Anti-de Sitter black holes and solitons with scalar hair

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Marcel Grossmann Meeting (MG13)
Stockholm, Sweden
5th July 2012

Collaborations and References

Work done in collaboration with:

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References:

Y. Brihaye and B. Hartmann, Phys. Rev. D 81 (2010) 126008
Y.Brihaye and B. Hartmann, Phys.Rev. D 84 (2011) 084008
Y. Brihaye and B. Hartmann, Phys. Rev. D (2012), in press

Outline

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1. Introduction
2. The model
3. Black holes with hyperbolic horizon ($k = -1$)
4. Black holes with planar horizon ($k = 0$)
5. Black holes with spherical horizon ($k = 1$)
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Introduction

- **Anti-de Sitter/Conformal fiel theory (AdS/CFT) correspondence** applied: use AdS black hole and soliton solutions of classical gravity theory to describe phenomena appearing in strongly coupled Quantum Field Theories $\Rightarrow$ holographic superconductors, quark-gluon plasma ...
- Instability of black holes in d-dimensional AdS_d (radius L) with respect to condensation of scalar field related to **Breitenlohner-Freedman bound** on mass m of scalar field (Breitenlohner & Freedman, 1982)

$$m^2 \geq m^2_{\mathrm{BF,d}} = -\frac{(d-1)^2}{4L^2} \Rightarrow \ AdS_d \ \textbf{stable}$$

Introduction

- Two type of instabilities
  - (Gubser, 2008) **scalar field charged** under $U(1)$, charge e

    $$m_{\rm eff}^2 = m^2 - e^2|g^{tt}|A_t^2$$

    if $m^2 \geq m^2_{\rm BF,d}$: asymptotic AdS_d stable
    $e^2|g^{tt}|$ large close to horizon of black hole $\Rightarrow m^2_{eff} < m^2_{\rm BF,d}$
    close to horizon $\Rightarrow$ black hole forms scalar hair
  - **uncharged** scalar field
    near-horizon geometry of **extremal** black holes given by $AdS_2 \times M_{d-2}$ (Robinson, 1959; Bertotti, 1959; Bardeen & Horowitz, 1999)
    if $m^2_{\rm BF,2} > m^2 > m^2_{\rm BF,d} \Rightarrow$ asymptotic AdS_d stable, but black hole forms scalar hair

Introduction

- gauge/gravity duality relates **strongly coupled** Quantum Field Theories (QFT) to **weakly coupled** gravity theories (and vice versa)
- Einstein gravity corresponds to large N limit on QFT side
- away from large N limit on QFT side: include “stringy” corrections on gravity side
- Gauss-Bonnet terms are corrections that appear in low energy effective action of String Theory

Outline

Action

Gauss-Bonnet gravity + scalar field ψ + U(1) gauge field A_μ

$$S = \frac{1}{16\pi G}\int d^5x\sqrt{-g}\,\Big(16\pi G\mathcal{L}_{\rm matter} + R - 2\Lambda + \frac{\alpha}{4}\left(R^{\mu\nu\lambda\rho}R_{\mu\nu\lambda\rho} - 4R^{\mu\nu}R_{\mu\nu} + R^2\right)\Big)$$

with matter Lagrangian

$$\mathcal{L}_{\rm matter} = -\frac{1}{4}F_{MN}F^{MN} - (D_M\psi)^* D^M\psi - m^2\psi^*\psi \ \ , \ \ M,N = 0,1,2,3,4$$

$F_{MN} = \partial_M A_N - \partial_N A_M$ field strength tensor
$D_M\psi = \partial_M\psi - ieA_M\psi$ covariant derivative
$\Lambda = -6/L^2$: cosmological constant
G: Newton's constant , α: Gauss–Bonnet coupling
e: gauge coupling , m^2: mass of the scalar field ψ

Ansatz for static solutions

- Metric

$$ds^2 = -f(r)a^2(r)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{L^2}d\Sigma^2_{k,3}$$

with

$$d\Sigma^2_{k,3} = \begin{cases} d\Xi^2_3 & \text{for } k=-1 \text{ hyperbolic horizon} \\ dx^2+dy^2+dz^2 & \text{for } k=0 \text{ flat horizon} \\ d\Omega^2_3 & \text{for } k=1 \text{ spherical horizon} \end{cases}$$

- Matter fields

$$A_M dx^M = \phi(r)dt \quad , \quad \psi = \psi(r)$$

Equations of motion

$$
\begin{aligned}
f' &= 2r\frac{k-f+2r^2/L^2}{r^2+2\alpha(k-f)} \\
&- \gamma\frac{r^3}{2fa^2}\left(\frac{2e^2\phi^2\psi^2+f(2m^2a^2\psi^2+\phi'^2)+2f^2a^2\psi'^2}{r^2+2\alpha(k-f))}\right) \\
a' &= \gamma\frac{r^3(e^2\phi^2\psi^2+a^2f^2\psi'^2)}{af^2(r^2+2\alpha(k-f))} \\
\phi'' &= -\left(\frac{3}{r}-\frac{a'}{a}\right)\phi'+2\frac{e^2\psi^2}{f}\phi \\
\psi'' &= -\left(\frac{3}{r}+\frac{f'}{f}+\frac{a'}{a}\right)\psi'-\left(\frac{e^2\phi^2}{f^2a^2}-\frac{m^2}{f}\right)\psi
\end{aligned}
$$

where $\gamma = 16\pi G$

Boundary conditions

- Regular horizon at $r = r_h > 0$

$$f(r_h) = 0 \quad , \quad a(r_h) \ \text{finite}$$

$$\phi(r_h) = 0 \ , \ \psi'(r_h) = \left. \frac{m^2\psi\left(r^2 + 2\alpha k\right)}{2rk + 4r/L^2 - \gamma r^3\left(m^2\psi^2 + \phi'^2/(2a^2)\right)} \right|_{r=r_h}$$

Behaviour on the AdS boundary

- Matter field

$$\phi(r \gg 1) = \mu - \frac{Q}{r^2} \quad , \quad \psi(r \gg 1) = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}$$

where

$$\lambda_{\pm} = 2 \pm \sqrt{4 + m^2 L_{\rm eff}^2}$$

with effective AdS radius

$$L_{\rm eff}^2 \equiv \frac{2\alpha}{1 - \sqrt{1 - 4\alpha/L^2}} \sim L^2 \left(1 - \alpha/L^2 + O(\alpha^2)\right)$$

Q: charge ($k = 1$); charge density ($k = -1$, $k = 0$)

Behaviour on the AdS boundary

Asymptotically Anti-de Sitter space-time

- Metric functions

$$
\begin{aligned}
f(r \gg 1) &= k + \frac{r^2}{L_{\rm eff}^2} + \frac{f_2}{r^2} + O(r^{-4}) \\
a(r \gg 1) &= 1 + \frac{a_4}{r^4} + O(r^{-6})
\end{aligned}
$$

f_2, a_4 constants (have to be determined numerically)

Properties of black holes

- **specific heat**

$$C = T_{\rm H}\left(\partial S/\partial T_{\rm H}\right)$$

in canonical ensemble: $C > 0 \Rightarrow$ black hole stable
$C < 0 \Rightarrow$ black hole unstable

- **Energy** E

$$\frac{16\pi G E}{V_3} = \sqrt{1 - \frac{\alpha}{L^2}}\left(-3 f_2 - 8\frac{a_4}{L^2_{\rm eff}}\right)$$

V_3: volume of the 3-dimensional space

- **Free energy** $F = E - T_{\rm H} S$ with **Hawking temperature** $T_{\rm H}$ and **entropy** S

$$T_{\rm H} = \frac{f'(r_h)a(r_h)}{4\pi} \quad , \quad \frac{S}{V_3} = \frac{r_h^3}{4G}\left(1 - \frac{6\alpha}{r_h^2}\right)$$

Black holes without scalar hair

(Boulware & Deser, 1982; Cai, 2003)

For $m^2 > m^2_{\mathrm{BF},2} \Rightarrow \psi(r) \equiv 0$ and

$$\phi(r) = \frac{Q}{r_h^2} - \frac{Q}{r^2}$$

$$f(r) = k + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 - \frac{4\alpha}{L^2} + \frac{4\alpha M}{r^4} - \frac{4\alpha\gamma Q^2}{r^6}}\right) \quad , \quad a(r) \equiv 1$$

M: mass parameter , Q: charge (density)
event horizon at $f(r_h) = 0$

Outline

Uncharged black holes for $k = -1$

- Uncharged black holes $Q = 0$; uncharged scalar field $e = 0$
- for $k = -1$ extremal solution with $f(r_h) = 0$, $f'(r)|_{r=r_h} = 0$ exists
- extremal solution has $T_{\rm H} = 0$, $r_h^{(\rm ex)} = L/\sqrt{2}$
- close to **extremality** horizon topology is $AdS_2 \times H^3$
(Astefanesei, Banerjee & Dutta, 2008)
- hyperbolic Gauss-Bonnet black holes in $d = 5$ have AdS_2 radius

$$R = \sqrt{L^2/4 - \alpha}$$

(Y. Brihaye & B.H., PRD 84, 2011)

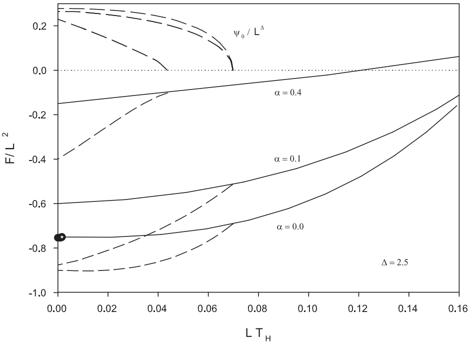
- asymptotic AdS_5 stable, near-horizon AdS_2 unstable for

$$m^2_{\rm BF,5} = -\frac{4}{L^2_{\rm eff}} \le m^2 \le -\frac{1}{4R^2} = m^2_{\rm BF,2}$$

Black holes with scalar hair, $\gamma \neq 0$, $\alpha \neq 0$

(Y. Brihaye & B.H., PRD 84, 2011)

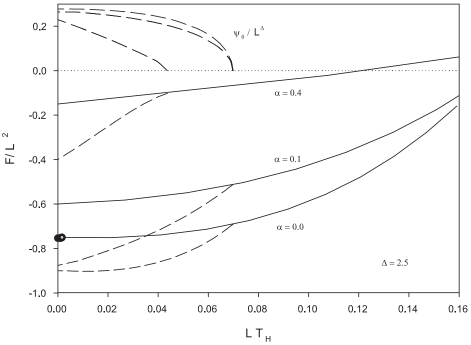
- black holes with scalar hair **thermodynamically preferred**



Black holes with scalar hair, $\gamma \neq 0$, $\alpha \neq 0$

(Y. Brihaye & B.H., PRD 84, 2011)

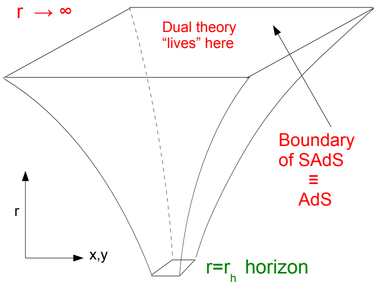
- the larger α the lower $T_{\rm H}$ at which instability appears



Outline

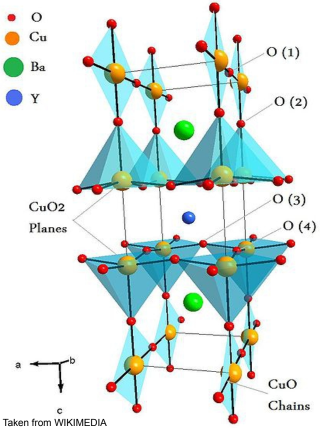
Black holes with planar horizon in AdS

- $k = 0$: planar horizon
- charged scalar field $e \neq 0$
- $r \to \infty$: planar AdS boundary



Taken from arxiv: 0808.1115

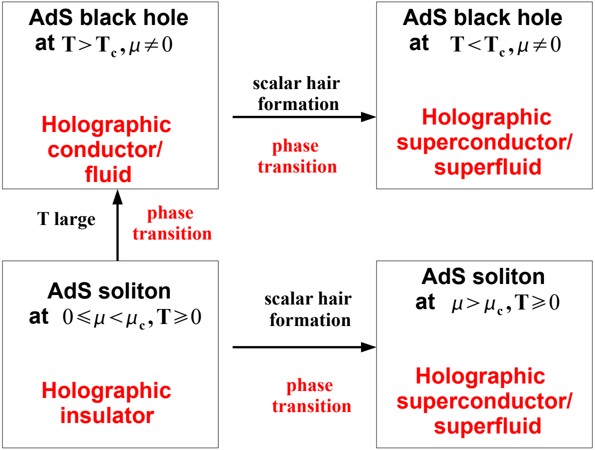
Applications to holographic superconductors



Taken from WIKIMEDIA

- Example of high temperature superconductor: Yttrium(Y)-barium(BA)-copper(Cu)-oxide(O)
- highest possible $T_c = 92$K
- **superconductivity associated to CuO_2-planes**
- BCS theory not valid
- **strongly interacting Quantum field theories**

Holographic phase transitions



Including Gauss-Bonnet corrections

- **Mermin-Wagner theorem**: spontaneous symmetry breaking forbidden in (2+1) dimensions at finite temperature, but holographic superconductors (in Einstein gravity) have been constructed (see e.g. (Hartnoll, Herzog & Horowitz, 2008))

Q: Can Gauss-Bonnet corrections suppress condensation?

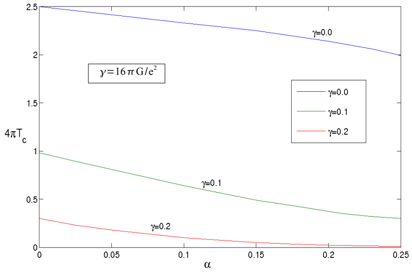
- for $G = 0$ (no backreaction): condensation can **not** be suppressed for (3+1)-dimensional Holographic Gauss-Bonnet superconductors (Gregory, Kanno & Soda, 2009)

Q: Can backreaction suppress condensation?

Including Gauss-Bonnet corrections

(Brihaye & B. Hartmann, Phys. Rev. D 81, 2010)

- Gauss-Bonnet coupling $0 \leq \alpha \leq L^2/4$

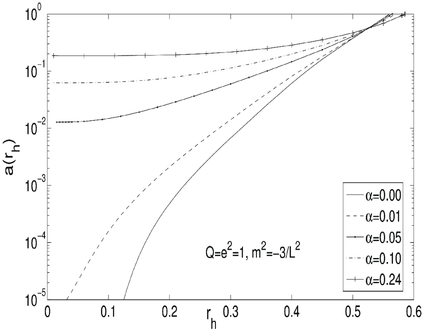


$\Longrightarrow$ **condensation gets harder for $\alpha > 0$, but not suppressed**

Outline

Charged black hole with scalar hair, $k = 1$

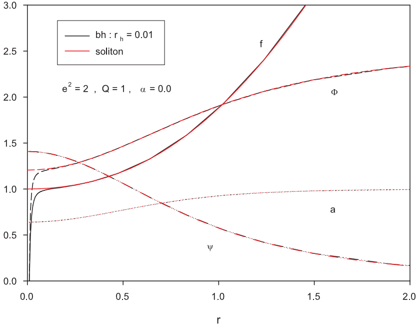
(Brihaye & B. Hartmann, Phys. Rev. D, in press)



- For small α: solution exists down to $r_h = 0$
  $\rightarrow$ **soliton?**
- For large α: solution has $a(r_h) \rightarrow 0$ for $r_h \rightarrow r_h^{(cr)} > 0$
  $\rightarrow$ **extremal black hole?**

Charged black hole with scalar hair, $k = 1$

(Brihaye & B. Hartmann, Phys. Rev. D, in press)

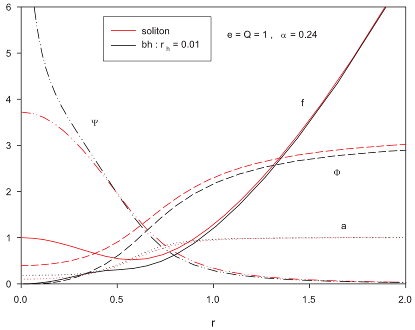


For $\alpha = 0$:

- Black hole tend to soliton solutions in the limit $r_h \to 0$

Charged black hole with scalar hair, $k = 1$

(Brihaye & B. Hartmann, Phys. Rev. D, in press)



$\alpha \neq 0$:

- Gauss-Bonnet solitons with scalar hair exist
- black holes with scalar hair do **not** tend to corresponding solitons for $r_h \to 0$

Charged black hole with scalar hair, $k = 1$

(Brihaye & B. Hartmann, Phys. Rev. D, in press)

There exist no extremal Gauss-Bonnet black holes with scalar hair.

Charged black hole with scalar hair, $k = 1$

Proof:

- assume near-horizon geometry to be $AdS_2 \times S^3$:

$$ds^2 = v_1 \left(-\rho^2 d\tau^2 + \frac{1}{\rho^2} d\rho^2\right) + v_2 \left(d\psi^2 + \sin^2\psi \left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right)$$

  v_1, v_2: positive constants

- Combination of equations of motion yields

$$0 = 16\pi G \left(\frac{\rho^2}{v_1}\psi'^2 + \frac{e^2\phi^2\psi^2}{\rho^2 v_1}\right)$$

  This leads to: $\psi' = 0$ and $\phi^2\psi^2 = 0$ in near horizon geometry

- $\phi^2 = 0$ ruled out $\rightarrow \psi \equiv 0$ **in hear horizon geometry** q.e.d.

Conclusions & Outlook

Conclusions

- Two mechanisms can make AdS black holes unstable to scalar condensation
  - **uncharged scalar field**: black holes with AdS_2 factor in near-horizon geometry (near-extremal black holes)
    Example in this talk: uncharged, static black holes with hyperbolic horizon ($k = -1$*)*
  - **charged scalar field**: coupling to gauge field lowers effective mass of scalar field
    Examples in this talk: charged, static black holes with flat or spherical horizon ($k = 0$*,* $k = 1$*)*
- Black holes with scalar hair thermodynamically preferred

Outlook

- Instabilities of other black holes in AdS_d
  - charged and rotating Einstein black holes:
    Y. Brihaye & B.H., JHEP, 2012
  - charged and rotating Gauss-Bonnet black holes:
    Y. Brihaye & B.H., in preparation
- Implications on CFT side?
- Implications for stability of String Theory/Supergravity black holes?