# 「Existence and some properties of Thermodynamic Black Di-rings」

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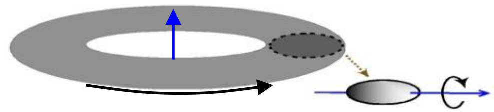
This poster contribution is based on the work in
Phys. Rev. D82, 084009(2010):arXiv:1008.4290v2 [hep-th]

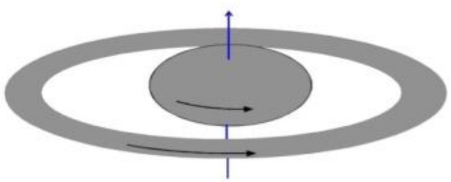
# I. Introduction

- Since the discovery of S^1 rotational black ring by Emparan and Reall several 5-dimensional black hole systems have been obtained using solitonic methods.

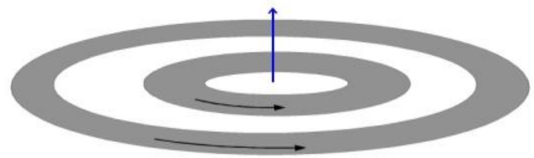
( asymptotically Minkowski cases )



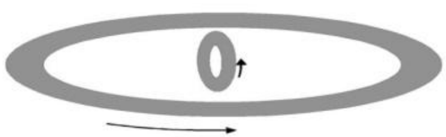
Black rings (full rotation)



Black Saturn



Black di-ring



Orthogonal black di-ring /Bi-ring

Black lense

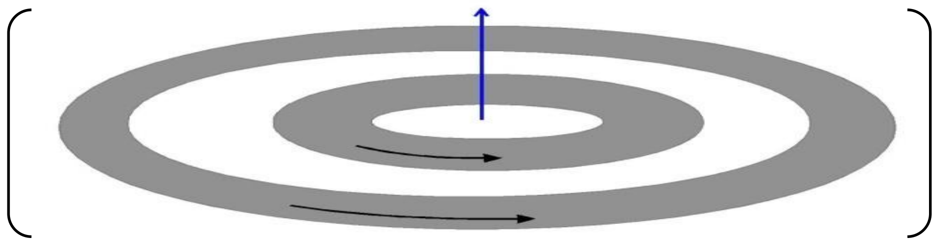
• • • •

- Previously we succeeded to superimpose two S^1-rotating black rings in concentric and regular way as simple multi-BH systems.
We call the solutions black di-rings.

**Here we concentrate the study on**

**black di-rings**

**( 5 dim. concentrically superimposed two S^1-rotating BRs )**



**< Two different solution-sets of black di-rings >**

**( diring I )**

**the Backlund-like transformation.**
**( Kramer-Neugebauer's -like method )**

**Iguchi & Mishima(I&M): hep-th/0701043**
**Phys. Rev. D75, 064018 (2007)**

**( diring II )**

**Inverse Scattering Method (ISM)**
**( Belinsky-Zakharov technique )**

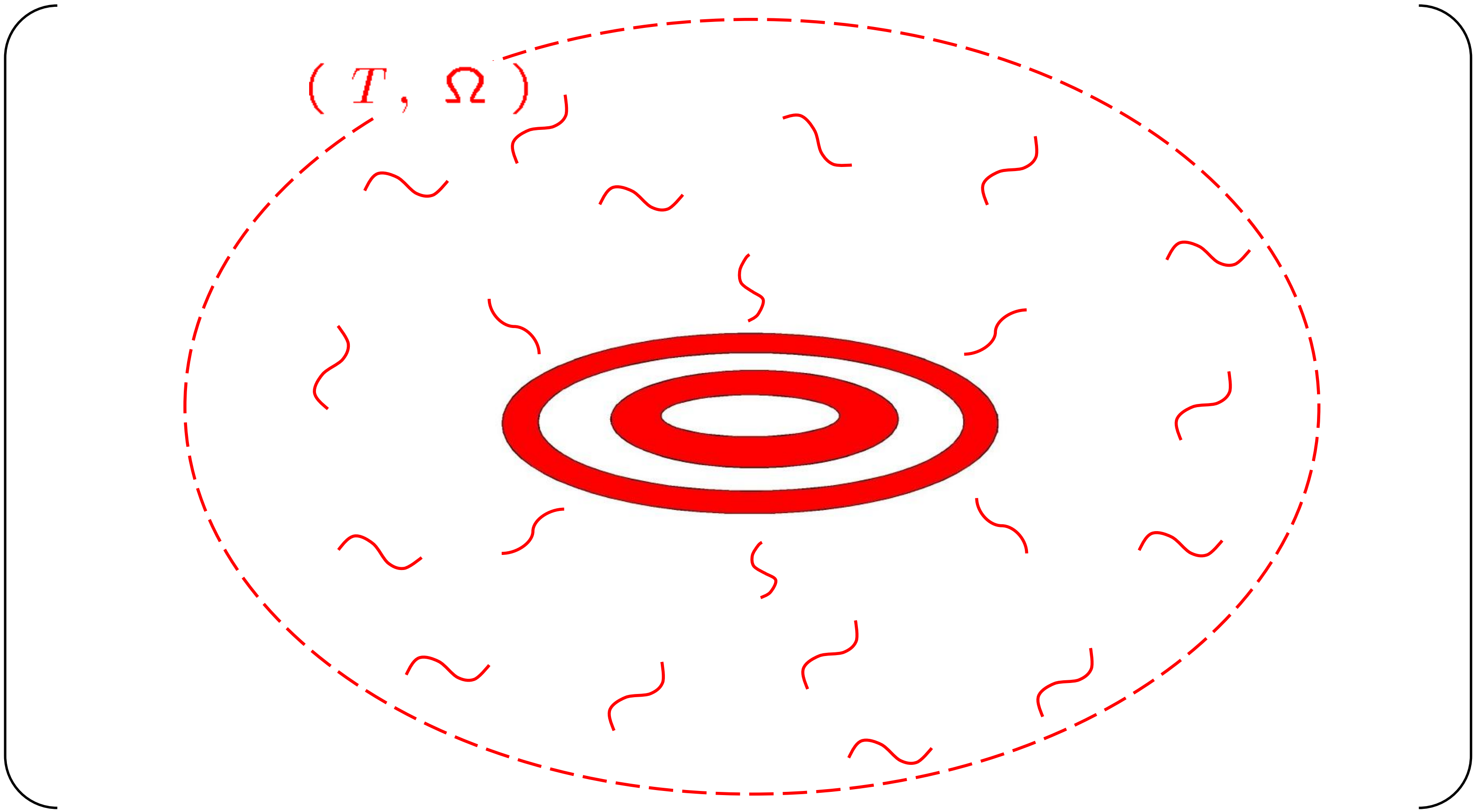
**Evslin & Krishnan(E&K): hep-th/0706.1231**
**CQG26:125018(2009)**

- **The above two solution-sets may be equivalent, confirmed with the aid of numerical methods.** ( See Phys. Rev. D82, 084009(2010): arXiv:1008.4290v2[hep-th] )
  **The choice depends on the problem.**

**< Purpose of this contribution >**

**We show some physical properties of the di-ring systems, especially the existence and properties of thermodynamic regular black di-ring systems ( globally defined thermal states of regular black di-rings ) .**

TBD



## II. Existence of Regular thermodynamic black di-rings

**Here we omit the explanation of the solution-generating methods and the expression of the di-ring solutions, and just show the results.**

(
For more details,
See Phys.Rev.D75,064018(2007) hep-th/0701043 (I&M)
CQG26:125018(2009) hep-th/0706.1231 (E&K)
)

**First some physical quantities are normalized using the ADM mass *M*.**

$$j^2 = \frac{27\pi}{32G}\frac{J^2}{M^3}, \quad a_h^2 = \frac{27}{256\pi}\frac{A_h^2}{(GM)^3},$$

$$\tau_h^2 = \frac{32\pi}{3}(GM)T_h^2, \quad \omega_h^2 = \frac{8}{3\pi}(GM)\Omega_h^2, \quad \cdots$$

(
***j*** **: total angular mom**
$a_h$ **: total horizon area**
$\tau_h$ **: horizon temperature**
$\omega_h$ **: horizon angular velocity**
…
)

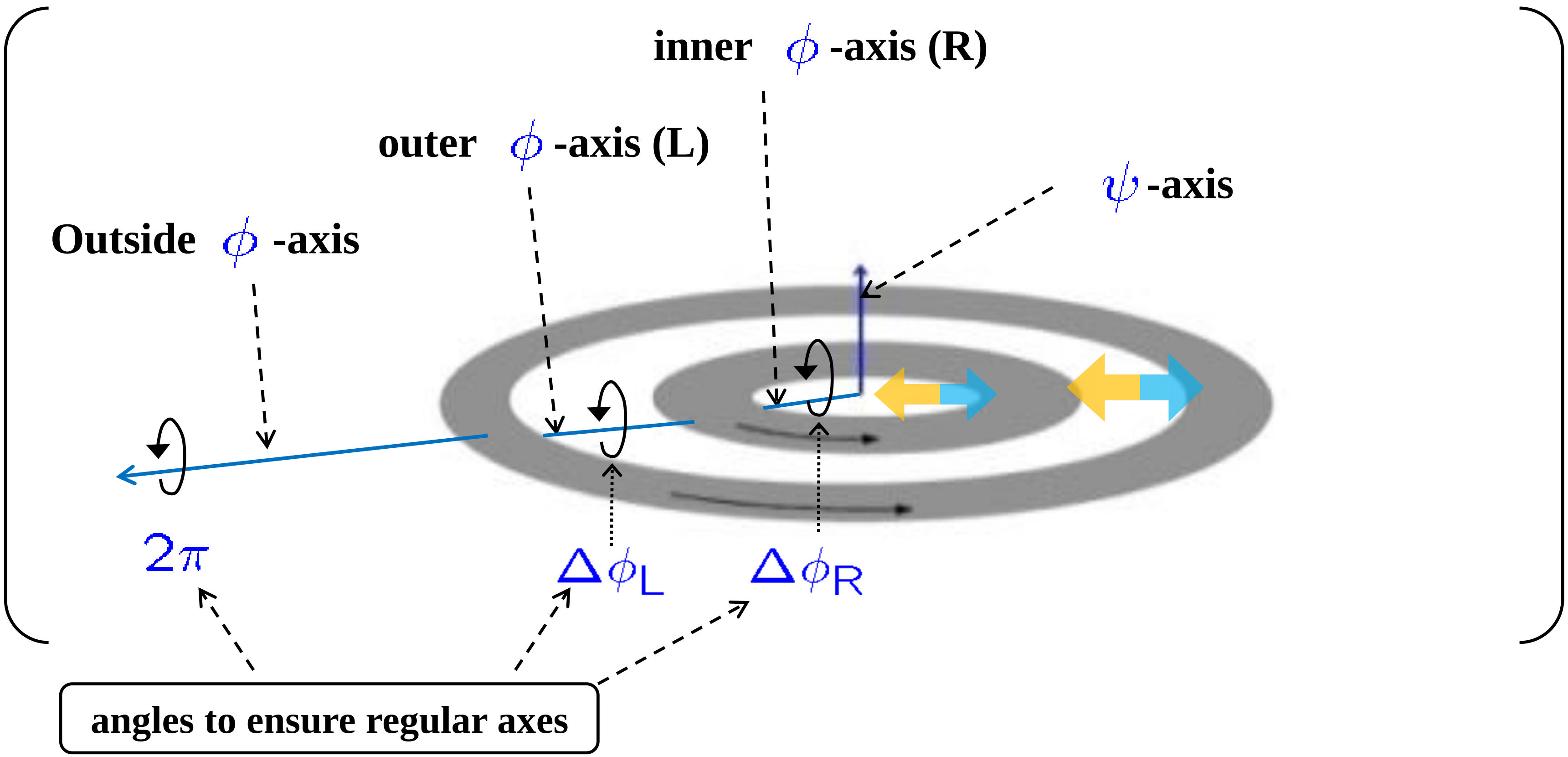
**Then the necessary conditions are given in the following.**

## 1. Regularity of black di-rings

**The condition of conical singularity-free :**

**( = the balance condition of gravitational attraction and centrifugal force )**

(1) $\Delta\phi_L = 2\pi$, (2) $\Delta\phi_R = 2\pi$

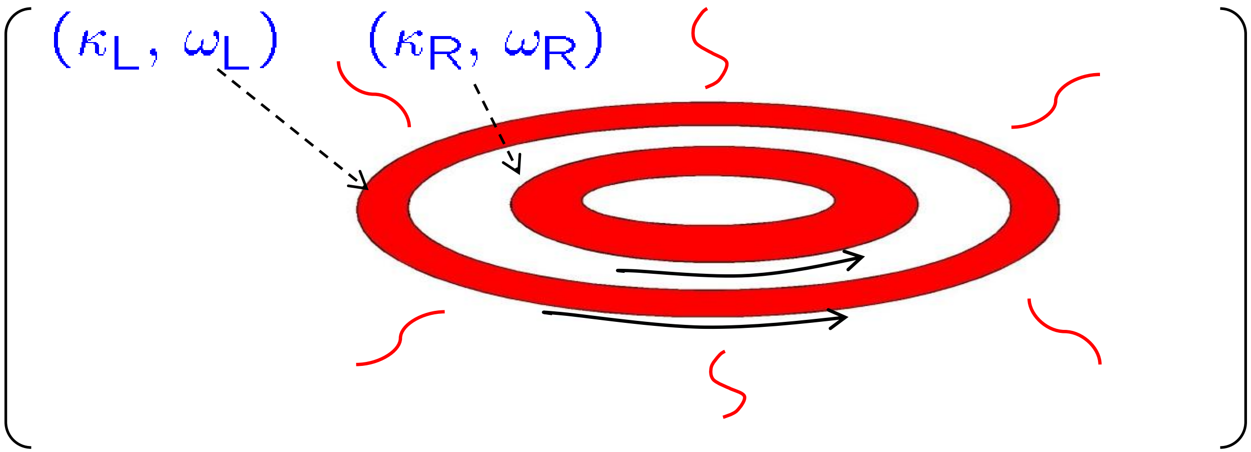


## 2. Global thermal equilibrium of black dirings

Surface gravity

In the thermal system, a temperature $(T \sim \kappa)$ and an angular velocity $(\omega)$ must be globally defined as intensive variables,

(3) $\kappa = \kappa_L = \kappa_R$, (4) $\omega = \omega_L = \omega_R$.



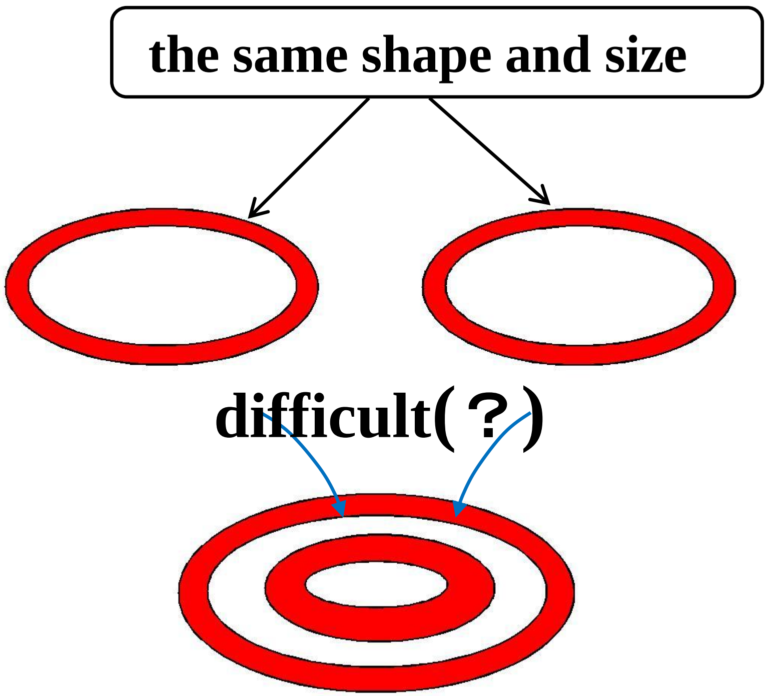

The first important problem is to determine

**whether the thermodynamic regular black diring systems exist or not.**

## < Existence is not so trivial >

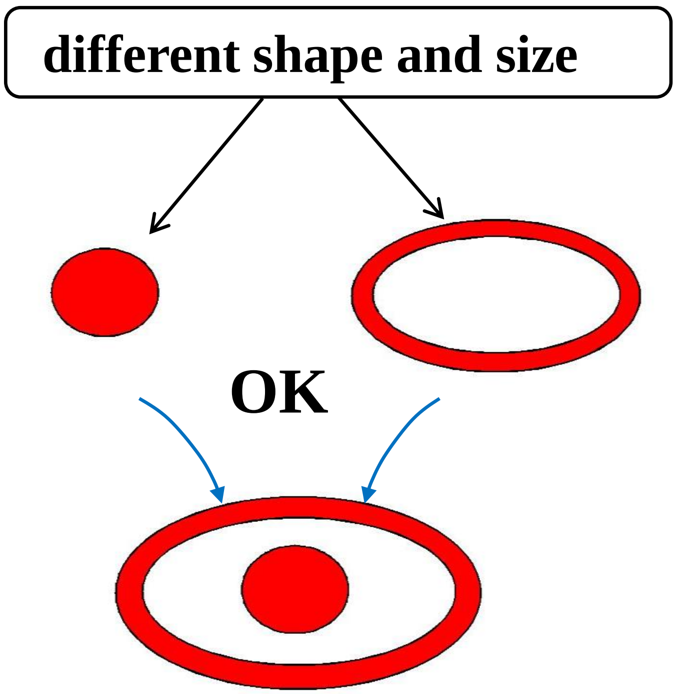
**Some plausible suggestion … ( Elvang, Emparan and Virmani JHEP09 (2008) 003 )**

**If two black rings have the same temperature and angular velocity, …**




**( Black di-ring )**

**cf.**




**( Black Saturn )**

**If the answer is yes, some mechanism is necessary … ?**

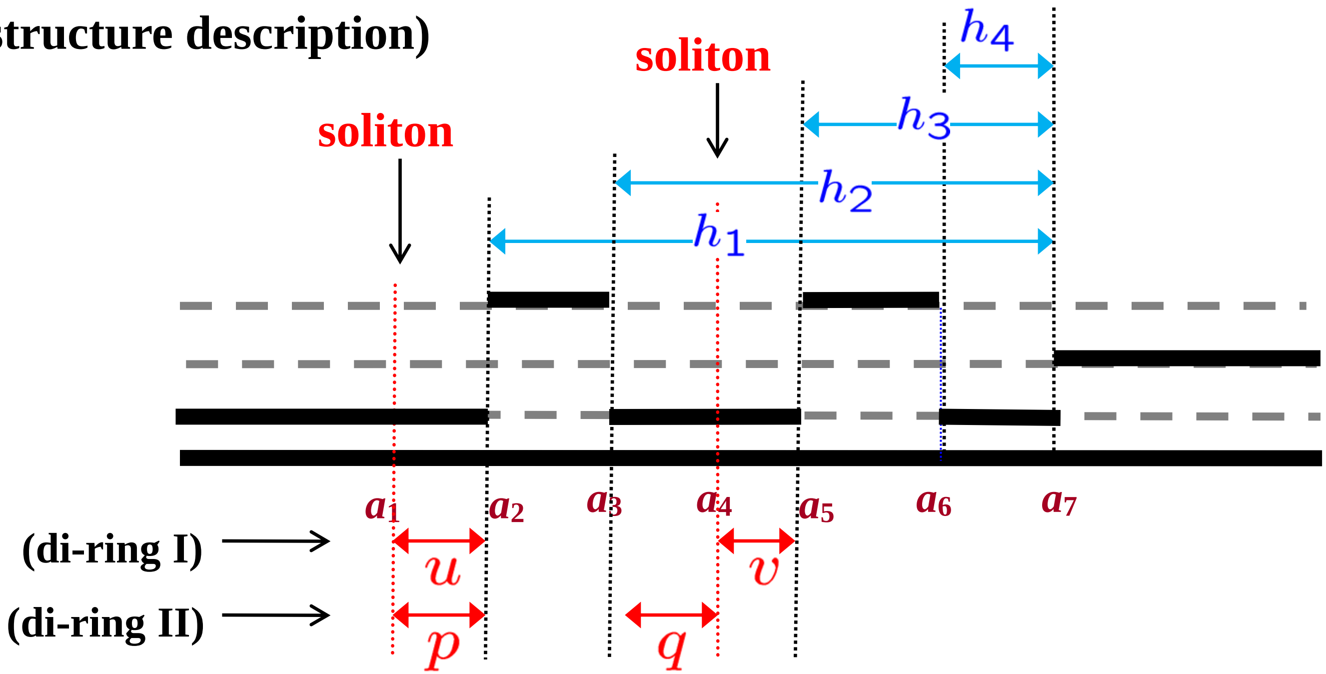
# < Remarkable reduction of the constraints for TBD >

**After describing the above four constrains from (1) to (4) of thermodynamic regular BD by appropriate six independent moduli-parameters, solve the constraints:**

**The following six moduli parameters are used.**

$$\begin{bmatrix} h_1 = a_{72}, & h_3 = a_{75}, & p = a_{21} \text{ (or } u = a_{21}), \\ h_2 = a_{73}, & h_4 = a_{76}, & q = a_{54} \text{ (or } v = a_{43}) \end{bmatrix}$$

**( Rod structure description)**



(I) & (II)

$$\bullet\ 0 = (4h_1^2 - 7h_1h_4 + 4h_4^2)\,h_3^3 - (8h_1^3 - 8h_1^2h_4 - h_1h_4^2 + 4h_4^3)\,h_3^2 + (5h_1^4 - 2h_1^3h_4 - 2h_1^2h_4^2 + h_1h_4^3 + h_4^4)\,h_3 - h_1^5$$

( cubic equation for $h_3$ )

$$\bullet\ h_2 = h_1 - h_3 + h_4 \quad (\sim d_1 = d_3)$$

+

(I)

$$\bullet\ u - v = \frac{(h_1 - h_3)(h_2^2 + 2h_1h_3 - h_3^2)}{h_3h_4}, \qquad \bullet\ (u + h_1)(h_2 - v) = \frac{h_1^2h_2^2}{h_3h_4}$$

or

(II)

$$\bullet\ p + q = \frac{(h_1 - h_2)(h_3^2 + 2h_1h_2 - h_2^2)}{h_2h_4}, \qquad \bullet\ (p + h_1)(q + h_2) = \frac{h_1^2h_3^2}{h_2h_4}$$

**Procedure to determine moduli-parameters of TBD**

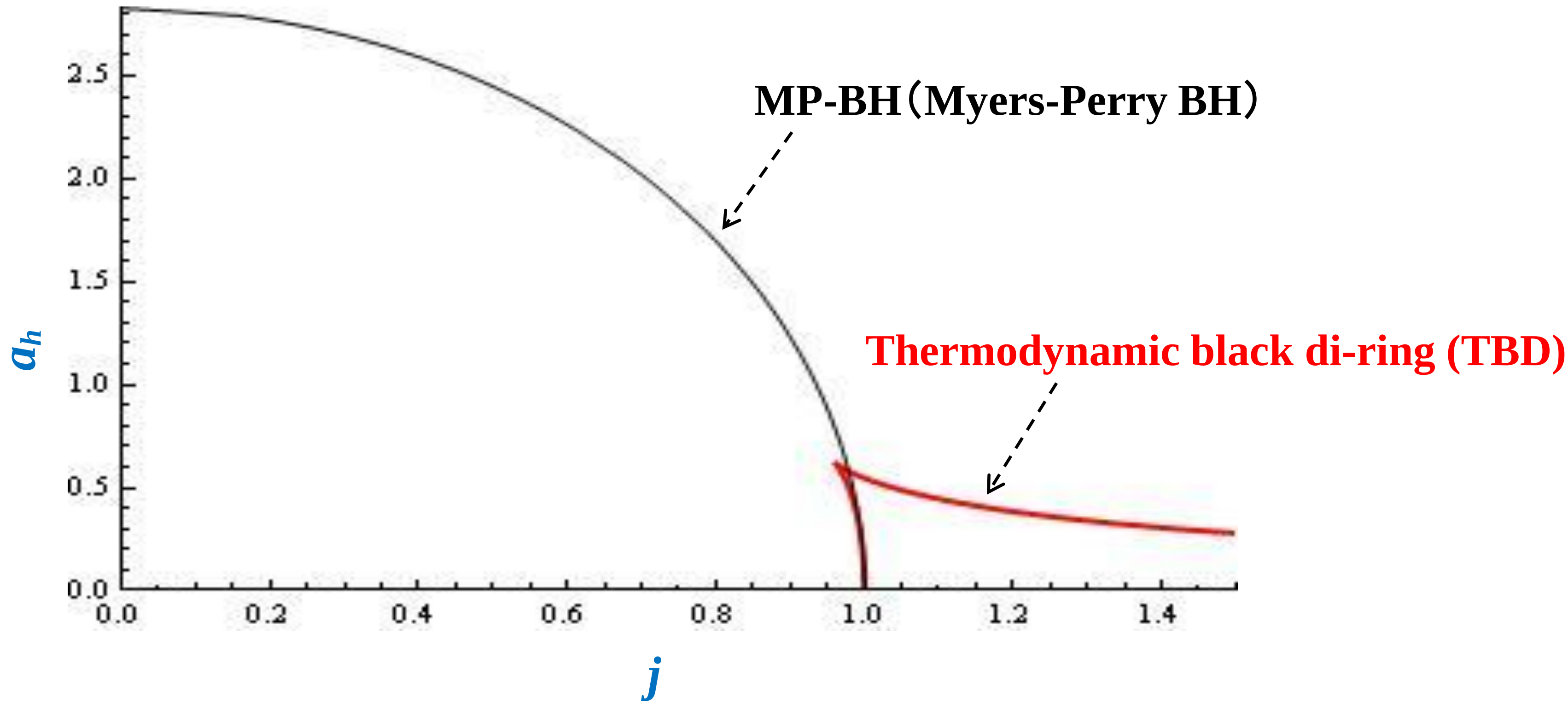
**(i) First $h_1$ is fixed to be 1 using the arbitrariness of global scaling.**

**(ii) $h_4$ given, $h_3$ is determined by solving the above cubic equation. Other parameters $h_2$ and ( $u, v$ ) or ( $p, q$ ) are determined by using the other equations.**

**(iii) Then we choose the set which satisfies $1 > h_2 > h_3 > h_4 > 0$ as physical one.**

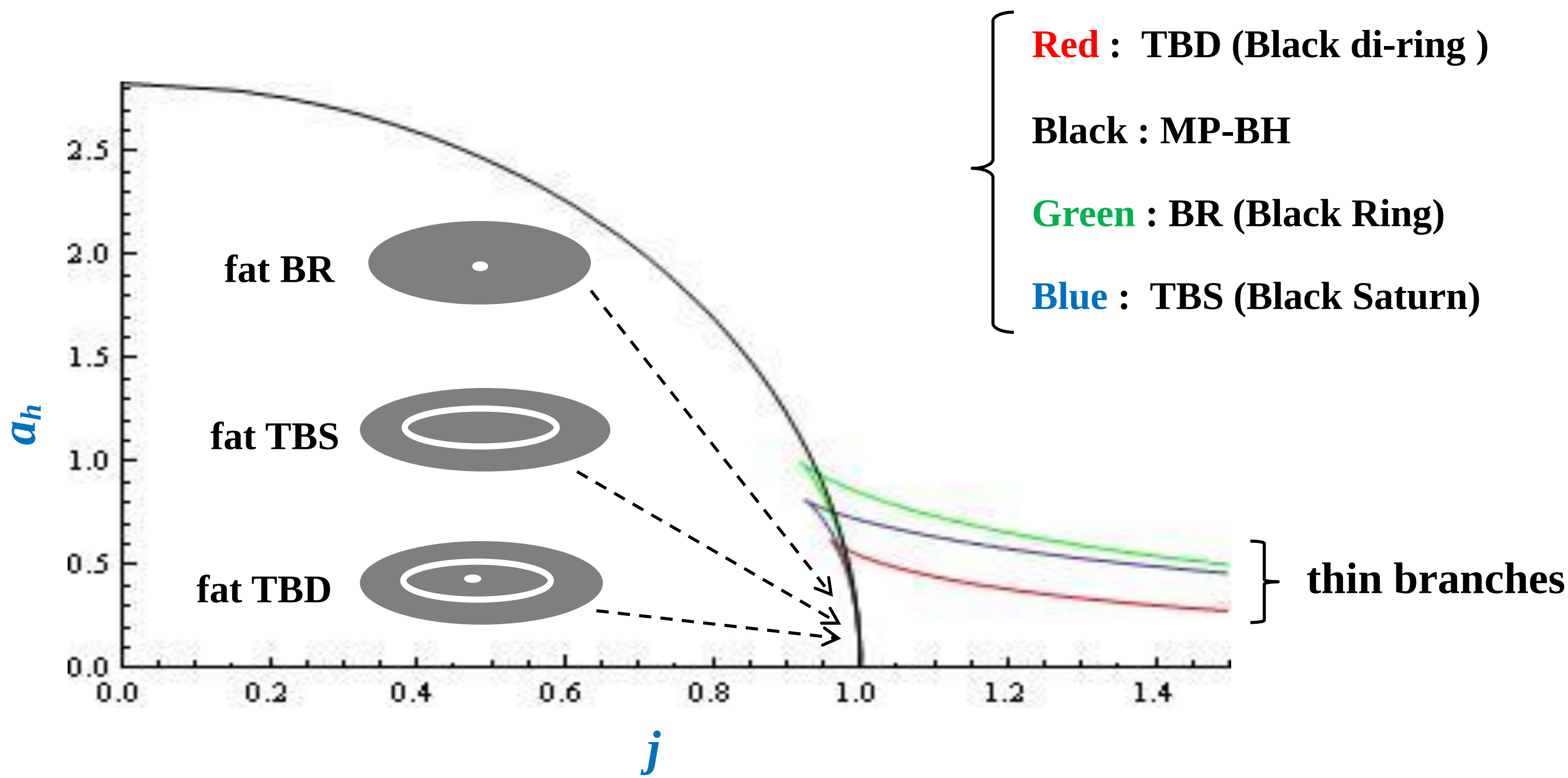
## < existence of TBD >

**Phase diagram of total areas ( $a_h$ ) against angular momentum ( $j$ )**




- **Thermodynamic black di-ring systems exist !**

# < Comparing thermodynamic black di-ring with other thermodynamic objects >



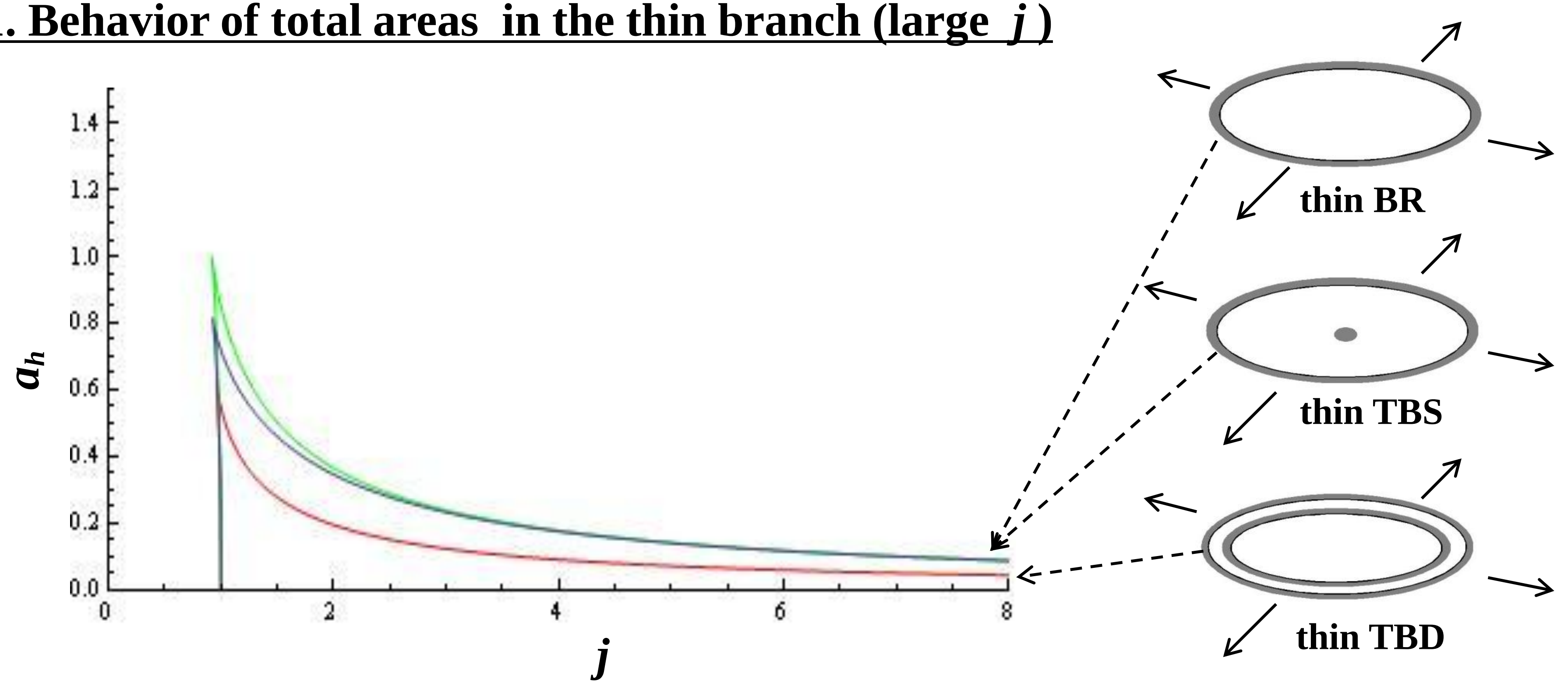

- The phase of thermodynamic di-ring has a similar behavior to the BR or TBS system.

a ‘fat ring ’ branch and a ‘thin ring’ branch appear.

# III. Some peculiarities of thermodynamic black di-rings

**To clarify the property of TBD, we contrast TBD with TBS .**

**1. Behavior of total areas in the thin branch (large *j* )**



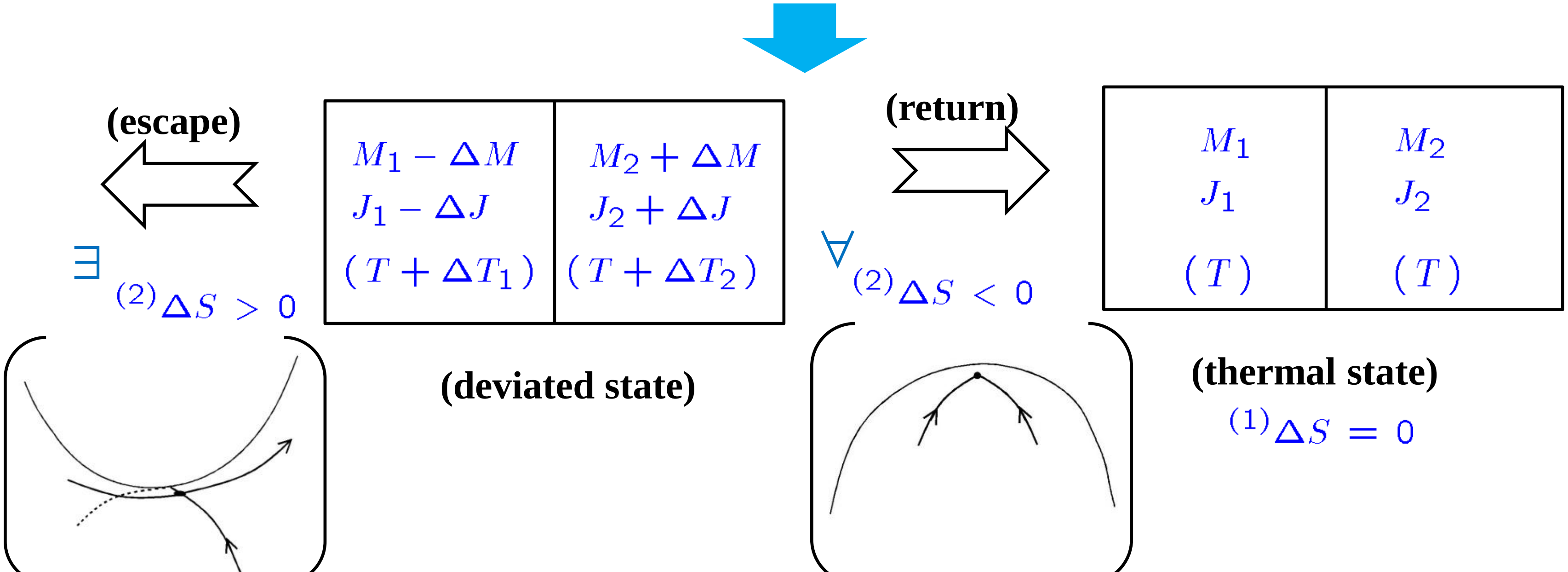

- **As *j* increases, TBS immediately approaches BR, while TBD does not approach the BR relatively.**
- **As *j* increases, in the TBS case the central black hole is just left, in the TBD case the inner ring always runs after the outer ring.**

# 2. Meta-stability of the thermodynamic di-ring system

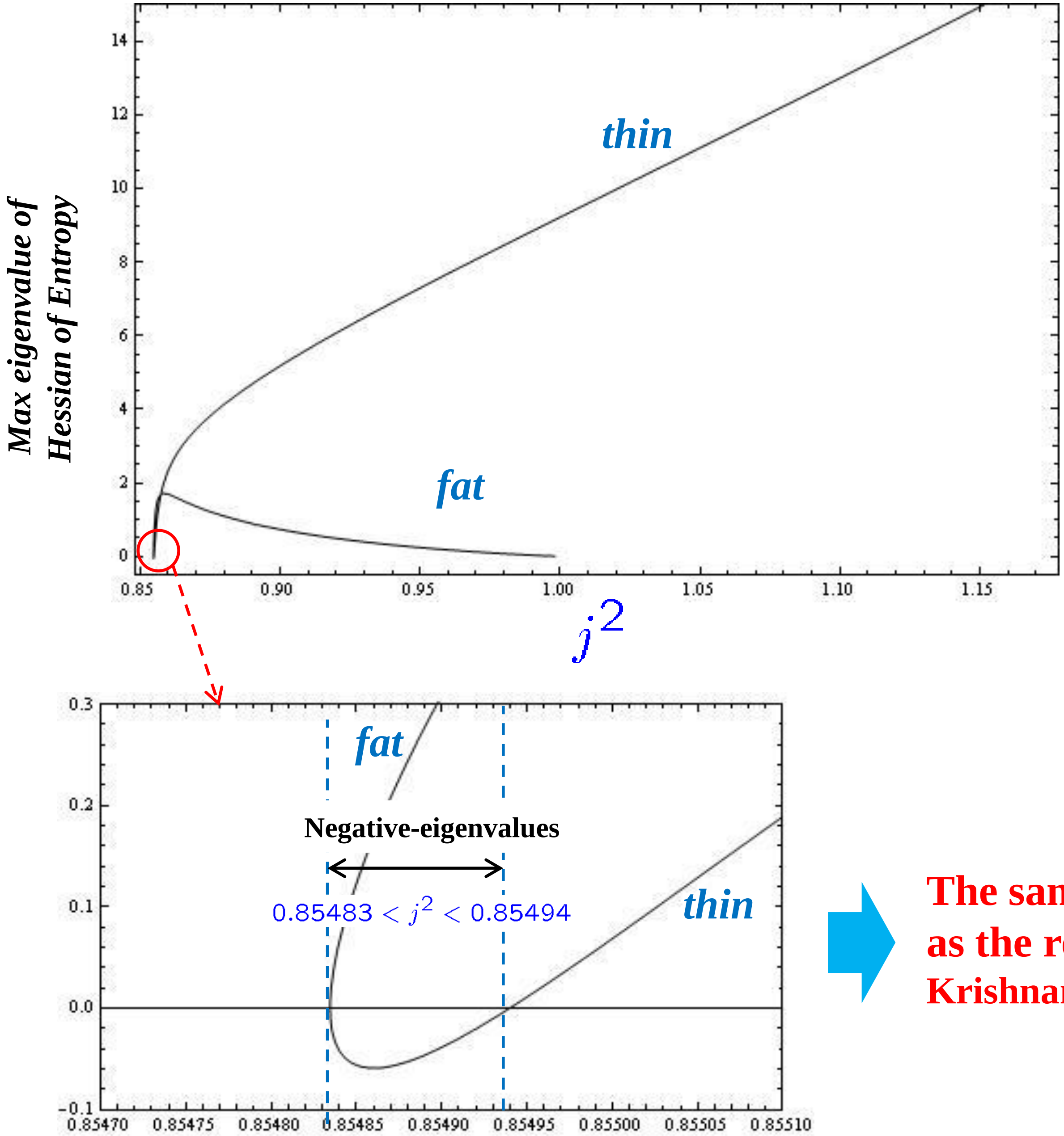
- **Next , as another peculiar property of the di-ring , we consider a certain kind of thermodynamic local stability following the discussion introduced by Evslin & Krishnan (JHEP 09(2008)003).**

- **To do this, under the condition of fixed mass and angular momentum , we shall search for local maxima of the corresponding entropy function, which is a function of appropriate moduli-parameters deduced from rod structure.**

- **If maximal eigenvalue of Hessian of the entropy function becomes negative, we can say that meta-stability occurs at this point.**

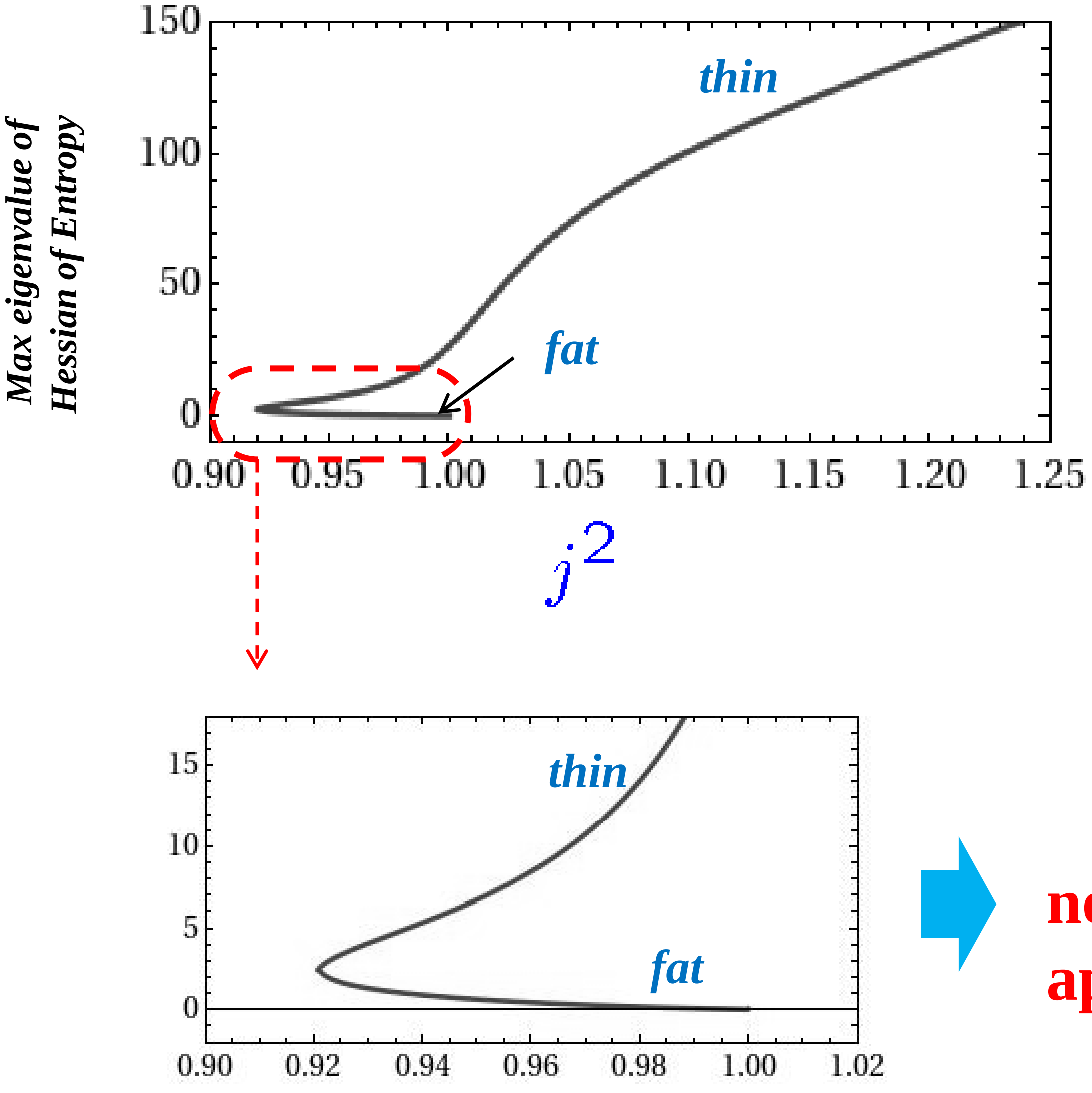


## (1) The behavior of the thermodynamic black Saturn as a demonstration of our method




**The same meta-stability as the result of Evslin and Krishnan is re-established!**

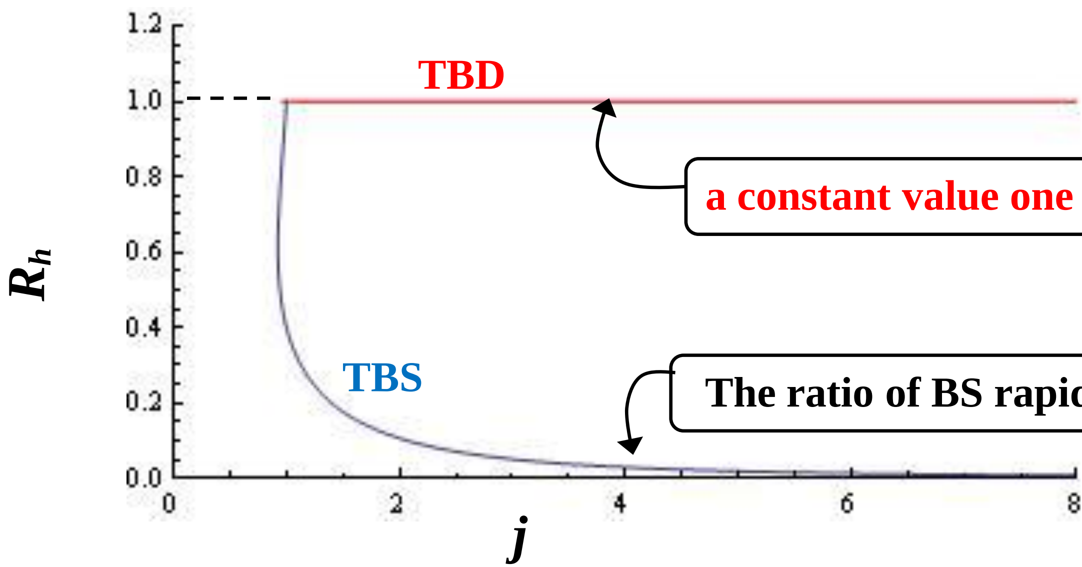
(2) The behavior of the thermodynamic black di-rings




**no meta-stability appears.**

# 3. Ratio of horizon areas of the inner BR(BH) and the outer BR

$$R_h = \frac{area\ of\ inner\ \mathrm{BH}\ or\ \mathrm{BR}}{area\ of\ outer\ \mathrm{BR}}$$




a constant value one

( Rod length of inner horizon )
||
( Rod length of outer horizon )

The ratio of BS rapidly decreases.

(TBS)

- **The influence of the inner BH quickly vanishes so that the BS behaves like a single BR.**

(TBD)

- **The interaction between two rings do not become so weak or their nonlinear intimate relation remains … ?**

## III. Summary

- **The thermodynamic systems of regular di-rings exist.**
- **The phase of thermodynamic di-ring has a 'fat ring' branch and a 'thin ring' branch.**
- **In the thin branch, the behavior of the phase of black di-ring is very different from that of black Saturn.**
- **No meta-stable state seems to be realized in the thermo-dynamic black di-ring, compared with the black Saturn.**
- **Entropies (i.e. horizon areas) of the inner ring and the outer ring are always equal. Some hidden mathematical or physical structure might be expected.**
  - ✓ **See Phys. Rev. D82, 084009(2010): arXiv:1008.4290v2[hep-th] for further results and discussion.**
  - ✓ **Some similar results are also obtained by Emparan and Figueras. ( JHEP11(2010)022: arXiv:1008.3234[hep-th] )**