Chaos in geodesic motion around a black ring

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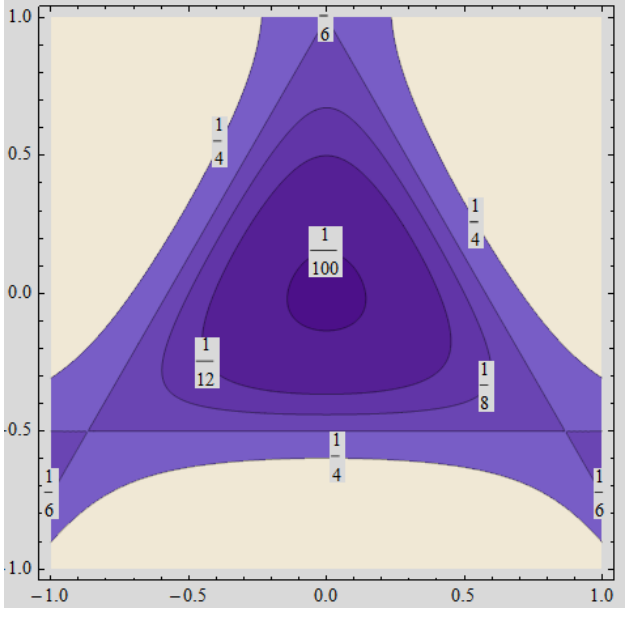
Introduction

non-integrable system

- **non-integrable system**
  - #(COM) < #(DOF)
- **Chaos**
  - sensitivity to initial cond.
  - density of periodic orbits
- **examples**
  - 3-body problem [Poincare(1890)]
  - Henon-Heiles system [Henon-Heiles(1964)]

$$V = \frac{1}{2}(x^2 + y^2 + 2x^2 y - \frac{2}{3}y^3)$$



Chaos in GR

- **two main issues**
  - chaotic oscillations near BKL singularity
  - chaotic particle dynamics
    - charged particle + mag. BH [Varvoglis-Papadopoulos(1992)]
    - particle + perturbed BH [Bombell-Calzetta(1992)]
    - spinning particle + BH [Suzuki-Maeda(1997)]
    - particle + multiple BHs [Contopoulos(1990)]

integrable mechanics

- **enough symmetries**
  - #(COM) ≧ #(DOF)
- **example [Kerr geodesic]**
  - Integrable (additional quadratic consts.)
    [Carter(1968), Frolov-Krtous-Kubiznak (2007)]
  - Killing tensors (hidden sym.)
    [Walker-Penrose(1972)]
- **question**
  - How about black ring geodesic??

Black Ring

geometry [Emparan-Reall(2002)]

metric

$$ds^2 = -\frac{F(y)}{F(x)}\left(dt - CR\frac{1+y}{F(y)}d\Psi\right)^2 + \frac{R^2}{(x-y)^2}F(x)\left[-\frac{G(y)}{F(y)}d\Psi^2 - \frac{dy^2}{G(y)} + \frac{dx^2}{G(x)} + \frac{G(x)}{F(x)}d\Phi^2\right]$$

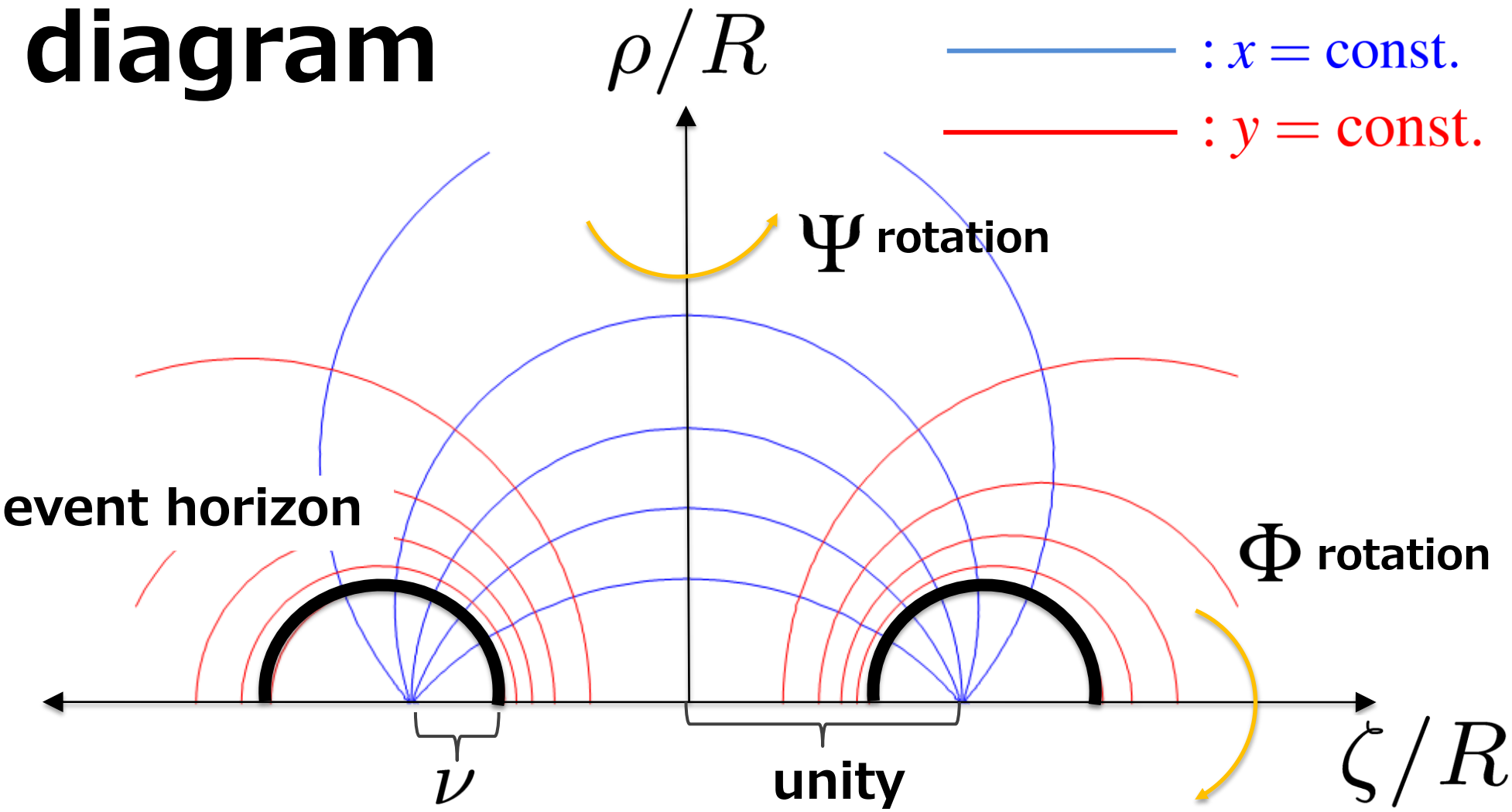
$$F(\xi) = 1+\lambda\xi, \quad G(\xi) = (1-\xi^2)(1+\nu\xi), \quad C = \sqrt{\lambda(\lambda-\nu)\frac{1+\lambda}{1-\lambda}}, \quad \lambda = \frac{2\nu}{1+\nu^2}$$

2-parameters

ν : thickness $(0 < \nu < 1)$ R : ring radius

3-Killings: t^a, Φ^a, Ψ^a

diagram



new coordinates

$$\zeta = R\frac{\sqrt{y^2-1}}{x-y} \qquad \rho = R\frac{1-x^2}{x-y}$$

geodesic motion

- **EOM**
  - 3-COMs (E:energy, ℓ_Φ, ℓ_Ψ :2-angular momenta)
  - constraint eq.

$$H = g^{ij} p_i p_j + E^2 \left(U + \frac{\kappa}{E^2} \right) = 0 \quad (i, j = \zeta, \rho)$$

  - 2d effective potential

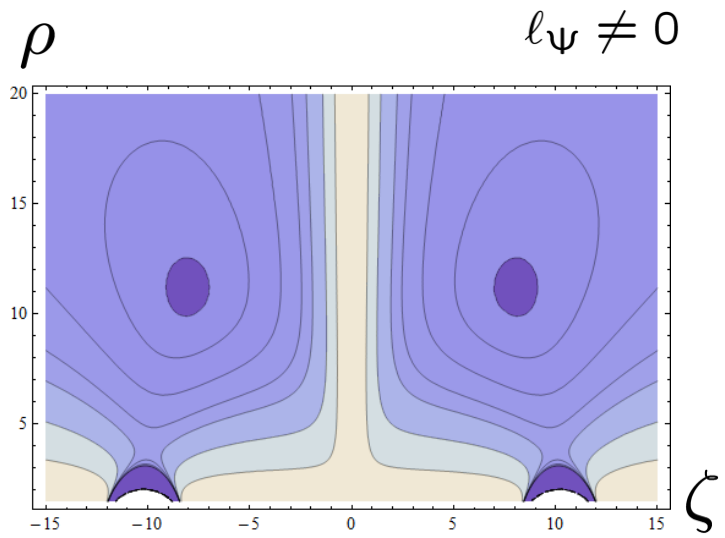
$$U = U(\zeta, \rho; \ell_\Phi, \ell_\Psi)$$

  - “non-separable” [Hoskisson(2007)]

bounded orbits

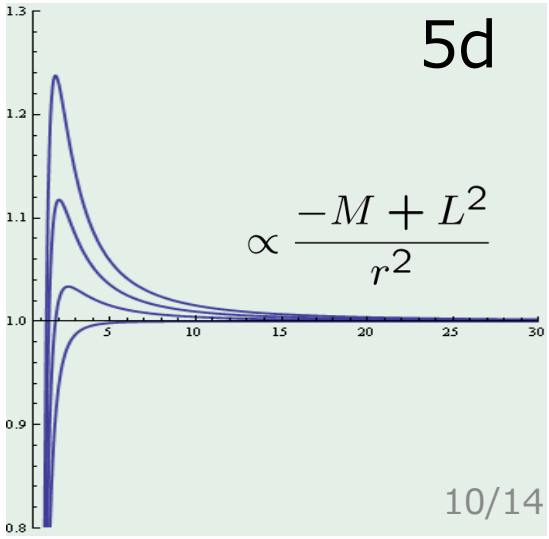
- **bound states** [TI-Ishihara-Takamori(2010)]
  - exist near the axis for $0 < \nu \leq \nu_0$
  - NOT exist for $\nu_0 < \nu \leq 1$
  - critical thickness

    $\nu_0 = 0.65379\cdots$ (the exact value exists)
  - photon has also bound states



- **difference from BH**
  - NOT exist in HDBH

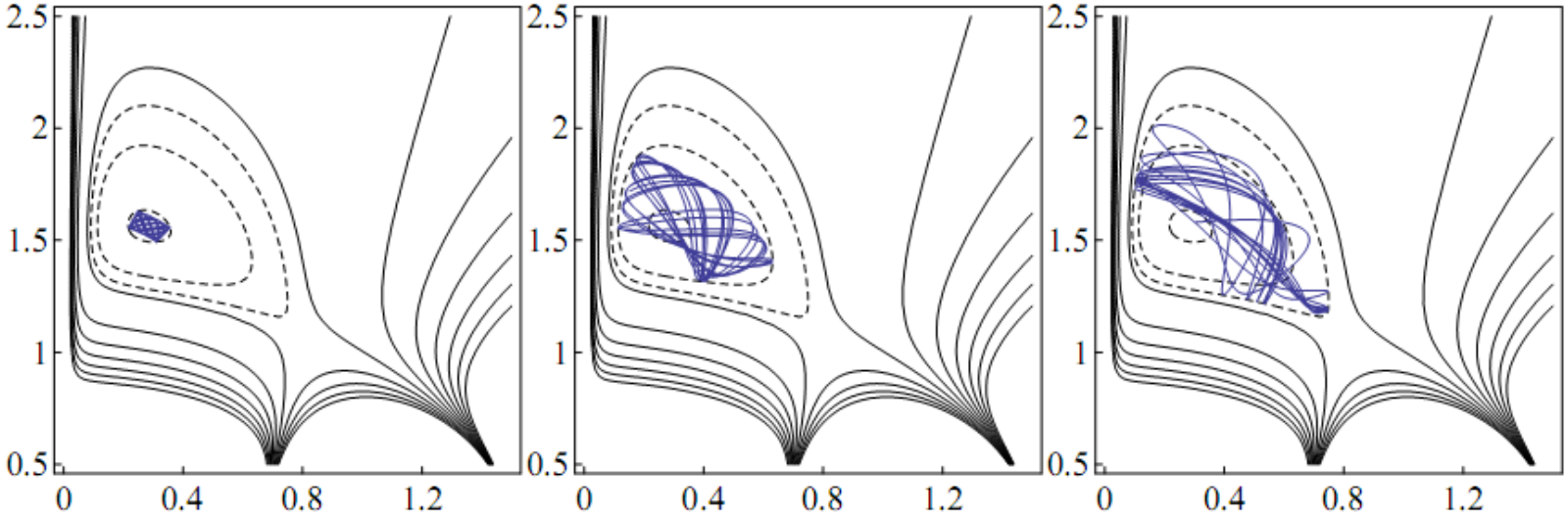
$$V = \frac{\kappa}{2} - \frac{\kappa M}{2r^{D-3}} + \frac{L^2}{2r^2} - \frac{ML^2}{2r^{D-1}},$$



results

chaotic orbits

- **bound orbits**
  - projected orbits for various energies
    - $\nu = 0.4 \quad \ell_\phi = 1.52,\ \ell_\psi = 0.02$



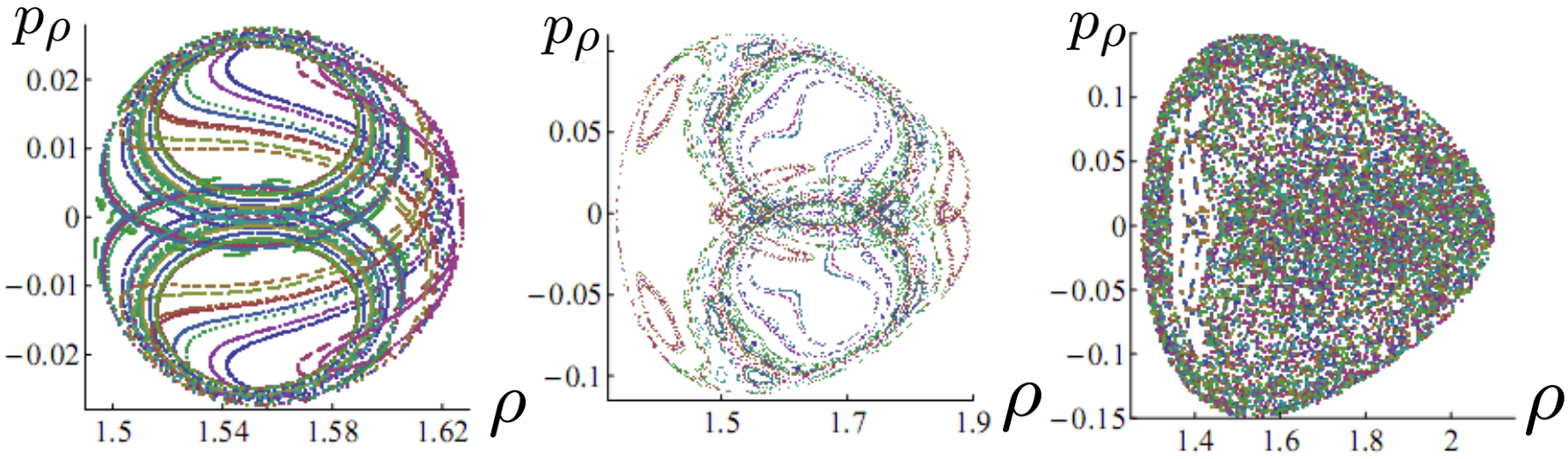
$E = E_1, E_2, E_3\ (E_1 < E_2 < E_3)$

  - complex orbits appear
  - chaotic orbits? → Poincare section

Characteristic of chaos

- **Poincare section**
  - exact periodic motion = phase space orbit is on a torus
  - characteristic of density of periodic orbits

➢ $\nu = 0.4 \quad \ell_\phi = 1.52,\ \ell_\psi = 0.02$



  - the scattered points imply the particle motion is chaos

conclusions

- **chaos appears in geodesic motion around a BR**

  - no additional const. (no Killing tensor)

future works

- thin ring case ($\nu \ll 1$) or fat ring case ($\nu \simeq 1$)
- black ring as photon storage
- null geodesic chaos