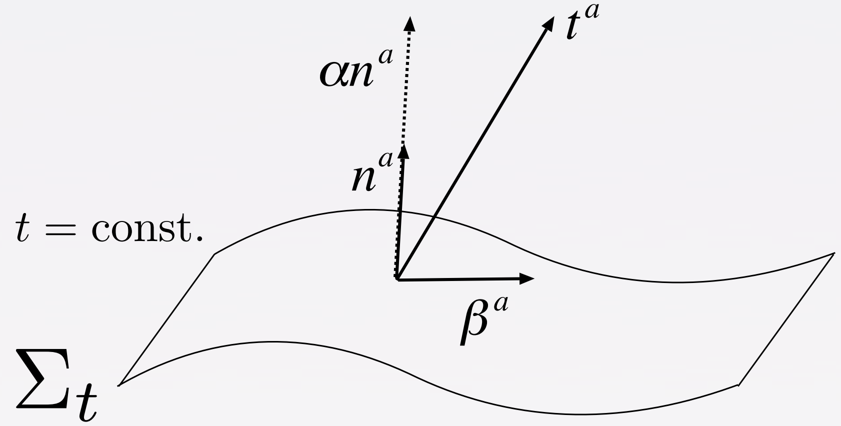

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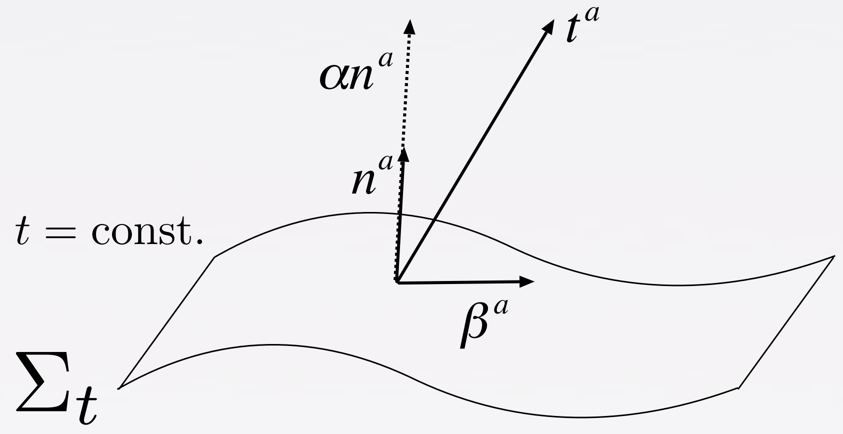

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- $$\begin{aligned}\partial_t \tilde{A}_{ij} &= \chi\left[-(D_iD_j\alpha)^{\mathrm{TF}} + \alpha\left(R_{ij}^{\mathrm{TF}} - 8\pi S_{ij}^{\mathrm{TF}}\right)\right] \\ &\quad +\alpha\left(K\tilde{A}_{ij} - 2\tilde{A}_{ik}\tilde{A}^k_j\right) + \beta^k\partial_k\tilde{A}_{ij} + \tilde{A}_{ik}\partial_j\beta^k + \tilde{A}_{kj}\partial_i\beta^k - \frac{2}{D-1}\partial_k\beta^k\tilde{A}_{ij}\end{aligned}$$

- $$\bar{R}_{ij} = -\frac{1}{2}\bar{\gamma}^{kl}\bar{\gamma}_{ij,kl} + \frac{1}{2}\left(\bar{\gamma}_{ki}\partial_j\bar{\Gamma}^k + \bar{\gamma}_{kj}\partial_i\bar{\Gamma}^k\right) + \cdots$$

- $\bar{\Gamma}^i = \bar{\gamma}^{jk}\bar{\Gamma}^i_{jk}$

- $$\begin{aligned}\partial_t\bar{\Gamma}^i &= -2\tilde{A}^{ij}\partial_j\alpha + 2\alpha\left[\bar{\Gamma}^i_{jk}\tilde{A}^{jk} - \frac{D-2}{D-1}\bar{\gamma}^{ij}K_{,j} - 8\pi\bar{\gamma}^{ij}j_j - \frac{(D-1)}{2}\frac{\chi_{,j}}{\chi}\tilde{A}^{ij}\right] \\ &\quad +\beta^j\partial_j\bar{\Gamma}^i - \bar{\Gamma}^j\partial_j\beta^i + \frac{2}{D-1}\bar{\Gamma}^i\partial_j\beta^j + \frac{D-3}{D-1}\bar{\gamma}^{ik}\beta^j_{,jk} + \bar{\gamma}^{jk}\beta^i_{,jk}\end{aligned}$$

Contents

Cartoon method in 4D

Alcubierre, Brandt, Brugmann, Holz, Seidel, Takahashi, and Thornburg, gr-qc/9908012.

- Axisymmetric system (x=y, z)

Cartoon method in 4D

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- Axisymmetric system (x=y, z)
  - cylindrical coordinates (ρ, ϕ, z)

$$x = \rho \cos \phi$$
$$y = \rho \sin \phi$$
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Cartoon method in 4D

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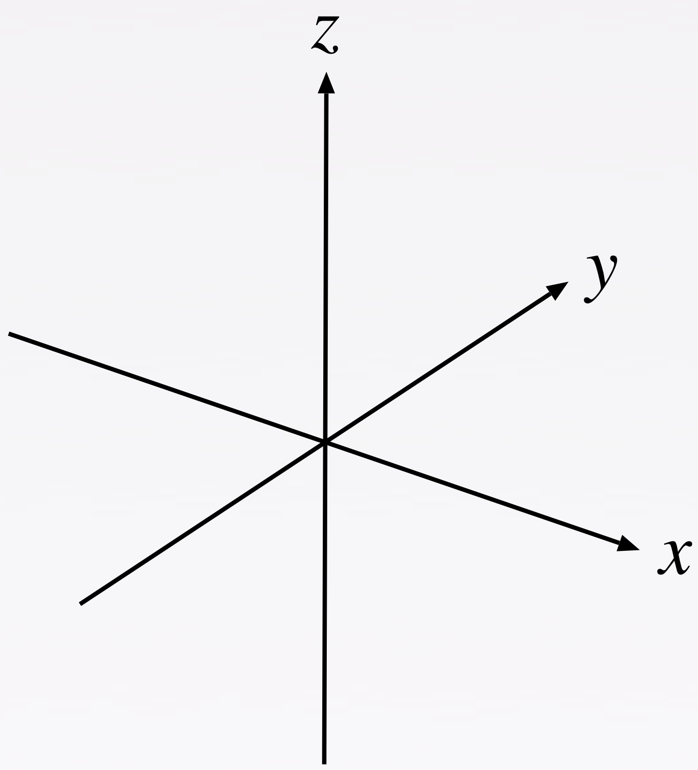

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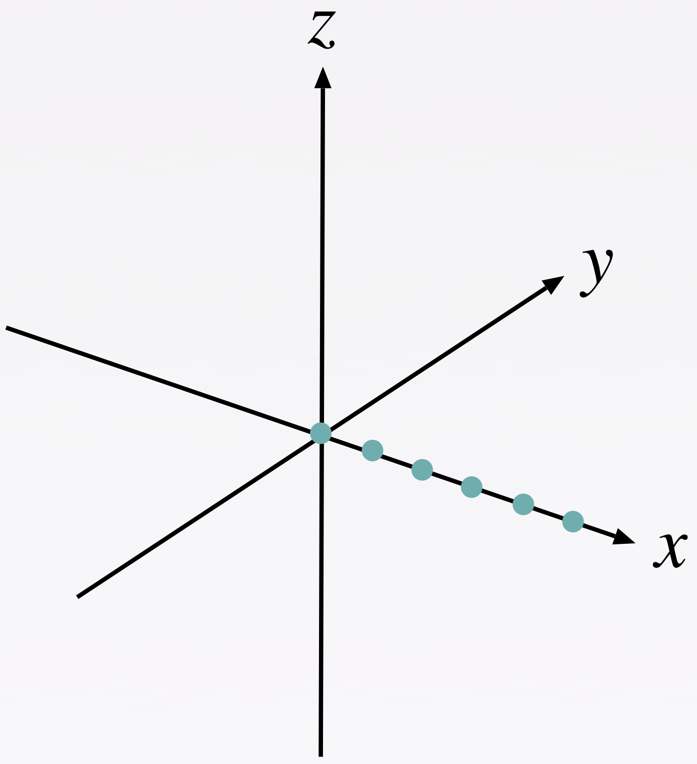

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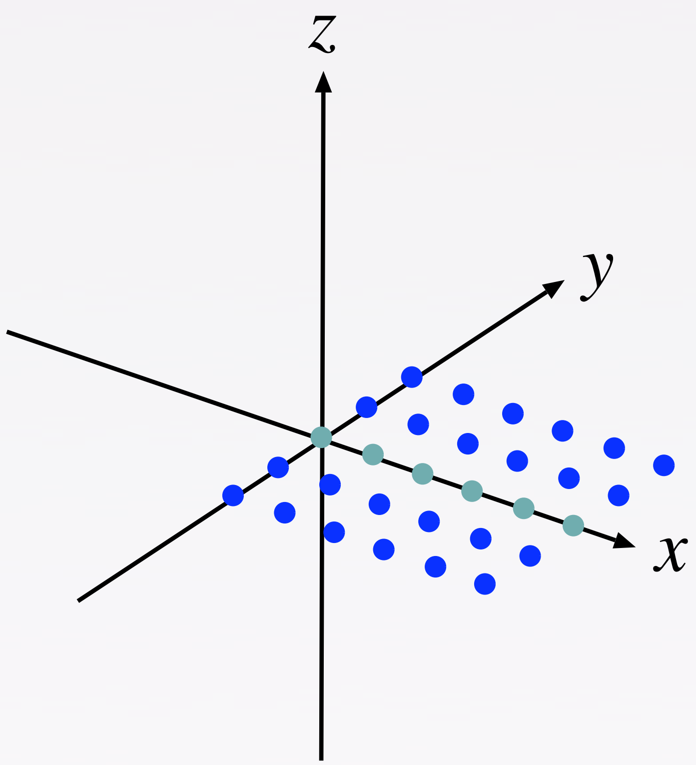

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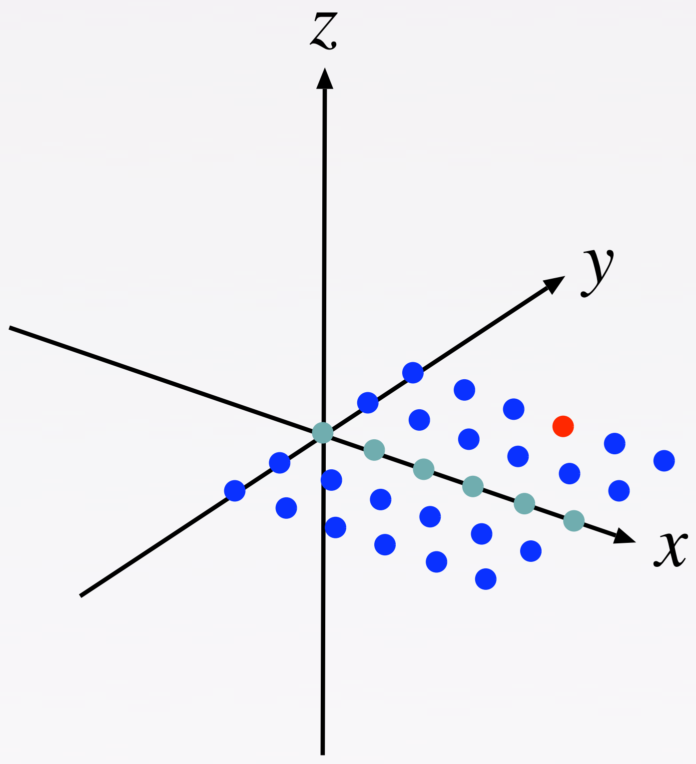

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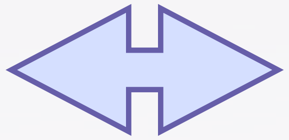

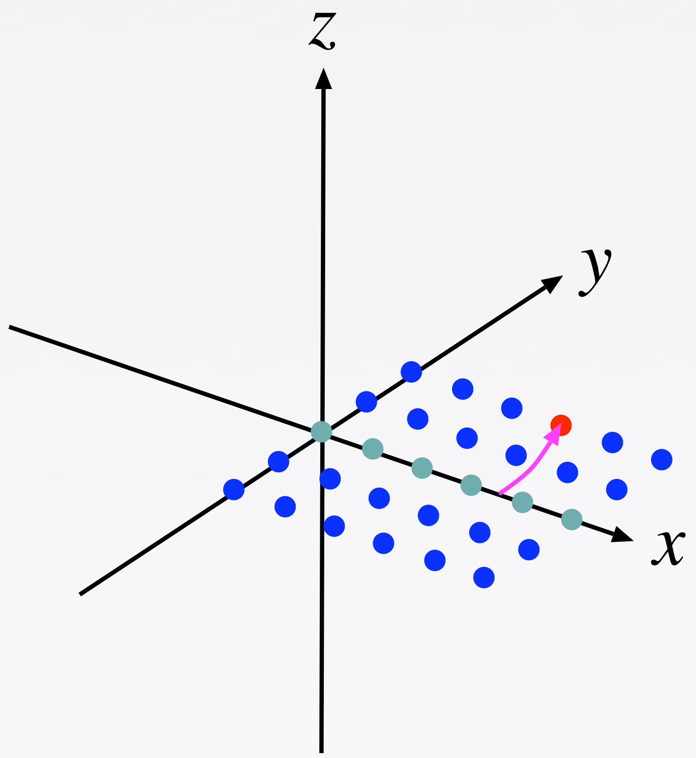

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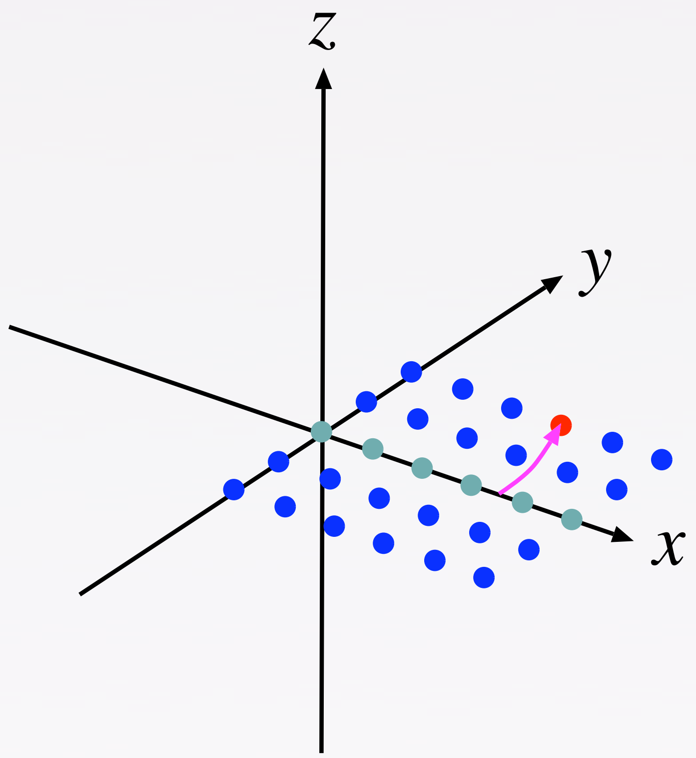

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\begin{aligned}
x &= \rho \cos\phi \\
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Scalar

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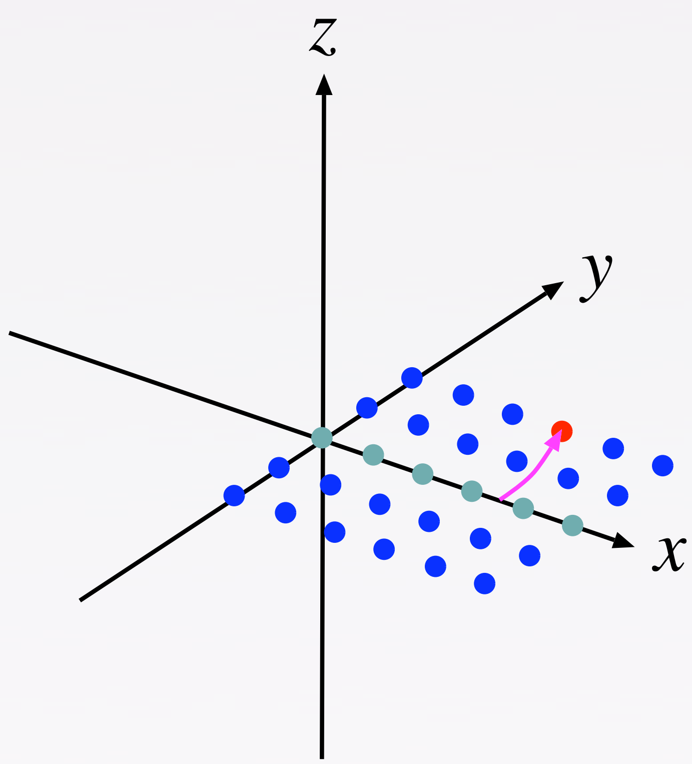

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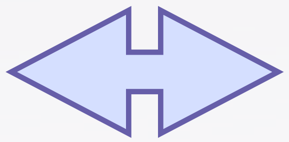

Vector

$$T^z(x,y,z) = T^z(\rho, 0, z)$$

$$T^x(x,y,z) = (x/\rho)T^x(\rho,0,z) - (y/\rho)T^y(\rho,0,z)$$
$$T^y(x,y,z) = (y/\rho)T^x(\rho,0,z) + (x/\rho)T^y(\rho,0,z)$$

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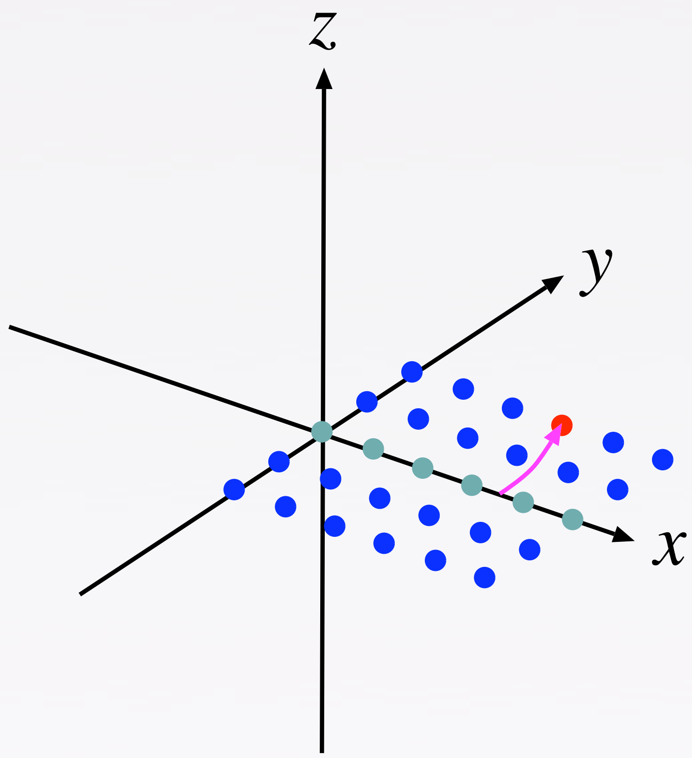

- Axisymmetric system (x=y, z)

cylindrical coordinates (ρ, ϕ, z)

$$x = \rho \cos\phi$$
$$y = \rho \sin\phi$$
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Symmetric tensor

$$S^{zz}(x,y,z) = S^{zz}(\rho,0,z)$$

$$S^{zx}(x,y,z) = (x/\rho)S^{zx}(\rho,0,z) - (y/\rho)S^{zy}(\rho,0,z)$$
$$S^{zy}(x,y,z) = (y/\rho)S^{zx}(\rho,0,z) + (x/\rho)S^{zy}(\rho,0,z)$$

$$S^{xx}(x,y,z) = (x/\rho)^2 S^{xx}(\rho,0,z) + (y/\rho)^2 S^{yy}(\rho,0,z) - (2xy/\rho^2)S^{xy}(\rho,0,z)$$
$$S^{yy}(x,y,z) = (y/\rho)^2 S^{xx}(\rho,0,z) + (x/\rho)^2 S^{yy}(\rho,0,z) + (2xy/\rho^2)S^{xy}(\rho,0,z)$$
$$S^{xy}(x,y,z) = (xy/\rho)[S^{xx}(\rho,0,z) - S^{yy}(\rho,0,z)] + [(x^2 - y^2)/\rho^2]S^{xy}(\rho,0,z)$$

Cartoon method in 5D

- Space is 4D (x, y, z, w)
  - U(1)×U(1) symmetry (x=y, z=w)
  - SO(3) symmetry (x=y=z, w)

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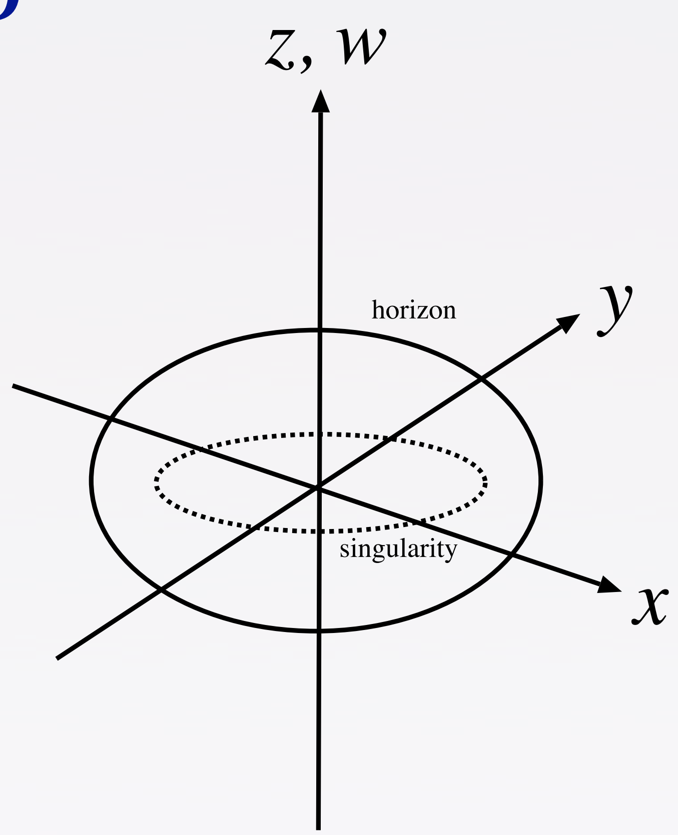

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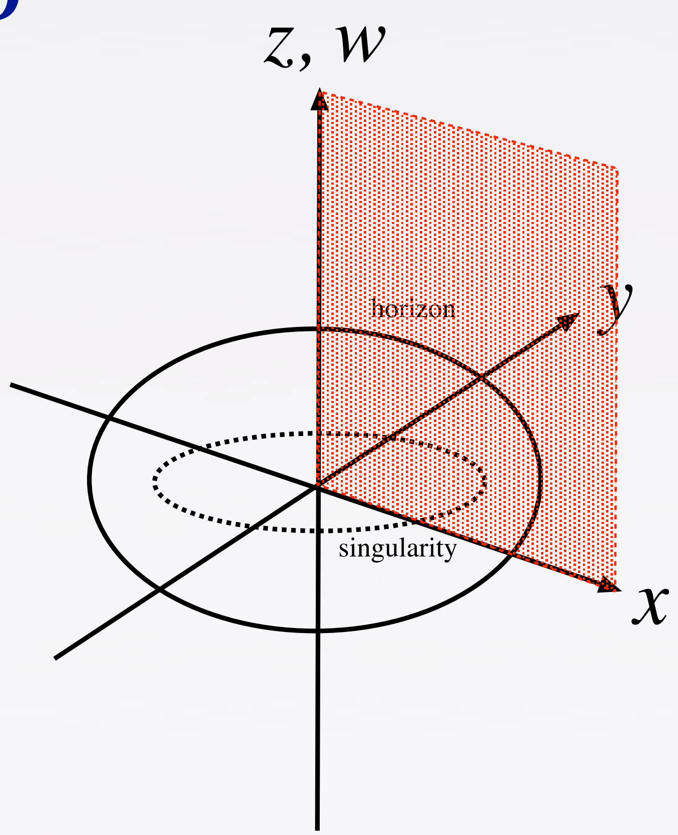
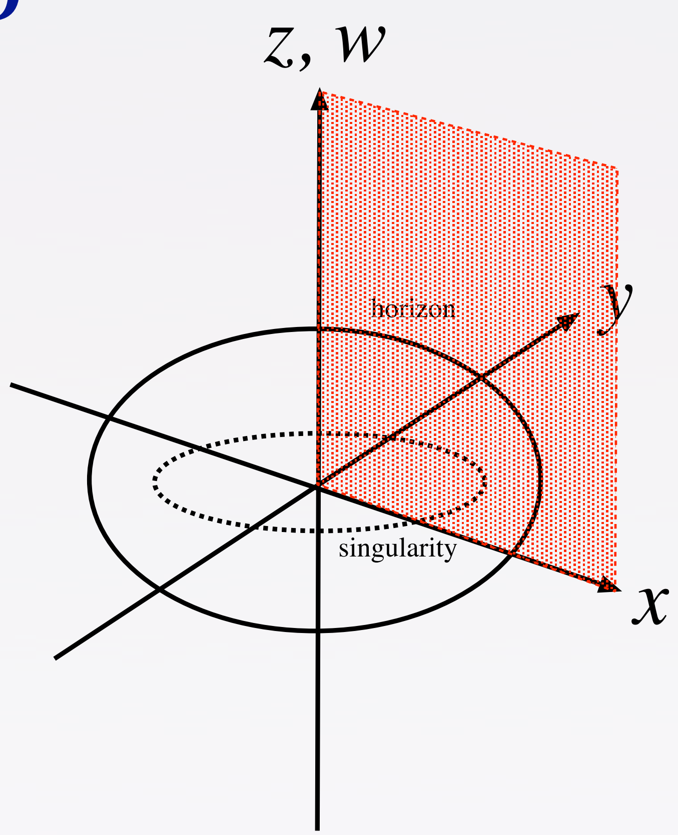


Data of (x, 0, z, 0)

Cartoon method in 5D

- Space is 4D (x, y, z, w)



- U(1)×U(1) symmetry (x=y, z=w)
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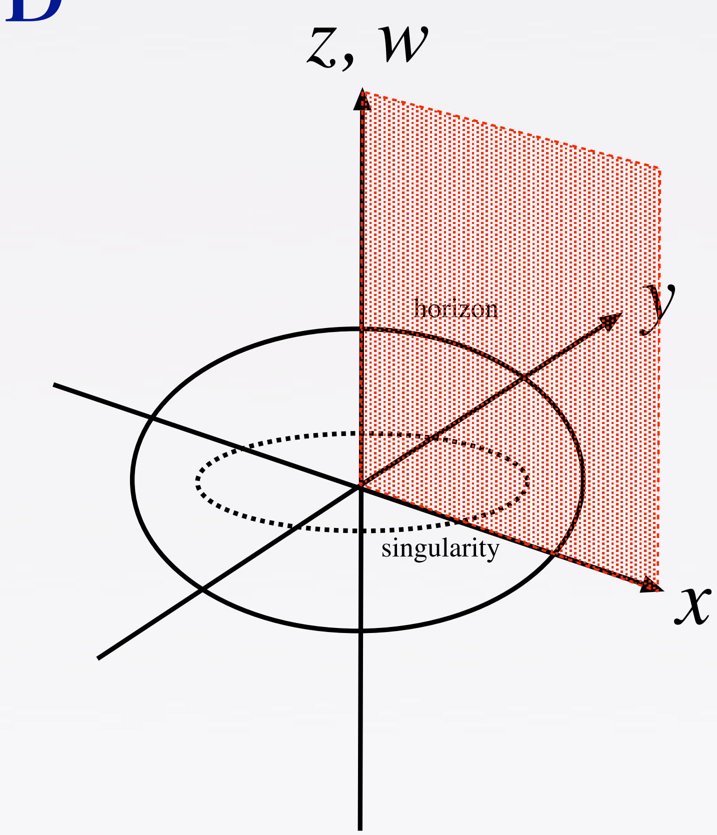
Data of (x, 0, z, 0) Data of (x, y, z, 0)

1st cartoon

Cartoon method in 5D



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Data of (x, 0, z, 0) → Data of (x, y, z, 0) → Data of (x, y, z, w)

1st cartoon

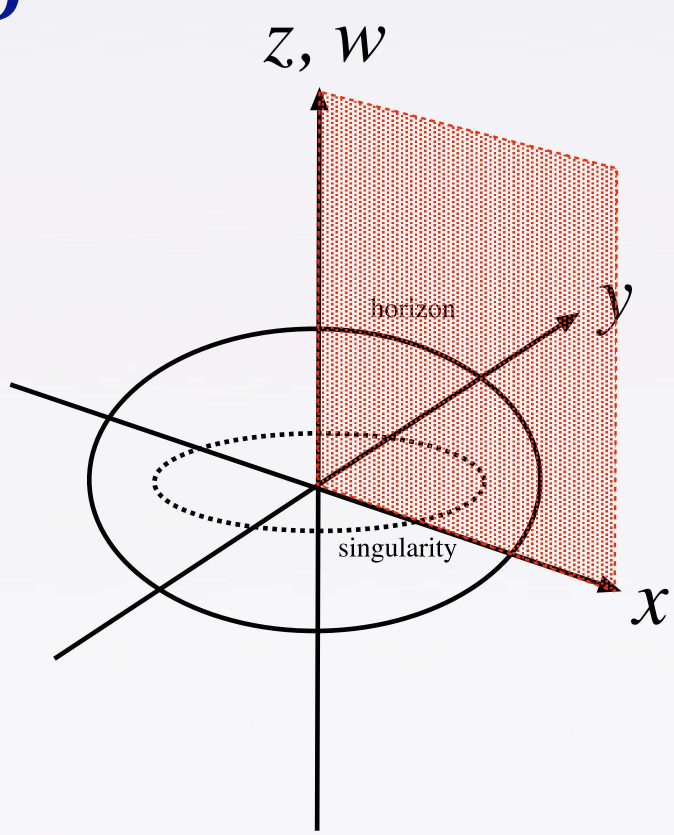
2nd cartoon

Cartoon method in 5D

- Space is 4D (x, y, z, w)

- U(1)×U(1) symmetry (x=y, z=w)

- SO(3) symmetry (x=y=z, w)



Data of (x, 0, z, 0) → Data of (x, y, z, 0) → Data of (x, y, z, w)

1st cartoon

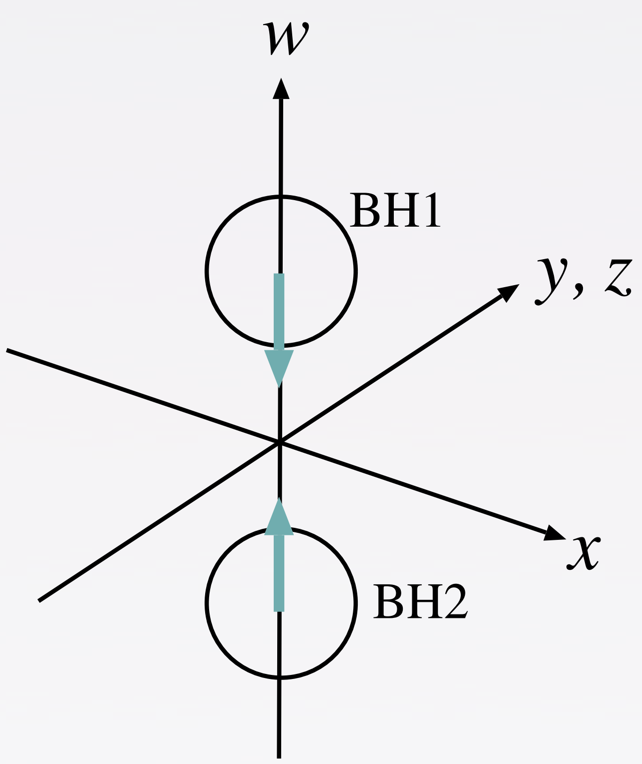
2nd cartoon

Double Cartoon method

Cartoon method in 5D

- Space is 4D (x, y, z, w)
  - U(1)×U(1) symmetry (x=y, z=w)
  - SO(3) symmetry (x=y=z, w)

Cartoon method in 5D



- Space is 4D (x, y, z, w)
  - U(1)×U(1) symmetry (x=y, z=w)
  - SO(3) symmetry (x=y=z, w)

Cartoon method in 5D

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  - U(1)×U(1) symmetry (x=y, z=w)
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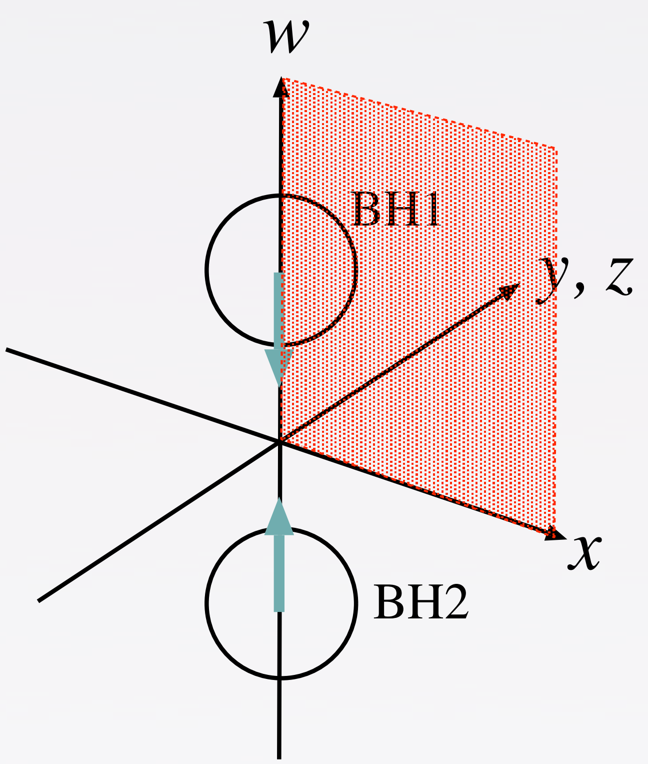

Cartoon method in 5D

- Space is 4D (x, y, z, w)
  - U(1)×U(1) symmetry (x=y, z=w)
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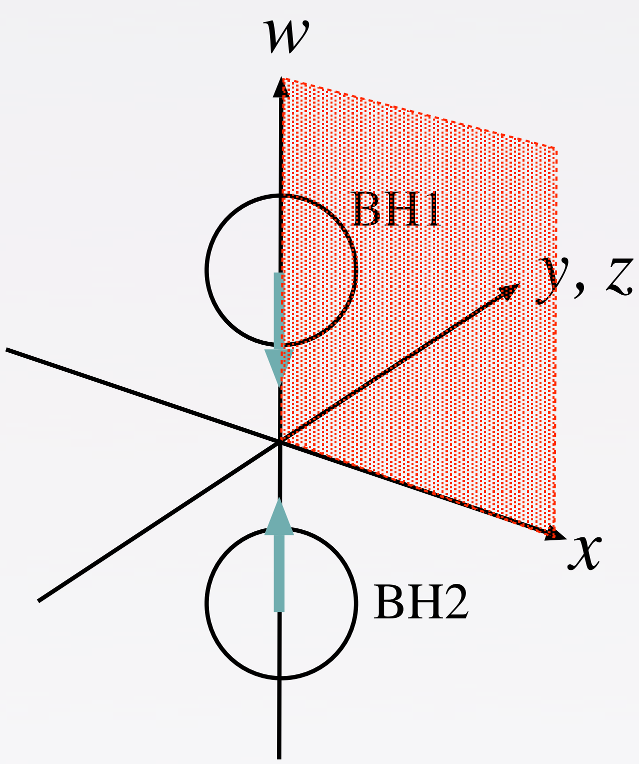

Scalar

$$\Psi(x,y,z,w) = \Psi(r,0,0,w)$$

Cartoon method in 5D

- Space is 4D (x, y, z, w)
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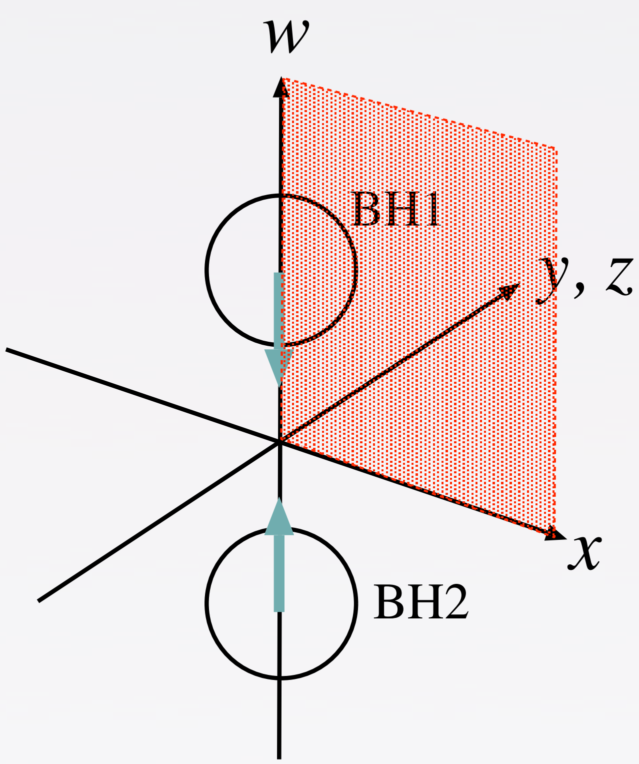


Vector

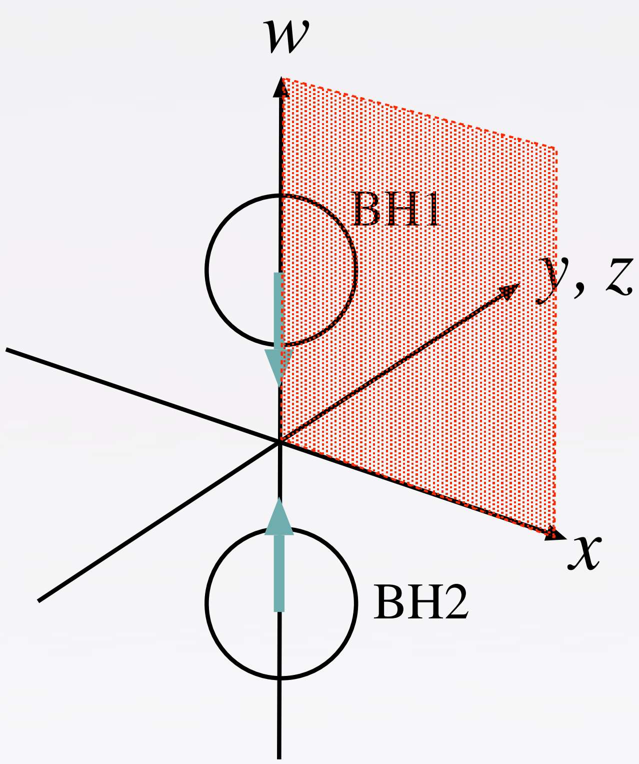
$$T^x(x,y,z,w) = (x/r)T^x(r,0,0,w)$$

$$T^y(x,y,z,w) = (y/r)T^x(r,0,0,w)$$

$$T^z(x,y,z,w) = (z/r)T^x(r,0,0,w)$$

$$T^w(x,y,z,w) = T^w(r,0,0,w)$$

Cartoon method in 5D



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Symmetric tensor

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$$S^{xw}(x,y,z,w) = (x/r)S^{xw}(r,0,0,w)$$
$$S^{yw}(x,y,z,w) = (y/r)S^{xw}(r,0,0,w)$$
$$S^{zw}(x,y,z,w) = (z/r)S^{xw}(r,0,0,w)$$

$$S^{xx}(x,y,z,w) = (x^2/r^2)S^{xx}(r,0,0,w) + \left(1 - x^2/r^2\right) S^{yy}(r,0,0,w)$$
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Contents

- Introduction
- BSSN formulation
- Cartoon method
- Codes
- Summary

Our code for 5D spacetimes

- My code
  - SO(4) symmetry (x=y=z=w) [1D code]
  - U(1)×U(1) symmetry (x=y, z=w) [2D code]
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- Scheme
  - Time direction: 4th-order Runge-Kutta
  - Space direction: 4th-order finite discretization
  - Courant number: C=0.5

Code tests

- Linear gravitational waves in a flat spacetime
  - (x=y, z=w)
    - scalar mode
    - vector mode
    - tensor mode
  - (x=y=z, w)
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- Energy extraction by the Landau-Lifshitz pseudo tensor
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  - Schwarzschild spacetime in geodesic slices
  - Long term evolutions in the 1+log slicing
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Linear gravitational wave in a flat spacetime

- (x=y, z=w)-symmetry
  - hyper-spherical coordinate (r, θ, ϕ, χ)

$$
\begin{aligned}
x &= r \sin\theta \cos\phi \\
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z &= r \cos\theta \cos\chi \\
w &= r \cos\theta \sin\chi
\end{aligned}
$$

$$
h_{ij} = H(t,r) r^2 \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sin^2\theta(1 - 3\sin^2\theta) & 0 \\ 0 & 0 & 0 & \cos^2\theta(3\sin^2\theta - 2) \end{pmatrix}
$$

$$
\boxed{H(t,r) = A_0 \omega_0 \int_0^{2\pi} d\theta \sin(3\theta) e^{-\omega_0^2 (t - r\sin\theta)^2/2}}
$$

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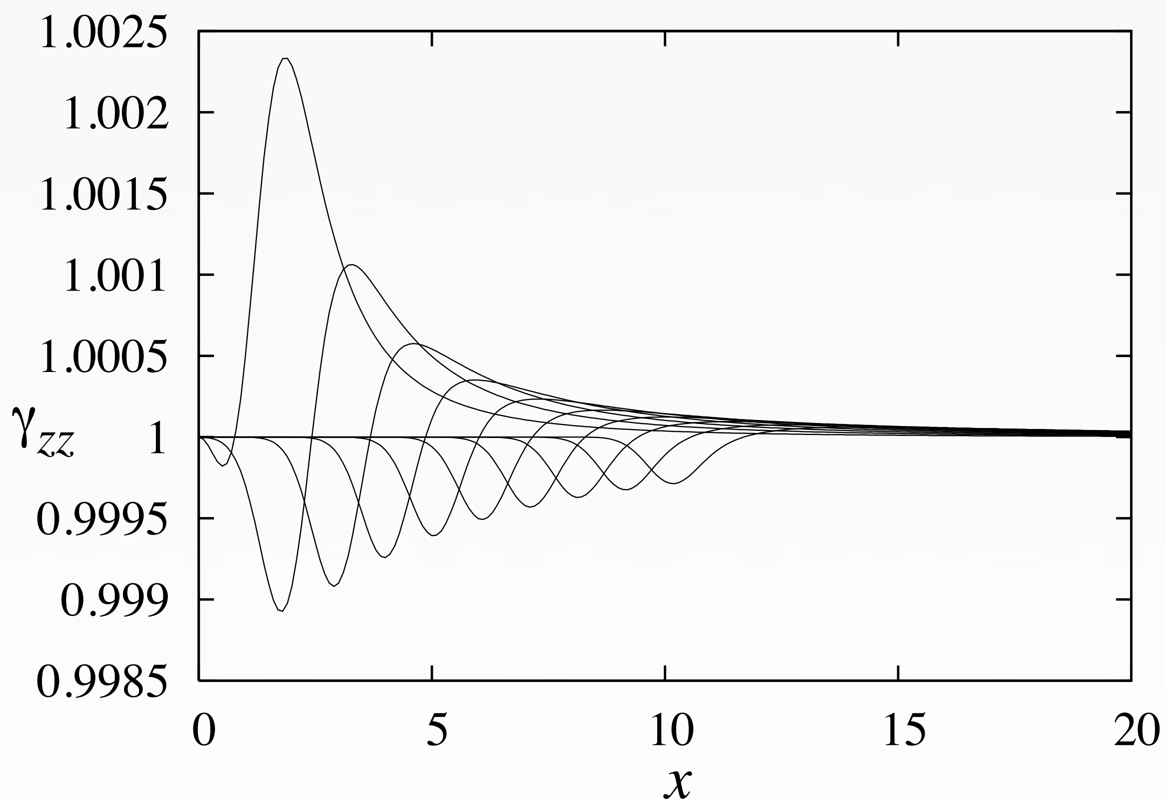

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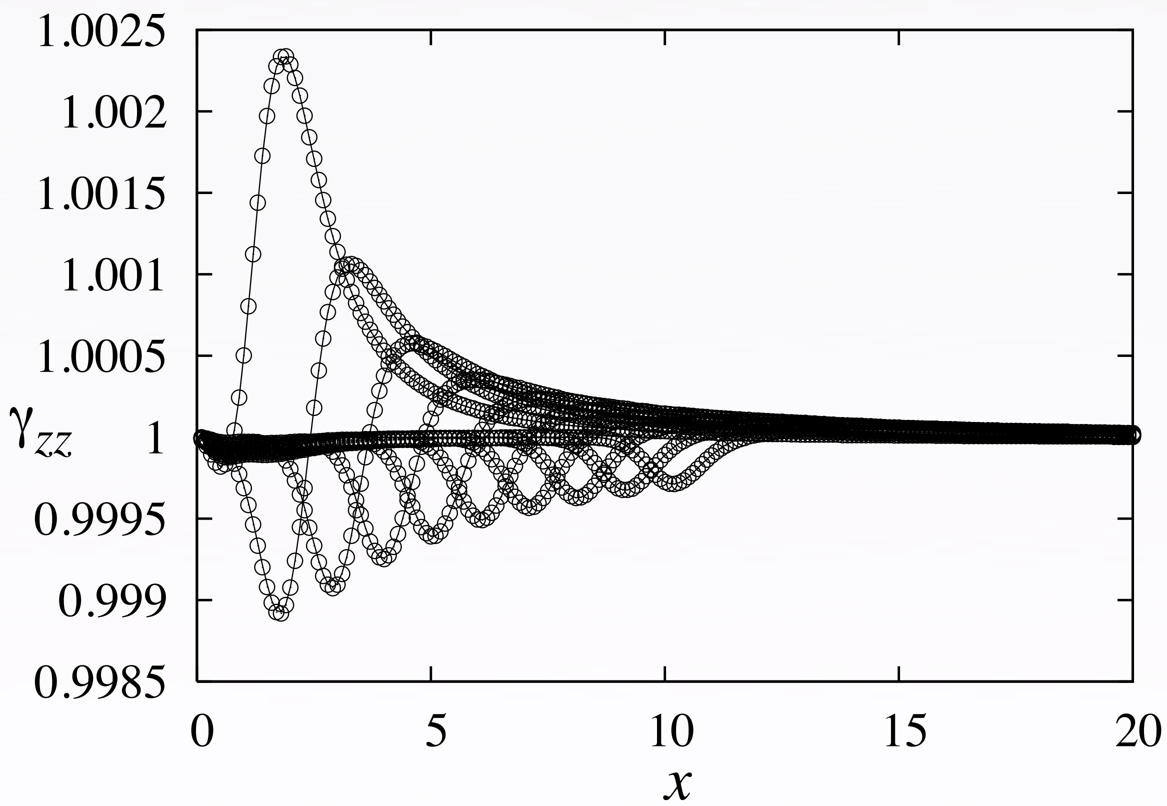

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Energy extraction by the Landau-Lifhistz pseudo tensor

- **super potential** $H^{\mu\alpha\nu\beta} = \tilde{g}^{\mu\nu}\tilde{g}^{\alpha\beta} - \tilde{g}^{\alpha\nu}\tilde{g}^{\mu\beta}$ $\quad \tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu},$
- **Landau-Lifshitz pseudo tensor** $16\pi G t^{\mu\nu}_{\rm LL} = (-g)^{-1} H^{\mu\alpha\nu\beta}{}_{,\alpha\beta} - (2R^{\mu\nu} - g^{\mu\nu}R)\,.$

$$\begin{aligned}16\pi G(-g)t^{\mu\nu}_{\rm LL} &= \tilde{g}^{\mu\nu}{}_{,\alpha}\tilde{g}^{\alpha\beta}{}_{,\beta} - \tilde{g}^{\mu\alpha}{}_{,\alpha}\tilde{g}^{\nu\beta}{}_{,\beta} + \frac{1}{2}g^{\mu\nu}g_{\alpha\beta}\tilde{g}^{\alpha\rho}{}_{,\sigma}\tilde{g}^{\sigma\beta}{}_{,\rho} \\ &- \left(g^{\mu\alpha}g_{\beta\rho}\tilde{g}^{\nu\rho}{}_{,\sigma}\tilde{g}^{\beta\sigma}{}_{,\alpha} + g^{\nu\alpha}g_{\beta\rho}\tilde{g}^{\mu\rho}{}_{,\sigma}\tilde{g}^{\beta\sigma}{}_{,\alpha}\right) + g_{\alpha\beta}g^{\rho\sigma}\tilde{g}^{\mu\alpha}{}_{,\rho}\tilde{g}^{\nu\beta}{}_{,\sigma} \\ &+ \frac{1}{4(D-2)}\left(2g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta}\right)\left[(D-2)g_{\rho\sigma}g_{\gamma\delta} - g_{\sigma\gamma}g_{\rho\delta}\right]\tilde{g}^{\rho\delta}{}_{,\alpha}\tilde{g}^{\sigma\gamma}{}_{,\beta}\end{aligned}$$

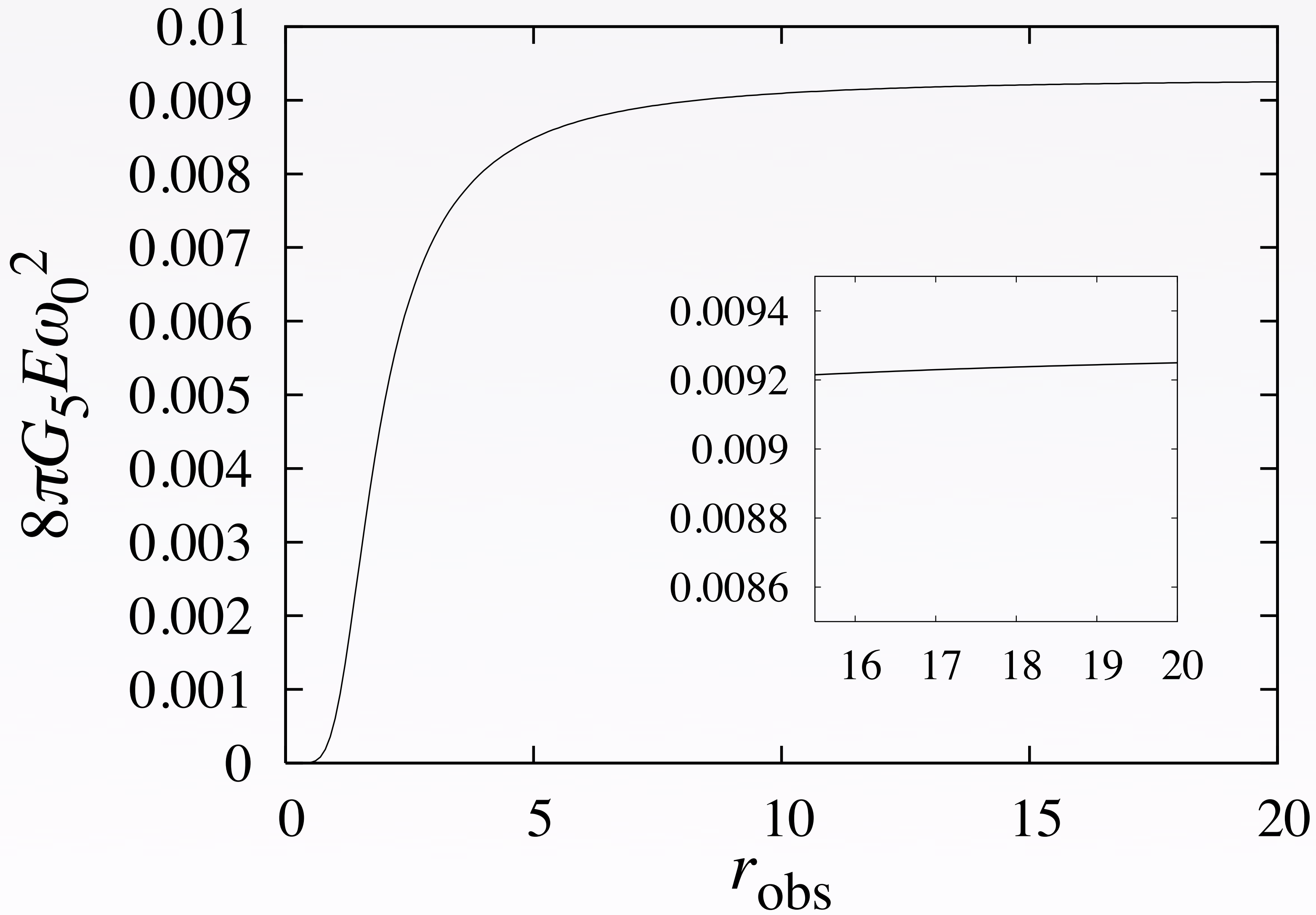
- **conservation:** $\left[(-g)\left(T^{\mu\nu} + t^{\mu\nu}_{\rm LL}\right)\right]_{,\nu} = 0$
- **For a perturbation of a flat spacetime,** $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $\hat{h}_{\mu\nu} := h_{\mu\nu} - (1/2)h\eta_{\mu\nu}$

$$\boxed{\begin{aligned}16\pi G t^{\mu\nu}_{\rm LL} &= \hat{h}^{\mu\nu}{}_{,\alpha}\hat{h}^{\alpha\beta}{}_{,\beta} - \hat{h}^{\mu\alpha}{}_{,\alpha}\hat{h}^{\nu\beta}{}_{,\beta} + \frac{1}{2}\eta^{\mu\nu}\hat{h}^{\alpha\rho}{}_{,\sigma}\hat{h}^{\sigma}{}_{\alpha,\rho} \\ &- \left(\hat{h}^{\mu\rho}{}_{,\sigma}\hat{h}_{\rho}{}^{\sigma,\nu} + \hat{h}^{\nu\rho}{}_{,\sigma}\hat{h}_{\rho}{}^{\sigma,\mu}\right) + \hat{h}^{\mu\alpha,\rho}\hat{h}^{\nu}{}_{\alpha,\rho} \\ &+ \frac{1}{2}\hat{h}^{\rho\sigma,\mu}\hat{h}_{\rho\sigma}{}^{,\nu} - \frac{1}{4}\eta^{\mu\nu}\hat{h}^{\rho\sigma,\alpha}\hat{h}_{\rho\sigma,\alpha} - \frac{1}{4(D-2)}\left(2\hat{h}^{,\mu}\hat{h}^{,\nu} - \eta^{\mu\nu}\hat{h}^{,\alpha}\hat{h}_{,\alpha}\right).\end{aligned}}$$

- **total radiated energy:** $\boxed{E_{\rm rad} = \int t^{0i}\hat{n}_i dS dt}$

Energy extraction by the Landau-Lifhistz pseudo tensor

$$E_{\mathrm{rad}}(r_{\mathrm{obs}}) = \int t^{0i}_{\mathrm{LL}} n_i dS dt$$



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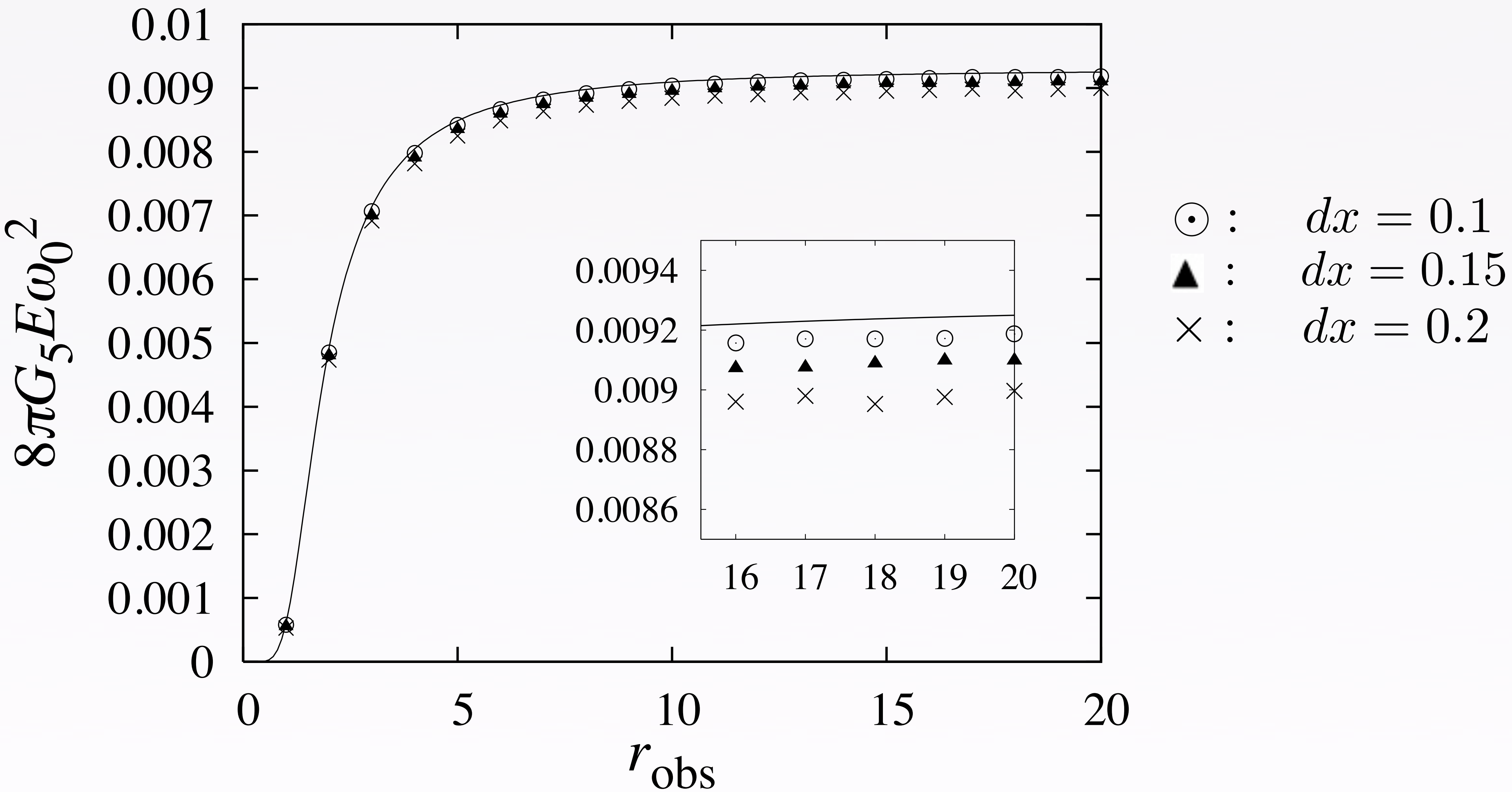

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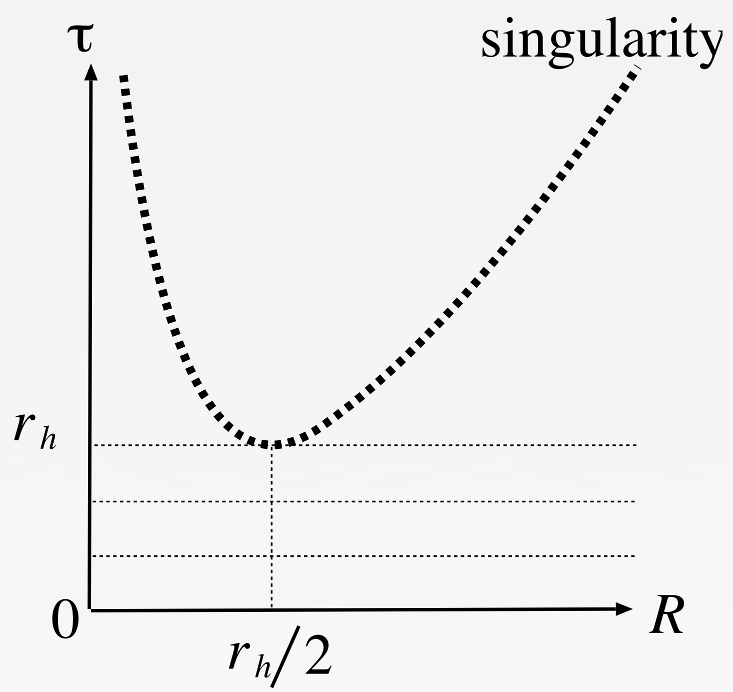
5D Schwarzschild spacetime in geodesic slices



$$ds^2 = -d\tau^2 + \frac{\left[r_0^2 + (r_h/r_0)^2\tau^2\right]^2}{\left[r_0^2 - (r_h/r_0)^2\tau^2\right]}\frac{dR^2}{R^2} + \left[r_0^2 - \left(\frac{r_h}{r_0}\right)^2\tau^2\right]d\Omega_3^2$$

$$r_0^2 = R^2\left(1 + \frac{r_h^2}{4R^2}\right)^2$$

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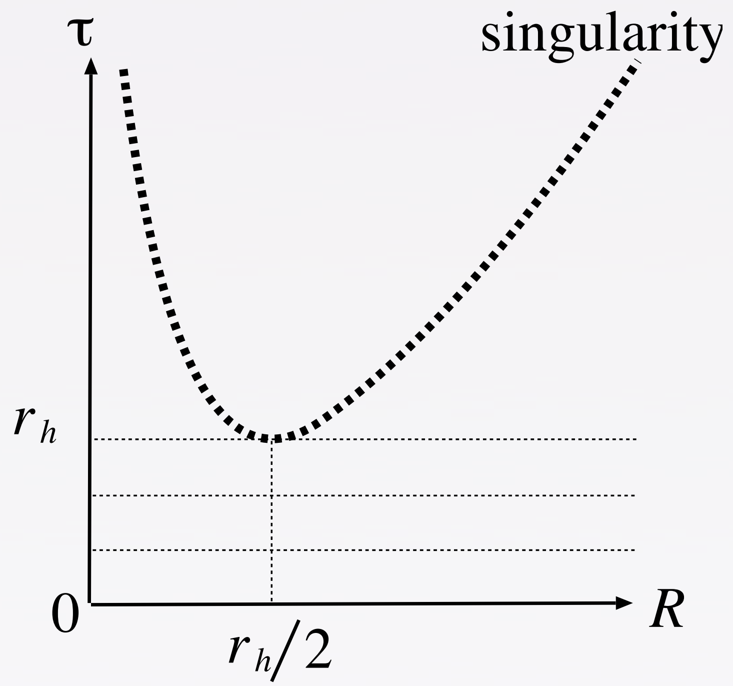

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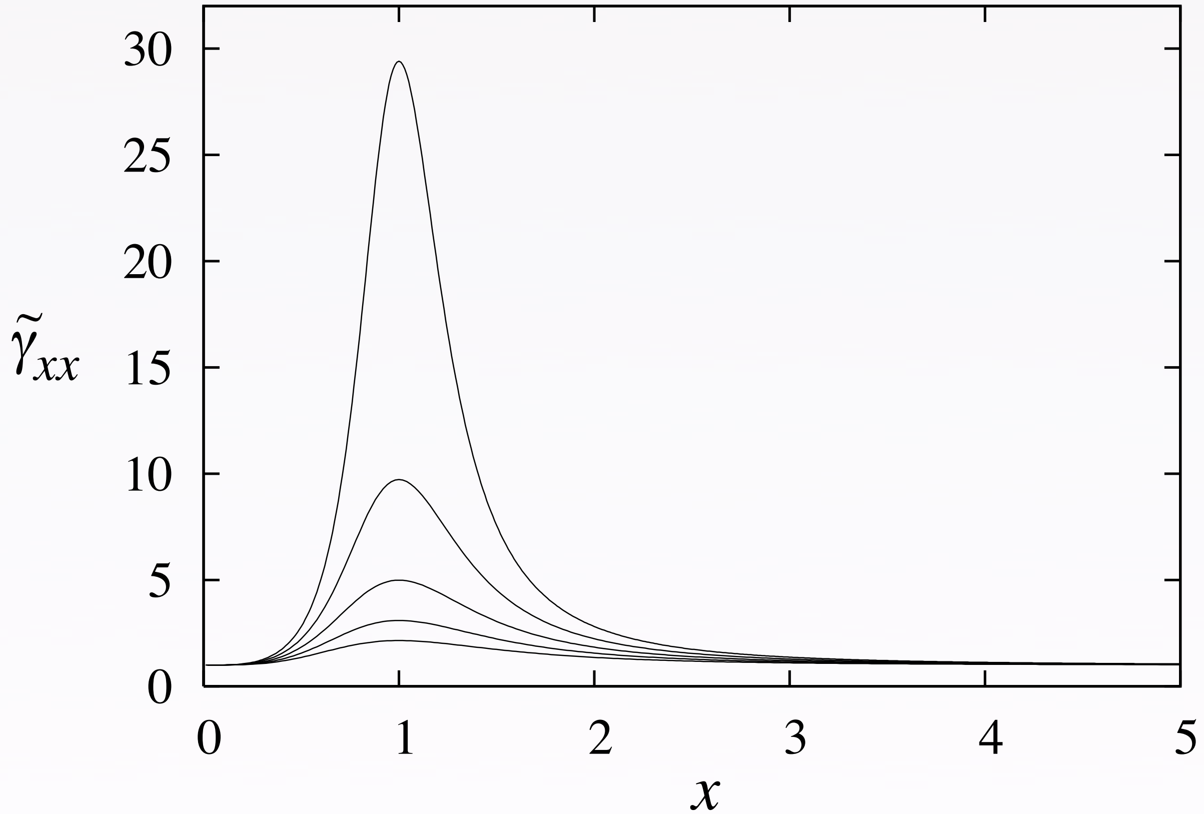
- **snapshots for** $\tau/r_h = 0.5, ..., 0.9$



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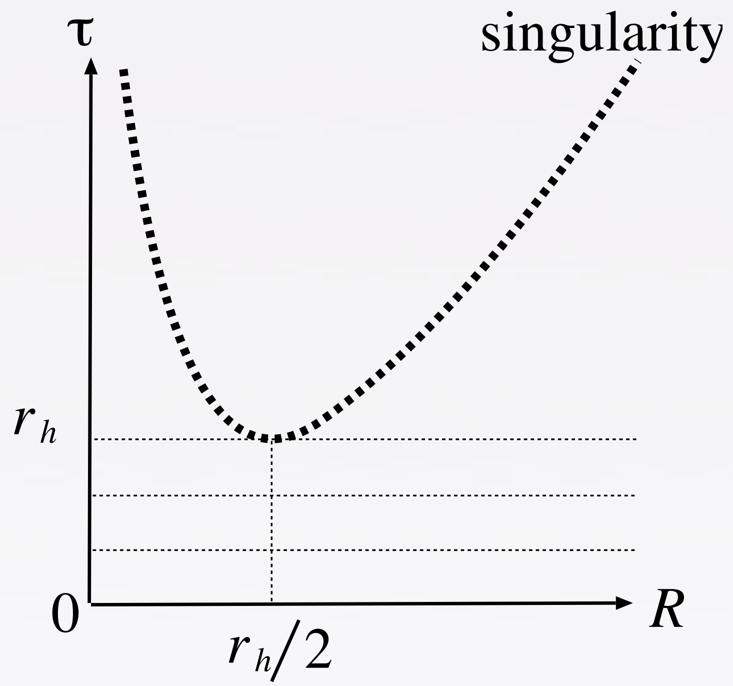

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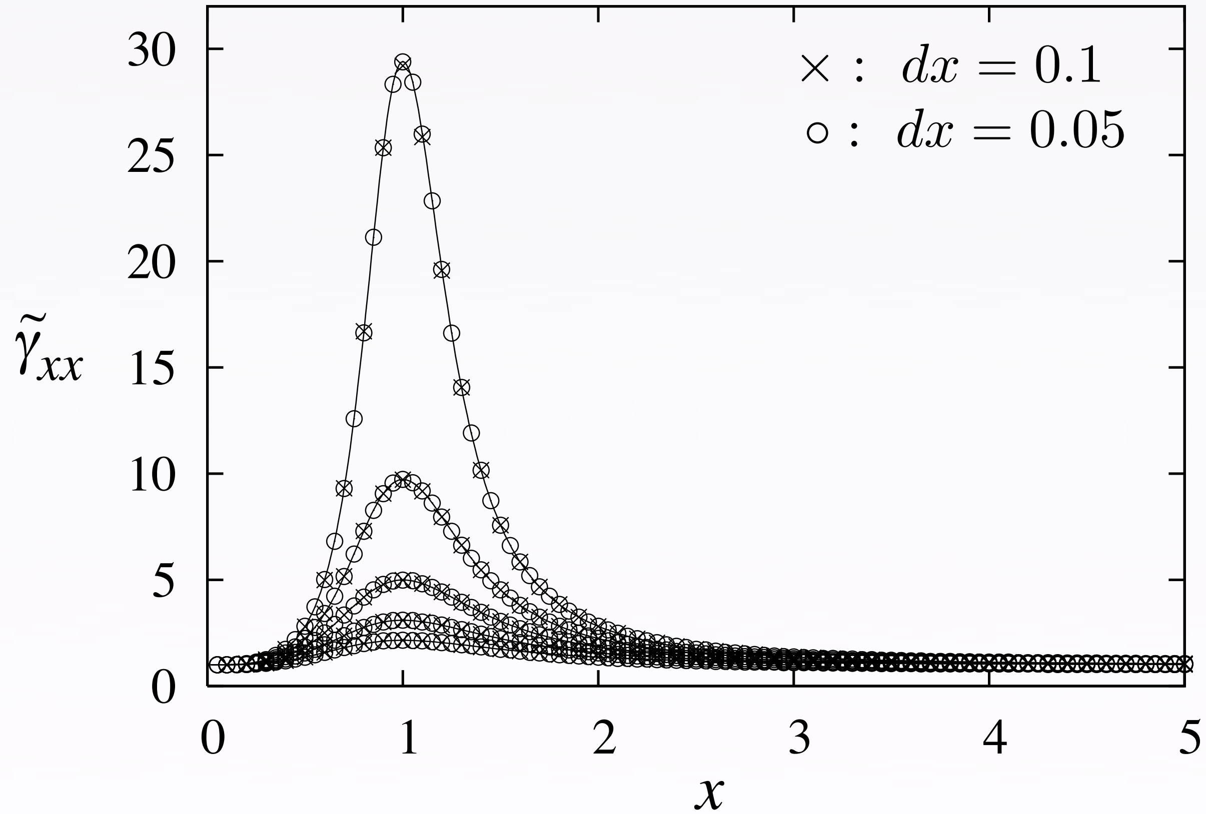

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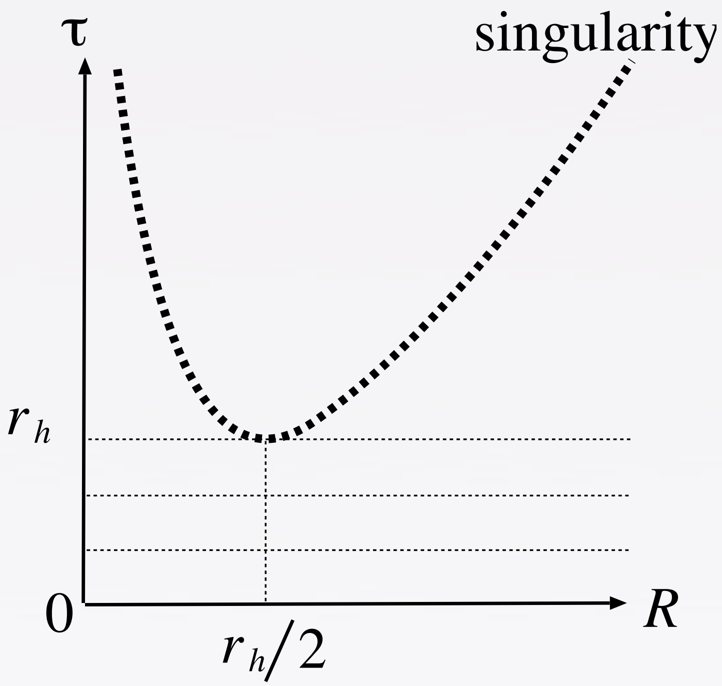

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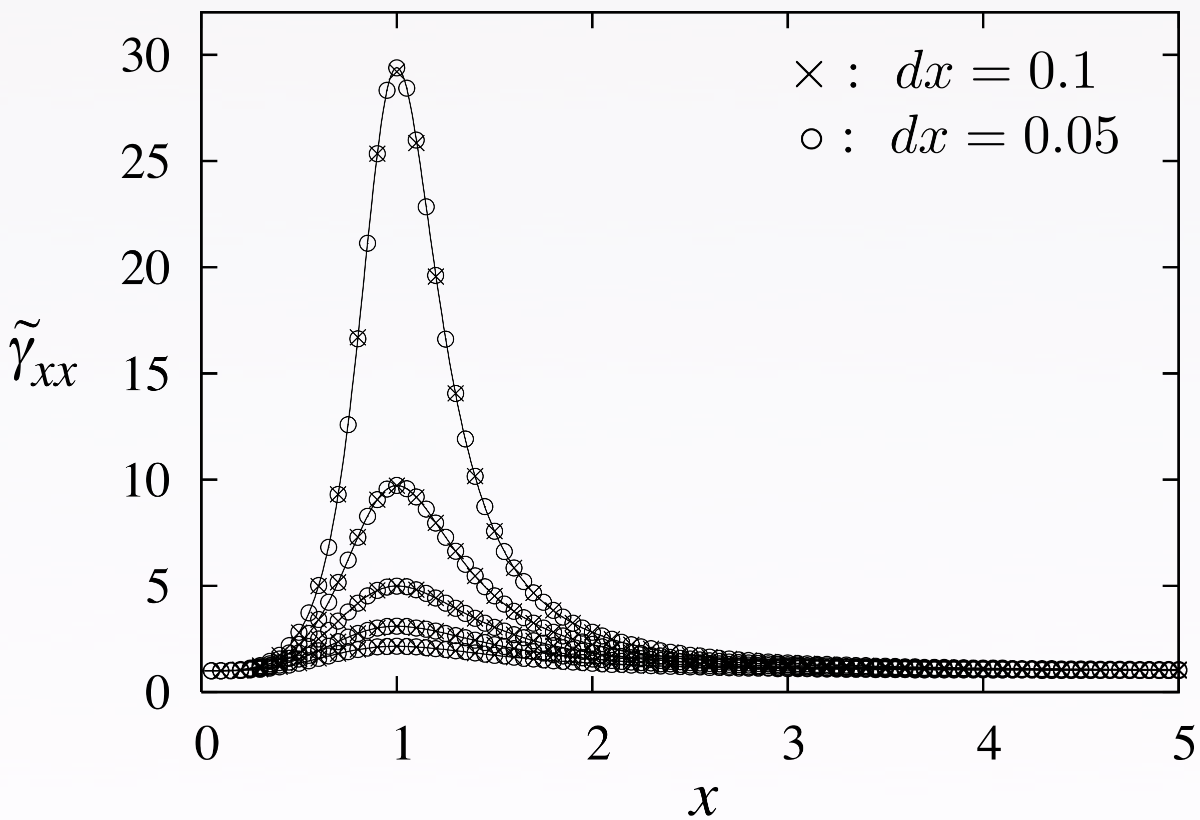

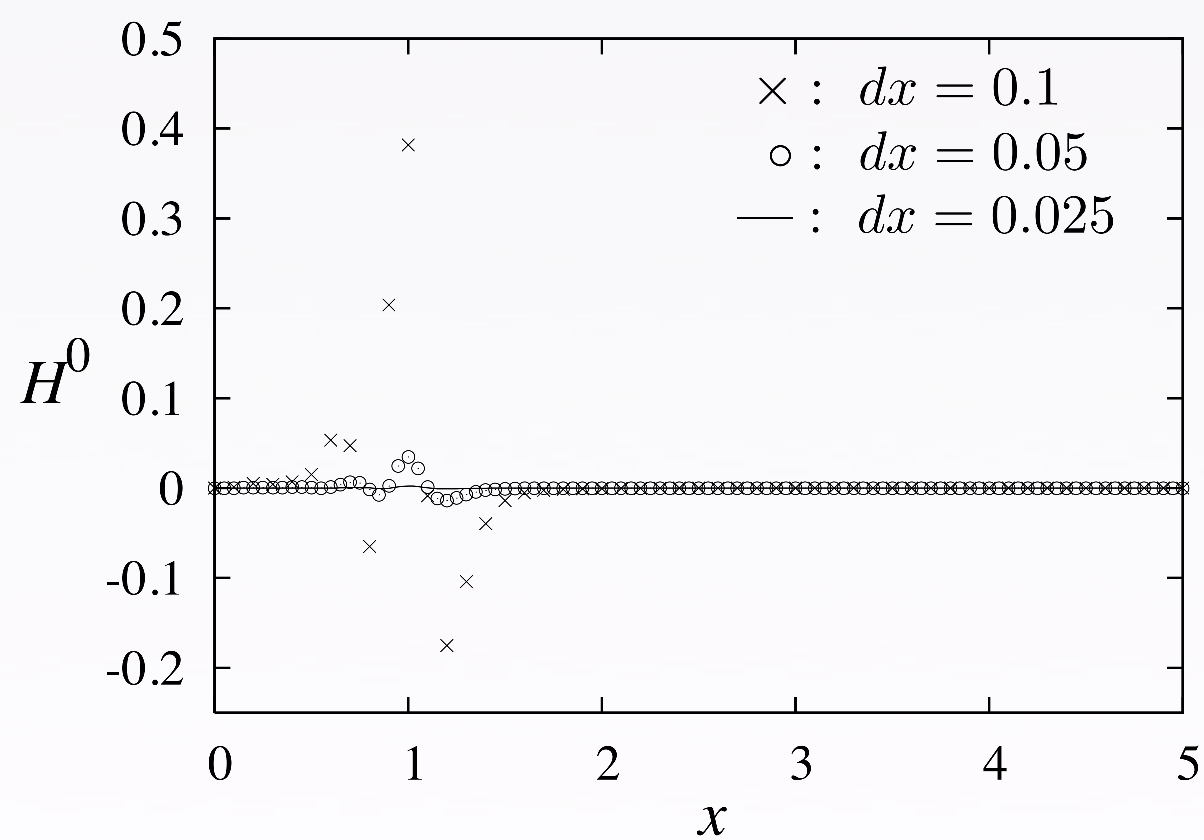

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Long-term evolution in the 1+log slicing

c.f. Hannam, Husa, Brugmann, Gonzalez, Sperhake and O'Murchadha, gr-qc/0612097 (4D case)

- **1+log slicing** $\partial_t \alpha = -2\alpha K$
- **Γ-driver condition** $\partial_t \beta^i = \frac{2}{3} B^i; \qquad \partial_t B^i = \partial_t \tilde{\Gamma}^i - \eta B^i$
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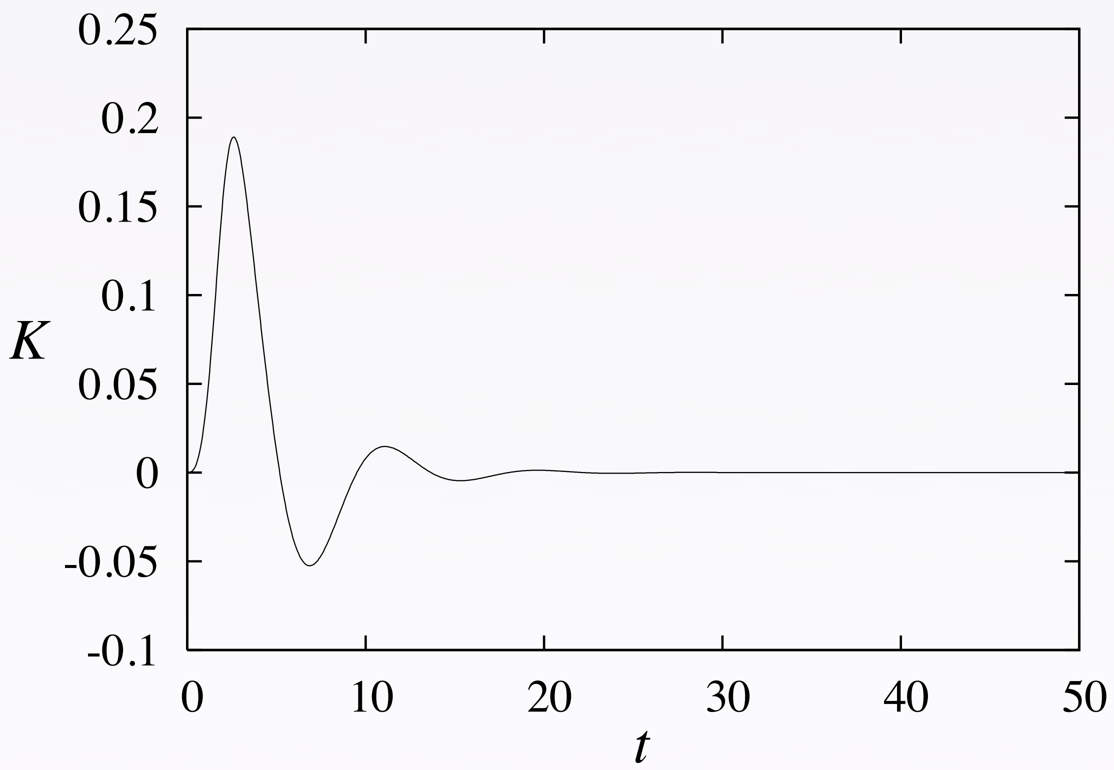

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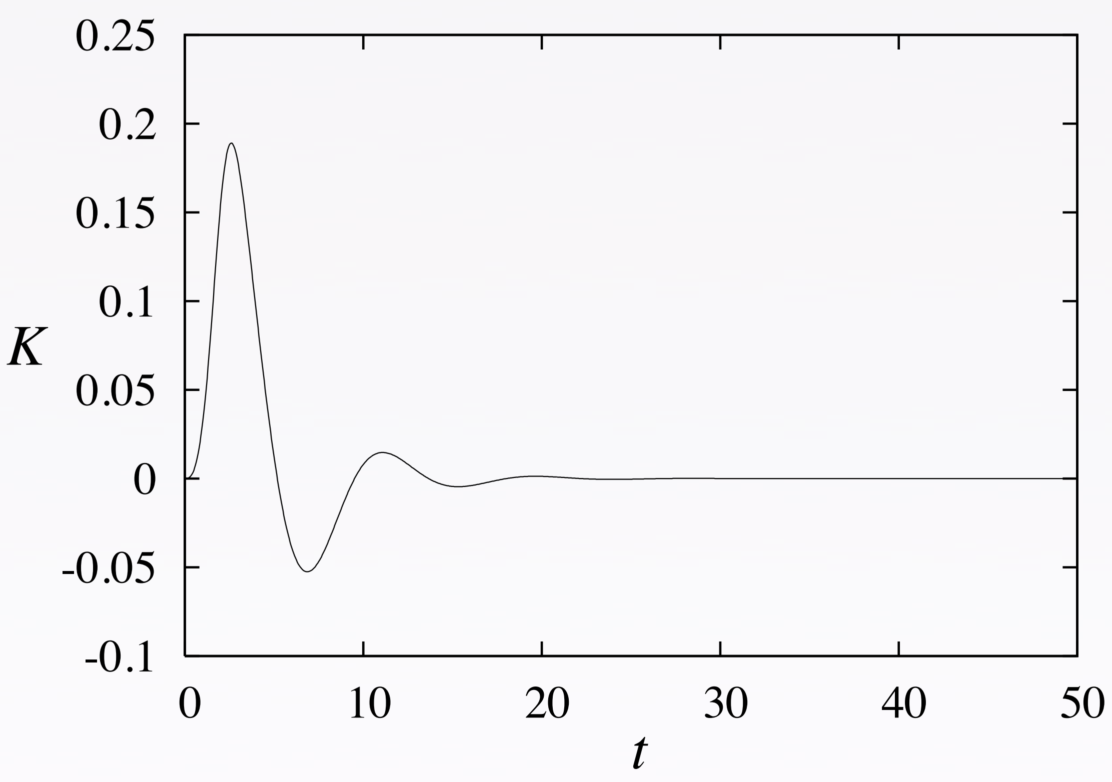

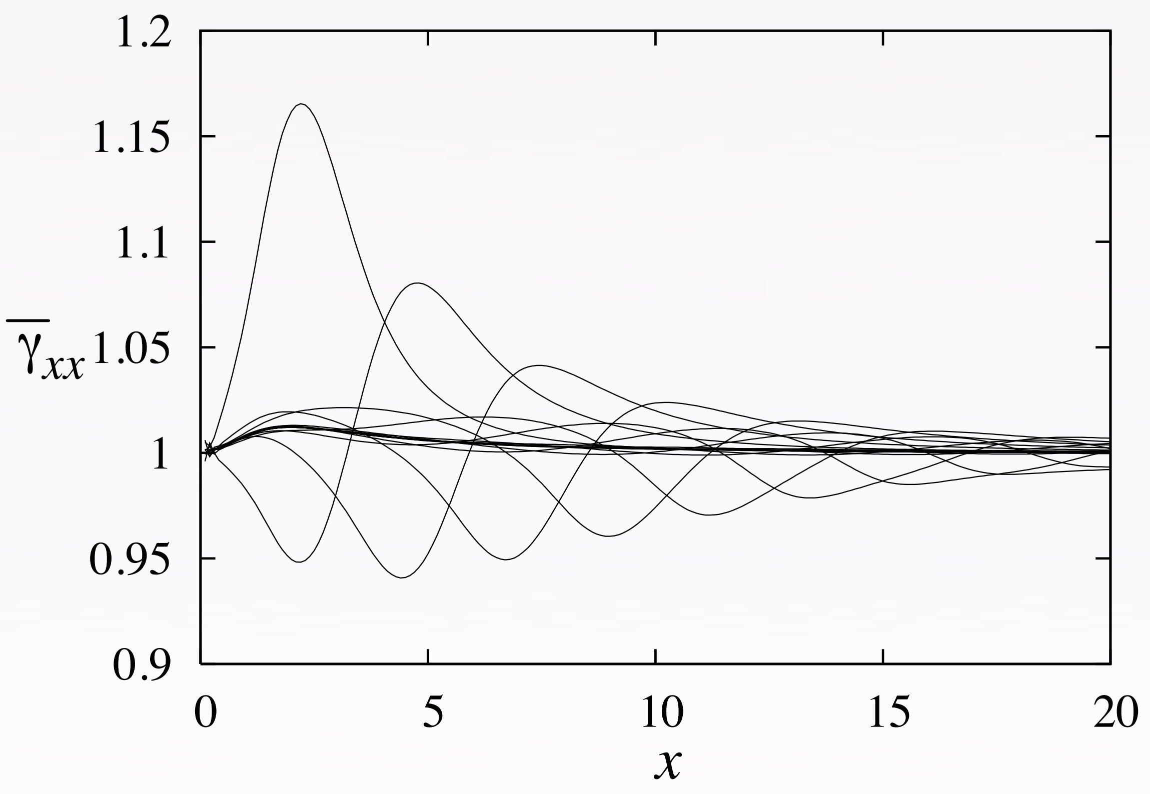

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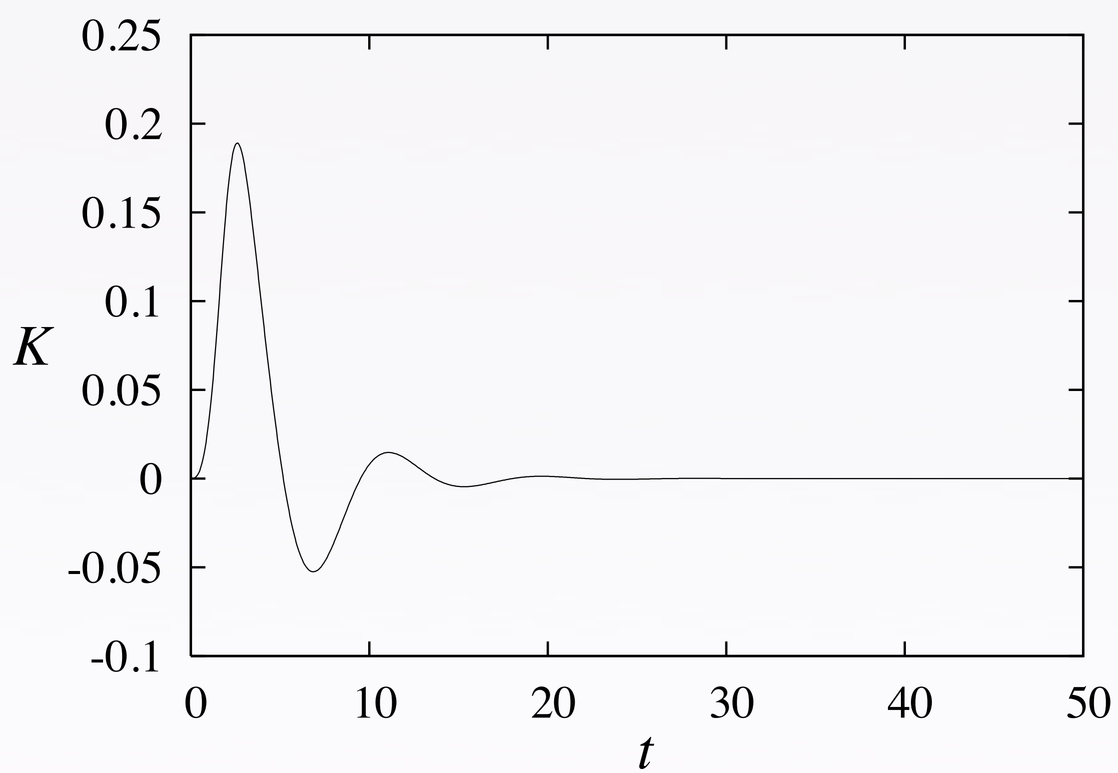

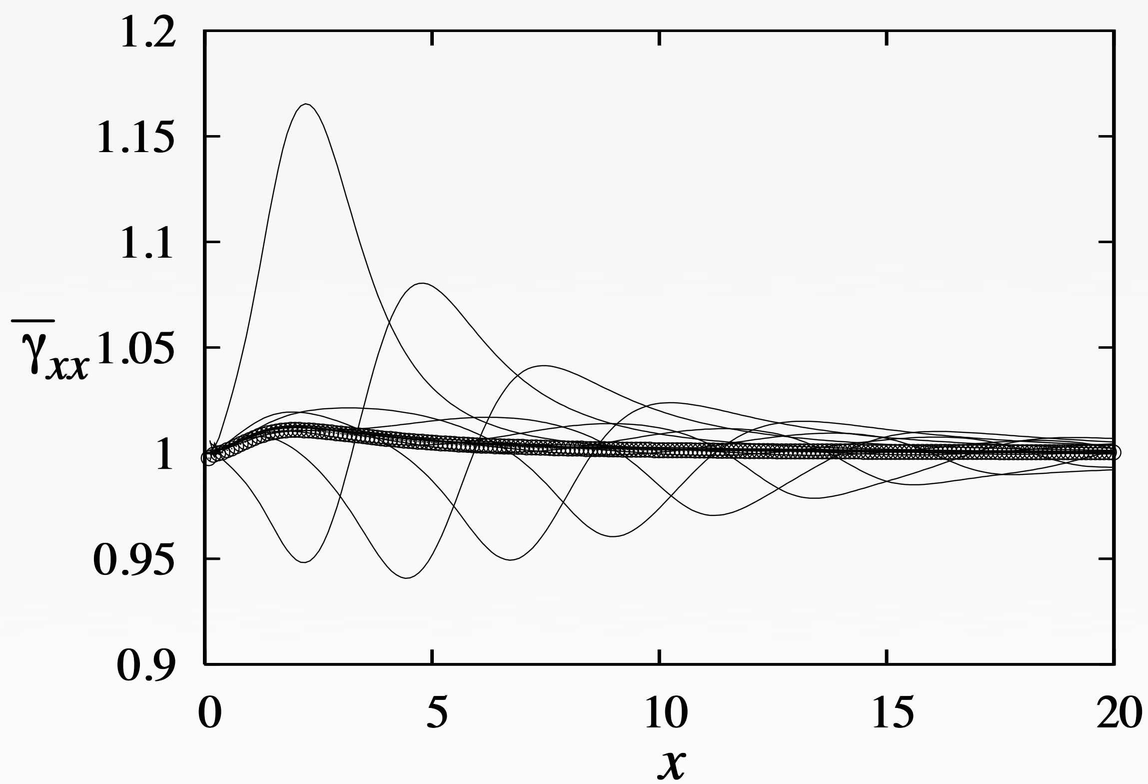

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Evolution starting from the limit surface

with Nakao, Abe, Shibata *in progress*

- Final state of the maximally sliced evolution, $K = 0$.

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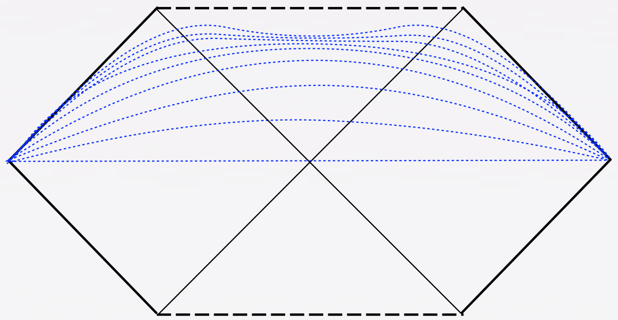

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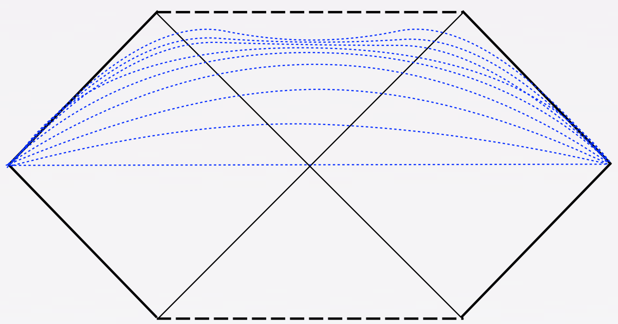

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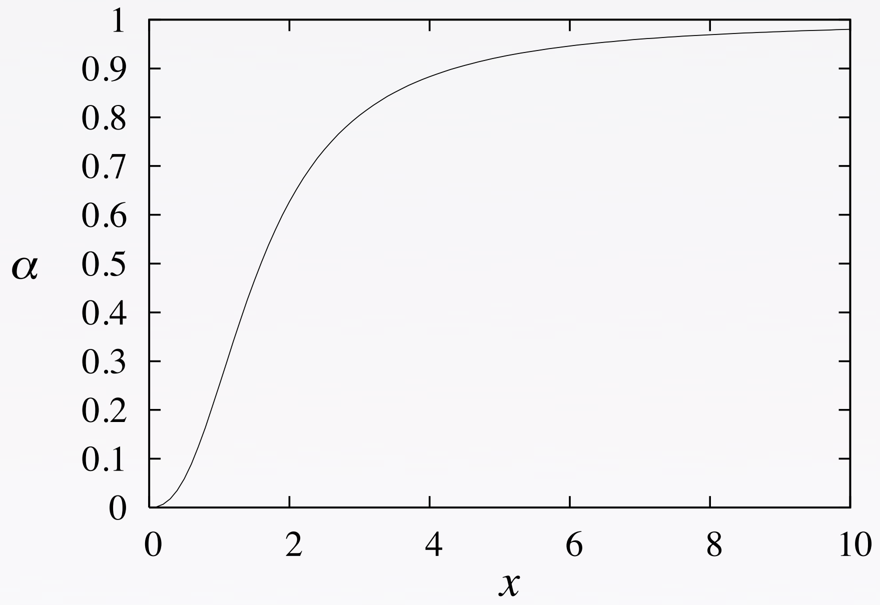

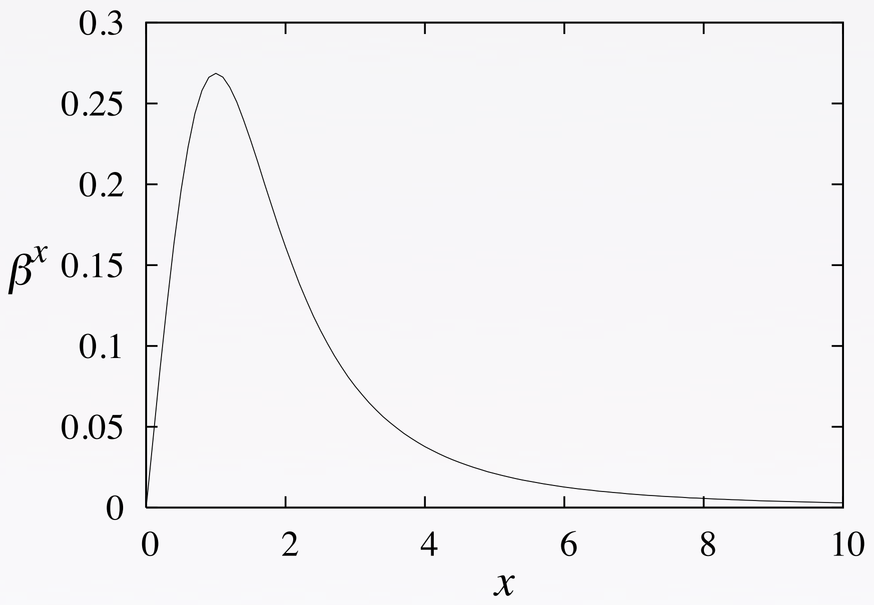

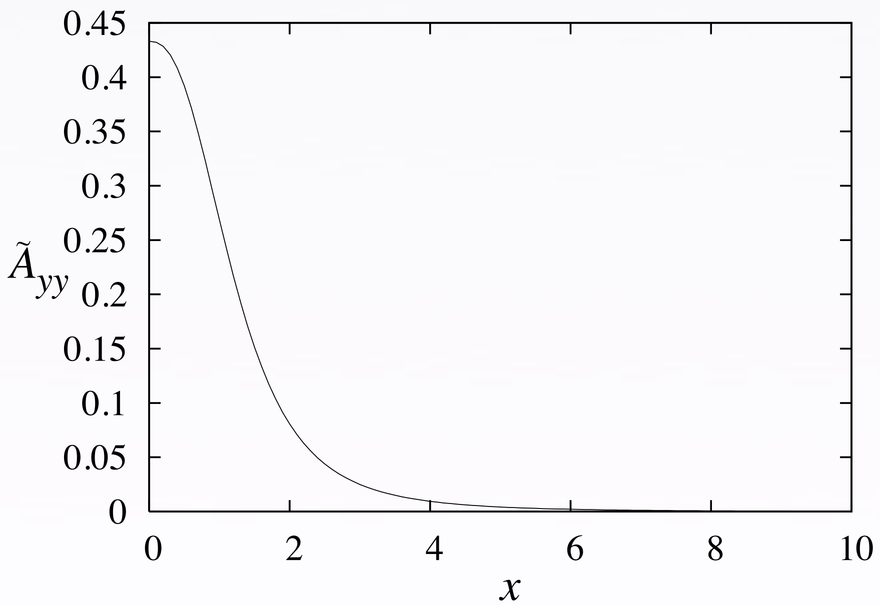

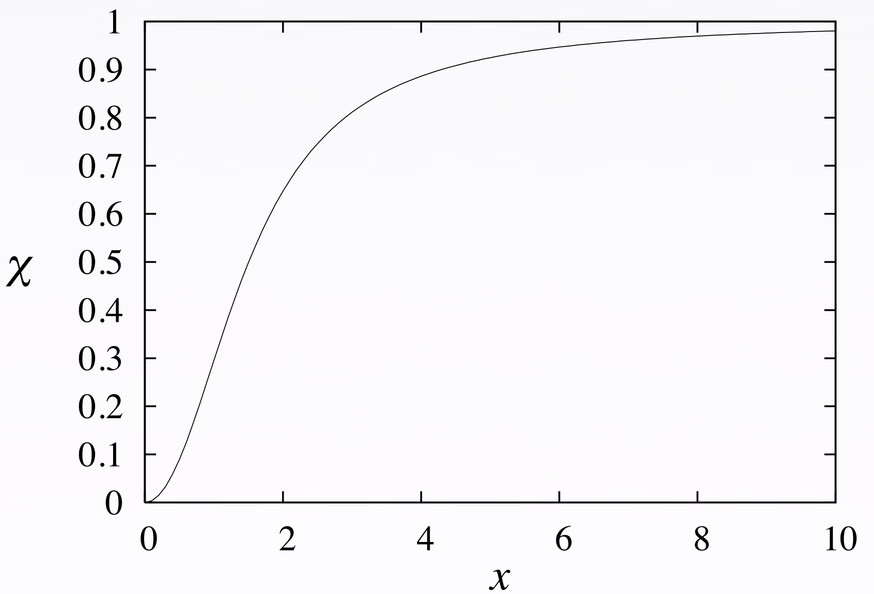

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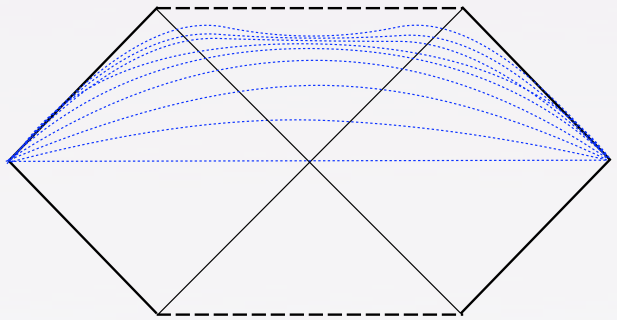

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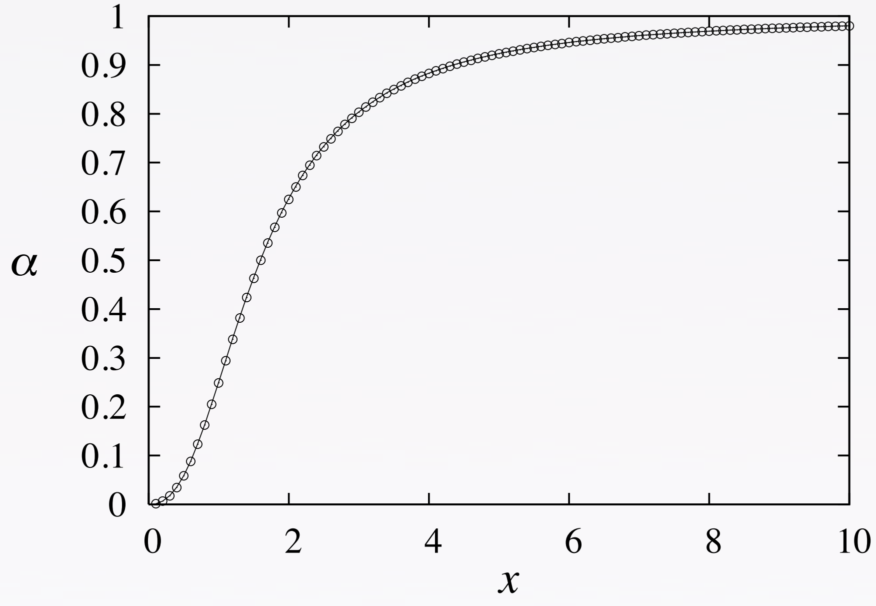

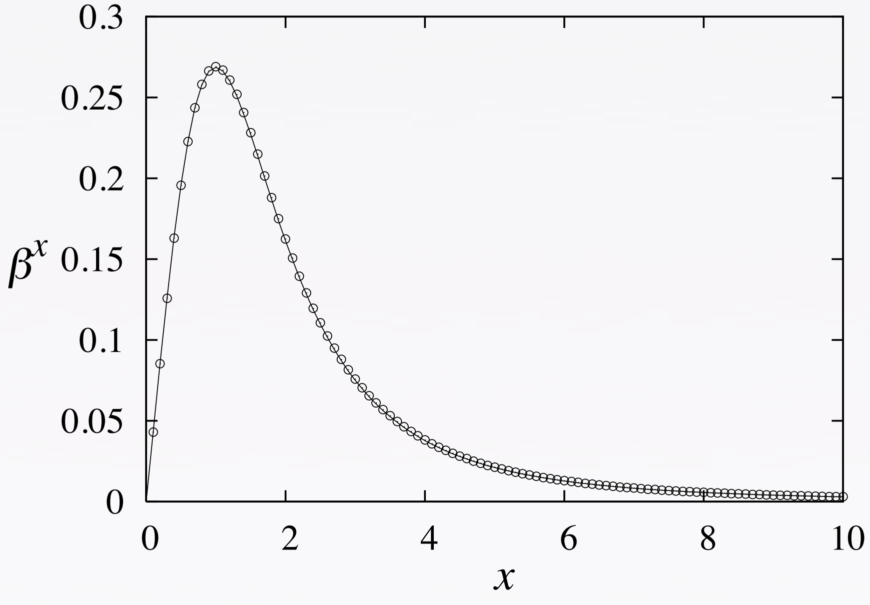

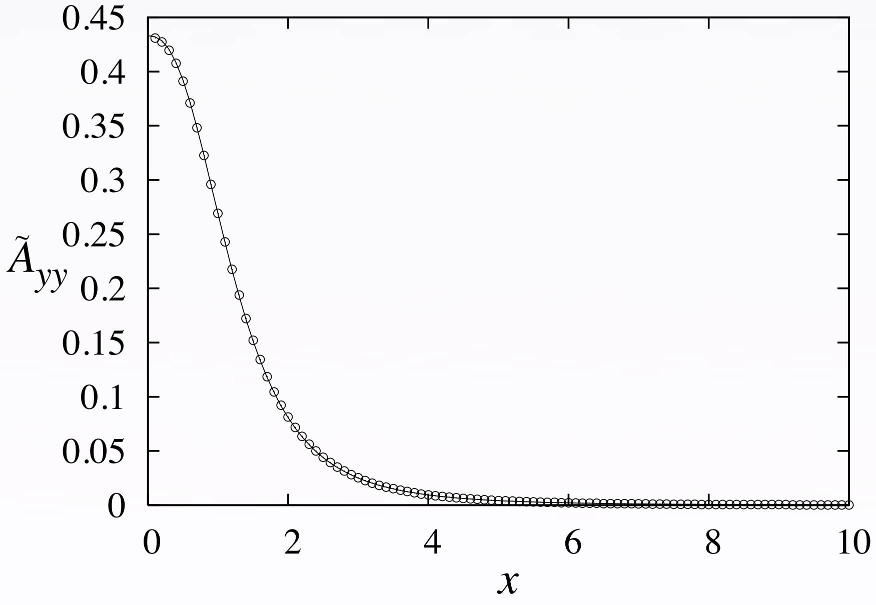

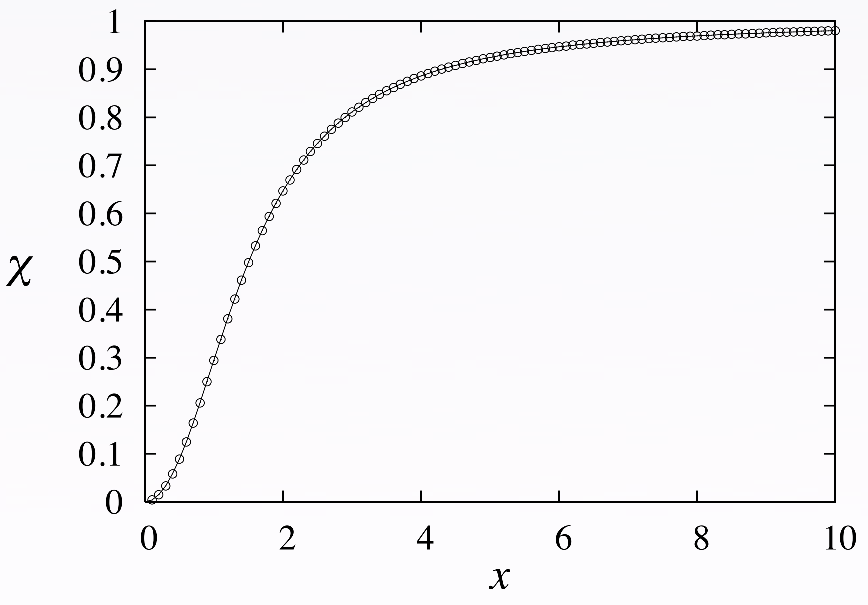

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- Introduction
- BSSN formulation
- Cartoon method
- Codes
- **Summary**

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We are ready now!

Appendix