Analyticity of event horizons of extremal Kaluza-Klein[KK] BHs

[Can we construct a regular extremal KKBH?]

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Introduction & Summary

We may live in a higher-dim space time with compactified extra-dims. (KK space time)

As a first step, It is important to construct and investigate a toy model of BH in KK space time (KKBH) to understand qualitative feature of KKBH.

In this talk, we discuss the analyticity of horizon of some toy model of KKBH and show that curvature tensor is always diverge at the horizon in $D \geq 6$.

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A toy model of KKBH

Multi black holes

(Majumdar-Papapetrou 1947)
(Myers 1987)

D dim Einstein-Maxwell system

$$S = \frac{1}{16\pi G_D}\int dx^D \sqrt{-g}(R - F_{\mu\nu}F^{\mu\nu})$$

$$\Longrightarrow R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 2\left(F_{\mu\lambda}F_{\nu}{}^{\lambda} - \frac{1}{4}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma}\right)$$

$$\nabla_\nu F^{\mu\nu} = 0$$

anzats

$$ds^2 = -H^{-2}dt^2 + H^{2/(D-3)}\left[dr^2 + r^2 d\Omega^2_{S^{D-2}}\right]$$

$$A_\mu dx^\mu = \pm\sqrt{\frac{D-2}{2(D-3)}}H^{-1}dt$$

Einstein eq. and Maxwell eq. reduce to

$\Longrightarrow \quad \triangle_{E^{D-1}} H = 0$ (Laplace eq. on E^{D-1})

$$(\ ds^2 = -H^{-2}dt^2 + H^{2/(D-3)}\left[dr^2 + r^2 d\Omega^2_{S^{D-2}}\right] \ )$$

Point source sols. are

$$H = 1 + \sum_{i=1}^{N} \frac{M_i}{|\vec{x} - \vec{x}_i|^{D-3}}$$

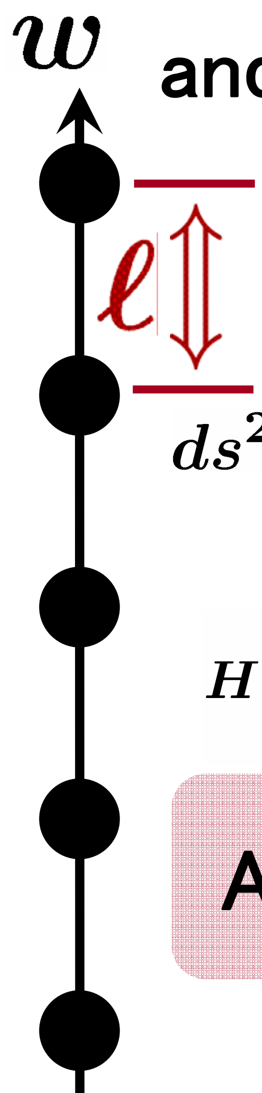
$$= 0 \iff g_{tt} = 0$$

Each black hole locates at $\vec{x} = \vec{x}_i$

This sol. describes multi black holes.

Kaluza-Klein black holes
(Myers 1987)

Let us consider to superpose BHs periodically and to take identification by the period.



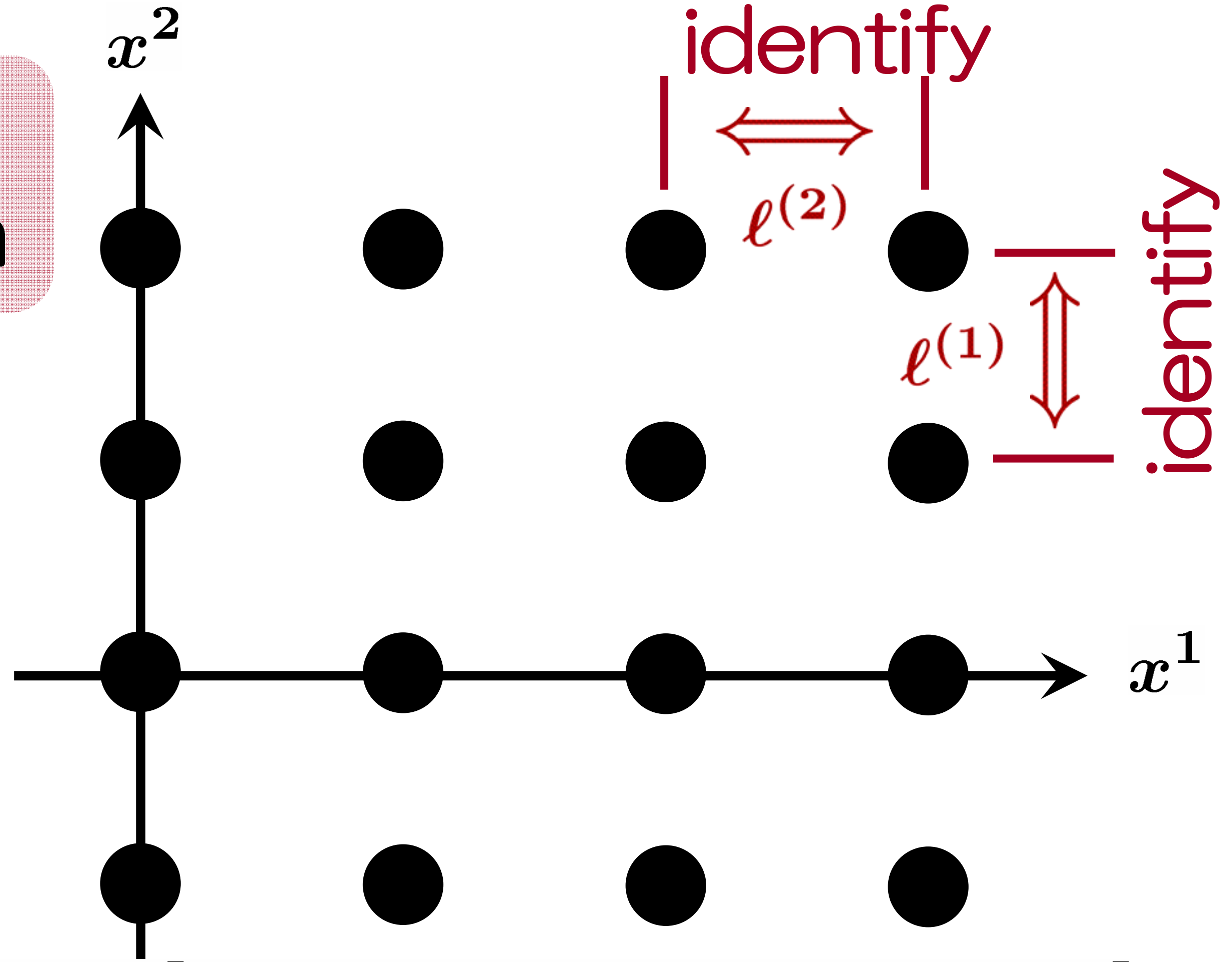
identify ℓ : size of extra dim.

$$ds^2 = -H^{-2}dt^2 + H^{2/(D-3)}\left[dx_1^2 + dx_2^2 + \cdots + dx_{D-2}^2 + dw^2\right]$$

$$H = 1 + \sum_{n=-\infty}^{\infty} \frac{M}{(x_1^2 + x_2^2 + \cdots + x_{D-2}^2 + (w + n\ell)^2)^{(D-3)/2}}$$

A toy model of KKBH with $\mathbf{S}^1$ extra dim

KKBH with T^N extra dim



$$ds^2 = -H^{-2}dt^2 + H^{2/(D-3)}\left[\sum_{IJ=1}^{N}\delta_{IJ}dx^I dx^J + \sum_{AB=N+1}^{D-1}\delta_{AB}dx^A dx^B\right]$$

$$H = 1 + \sum_{\{n_1 \sim n_N\}} \frac{M}{\left[\sum_{IJ=1}^{N}\delta_{IJ}(x^I - n_I\ell^{(I)})(x^J - n_J\ell^{(J)}) + \sum_{AB=N+1}^{D-1}\delta_{AB}dx^A dx^B\right]^{(D-3)/2}}$$

Analyticity at the event horizon

Analyticity of event horizon
[two BHs case]

First, we discuss the case of two BHs.

$$ds^2 = -H^{-2}dt^2 + H^{2/(D-3)}\left[dr^2 + r^2 d\Omega^2_{S^{D-2}}\right]$$

$$H = 1 + \frac{M_1}{r^{D-3}} + \frac{M_2}{(r^2 - 2ar\cos\theta + a^2)^{(D-3)/2}}$$

To find an extension across the horizon we introduce new coord. u.

$$dt = du - H^{(D-2)/(D-3)}dr + W d\theta$$

$$W = -\int \partial_\theta H^{(D-2)/(D-3)} dr$$

Near $r = 0$

$$ds^2 \simeq -r^{2(D-3)}du^2 - 2r^{(D-4)}drdu + r_{\mathrm{H}}^2 d\Omega_{S^{D-2}}^2$$

To avoid coord. sing., we should further introduce new radial coord. $\tilde{r}$.

$$r^{(D-4)}dr = d\tilde{r} \to r \propto \tilde{r}^{1/(D-3)}$$

However $H = 1 + \frac{M_1}{r^{D-3}} + \frac{M_2}{(r^2 - 2ar\cos\theta + a^2)^{(D-3)/2}}$

$r = \tilde{r}^{1/(D-3)}$

H contains fractional power of $\tilde{r}$ if $D \geq 5$.

Naive extension is not analytic in the case of higher dimensional multi BHs.

In fact, we can show that there is no extension where the metric function is analytic at horizon.

(Welch1995)

(Candlish, Reall 2007)

$D = 5$ case：horizon is C^2 but not C^3

$$R_{\mu\nu\rho\sigma}(\tilde{r} \to 0) = \text{finite} \qquad \nabla_{\tilde{r}} R_{\tilde{r}u\theta\tilde{r}}(\tilde{r} \to 0) \propto \frac{1}{\sqrt{\tilde{r}}}$$

(measured by a regular freefall observer)

$D \geq 6$ case：horizon is C^1 but not C^2

$$R_{\theta\tilde{r}\theta\tilde{r}}(\tilde{r} \to 0) \propto \frac{1}{\tilde{r}^{(D-5)/(D-3)}}$$

(measured by a regular freefall observer)

Analyticity of event horizon
[KK BHs case]

Whether the harmonics H contains fractional power of $\tilde{r}$ or not seems to be strongly related to non analyticity of the horizon.

【 S^1 case】

$$H = 1 + \sum_{n=-\infty}^{\infty} \frac{M}{(x_1^2 + x_2^2 + \cdots + x_{D-2}^2 + (w + n\ell)^2)^{(D-3)/2}}$$

$$= 1 + \sum_{n=-\infty}^{\infty} \frac{M}{(r^2 \sin^2\theta + (r\cos\theta + n\ell)^2)^{(D-3)/2}}$$

$$=: Y$$

$$= 1 + \frac{M}{r^{D-3}} + \sum_{n=-\infty (n\neq 0)}^{\infty} \frac{M}{(r^2 + 2rn\ell\cos\theta + n^2\ell^2)^{(D-3)/2}}$$

$$= Y(-r)$$

$$= 1 + \frac{M}{r^{D-3}} + \sum_{n=-\infty (n\neq 0)}^{\infty} \frac{M}{(r^2 + 2r(-n)\ell\cos\theta + n^2\ell^2)^{(D-3)/2}}$$

Y formally have $Y(r) = Y(-r)$ symmetry.

So we can expand H as

$$H = 1 + \frac{M}{r^{D-3}} + \sum_{n=1}^{\infty} Q_n(\theta) r^{2n}$$

$$= 1 + \frac{M}{\tilde{r}} + \sum_{n=0}^{\infty} Q_n \tilde{r}^{2n/(D-3)} \qquad (r \propto \tilde{r}^{1/(D-3)})$$

$$= 1 + \frac{M}{\tilde{r}} + Q_0 + Q_2(\theta)\tilde{r}^{2/(D-3)} + Q_4(\theta)\tilde{r}^{4/(D-3)} + \cdots$$

H contain fractional power of $\tilde{r}$ if $D \geq 6$.

Amazingly in $D = 5$ case, H does not contain fractional power of $\tilde{r}$ unlike the case of two BHs.

【 T^N case】

After some calculations, we can also show

$$H = 1 + \frac{M}{\tilde{r}} + Q_0 + Q_2(\theta_i)\tilde{r}^{2/(D-3)} + Q_4(\theta_i)\tilde{r}^{4/(D-3)} + \cdots$$

in the case of T^N extra dims.

PP (parallelly propagated) singularity

We consider the space time

$$ds^2 = -H^{-2}dt^2 + H^{2/(D-3)}\left[dr^2 + r^2 d\Omega^2_{S^{D-2}}\right]$$

$$H = 1 + \frac{M}{r^{D-3}} + Q_0 + Q_2(\theta)r^2 + Q_4(\theta)r^4 + \cdots$$

and the regular (nearly) free fall observer frame $(e_{(\mu)})^a$

$$(e_{(0)})^a = A\left(\frac{\partial}{\partial t}\right) + B\left(\frac{\partial}{\partial r}\right)$$

$$(e_{(2)})^a = C\left(\frac{\partial}{\partial \theta}\right)$$

$\cdots \quad \cdots$

where A, B, C should be chosen appropriately
so that each $(e_{(\mu)})^a$ is regular at the horizon $r = 0$.

we can show the leading behavior of the Riemann tensor measured by $(e_{(\mu)})^a$ as

$$R_{(0)(2)(0)(2)} \propto \frac{2(D-1)Q_2 + (D-2)\partial_\theta \partial_\theta Q_2}{r^{D-5}} \to \infty$$

Summary and Discussion

	two BHs	S^1 or T^N compactification
$D = 5$	C^2 (not C^3)	analytic
$D \geq 6$	C^1 (not C^2)	C^1 (not C^2)

- In ordinary sense, if $\ell_{(i)} \ll r_{\mathrm{BH}}$ we want to consider that BH can be treat as an isolated system. However this picture may be broken in extremal BH case in $D \geq 6$.

- Squashed KKBH also have same property in $D \geq 7$ (odd dimension). (Tatsuoka, Ishihiara, Kimura, Matuno, 2011)

Thank you

- The singularity is not much strong so that an observer can traverse the horizon.
  (The tidal force diverges but
  its integration is finite.)

- Each BH is maximally charged.

- Solutions are static because of a force balance between the gravitational attractive force and Coulomb repulsive force.