Black holes, wormholes and particle-like solutions in EsGB theories

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Outline

1. Black Holes
2. Wormholes
3. Particle-like ECOs
4. Conclusions

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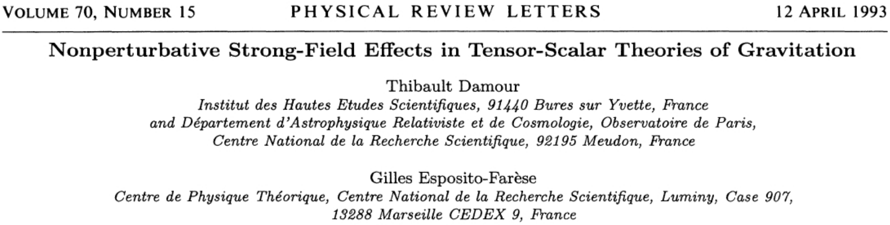
Matter-induced spontaneous scalarization (STTs)


VOLUME 70, NUMBER 15 PHYSICAL REVIEW LETTERS 12 APRIL 1993

Nonperturbative Strong-Field Effects in Tensor-Scalar Theories of Gravitation

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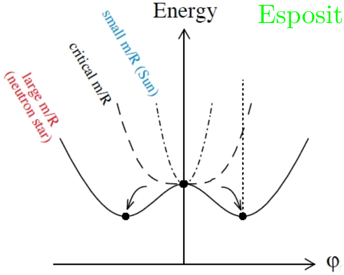
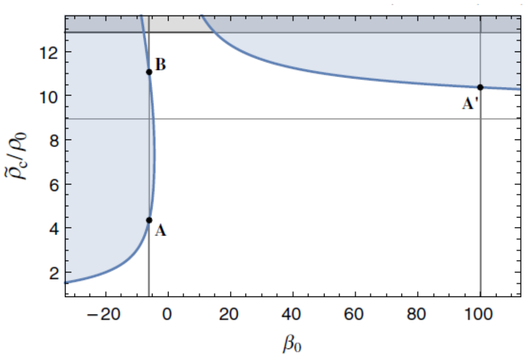
Esposito-Farese (lecture)



"spontaneous scalarization"

Matter-induced spontaneous scalarization (STTs)

Mendes et al. 1604.04175



1. spontaneous scalarization: $\beta_0 < 0,\, T < 0 \Longrightarrow \beta_0 T > 0$
2. spontaneous scalarization: $\beta_0 > 0,\, T > 0 \Longrightarrow \beta_0 T > 0$

Einstein-scalar-Gauss-Bonnet Theories

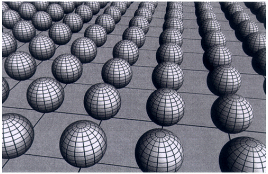
EsGB action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}(\partial_\mu\varphi)^2 + f(\varphi)R^2_{\rm GB}\right]$$

Gauss-Bonnet term: quadratic in the curvature

$$R^2_{\rm GB} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

coupling function $f(\varphi)$



In 4 spacetime dimensions the coupling to the scalar is needed.

The resulting set of equations of motion are of second order (Horndeski).

Einstein-scalar-Gauss-Bonnet Theories

Gregory Horndeski, ‘Horndeski Scalar Theory, Past, Present and Future’

Einstein-scalar-Gauss-Bonnet Theories

generalized Einstein equations

$$\begin{aligned} G_{\mu\nu} &= -\frac{1}{4}g_{\mu\nu}\partial_\rho\varphi\partial^\rho\varphi + \frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi \\ &- \frac{1}{2}\left(g_{\rho\mu}g_{\lambda\nu} + g_{\lambda\mu}g_{\rho\nu}\right)\eta^{\kappa\lambda\alpha\beta}\tilde{R}^{\rho\gamma}_{\alpha\beta}\nabla_\gamma\partial_\kappa f(\varphi) \end{aligned}$$

scalar equation

$$\nabla_\mu\nabla^\mu\varphi + \frac{df}{d\varphi}R^2_{\rm GB} = 0$$

crucial: choice of coupling function $f(\varphi)$

- GR black hole solutions do not remain solutions
  $\Longrightarrow$ only hairy black holes result
- GR black hole solutions do remain solutions
  $\Longrightarrow$ in addition spontaneously scalarized black holes emerge

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EdGB black holes

Kanti et al. hep-th/9511071, Torii et al. gr-qc/9606034

coupling function

$$f(\phi) = \frac{\alpha}{4} e^{-\gamma\phi}$$

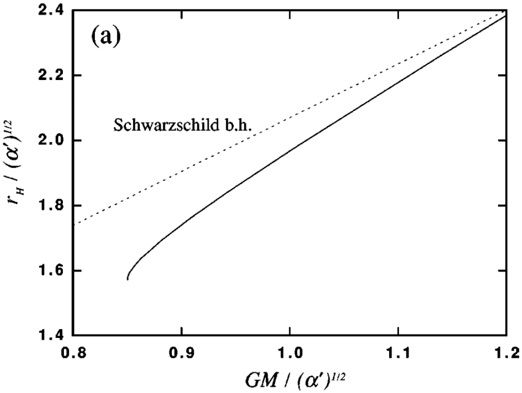
static black holes

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4} e^{2\phi_h}}$$

lower bound
on the horizon size
for fixed α'



lower bound on the mass

EdGB black holes

Kanti et al. hep-th/9511071, Antoniou et al. 1711.03390

coupling function

$$f(\phi) = \frac{\alpha}{4} e^{-\gamma\phi}$$

static black holes

critical black holes:

horizon expansion

$$\sqrt{1 - 6\frac{\alpha'^2}{r_h^4} e^{2\phi_h}}$$

lower bound
on the horizon size
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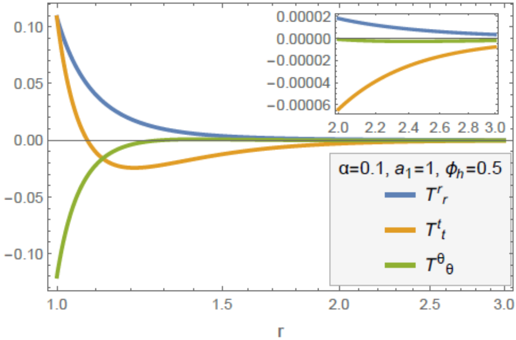
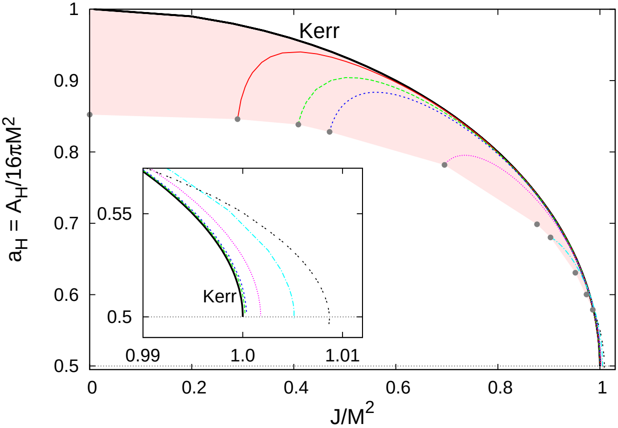


negative effective energy density

EdGB black holes

Kleihaus et al. 1101.2868

horizon area versus angular momentum



EdGB black holes

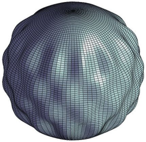
perturbation theory: damped oscillations

metric

$$g_{\mu\nu} = g^{(0)}_{\mu\nu}(r) + \epsilon h_{\mu\nu}(t, r, \theta, \varphi)$$

scalar

$$\phi = \phi_0(r) + \epsilon\delta\phi(t, r, \theta, \varphi)$$



polar modes: even-parity perturbations

axial modes: odd-parity perturbations (pure space-time modes)

master equation: Schrödinger-like equation

eigenvalue ω

$$\omega = \omega_R + i\omega_I$$

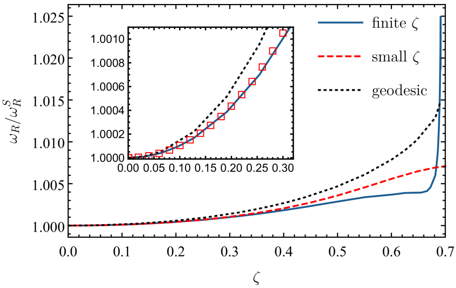
frequency: ω_R decay time: $\tau = 1/\omega_I$

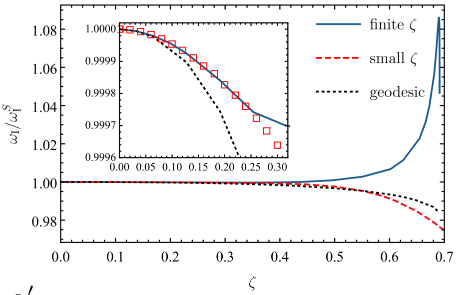
EdGB black holes

Blazquez-Salcedo et al. 1609.01286

quasi-normal mode (axial $l = 2$) versus coupling constant

normalized to the Schwarzschild values





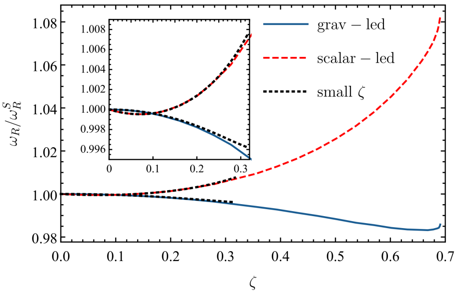
real part $\qquad \zeta = \dfrac{\alpha'}{M^2} \qquad$ imaginary part

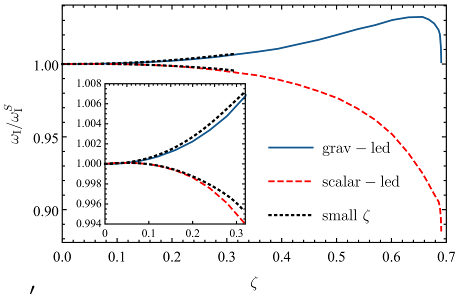
EdGB black holes

Blazquez-Salcedo et al. 1609.01286

quasi-normal mode (polar $l = 2$) versus coupling constant

normalized to the Schwarzschild values





real part $\qquad \zeta = \frac{\alpha'}{M^2} \qquad$ imaginary part

Static curvature induced scalarized black holes

Doneva et al. 1711.01187, Silva et al. 1711.02080, Antoniou et al. 1711.03390

curvature induced scalarized black holes

Einstein equations

$$G_{\mu\nu} = T_{\mu\nu}$$

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

GR solutions remain solutions: $\varphi = 0, \ \dfrac{df(\varphi)}{d\varphi} = 0$

tachyonic instability

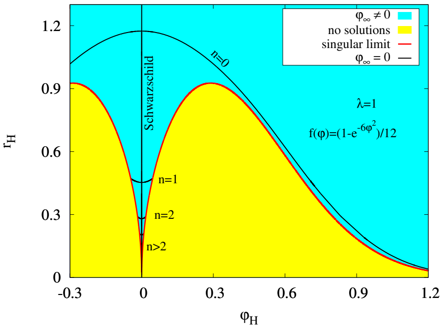
effective mass

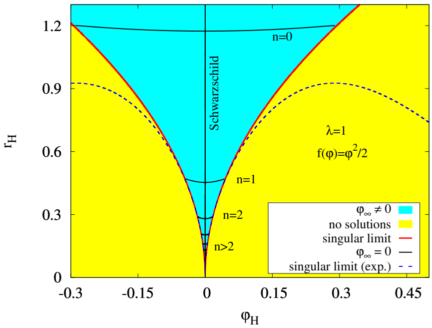
$$m^2_{\rm eff} = -\eta R^2_{\rm GB} < 0 \ , \quad \text{if } \eta > 0$$

Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755

domain of existence of spontaneously scalarized static black holes



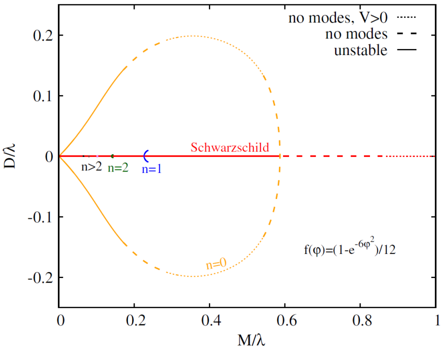


$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \qquad f(\varphi) = \frac{\lambda^2}{2}\varphi^2$$

spontaneously scalarized black holes, $\varphi_\infty \neq 0$, radicand negative

Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 1805.05755



solutions

Schwarzschild red

scalarized $n = 0$ orange

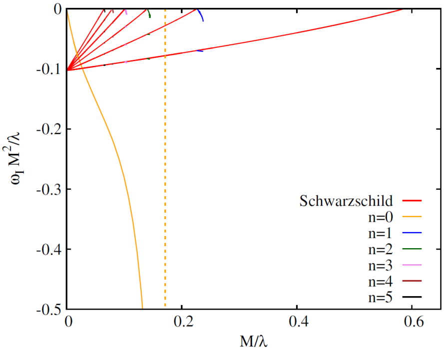
scalarized $n > 0$ ...

unstable radial modes

Schwarzschild red

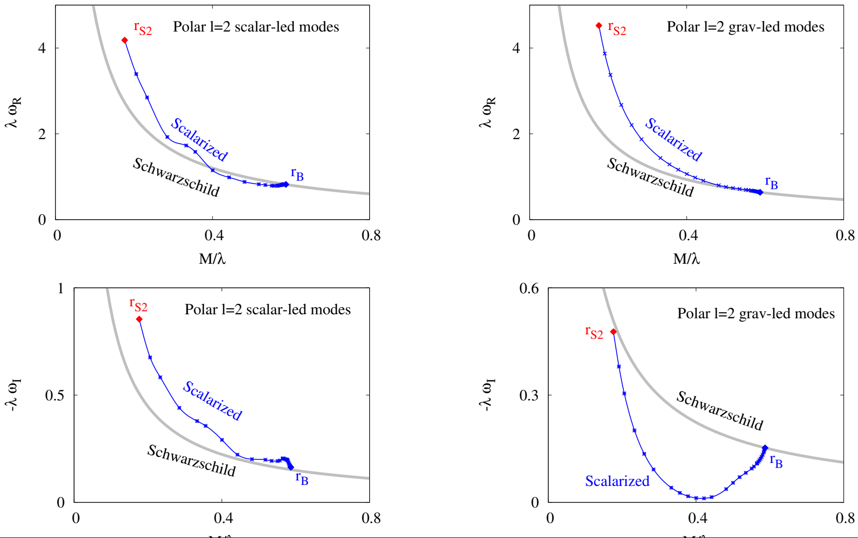
scalarized $n = 0$ orange

scalarized $n > 0$ ...



Static curvature induced scalarized black holes

Blazquez-Salcedo et al. 2006.06006



Rotating curvature induced scalarized black holes

Cunha et al. 1904.09997, Collodel et al. 1912.05382, Dima et al. 2006.03095

scalar equation

$$\nabla_\mu \nabla^\mu \varphi + \frac{df}{d\varphi} R^2_{\rm GB} = 0$$

Gauss-Bonnet: Kerr

$$R^2_{\rm GB} = \frac{48M^2}{(r^2+\chi^2)^6}\left(r^6 - 15r^4\chi^2 + 15r^2\chi^4 - \chi^6\right) \ , \quad \chi = a\cos\theta$$

effective mass

$$m^2_{\rm eff}(r) = -\eta R^2_{\rm GB} < 0$$

- $\eta > 0$

  $\Longrightarrow$ spin suppresses scalarization

- $\eta < 0$

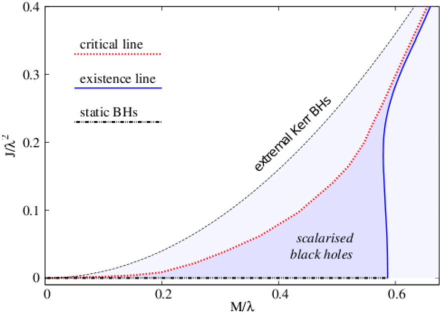
  $\Longrightarrow$ spin induces scalarization

Rotating curvature induced scalarized black holes

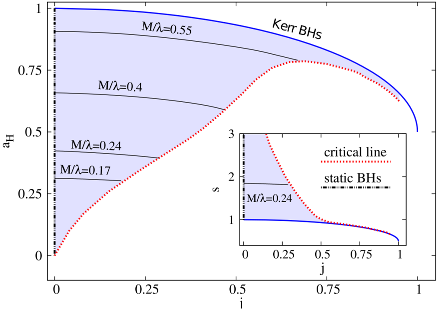
Cunha et al. arXiv:1904.09997

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \ , \qquad V(\varphi) = 0$$



angular momentum vs mass



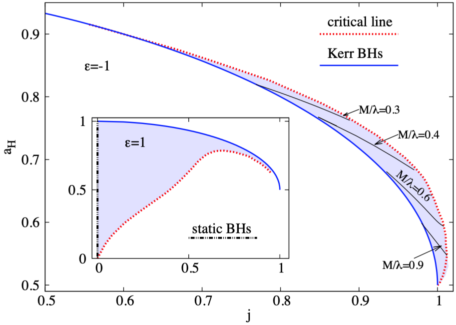
area/entropy vs angular momentum

Rotating curvature induced scalarized black holes

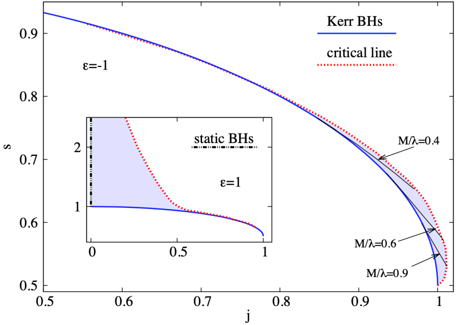
Herdeiro et al. arXiv:2009.03904, Berti et al. arXiv:2009.03905

coupling function

$$f(\varphi) = \frac{\lambda^2}{12}\left(1 - e^{-6\varphi^2}\right) \ , \qquad V(\varphi) = 0$$



area vs angular momentum



entropy vs angular momentum

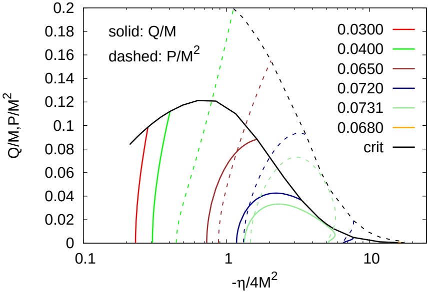
even scalar field

Rotating curvature induced scalarized black holes

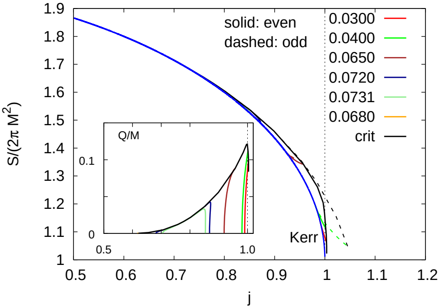
Herdeiro et al. arXiv:2009.03904, Berti et al. arXiv:2009.03905

coupling function

$$f(\varphi) = \frac{\eta}{8}\varphi^2 \,, \quad \eta < 0 \,, \qquad V(\varphi) = 0$$



charge/dipole moment vs coupling



entropy vs angular momentum

even (Q) and odd (P) scalar field

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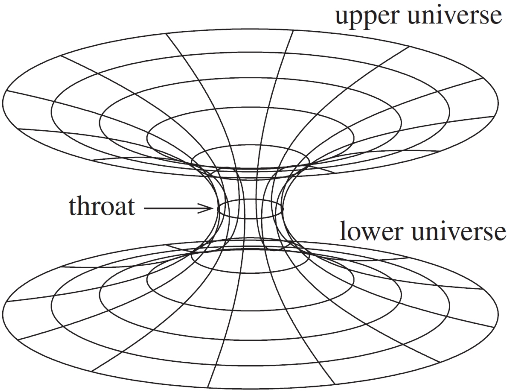
Wormholes



embedding diagram

- 2 asymptotically flat regions
- sphere of minimal surface/radius
- no horizon
- no singularity

violation of the energy conditions

Static Dilatonic EGB Wormholes

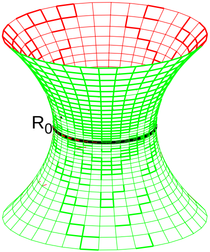
Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091

static spherically symmetric wormholes

$$ds^2 = -e^{F_0} dt^2 + e^{F_1} \left[d\eta^2 + h \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]$$

$$h = \eta^2 + \eta_0^2$$

$$-\infty < \eta < \infty$$



embedding of the throat of the wormhole

Static Dilatonic EGB Wormholes

Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091

line element

$$ds^2 = -e^{F_0} dt^2 + e^{F_1} \left[d\eta^2 + h \left(d\theta^2 + \sin^2\theta d\varphi^2 \right) \right]$$
$$h = \eta^2 + \eta_0^2$$

global charges

mass M, dilaton charge D

$$F_0 \to -\frac{2M}{\eta}, \qquad \phi \to -\frac{D}{\eta}.$$

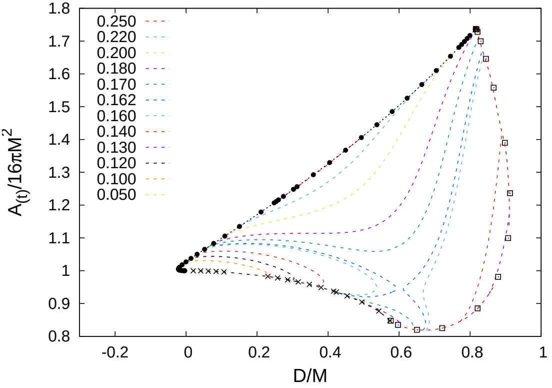
properties of the throat

circumferential radius R:	R^2	$R^2 = e^{F_1} h$
throat radius R_0:	R_0^2	$\mathrm{Min}\left(R^2\right)$
throat area:	A_{Th}	$= \int_\Sigma d^2x \sqrt{g_2}$

g_2 determinant of the metric on the throat

Static Dilatonic EGB Wormholes

Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091



throat area vs dilaton charge

domain of existence

- dashed lines $\bar{\alpha} = \frac{\alpha}{\eta_0^2} = \text{const}$
- lower boundary: black hole
- right boundary: singularity
- left boundary: $R' = 0, \; R'' = 0$

Static Dilatonic EGB Wormholes

Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091

Smarr like mass relation

$$M = 2S_{\rm th}\frac{\kappa}{2\pi} - \frac{D}{2\gamma} + \frac{D_{\rm th}}{2\gamma}$$

with

$$S_{\rm th} = \frac{1}{4}\int \sqrt{g_2}\left(1 + 2\alpha e^{-\gamma\phi}\tilde{R}\right) d^2x$$

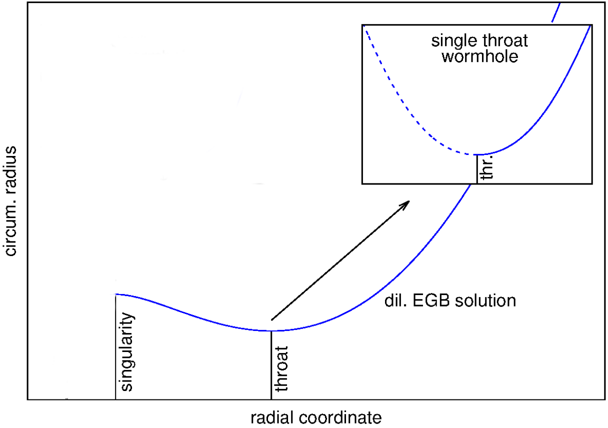
throat surface gravity κ

throat dilaton charge

$$D_{\rm th} = \frac{1}{4\pi}\int \sqrt{g_2} e^{F_0/2} n_0^\mu \partial_\mu \phi \left(1 + 2\alpha\gamma^2 e^{-\gamma\phi}\tilde{R}\right) d^2x$$

Static Dilatonic EGB Wormholes

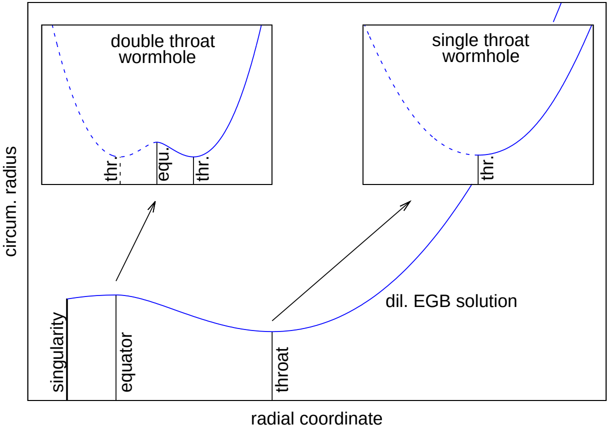
Kanti et al. 1108.3003, 1111.4049, Antoniou et al. 1904.13091



junction conditions: thin shell of ordinary matter needed

Double Throat Dilatonic EGB Wormholes

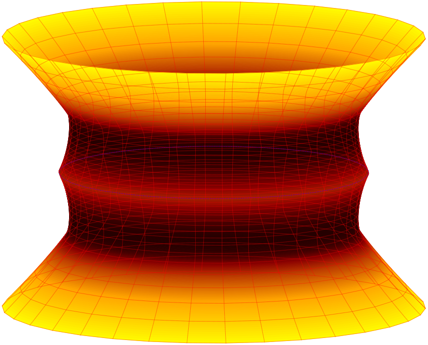
Antoniou et al. 1904.13091, in preparation



junction conditions: thin shell of ordinary matter needed

Double Throat Dilatonic EGB Wormholes

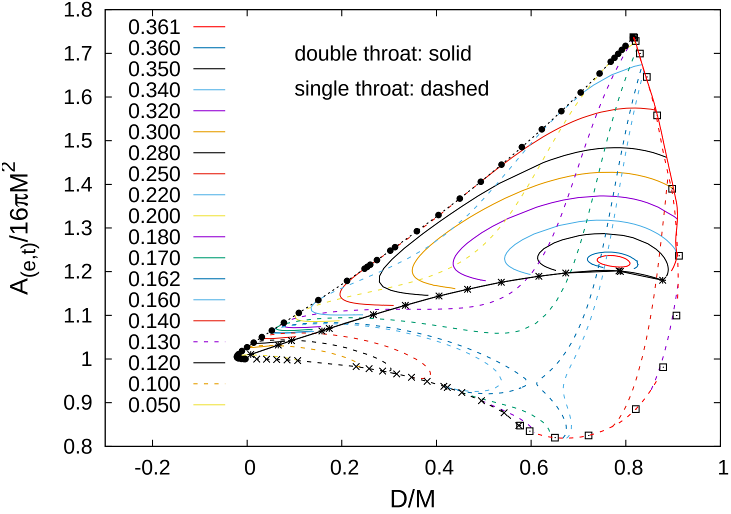
Antoniou et al. 1904.13091, in preparation



embedding of double throat wormhole

Double Throat Dilatonic EGB Wormholes

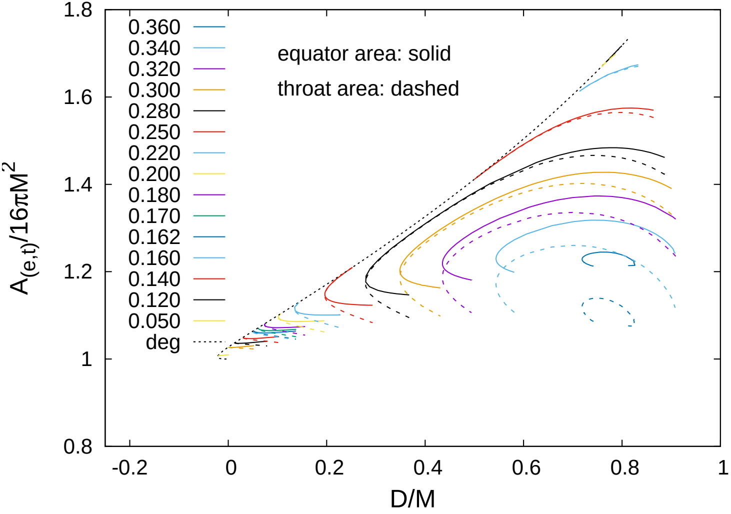
Antoniou et al. 1904.13091, in preparation



- stars
  singularity
  equator
- squares
  cusp
  singularity
  elsewhere

Double Throat Dilatonic EGB Wormholes

Antoniou et al. 1904.13091, in preparation



- equator

  vs

- throat

  area

Slowly Rotating Dilatonic EGB Wormholes

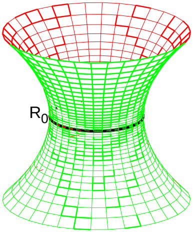
Antoniou et al. in preparation

rotating wormholes

$$ds^2 = -e^{F_0}dt^2 + e^{F_1}\left(e^{F_2}\left(d\eta^2 + hd\theta^2\right) + h\sin^2\theta\left(d\varphi - \Omega dt\right)^2\right)$$

$$h = \eta^2 + \eta_0^2$$

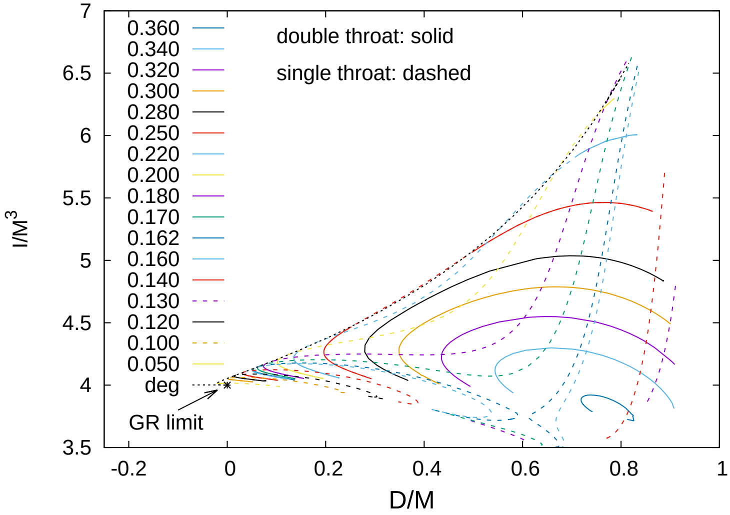
$$-\infty < \eta < \infty$$



lowest order perturbation theory

Slowly Rotating Dilatonic EGB Wormholes

Antoniou et al. in preparation



- moment of inertia

  vs

- charge

Geodesics of Dilatonic EGB Wormholes

Kanti et al. 1108.3003, 1111.4049

- geodesics from Lagrangian

$$\mathcal{L} = \frac{1}{2} e^{-2\beta\phi} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu$$

$\beta = const.$ (= 1/2 for heterotic string theory)

- conjugate momenta $p_\mu = \frac{\partial \mathcal{L}}{\partial \dot{x}^\mu}$

$$p_t = -e^{-2\beta\phi} e^{F_0} \dot{t} \ , \quad p_\varphi = e^{-2\beta\phi} e^{F_1} (\eta_0^2 + \eta^2) \dot{\varphi}$$

$$p_\eta = e^{-2\beta\phi} e^{F_1} \dot{\eta}$$

- first integrals

$$p_t = const. = -E \ , \quad p_\varphi = const. = L$$

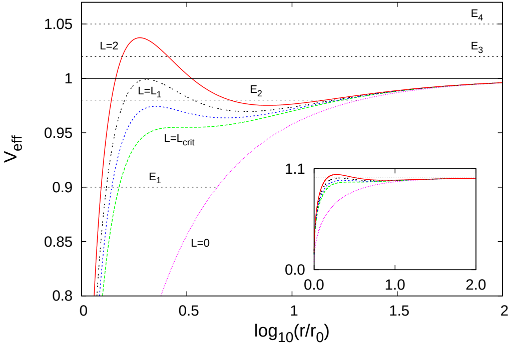
- time-like geodesics

$$2\mathcal{L} = -e^{2\beta\phi} e^{-F_0} E^2 + e^{-2\beta\phi} e^{F_1} \dot{\eta}^2 + e^{2\beta\phi} e^{-F_1} \frac{L^2}{\eta_0^2 + \eta^2} = -1$$

Geodesics of Dilatonic EGB Wormholes

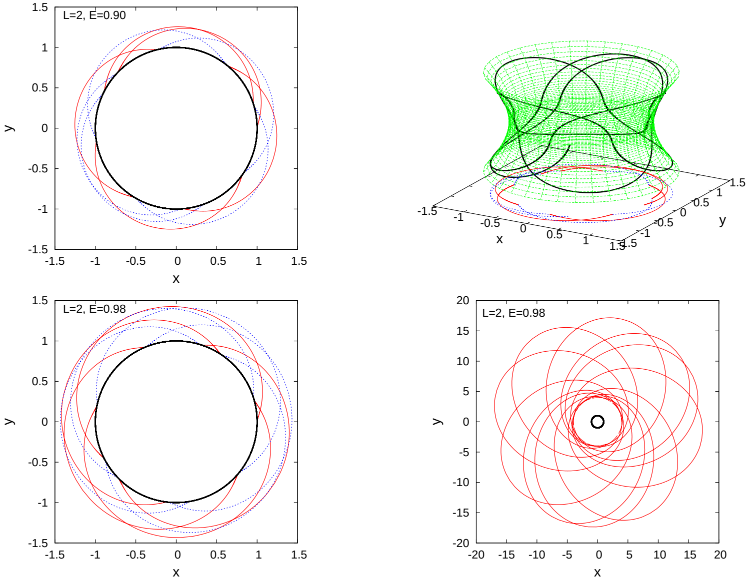
Kanti et al. 1108.3003, 1111.4049

- radial equation: $\dot{\eta}^2 = e^{4\beta\phi} e^{-F_0 - F_1} \left[E^2 - V_{\text{eff}}^2(\eta, L) \right]$
- effective potential: $V_{\text{eff}}^2(\eta, L) = e^{F_0} \left(e^{-2\beta\phi} + e^{-F_1} \frac{L^2}{\eta_0^2 + \eta^2} \right)$



- $E^2 \geq V_{\text{eff}}^2(\eta, L)$
- turning points η_i: $E^2 - V_{\text{eff}}^2(\eta_i, L) = 0$
- no horizon
- bound orbits:
  motion around the throat
  motion across the throat

Geodesics of Dilatonic EGB Wormholes



Traversable Dlatonic EGB Wormholes?

Kanti et al. 1108.3003, 1111.4049

acceleration of a traveler at the throat?

- string theory
  $\alpha \sim \ell_P^2 \Longrightarrow r_0 \sim 10\,\ell_P$
  acceleration $(10^{51} - 10^{52})\, g_\oplus$
  $g_\oplus$: acceleration of gravity at the surface of the earth

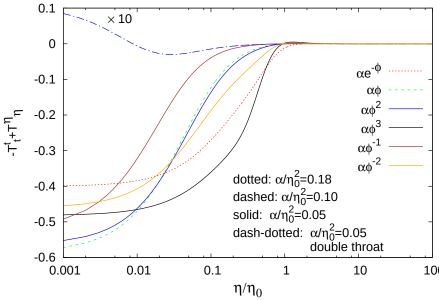
- acceleration on the order of $g_\oplus$:
  throat radius $(10 - 100)$ light-years

Cuyubamba et al. 1804.11170

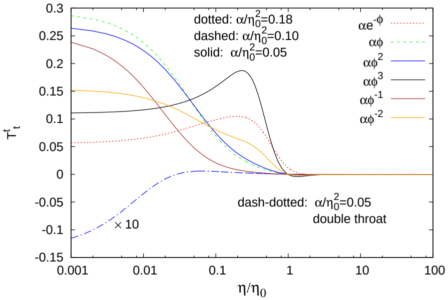
Scalar EGB Wormholes with Other Coupling Functions

Antoniou et al. 1904.13091

no spontaneous scalarization: no GR wormholes



null



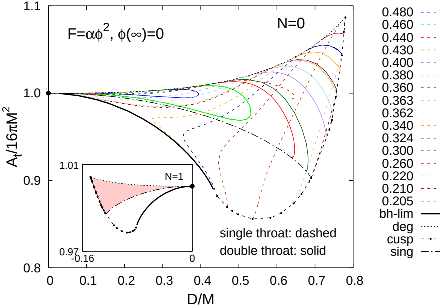
weak

violation of the energy conditions

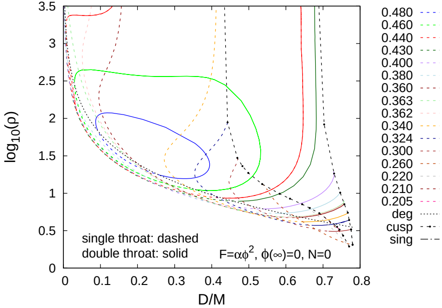
Scalar EGB Wormholes with Other Coupling Functions

Antoniou et al. 1904.13091

no spontaneous scalarization: no GR wormholes



domain of existence



matter at throat

$$F(\varphi) = \alpha\varphi^2 \ , \quad \varphi(\infty) = 0$$

Scalar EGB Wormholes with Other Coupling Functions

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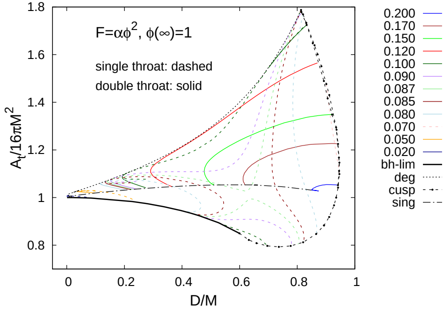


domain of existence

$$F(\varphi) = \alpha\varphi^2 \ , \quad \varphi(\infty) = 0$$

Outline

1 Black Holes

2 Wormholes

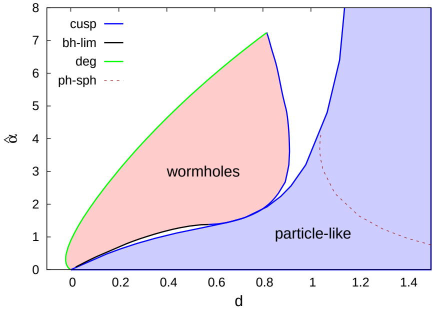
3 Particle-like ECOs

4 Conclusions

Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

globally regular metric, regular stress-energy tensor, diverging scalar (origin)



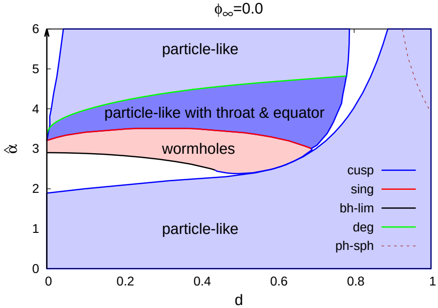
domain of existence: $\hat{\alpha} = \dfrac{\alpha}{M^2}$ vs $d = \dfrac{D}{M}$

$$F(\varphi) = \alpha e^{-\varphi} \;, \quad \varphi(\infty) = 0$$

Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

globally regular metric, regular stress-energy tensor, diverging scalar (origin)



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Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

$$F(\varphi) = \alpha\varphi^2$$

embeddings

Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

metric

$$ds^2 = -e^{f_0} dt^2 + e^{f_1} \left[dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

coupling $F(\phi) = \alpha\phi^n$, $n \geq 2$

expansion at origin

$$\begin{aligned}
f_0 &= f_{0c} + \frac{e^{f_{1c}}}{64\alpha} r^2 + \frac{e^{f_{1c}}\phi_c}{96\alpha c_0} r^3 + \mathcal{O}(r^4) \\
f_1 &= f_{1c} + \frac{e^{f_{1c}}}{64\alpha} r^2 + \frac{\nu_3}{6} r^3 + \mathcal{O}(r^4) \\
\phi &= -\frac{c_0}{r} + \phi_c - \frac{e^{f_{1c}} c_0}{256\alpha} r + \frac{32\alpha c_0 \nu_3 - e^{f_{1c}}\phi_c}{768\alpha} r^2 + \mathcal{O}(r^3)
\end{aligned}$$

f_{0c}, f_{1c}, ν_3, ϕ_c, and c_0 are constants

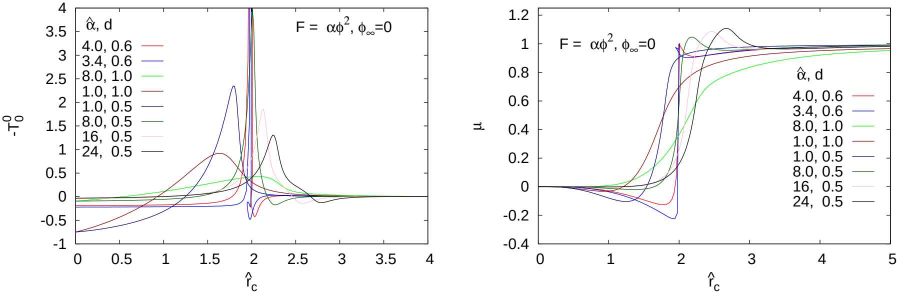
stress-energy tensor $(n = 2)$

$$T_t^t(0) = \frac{3}{32\alpha} \ , \quad T_r^r(0) = T_\theta^\theta(0) = T_\varphi^\varphi(0) = \frac{2}{32\alpha}$$

Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

ECOs and UCOs



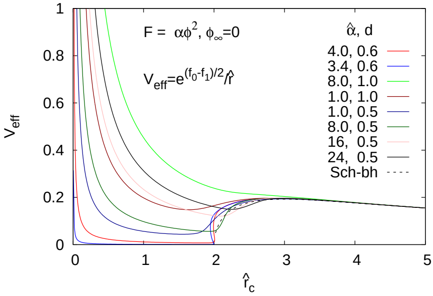
energy density $\rho = -T_0^0$ mass function $\mu(\hat{r}_c)$

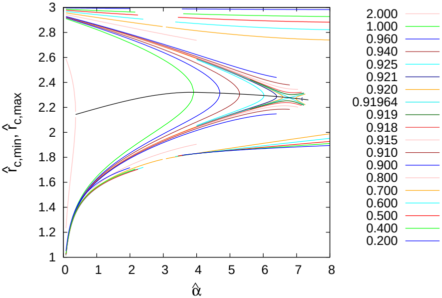
vs circumferential radius $\hat{r}_c = r_c/M$

Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650

UCOs: pairs of light-rings





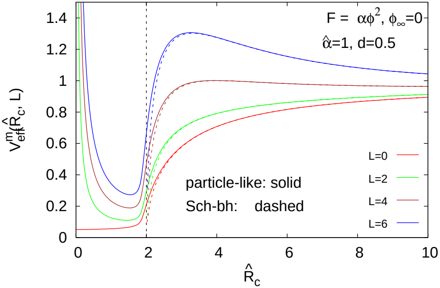
photon effective potential V_{eff} local extrema $\hat{r}_{\text{c,max}}$ and $\hat{r}_{\text{c,min}}$

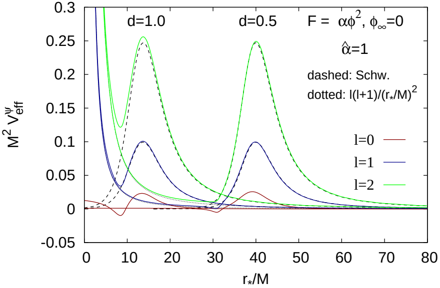
vs circumferential radius $\hat{r}_{\text{c}} = r_c/M$

Cardoso et al. 1406.5510 , Cunha et al. 1708.04211

Scalar EGB Particle-like Solutions

Kanti et al. 1910.02121, 2005.07650





effective potential $V_{\rm eff}^{m}$

(massive particles)

vs circumferential radius $\hat{R}_c = R_c/M$

effective potential $V_{\rm eff}^{\psi}$

(test scalar particle)

vs tortoise coordinate r^*/M

Cardoso et al. 1608.08637 echoes of ECOs

Outline

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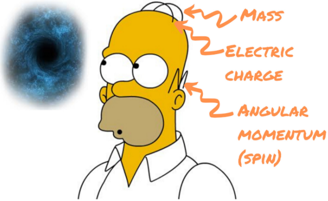
Conclusions

Beyond General Relativity: Scalar EGB Theories

black holes

- dilatonic
- spontaneously scalarized
  - curvature induced
  - spin induced



wormholes

- static
  - single throat
  - double throat
- slowly rotating
- geodesics

particle-like solutions

- regular metric, regular $T_{\mu\nu}$
- UCOs with pairs of light-rings
- echoes of ECOs

THANKS