New squashed black hole solutions with electromagnetic field

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Black hole spacetimes with compact extra dimensions

- Higher dimensional spacetimes with compact extra dimensions are much more realistic than the asymptotically flat spacetimes
- The spectrum of black solutions with compact extra dimensions is much richer than in the asymptotically flat case
- In 5D we can consider spacetimes with Kaluza-Klein asymptotic $S^2 \times S^1$ or more generally with twisted asymptotic $S^1 \hookrightarrow S^2$ (S^1 fibration over S^2)
- Black hole spacetimes with $S^1 \hookrightarrow S^2$ asymptotic, in most of the cases, can be interpreted as black holes sitting on gravitational instantons
- Explicit exact solutions with twisted asymptotic have been found - Ishihara and Matsuno (2005), Ishihara et al (2006, 2007), Wang (2006), S.Y. (2006), Chen and Teo (2010), Nedkova and S. Y. (2011, 2012)

EMD gravity

- Einstein-Maxwell-dilaton gravity

$$S = \int d^5x\sqrt{-g}\left[R - 2g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - e^{-2\alpha\varphi}F^{\mu\nu}F_{\mu\nu}\right] \qquad (1)$$

- The case $\alpha = \sqrt{8/3}$ corresponds to the Kaluza-Klein reduction of the 6D vacuum Einstein gravity

Exact solution

5D metric

$$ds^2 = Y^{-1/3}(r)\Big[-(1-\frac{r_+}{r})dt^2 + \frac{r}{r+r_0}\left(d\psi + r_\infty \cosh\gamma \cos\theta d\phi\right)^2\Big]$$
$$+Y^{2/3}(r)\left[\frac{r+r_0}{r-r_+}dr^2 + r(r+r_0)\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] \quad (2)$$

gauge potential 1-form

$$A = \frac{\sinh\gamma r_\infty}{2(r+r_0)}d\psi + \frac{\sinh\gamma\cosh\gamma}{2}\Big[-(r_+ + r_0) + \frac{r_\infty^2}{r+r_0}\Big]\cos\theta d\phi$$

scalar field

$$e^{\sqrt{2/3}\varphi} = Y^{1/3}(r) \quad (3)$$

$$Y(r) = \cosh^2\gamma - \frac{r-r_+}{r+r_0}\sinh^2\gamma, \quad r_\infty = \sqrt{r_0(r_+ + r_0)} \quad (4)$$

Limits of the solution

- $\gamma = \mathbf{0}$ The solution reduces to the squashed vacuum black hole of Ishihara-Matsuno
- $\mathbf{r_0} = \mathbf{0}$ The solution reduces to the dipole black string solution in EMD gravity with $\alpha = \sqrt{8/3}$
- $\mathbf{r}_+ = \mathbf{0}$ The solution reduces to a regular horizonless EMD solution which can be viewed as an EMD generalization of Gross-Perry-Sorkin Kaluza-Klein monopole

Interval structure and magnetic flux

- According to the uniqueness theorem in EMD gravity in spacetimes with compact extra dimesnions (S.Y. 2010), the solution is fully characterized by its interval structure (with respect to canonical basis**!**) and appropriately defined magnetic fluxes
- **interval structure**
  1) a semi-infinite interval located at $r \geq r_+, \theta = 0$ with direction $(1, 1)$
  2) an finite interval $r = r_+, 0 \leq \theta \leq \pi$ with direction $(0, 0)$ corresponding to the horizon
  3) a semi-infinite interval located at $r \geq r_+, \theta = \pi$ with direction $(-1, 1)$
- **magnetic flux** through the base space S^2_∞ of the S^1–fibration at infinity, $\mathcal{F} = \cosh\gamma \sinh\gamma (r_+ + r_0)$

Physical quantities

- According to the topology theorem (Hollands and S. Y. 2007) the horizon topology is determined by intervals structure and in our case it is S^3 (squashed S^3)
- Mass and tension

$$M = \frac{L}{4}\left[r_+ + (r_+ + r_0)\cosh^2\gamma\right], \ \mathcal{T} = \frac{L}{4}\left[r_0 + (r_+ + r_0)\cosh^2\gamma\right] \quad (5)$$

- NUT-charge

$$N = -\frac{1}{8\pi}\int_{C^2} d\frac{K}{\mathcal{V}} = \frac{L}{8\pi} = \cosh\gamma\frac{r_\infty}{2} \quad (6)$$

Black Lenses in equilibrium in EMD-gravity

$$ds^2 = V^{-1/3}\left\{-\Gamma^{-2}dt^2 + 2\left[(\Gamma^{-1}-1)dt + \frac{L}{2\sqrt{2}}d\Psi + \cosh\gamma\Omega\right]^2\right\} + V^{2/3}\Gamma^2\left(dx^2+dy^2+dz^2\right) \quad (7)$$

$$A = A_\mu dx^\mu = -\frac{\sinh\gamma}{\sqrt{2}\Gamma}d\Psi - \cosh\gamma\sinh\gamma\frac{\Omega}{\Gamma} \quad (8)$$

$$V = \cosh^2\gamma - \Gamma^{-2}\sinh^2\gamma,\ \Gamma = 1 + \sum_i \frac{m_i}{|R-R_i|} \quad (9)$$

$$\Omega = \sum_i m_i \frac{z-z_i}{|R-R_i|}\frac{(x-x_i)dy-(y-y_i)dx}{(x-x_i)^2+(y-y_i)^2} \quad (10)$$

Black Lenses in equilibrium in EMD-gravity

- A particular case (for $\gamma = 0$) of the solution is the recently published vacuum solution of Matsuno-Ishihara-Kimura-Tatsuoka
- The solution describes N black lenses (S^3/Z_{n_i}) in equilibrium in 5D EMD gravity
- The electric charge of each black lense is zero, $Q_i = 0$
- The gravitational force between the black lenses is balance by the tension of the compact dimension and the repulsive spin-spin interaction
- Kaluza-Klein uplifting $\rightarrow$ new 6D multi-black hole solution. Horizon topology - T^2 fibration over S^2

THANK YOU FOR THE ATTENTION!