Rotating black rings on Taub-NUT

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Introduction

- $5D$ asymptotically flat vacuum black holes/rings:
  - BH: Myers-Perry;
  - BRs: Emparan-Reall, Pomeransky-Sen'kov;
  - Multi-BH/BRs: Black Saturn, Black di-ring, Black bi-ring....
- BHs/BRs on Taub-NUT are also interesting because
  - interacting $D0$-$D6$ branes in string theory;
  - interesting interpretations in KK theory;
  - interesting in their own right as 5D Ricci-flat space-times.

Taub-NUT geometry

Metric:

$$\mathrm{d}s^2 = H^{-1}(\mathrm{d}\psi + 2n\cos\theta\,\mathrm{d}\phi)^2 + H(\mathrm{d}r^2 + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\,\mathrm{d}\phi^2)\,, \quad H = 1 + \frac{2n}{r}.$$

Properties:

- asympt. geometry: twisted S^1 over E^3.
- n: NUT charge, the size of S^1 at infinity.

Two viewpoints to visualize:

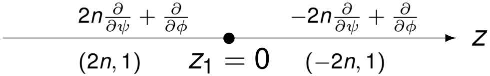
- S^1 (ψ) over $\mathbb{R}^3$ (r, θ, ϕ) with 1 fixed pt$\rightarrow$ nut.
- $S^1 \times S^1$ (ψ, ϕ) over upper-half complex plane (r, θ) with some exceptional pts $\rightarrow$ Rod structure.

Rod structure of Taub-NUT space

Rod structure: sequence of intervals along z-axis, each with a (normalised) direction, for a solution with $U(1) \times U(1)$ isometry. Two neighboring rods meet at a turning point.

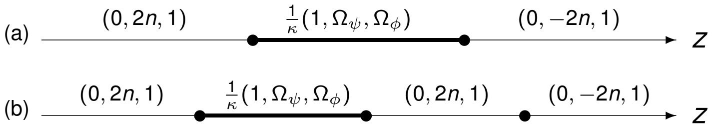
- Turning points: fixed-points for $U(1) \times U(1)$.
- Rod i: $[z_i, z_{i+1}]$
- Direction ℓ_i: A rod is the fixed points for the $U(1)$ subgp generated by ℓ_i.

Rod structure of Taub-NUT space:



A review of vacuum BHs/BRs on Taub-NUT

Rod structures of (a) BHs and (b) BRs on Taub-NUT:



- Black holes on Taub-NUT:

  KK black holes, Ishihara-Matsuno, Wang

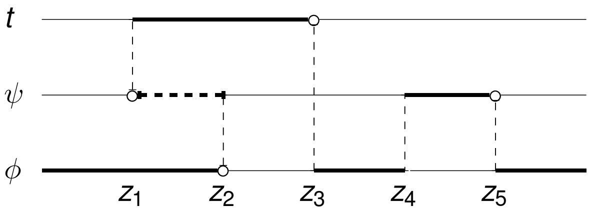
- Black rings on Taub-NUT:

  Ford et al. (static, $SL(3,\mathbb{R})$+ISM)

  Camps et al. (one rotational parameter, both S^1 and S^2-rotating)

Construction: inverse-scattering method

- ISM of black ring (3-soliton)+ISM of Taub-NUT (1-soliton)



Two steps:

- Remove a soliton at each of $z_{1,2,3,5}$, with trivial BZ vectors;
- Add back a soliton at each of $z_{1,2,3,5}$, with non-trivial BZ vectors, introducing four BZ parameters $C_{1,2,3,5}$.

A useful trick: Eliminate a turning point by joining up two neighboring rods (requiring their directions to be the same).

- Join up rods 1 and 2 by fixing C_1: adding rotation;
- Join up rods 5 and 6 by fixing C_5: adding nut charge.

Emparan-Reall black ring on Taub-NUT

- Set $C_2 = C_3 = 0$: 2-soliton transformation.
- Cast to C-metric coord (3 genuine turning points), and look for a simple form (difficult task). 4-parameter final solution:

$$
\begin{aligned}
\mathrm{d}s^2 &= -\frac{K(x,y)}{H(x,y)}(\mathrm{d}t + \omega_1\,\mathrm{d}\psi + \omega_2\,\mathrm{d}\phi)^2 + \frac{F(x,y)}{K(x,y)}(\mathrm{d}\psi + \omega_3\,\mathrm{d}\phi)^2 \\
&\quad -\frac{4\varkappa^4 G(x)G(y)H(x,y)}{(x-y)^4 F(x,y)}\,\mathrm{d}\phi^2 + \frac{4\varkappa^4(1-c)^2 H(x,y)}{(a-c)(1-b)\Psi(x-y)^3}\left(\frac{\mathrm{d}x^2}{G(x)} - \frac{\mathrm{d}y^2}{G(y)}\right), \\
\omega_1 &= \frac{\sqrt{b(1+b)(b-c)\Psi}}{K(x,y)}(x-y)J_2(x,y)\,, \\
\omega_2 &= \frac{\varkappa^2(a-1)(a+c)\sqrt{b(1-b^2)(b-c)}}{\sqrt{a-c}\,(x-y)K(x,y)}[(1+y)(x-y)J_1(x,y) - (1+x)(1+cx)J_3(x \\
\omega_3 &= \frac{\varkappa^2(a-1)(a+c)\sqrt{1-b}}{\sqrt{(a-c)\Psi}(x-y)F(x,y)}\Big\{2(a-1)(b-c)(a-c)(x-1)(1+by)(x+y+2) \\
&\quad +(1+by)[(1-b)(a-c)(1-cx) - 2(1-c)(a+bcx)]J_3(x,y) \\
&\quad +(1+b)(b-c)(1+y)(x-y)J_2(x,y)\Big\}\,,
\end{aligned}
$$

where G, H, F, K, $J_{1,2,3}$ are some polynomials and Ψ is a constant.

Some properties

It generalises the Emparan-Reall BR to Taub-NUT space:

- Correct rod structure: $S^1 \times S^2$ horizon topology.
- Background space: Taub-NUT.
- Purely S^1-rotating: $J_{S^2} = 0$.
- $n \to \infty$ recovers the Emparan-Reall black ring.
- Completely regular if balance is imposed. No naked singularities, no CTCs, no Dirac-Misner.

Interpretation in Kaluza-Klein theory

KK reduction along $\frac{\partial}{\partial\psi}$ yields:

- nut $\rightarrow$ magnetic monopole +
- S^1-rotating BR $\rightarrow$ static electric BH

Some properties:

- $J = PQ$: arising from $E \times B$, although the monopole and electric BH are separately (intrinsically) static.
- Extremal limit: extremal electric BH + magnetic monopole, with a repulsive net force!

PS black rings on Taub-NUT

- ISM: $C_{2,3} \neq 0$ to add S^2-rotation, 2-soliton on ER on TN.
- Background: Taub-NUT; $n \to \infty$ recovers PS.
- 4-parameter family. Complicated explicit form in C-metric coord.
- KK reduction: rotating electric BH + monopole.
- Two classes of extremal limit: positive/negative S^2 rotations.

Summary and outlook

Summary:

- ER and PS black rings have been generalised to TN space.

Outlook:

- Black Saturn on TN $\rightarrow$ (non-extremal) EM dihole.
- BHs/BRs on other gravitational instantons, e.g., Euclidean Kerr and Taub-bolt: Add an extra soliton at z_4.