The Many Faces of Black Holes

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Outline

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1. Introduction
   - General Relativity
   - Schwarzschild Black Holes
   - Kerr Black Holes
2. Astrophysical Black Holes
   - Stellar Mass Black Holes
   - Galactic Black Holes
3. Microscopic Black Holes
   - The Standard Model
   - Kerr–Newman Black Holes
   - Static Non-Abelian Black Holes
   - Rotating Einstein–Maxwell–Dilaton Black Holes
4. Conclusions and Outlook

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4. Conclusions and Outlook

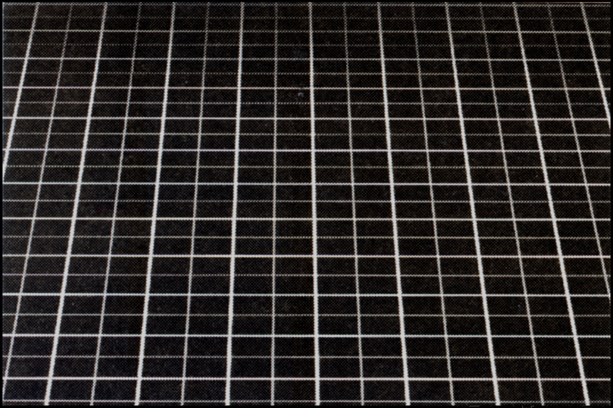
Flat Space–Time

- metric of Euclidean space

$$ds^2 = dx^2 + dy^2 + dz^2$$

- metric of Minkowski space-time

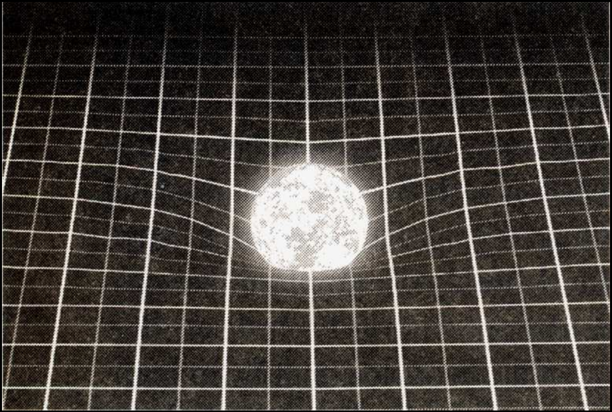
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$



Curved Space–Time

- metric of curved space-time

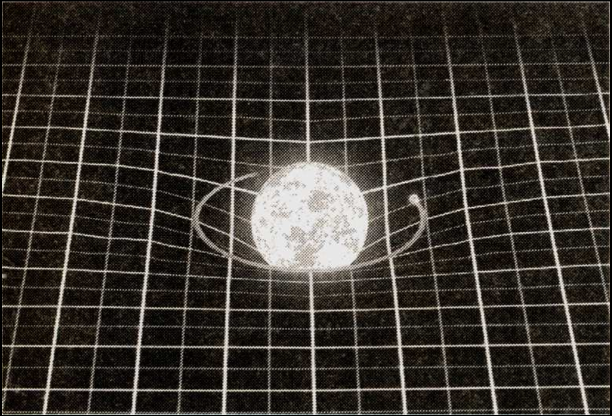
$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



Motion in Curved Space–Time

- motion in curved space–time

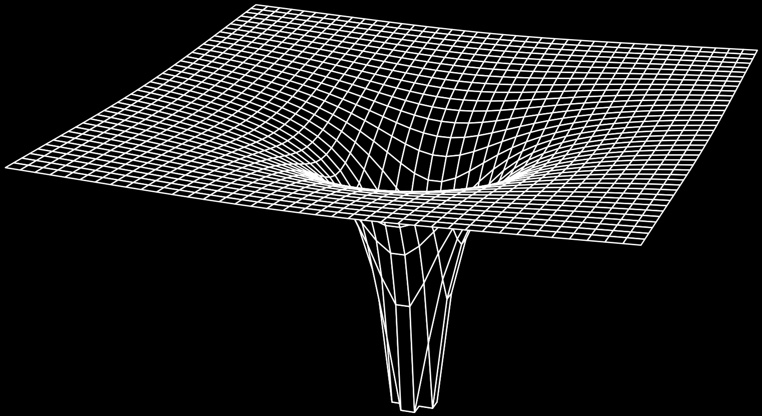
$$0 = \frac{d^2 x^\mu}{ds^2} + \left\{ {\mu \atop \rho\sigma} \right\} \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds}$$



Strongly Curved Space–Time

- metric of curved space–time

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$



Einstein Equations

- metric

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu$$

- Einstein equations

  matter tells space how to curve

$$\boxed{G_{\mu\nu} = \frac{8\pi G}{c^4}\, T_{\mu\nu}}$$

  $G_{\mu\nu}$: Einstein tensor

  $T_{\mu\nu}$: energy-momentum tensor

- equations of motion for matter/radiation

  metric $g_{\mu\nu}$ tells matter how to move

Einstein Equations

Outline

Schwarzschild Metric

Schwarzschild 1916

- space-time outside a star: $T_{\mu\nu} = 0$

$$ds^2 = -N(r)\, c^2 dt^2 + \frac{1}{N(r)} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

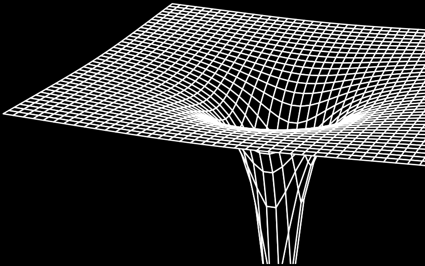
$$N(r) = 1 - \frac{2GM}{c^2 r}$$

static spherically symmetric metric

remark: Minkowski space-time has $N(r) = 1$

- space–time inside a star: $T_{\mu\nu} \neq 0$

Karl Schwarzschild 1873 — 1916



Schwarzschild Singularity

- Schwarzschild space-time

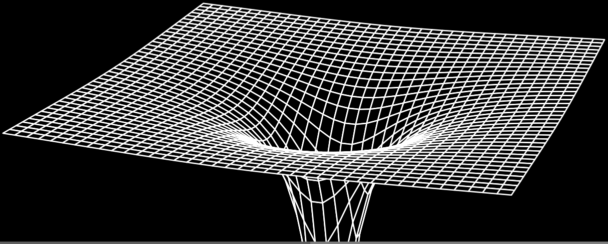
$$ds^2 = -N(r)\,c^2 dt^2 + \frac{1}{N(r)}\,dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

$$N(r) = 1 - \frac{2GM}{c^2 r} = 1 - \frac{r_{\rm H}}{r}$$

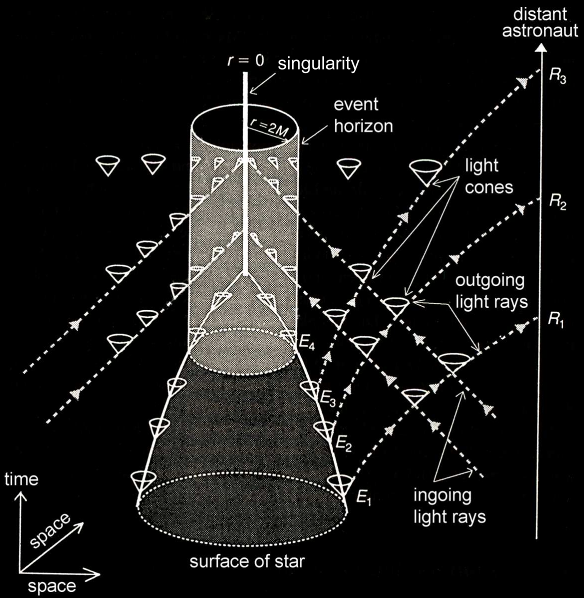
- black holes: M
- Schwarzschild radius $r_{\rm H}$

$$N(r_{\rm H}) = 0: \ r_{\rm H} = \frac{2GM}{c^2}$$

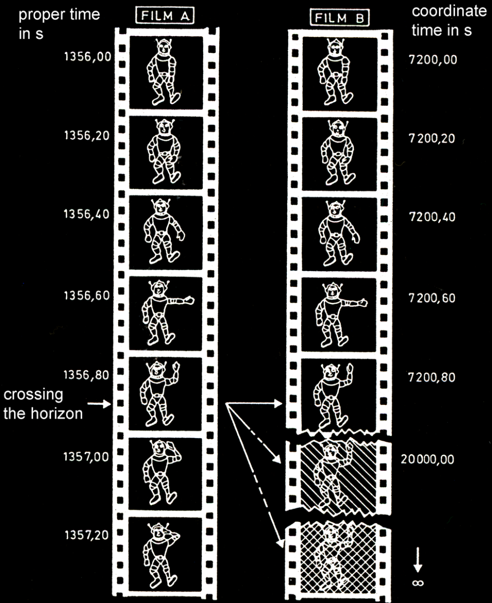
- event horizon
- coordinate singularity
- true singularity $r = 0$



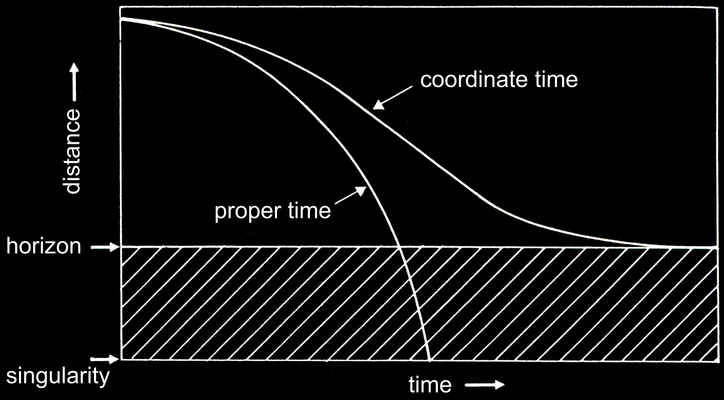
Formation of a Black Hole



Event Horizon



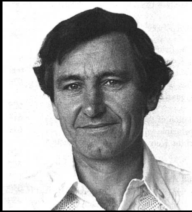
Event Horizon



Outline

Rotating Black Holes

rotating generalization of the Schwarzschild black holes: Kerr (1965)



Roy Kerr *1934

Kerr metric in Boyer–Lindquist coordinates

$$ds^2 = -\frac{\Delta}{\rho^2}\left(dt - a\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left(adt - \rho_0^2 d\phi\right)^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta\ , \quad \rho_0^2 = r^2 + a^2\ , \quad \Delta = r^2 - 2Mr + a^2$$

a is the specific angular momentum: $a = \frac{J}{M}$

$a = 0$: Schwarzschild

Kerr Black Holes in the Equatorial Plane

metric in Boyer–Lindquist coordinates:
equatorial plane: $\theta = \pi/2$
through center of black hole, perpendicular to the spin axis

$$ds^2 = -\left(1-\frac{2M}{r}\right)dt^2 - \frac{4Ma}{r}dtd\phi + \frac{dr^2}{1-\frac{2M}{r}+\frac{a^2}{r^2}} + \left(1+\frac{a^2}{r^2}+\frac{2Ma^2}{r^3}\right)r^2d\phi^2$$

comparison with Schwarzschild ($a \neq 0$)

- dt^2 Term: static limit
- $dt\,d\phi$ Term: frame dragging and Lense–Thirring
- dr^2 Term: event horizon

Event Horizon of Kerr Black Holes

First new feature

coordinate singularity:

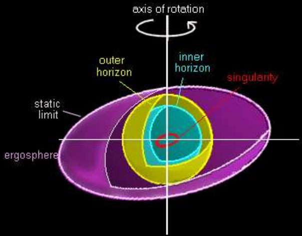
$$1 - \frac{2M}{r} + \frac{a^2}{r^2} = 0$$

radial coordinate of the horizon $r_{\rm H}$

$$r_{\rm H} = M \pm \sqrt{M^2 - a^2}$$

- $a < M$
  - $+$: event horizon of the black hole
  - $-$: inner horizon

  maximal angular momentum $a = M$: extremal black hole
- $a > M$: naked singularity (Cosmic Censorship)



black hole with horizons

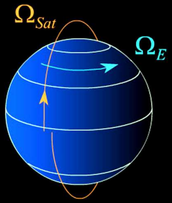
Sir
Roger Penrose
*1931

Gravitomagnetism

Second new feature

- The product $dt\,d\phi$ implies that the coordinates t and ϕ are intimately linked.
- The Kerr metric predicts Lense–Thirring effect and frame dragging.

What does Lense–Thirring mean?



- Schwarzschild: radial rocket thrust is required to keep a stationary observer at a fixed radius
- Kerr: additional tangential rocket thrust is required to keep an observer at a stationary position, i.e., a position from which the fixed stars do not appear to move

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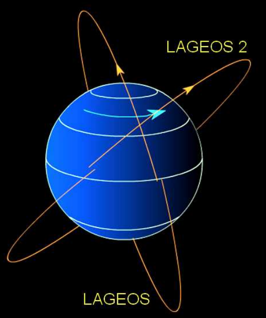

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Gravitomagnetism

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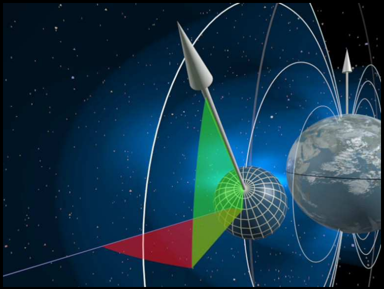
What does frame dragging mean?



- Schwarzschild: gyroscopes always point in the same direction
- Kerr: gyroscpes start to precess, i.e., the direction with respect to distant stars changes

Static Limit of Kerr Black Holes

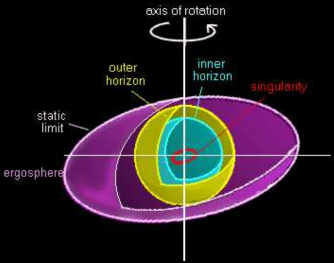
Third new feature

The coefficient of dt^2 goes to zero at the static limit

- in the equatorial plane $r_\mathrm{S} = 2M$
- for radii smaller than r_S (but greater than r_H) an observer cannot remain at rest

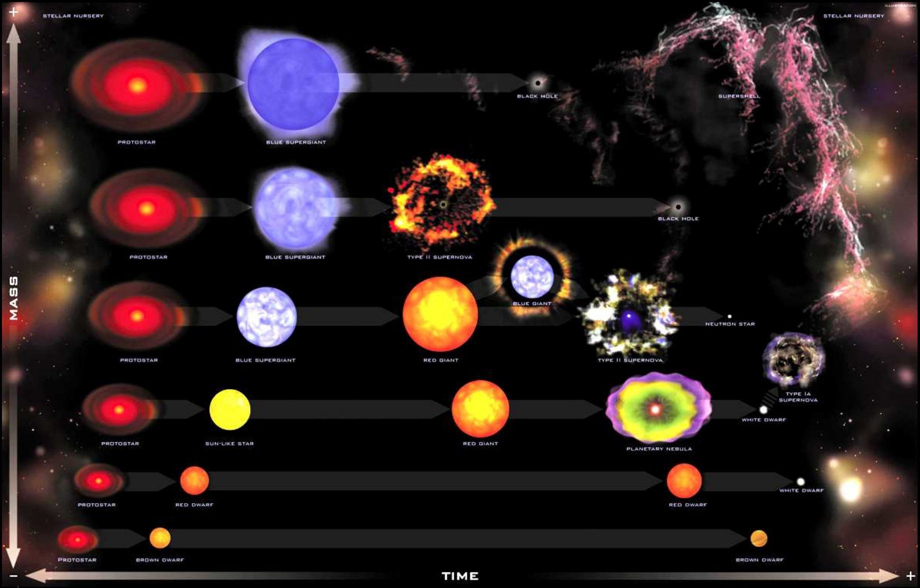
- the space between the static limit and the event horizon is called ergosphere
- inside the ergospere an observer is inexorably dragged along in the direction of rotation of the black hole
- horizon velocity:

$$\Omega = \frac{a}{r_+^2 + a^2}$$

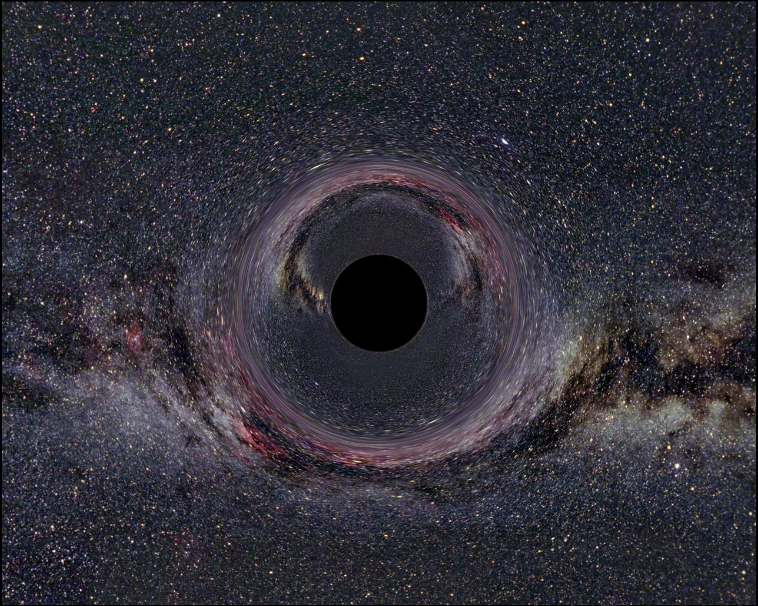


Outline

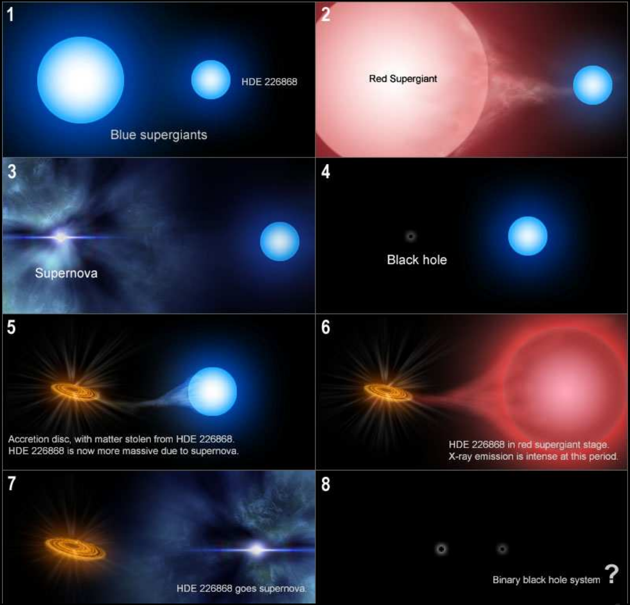
Stellar Evolution



Detection of Stellar Mass Black Holes

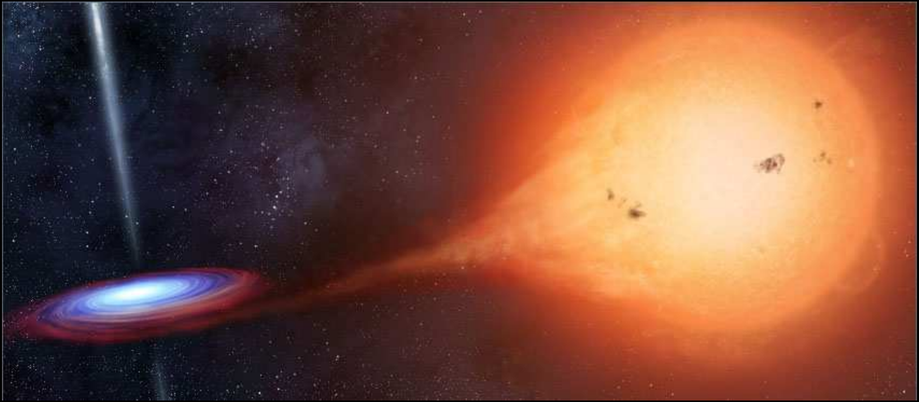


Artist's View of a Binary System



Formation of
Cygnus X-1
and its possible future

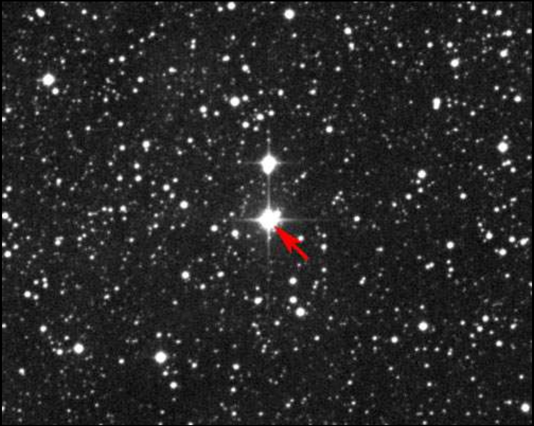
Artist's View of a Binary System



Cygnus X-1:

- O9-B0 supergiant ($T = 31,000$ K) and a compact object.
- mass of the supergiant $\sim 20 - 30\ M_\odot$.
- mass of compact object $\sim 7 - 13\ M_\odot$
- distance to Cygnus X-1 ~ 2500 parsec ~ 8000 ly

Candidates



Cygnus X-1

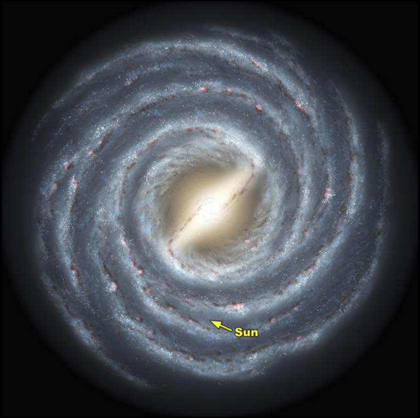
Evidence for stellar black holes

Stellar Black Hole Candidates in the Milky Way (R. Blandford & N. Gehrels 1999)

X-Ray Source Name	**Mass of Companion** (in $M_\odot$)	**Mass of Black Hole** (in $M_\odot$)
Cgnus X-1	$24-42$	$11-21$
V404 Cygni	~ 0.6	$10-15$
GS 2000+25	~ 0.7	$6-14$
H 1705-250	$0.3-0.6$	$6.4-6.9$
GRO J1655-40	2.34	7.02
A 0620-00	$0.2-0.7$	$5-10$
GS 1124-T68	$0.5-0.8$	$4.2-6.5$
GRO J042+32	~ 0.3	$6-14$
4U 1543-47	~ 2.5	$2.7-7.5$

Outline

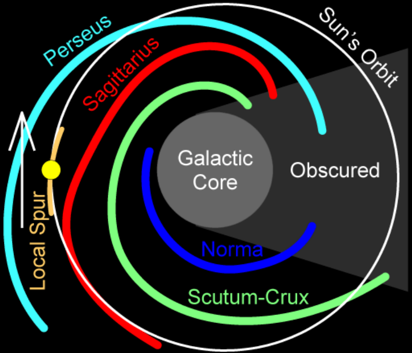
Milky Way



The Milky Way

- Diameter: $100,000 \text{ ly} = 30 \text{ kpc}$
- Thickness: $3,000 \text{ ly} = 0.92 \text{ kpc}$
- Thickness of bulge: $16,000 \text{ ly} = 5 \text{ kpc}$
- Mass: $1.9 \cdot 10^{12}\, M_{\odot}$
- Number of stars: $3 \cdot 10^{11}$
- Rotation velocity of sun: 1 orbit per $\sim 240 \cdot 10^{6}$ y

Milky Way



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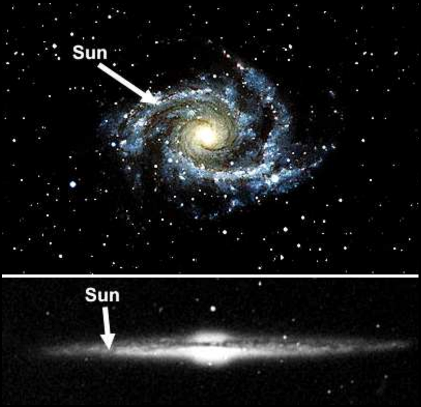

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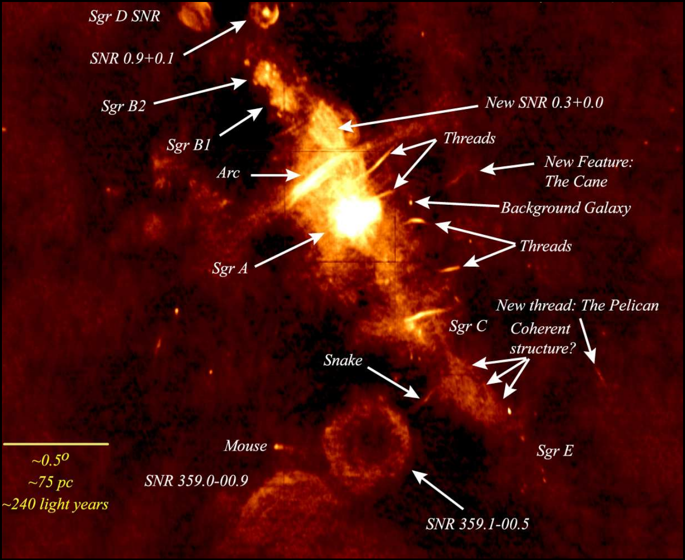
Black Hole at the Center of the Milky Way



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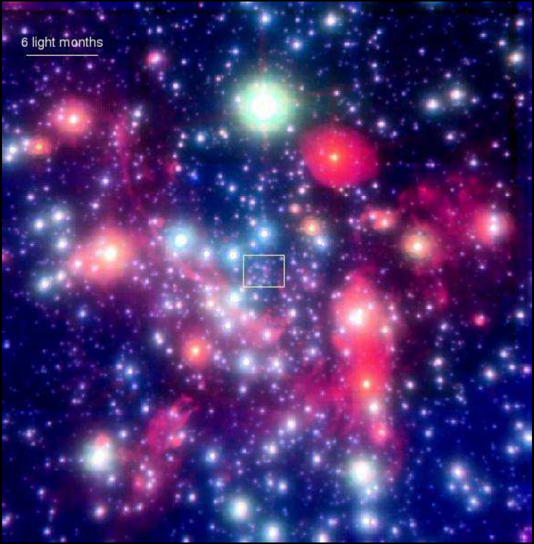

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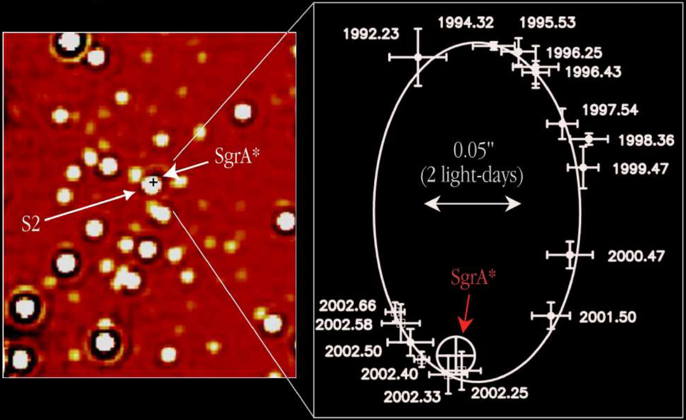

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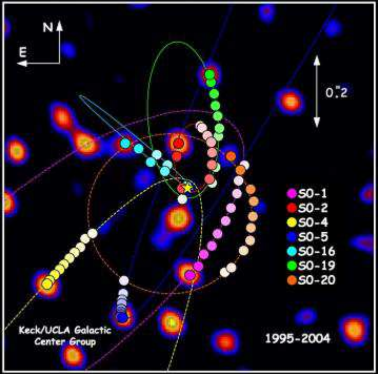


Black Hole: mass $3.7 \cdot 10^6\ M_\odot$ (Yusuf–Zasdeh *et al. Astrophys. J.* **644**, 198 (2006))
angular velocity $\sim 1/17\ \mathrm{min}$

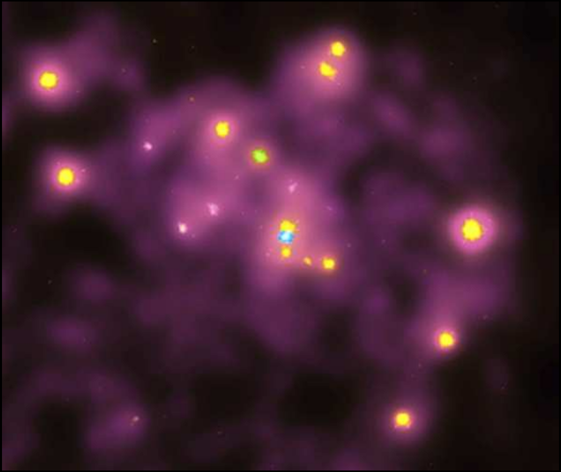
R. Genzel (1995 – 2006)

Andromeda Galaxy

Andromeda = M31 = NGC 224

- Location: Constellation Andromeda
- Distance = $2.5 \cdot 10^6$ ly $= 0.75$ Mpc

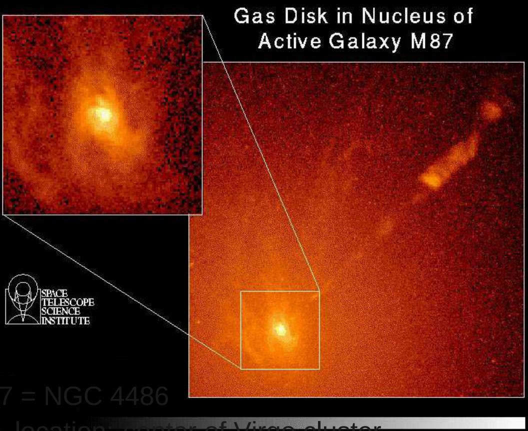
Andromeda Galaxy

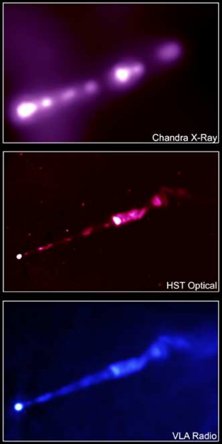


Andromeda

- Supermassive black hole $M = 30 \cdot 10^6\, M_\odot$
- Measuring method: X–rays

Elliptical Galaxy

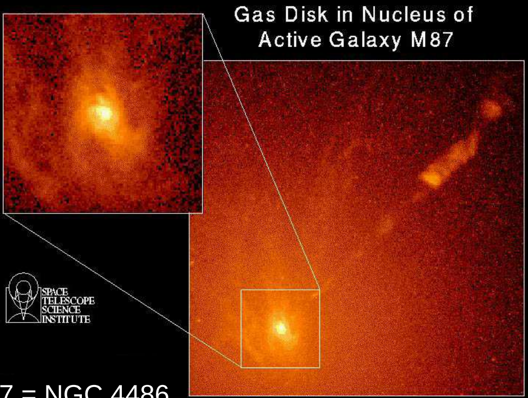


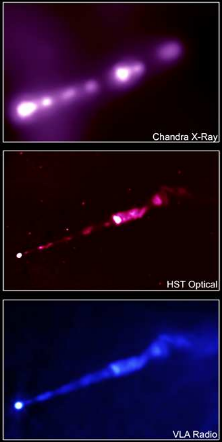


M87 = NGC 4486

- location: center of Virgo cluster
- distance = $6 \cdot 10^7$ ly $= 18$ Mpc
- mass = $2.7 \cdot 10^{12}\ M_\odot$
- supermassive black hole $M = 3 \cdot 10^9\ M_\odot$
- measuring method: motion of gas

Elliptical Galaxy





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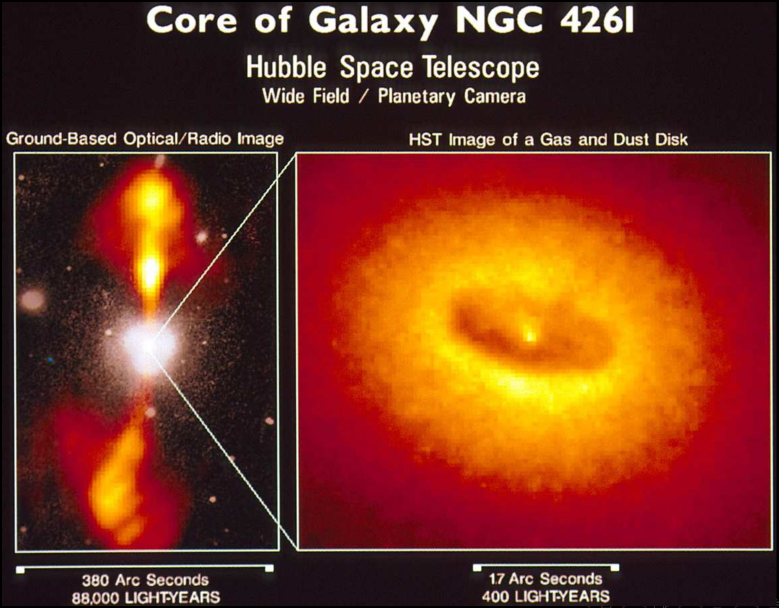
Accretion Disk

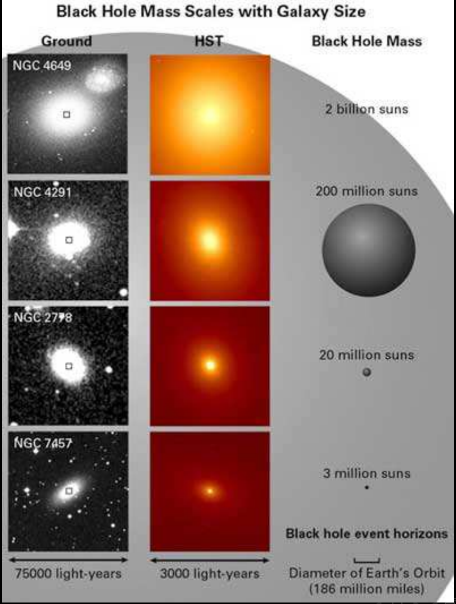


Evidence for black holes

Supermassive Black Hole Candidates (M.J. Rees 1997)

Name	**Mass** (in $M_\odot$)	**Method**
M87	$3 \cdot 10^9$	stars + optical disc
NGC 3115	10^9	stars
NGC 4486 B	$5 \cdot 10^8$	stars
NGC 4594 (Sombrero)	$5 \cdot 10^8$	stars
NGC 3377	$8 \cdot 10^7$	stars
NGC 3379	$5 \cdot 10^7$	stars
NGC 4258	$4 \cdot 10^7$	masing H_2O disc
M 31 (Andromeda)	$3 \cdot 10^7$	stars
M 32	$3 \cdot 10^6$	stars
Galactic Centre	$2.5 \cdot 10^6$	stars + (3D motions)

Correlation of Black Hole and Galaxy Size



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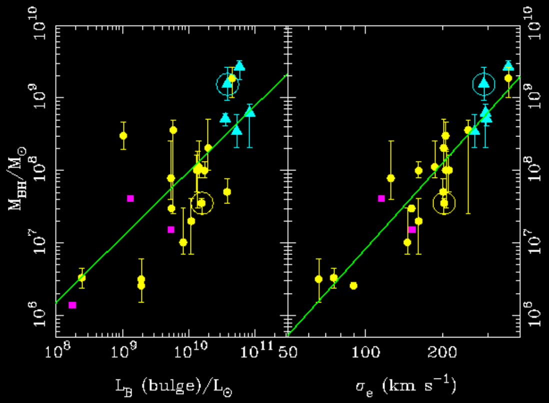

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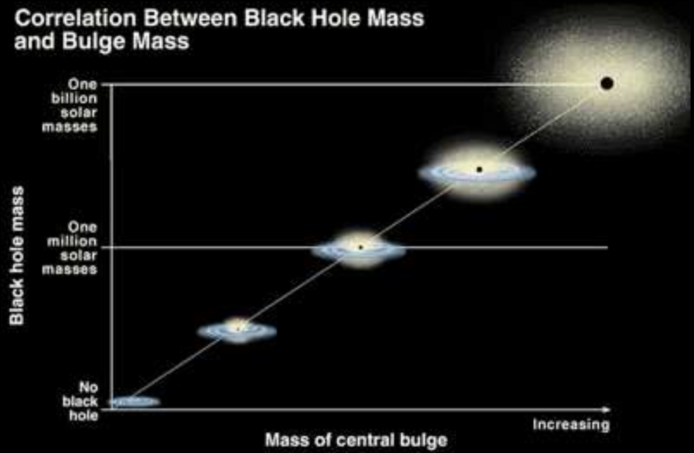
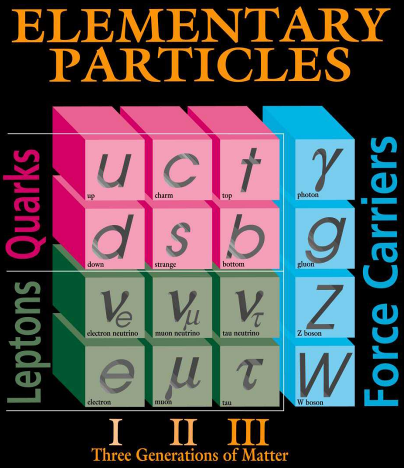


Outline

Standard Model of Particle Physics

- What types of black holes do current particle physics theories predict?
- Standard Model is modelled after Maxwell's Theory of Electromagnetism
- Standard Model: gauge field theory



Maxwell Theory

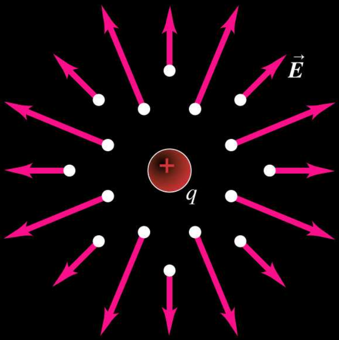
electric field $\vec{E}$, magnetic field $\vec{B}$
electromagnetic potential A^μ: $(\Phi, \vec{A})$

$$\vec{E} = -\nabla\Phi - \partial_t \vec{A}\ , \quad \vec{B} = \nabla \times \vec{A}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

the field strength tensor $F_{\mu\nu}$ is gauge invariant, while $A_\mu \to A_\mu + \partial_\mu \chi$

- the electromagnetic field can carry energy
- the electromagnetic field can carry momentum
- the electromagnetic field can carry angular momentum



Coulomb field

Outline

Einstein–Maxwell Equations

Einstein–Maxwell theory

- Einstein equations

Einstein tensor $\longrightarrow$ $G_{\mu\nu} = 8\pi G\, T_{\mu\nu}$ $\longleftarrow$ stress–energy tensor

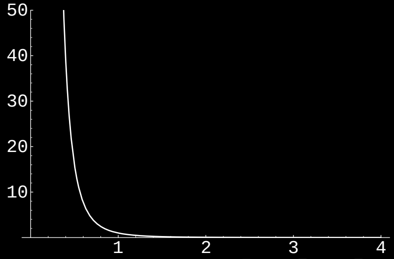
- Maxwell field equations

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}F^{\mu\nu}\right) = 0$$

For spherical symmetry:

$$T_{00} \sim E^2 \sim \frac{1}{r^4}$$

what are the properties of charged black holes?



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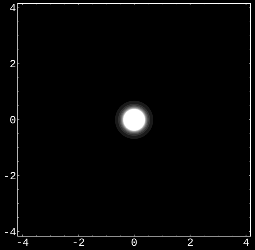
what are the properties of charged black holes?



Reissner–Nordström Black Holes

1916: H. Reissner, 1918: G. Nordström
charged static spherically symmetric black holes:

$$N(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$$

- electrically charged black hole: M, Q
  - horizons: $r_\pm = M \pm \sqrt{M^2 - Q^2}$
  - event horizon: $r_\mathrm{H} = M + \sqrt{M^2 - Q^2}$
- magnetically charged black hole: M, P

energy density outside the horizon due to the Coulomb field of the charge Q

$$M = M_\mathrm{H} + M_\mathrm{outside} = M_\mathrm{H} + 2\Phi_\mathrm{H} Q$$

Hans J. Reissner
1874 – 1967

Gunnar Nordström
1881 – 1923

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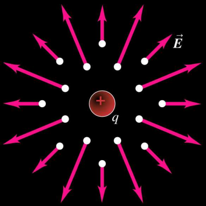

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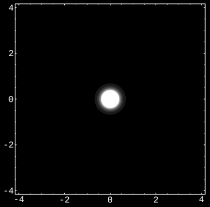
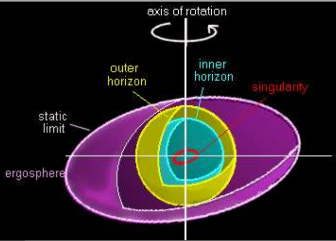


Kerr–Newman Black Holes

charged rotating black holes: adding charge to the Kerr solution

global charges:	$M,\ J = aM,\ Q,\ P$	
dipole moments:	$\mu_{\rm mag} = g_{\rm mag} \dfrac{Q}{2M} J, \quad \mu_{\rm el} = g_{\rm el} \dfrac{P}{2M} J$	$(g_{\rm Dirac} = 2)$
horizons:	$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2 - P^2}$	$(\Delta = 0)$
static limit:	$r_0 = M + \sqrt{M^2 - a^2 \cos^2\theta - Q^2 - P^2}$	$(g_{tt} = 0)$
horizon velocity:	$\Omega = \dfrac{a}{r_+^2 + a^2}$	



Rotation and Deformation

- Kerr–Newman black holes co–rotate

$$\Omega > 0 \quad \Leftrightarrow \quad J > 0$$

they do not counter-rotate:

$$\Omega > 0 \ , \ J < 0$$

- a static horizon implies vanishing angular momentum

$$\Omega = 0 \quad \Leftrightarrow \quad J = 0$$

there are no black holes with

$$\Omega = 0 \ , \ J \neq 0 \qquad \text{or} \qquad \Omega \neq 0 \ , \ J = 0$$

- rotation implies an oblate deformation of the horizon

Summary: Einstein–Maxwell Black Holes

	static	rotating
spherically symmetric	Schwarzschild (M) Reissner-Nordström (M, Q, P)	–
axially symmetric	–	Kerr (M, J) Kerr–Newman (M, Q, P, J)

- Uniqueness theorem

  black holes are uniquely determined by their mass M, angular momentum J, charges Q and P
- Israel's theorem

  static black holes are spherically symmetric
- Staticity theorem

  stationary black holes with non-rotating horizon are static

Werner Israel

*1931

Outline

Non–Abelian Fields

- standard model
  - QCD: gluons $a = 1, \ldots, 8$
  - WS: $W^{\pm}$, Z^0

    non-Abelian gauge fields

    non-linearity in field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

- effective theories
  - Skyrme model of strong interactions

    non–Abelian scalar and pseudo–scalar fields

$$\sigma^2 + \vec{\pi}^2 = f^2$$

What are the consequences of the presence of non–Abelian fields?

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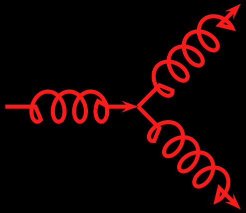

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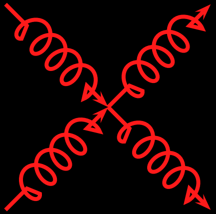
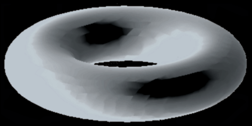


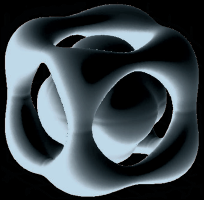
What are the consequences of the presence of non–Abelian fields?

Regular Solutions: Solitons

solitons in flat space: extended solutions with finite energy
examples: nucleons, nuclei, magnetic monopoles, etc.

- static spherically symmetric solitons
- static axially symmetric solitons
- static platonic solitons





coupling to gravity: gravitating solitons

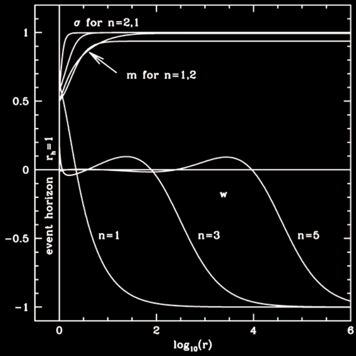
Non–Uniqueness of Black Holes

black hole solutions: Volkov, Gal'tsov 1989, et al.

metric: $$ds^2 = -A^2(r)N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\Omega^2$$

gauge potential: $$A_\mu dx^\mu = \frac{1 - w_k(r)}{2}\left[\tau_\varphi d\theta - \sin\theta \tau_\theta d\varphi\right]$$

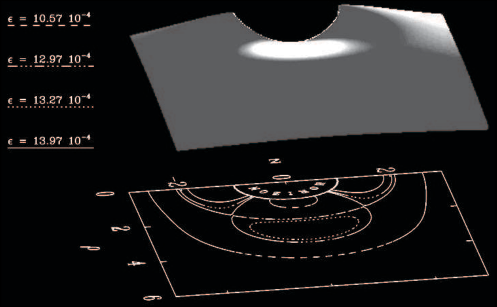
- regular at $r = r_{\rm H}$
- asymptotically flat
- node number k
  $k = 1, ..., \infty$
- limiting solution
  $k \to \infty$: RN
- no charge
- no uniqueness

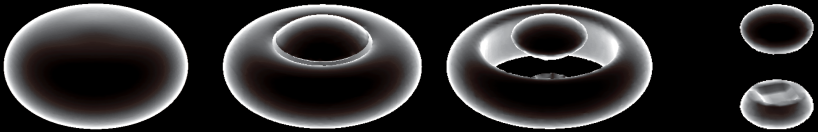


Static Axially Symmetric Black Holes

black hole solutions: Kleihaus, Kunz 1997

- regular horizon
- asymptotically flat
- no charge
- no uniqueness
- static but axially symmetric
- deformed horizon





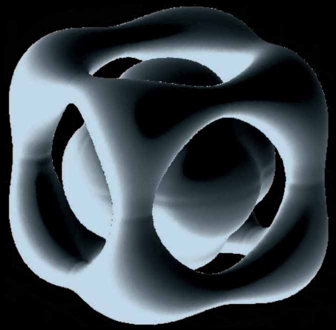
$\epsilon = 10.57\ 10^{-4}$ $\epsilon = 12.97\ 10^{-4}$ $\epsilon = 13.27\ 10^{-4}$ $\epsilon = 13.97\ 10^{-4}$

Platonic Black Holes

Challenge:
Are there black holes with only discrete symmetries?

Theodora Ioannidou,
Burkhard Kleihaus,
Jutta Kunz,
Kari Myklevoll

work in progress



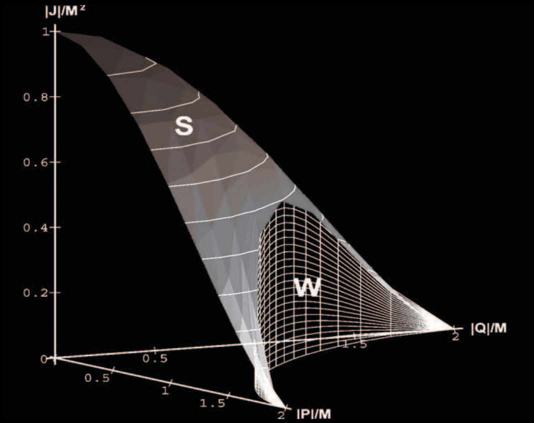
platonic black holes?

Outline

Inclusion of a Scalar Field: Dilaton with $\gamma = \sqrt{3}$

Rasheed 1995

domain of existence of Einstein–Maxwell–dilaton black holes: $\gamma = \sqrt{3}$



vertical wall W:

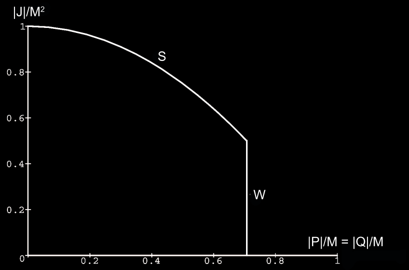
stationary black holes
with static horizon:
$\Omega = 0$, $J \neq 0$

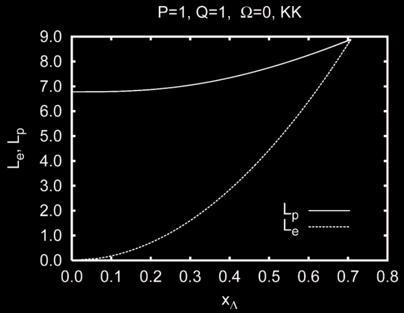
J increases, $M =$ const

- frame dragging?
- effect of rotation?

Angular Momentum and Deformation

extremal $|P| = |Q|$ solutions





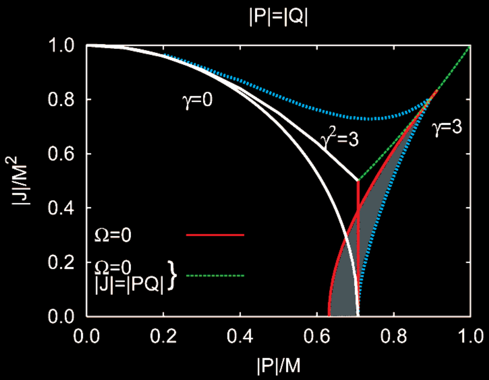
vertical wall W:
$\Omega = 0$, $J \neq 0$ black holes

horizon circumferences:
L_e and L_p

- a negative fraction of J resides behind the horizon: $J_\mathrm{H} < 0$
- effect of the rotation: prolate deformation of the horizon

EMD Black Holes: $\gamma > \sqrt{3}$

Kleihaus, Kunz, Navarro-Lérida 2004



extremal: $|P| = |Q|$

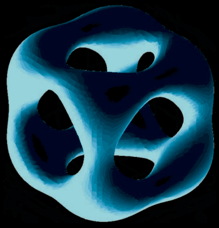
stationary: $\Omega = 0$

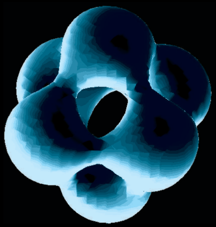
stationary: $\Omega = 0$,
$J = PQ$

what is in the shaded region?
counterrotating black holes

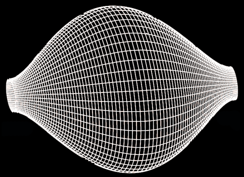
Conclusions: Surprises with Microscopic Black Holes

- Kerr black holes: astrophysical black holes
- Kerr-Newman black holes:
  uniqueness theorem
  Israel's theorem
- static non-Abelian black holes:
  no uniqueness in terms of global charges
  not spherically symmetric static black holes
  platonic black holes?
- rotating EMD black holes:
  $\Omega = 0,\ J \neq 0$
  $\Omega < 0,\ J > 0$
  prolate deformation





Outlook: Further Surprises?



Higher dimensions:

- black holes
  - non-uniqueness of black holes
  - $\Omega \neq 0$, $J = 0$ black holes
  - non-static $\Omega = 0$, $J = 0$ black holes
  - black holes with negative horizon mass
  - ???
- black rings
- black strings
- ???