Measurement of Ohmic Resistances, Bridge Circuits and Internal Resistances of Voltage Sources

Keywords:
OHM's law, KIRCHHOFF's laws (KIRCHHOFF's current and voltage laws), internal resistances of measuring instruments, WHEATSTONE bridge, bridge circuit, voltage source, internal resistances of voltage sources, terminal voltage, strain gauge.

Measuring program:
Measurement of resistances with different ohmmeters, determination of a resistance by means of a current / voltage measurement, WHEATSTONE bridge, internal resistance of a function generator, specific resistance of tap water, bridge circuit for measuring alterations to resistances.

References:
/1/ SCHENK, W., KREMER, F. (HRSG.): „Physikalisches Praktikum", Vieweg + Teubner Verlag, Wiesbaden
/2/ WALCHER, W.: „Praktikum der Physik" Teubner Studienbücher, Teubner-Verlag, Stuttgart
/3/ EICHLER, H.J., KRONFELDT, H.-D., SAHM, J.: „Das Neue Physikalische Grundpraktikum", Springer-Verlag, Berlin among others

1 Introduction

In the first part of this experiment, an insight into the different methods of measuring ohmic[1] resistances will be given. In particular, it will be shown to which degree real properties of measuring instruments affect the results of a measurement, and which methods yield the best results in individual cases.

In the second part of the experiment, the characteristics of real voltage sources are investigated. The main question here is how an important characteristic of such voltage sources, namely their internal resistance, can be measured.

Additionally, the specific resistance of tap water is measured, and the linear relationship between changes in resistance and voltage in a bridge circuit is analysed.

2 Theory

2.1 Kirchhoff's Laws

Knowledge of KIRCHHOFF's laws[2] is a prerequisite for analysing electric networks (circuits) (see Fig. 1).

KIRCHHOFF´s first law (*current law*) reads:

> The sum of all currents at a branch point equals zero.

For this law the sign convention is valid that currents towards and off a network node are marked by contrary signs. It is of no importance, whether the affluent currents are marked positively and the effluent currents are marked negatively.

Applied to the circuit in Fig. 1, KIRCHHOFF's first law at the nodes A and B reads as follows:

(1)   A: $I_1 - I_2 - I_3 = 0$ B: $I_2 + I_3 - I_1 = 0$

KIRCHHOFF´s second law (*voltage law*) reads:

> In a closed loop of a network the sum of all components of the voltage equals zero.

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[1]   Named after GEORG SIMON OHM (1789 - 1854)
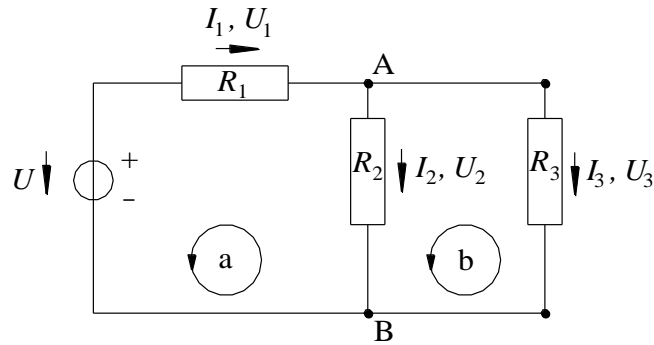[2]   GUSTAV ROBERT KIRCHHOFF (1824 – 1887)

Fig. 1:   Circuit with a direct voltage source with terminal voltage U, the resistors R_1,...,R_3 as well as two nodes A and B and two loops a and b.

For the application of this law a sign convention is essential as well. It reads:

a)   A direction („counting arrow") is assigned to each voltage running from the positive to the negative terminal (e.g. voltage source).

b)   A direction („counting arrow") is assigned to each current, which marks the direction of movement of the positive charge carriers, i.e., the current flows from the positive to the negative pole per definition. According to OHM's law, the direction of the voltage U_R over a resistance R corresponds to the direction of the current I_R, flowing through R and causing the voltage drop U_R.

c)   For the application of the voltage law a direction of rotation has to be fixed (i.e., clockwise or counter-clockwise). Voltages with counting arrows corresponding with the direction of rotation are counted as positive, the others as negative.

Applied to the circuit in Fig. 1, the KIRCHHOFF's second law in the loops a and b for a counter clockwise direction of rotation reads:

(2)   a : $U - U_2 - U_1 = 0$ b : $U_2 - U_3 = 0$

With KIRCHHOFF's two laws and the related sign conventions all electric networks which are employed in the course of the introductory laboratory course can be described.

Question 1:
-   How can the formulas for the parallel and series connection of ohmic resistances be derived from KIRCHHOFF's laws? Which are the corresponding relationships?

It is not always easy to recognize whether resistances and other components in a network are in parallel or in series connections. Two rules derived from KIRCHHOFF's laws can be useful in making that decision:

➢ Resistances are in parallel if they show the same voltage drop.
➢ Resistances are in series if the same current flows through them.

2.2   Methods for Measuring Ohmic Resistances

2.2.1 Determining the Resistance from the Markings

Fig. 2 depicts some common retail versions of resistors which are labeled with various kinds of markings (letterings or color codes). In the simplest case, the value of the resistor is printed directly on the casing. Some inscriptions commonly used for this purpose are "120R" for 120 Ω, "4R7" for 4.7 Ω, "3k3" for 3.3 kΩ, or "5M6" for 5.6 MΩ.
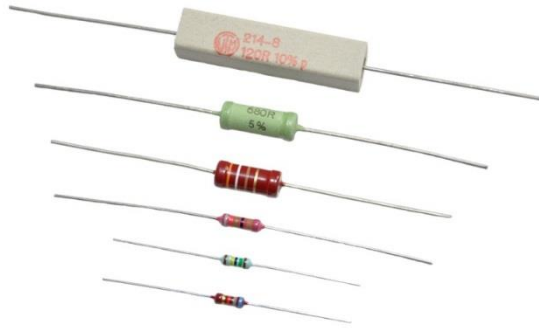
Fig. 2:   Common retail versions of resistors with various types of markings. Top *load resistors* (power rating of several W), bottom resistors for low power applications (< 1 W).

Equally simple is reading the colour key printed on most types of resistors. This colour key on the resistor consists of chromatic rings, which are always arranged in such a way that the first chromatic ring is closer to the one end of the resistor than the last chromatic ring is to the other end. Table 1 shows how the value of a resistance can be determined by means of the colour coding.

Table 1: Colour key for ohmic resistances.

3 - 4 rings	1st ring	2nd ring		3rd ring	4th ring	
5 - 6 rings	1st ring	2nd ring	3rd ring	4th ring	5th ring	6th ring
Colour ↓	1st number	2nd number	3rd number	Multiplier / Ω	Tolerance / %	Temp. coeff. / $10^{-6}\Omega K^{-1}$
black		0	0	1		$\pm\,250$
brown	1	1	1	10	$\pm\,1$	$\pm\,100$
red	2	2	2	10^2	$\pm\,2$	$\pm\,50$
orange	3	3	3	10^3		$\pm\,15$
yellow	4	4	4	10^4		$\pm\,25$
green	5	5	5	10^5	$\pm\,5^{*)}$	$\pm\,20$
blue	6	6	6	10^6		$\pm\,10$
violet	7	7	7	10^7		$\pm\,5$
grey	8	8	8	$10^{-2*)}$		$\pm\,1$
white	9	9	9	$10^{-1*)}$	$\pm\,10^{*)}$	
none					$\pm\,20$	
silver				10^{-2}	$\pm\,10$	
gold				10^{-1}	$\pm\,5$	

$^{*)}$ where the conductivity of gold and silver enamel interferes

Question 2:
-   What is the value of a resistor for the colour coding Red (first ring) - Violet - Brown - Gold?
-   What is the colour coding for a resistor of (3.9 kΩ $\pm$ 10 %)?

2.2.2 Determining the Resistance by Means of a Current / Voltage Measurement

When the two ends of an ideal voltage source (cf. Chapter 2.3), which delivers an adjustable terminal voltage *U*, are connected with the connecting wires of a resistor *R*, the current *I* flows through the resistor and OHM's law reads:

(3) $R = \dfrac{U}{I}$

By measuring the voltage *U* using a voltmeter and the current *I* using an ammeter, *R* can thus be determined. Such a measurement can be carried out using the two circuits A and B according to Fig. 3.
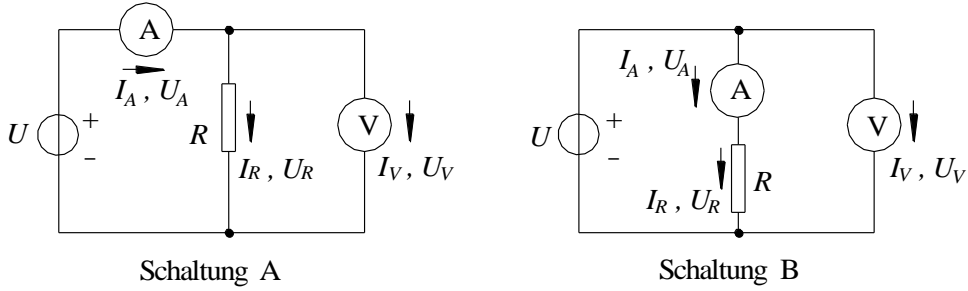
Fig. 3: Two possible circuits for measuring the resistance R by means of a current/voltage measurement. R is connected with a direct voltage source with terminal voltage U. The current is measured with the ammeter A, the voltage is measured with the voltmeter V.

If ideal measuring instruments were available, i.e., ammeter with a negligible internal resistance and a voltmeter with an infinitely high internal resistance, both circuits would yield the same result. However, an ammeter has an internal resistance of $R_A > 0$ and a voltmeter has an internal resistance of $R_V < \infty$. Consequently, a value for the resistance R with an error ΔR is determined with each circuit. We will now determine the relative error $\Delta R/R$ for both circuits.

Let I_A be the current through the ammeter, I_R the current through the resistor, and I_V the current through the voltmeter. U_A is the voltage drop across the ammeter, U_R is the voltage drop across R and U_V is the voltage drop across the voltmeter. According to KIRCHHOFF's current law we then obtain for the circuit A:

$$(4) \qquad I_A - I_R - I_V = 0$$

and thus

$$(5) \qquad I_A = I_R + I_V$$

Using this circuit, the value determined for the resistor R_M is given by

$$(6) \qquad R_M = \frac{U_V}{I_A} = \frac{U_V}{I_R + I_V}$$

The deviation ΔR from the actual value R

$$(7) \qquad R = \frac{U_R}{I_R}$$

is hence given by:

$$(8) \qquad \Delta R = \left| R - R_M \right|$$

Inserting Eq. (6) into Eq. (8) and using circuit A, we obtain after some rearrangement and taking into consideration $U_V = U_R$ (voltage law) for the relative error:

$$(9) \qquad \frac{\Delta R}{R} = \frac{R}{R + R_V}$$

According to KIRCHHOFF's voltage law we obtain for circuit B:

$$(10) \qquad U_V - U_A - U_R = 0$$

and thus

(11) $U_V = U_A + U_R$

Considering $I_A = I_R$, the measured resistance R_M is:

(12) $R_M = \dfrac{U_V}{I_A} = \dfrac{U_V}{I_R} = \dfrac{U_A + U_R}{I_A} = R_A + R$

For the actual resistance R we refer again to Eq. (7). Inserting Eq. (12) into Eq. (8), we obtain for the relative error, if using circuit B:

(13) $\dfrac{\Delta R}{R} = \left| -\dfrac{R_A}{R} \right|$

Question 3:
- Sketch the graph of the relative error as a function of the resistance R for circuits A and B in a diagram. Is one of the circuits better than the other one, in principle? If not: in which case would the individual circuits be preferred?
- The two circuits are called „current-correct" and „voltage-correct" circuits, respectively. Which name belongs to which circuit? Why?

2.2.3 Measurement of Resistance Using an Ohmmeter

Instead of determining the resistance from a current/voltage measurement, it can also be directly measured using an analogous ohmmeter (pointer instrument). In the simplest case such an ohmmeter consists of a voltage source (battery), to which the resistance R is connected, a variable internal resistance R_i and an ammeter, by means of which the current through the resistance R is determined. This current causes a needle deflection, which is then read off an appropriate OHM scale. This scale is inversely proportional to the current scale due to the relationship $R = U/I$.

Since the voltage source does not always provide the same voltage (ageing of battery), the ohmmeter has to be calibrated by adjusting R_i before beginning the measurement. For this purpose the contacts are shorted out and the needle deflection is adjusted to 0 Ω using a regulating screw.

Modern digital ohmmeters are structured differently. Generally, they are integrated into *multimeters*. Such instruments contain complex electronic circuits with integrated microprocessors for measuring the required parameters (current, voltage, resistance, frequency among others) and LCD elements displaying the measured values.

2.2.4 Measurement of Resistance Using the WHEATSTONE Bridge

By means of a WHEATSTONE bridge[3] the value of a resistance R can be determined without errors being caused by inadequate (real) measuring instruments for the current, voltage or resistance; however, a gauged comparative resistance is required for this purpose.

We consider a WHEATSTONE bridge-circuit like the one in Fig. 4. A homogenous resistance wire, usually of constantan[4] with the specific resistance ρ ($[\rho] = \Omega$m), the total length $l = l_1 + l_2$, and the cross-sectional area A is connected to the resistor R to be measured and a gauged comparative resistor R_3 as shown in the figure. The resistances of the two wires are:

(14) $R_1 = \rho \dfrac{l_1}{A}$ and $R_2 = \rho \dfrac{l_2}{A}$

The voltage U, taken from a direct-voltage source, is connected to this resistance network. The current flowing between the point P and the shiftable tapping point Q along the resistance wire is measured using an ammeter A.

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[3]   CHARLES WHEATSTONE (1802 – 1875)
[4]   Constantan is an alloy consisting of approx. 60 % copper and approx. 40 % nickel, the specific resistance of which is nearly constant across a wide temperature range ($\rho \approx 45 \times 10^{-8}$ Ωm at 20 °C).
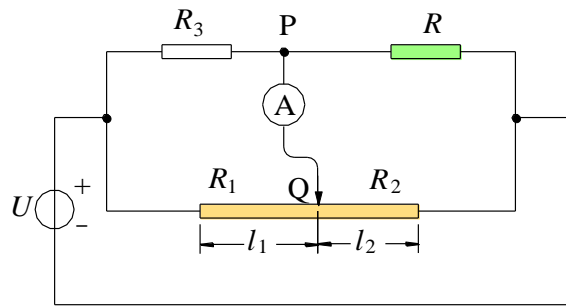
Fig. 4: Wheatstone bridge with constantan resistance wire (yellow). R (green) is the resistor to be measured, R_3 is the resistor used for comparison.

There is a position of the tapping point Q, where no voltage is found between P and Q and therefore no current flows. In that case the voltages over R_3 and R_1 as well as at R and R_2 are equal. Such a WHEATSTONE bridge is called „adjusted" and we obtain:

$$(15) \qquad \frac{R_3}{R} = \frac{R_1}{R_2} = \frac{l_1}{l_2}$$

and thus

$$(16) \qquad R = R_3 \frac{l_2}{l_1}$$

Thus, in the case of an adjusted WHEATSTONE bridge, the resistance R can be determined by measuring the lengths l_1 and l_2 and knowing the gauged resistance R_3 from Eq. (16); insufficiencies of electric instruments are then of no importance. This is the advantage of this measuring method, a so-called *compensation method*.

Question 4:
- In Fig. 4, enter all the points where the circuit branches in the unbalanced WHEATSTONE bridge as well as the currents that flow, including their signs.
- In Fig. 4, draw all of the loops of the unbalanced WHEATSTONE bridge as well as the voltages in these loops, including their signs.

2.2.5 Bridge Circuit for Measuring Alterations to Resistance

Bridge circuits are, among other applications, used to convert small alterations to resistance ΔR into proportional voltages. This is a standard method in many areas of sensor-measuring-technology.

We consider, for example a bridge circuit with *strain gauges* (SG). SGs may be used for constructing force sensors. We will get to know such a force sensor in the experiment "*Sensors...*". Its theoretical foundation, however, shall already be described here.

The principle of a strain gauge is the elongation of a thin electrical conductor of length l by means of an external force $\mathbf{F}$[5], which simultaneously causes a decrease of its cross-sectional area A (Fig. 5). This alters its resistance R, which is, according to Eq. (14), given by:

$$(17) \qquad R = \rho \frac{l}{A}$$

For a conductor with a circular cross-section of diameter d we get:

$$(18) \qquad R = \rho \frac{4\,l}{\pi\,d^2}$$

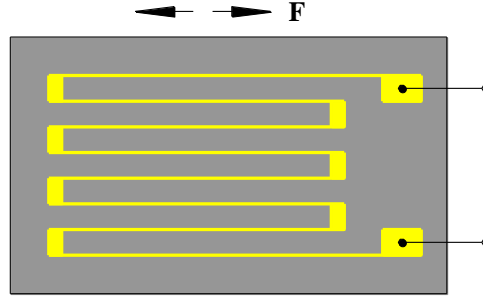[5] A *positive elongation* is a stretching, a *negative one* a compression.

Fig. 5: Diagram of a strain gauge (SG) on metal foil basis. A thin metal foil (yellow) is plated on the carrier foil (gray) in meander form in order to increase the effective length of the conductor while keeping the size of the SG small. The carrier foil is pasted on the work piece to be investigated and follows its deformations upon application of a force **F**.

The elongation changes the length l by Δl, the diameter d by Δd, and, depending on the material, possibly the specific resistance ρ by $\Delta\rho$. The resulting change in the ohmic resistance is given by the total differential ΔR:

$$(19) \qquad \Delta R = \frac{\partial R}{\partial \rho}\,\Delta\rho + \frac{\partial R}{\partial l}\,\Delta l + \frac{\partial R}{\partial d}\,\Delta d = \frac{1}{\pi}\left(\frac{4l}{d^2}\,\Delta\rho + \rho\,\frac{4}{d^2}\,\Delta l - 2\,\rho\,\frac{4l}{d^3}\,\Delta d \right)$$

The relative change in resistance is thus:

$$(20) \qquad \frac{\Delta R}{R} = \frac{\Delta\rho}{\rho} + \frac{\Delta l}{l} - 2\,\frac{\Delta d}{d}$$

The relative change in length $\Delta l/l$ is defined as the *elongation ε*.

$$(21) \qquad \varepsilon = \frac{\Delta l}{l}$$

The POISSON-number[6] μ is defined as the negative quotient of the relative change of the cross section $\Delta d/d$ and the relative change of length $\Delta l/l$, thus:

$$(22) \qquad \mu = -\frac{\dfrac{\Delta d}{d}}{\dfrac{\Delta l}{l}} := -\frac{\dfrac{\Delta d}{d}}{\varepsilon}$$

Factoring out the value of $\varepsilon = \Delta l/l$ in Eq. (20) and substituting Eq. (21) and Eq. (22) into Eq. (20) gives:

$$(23) \qquad \frac{\Delta R}{R} = \left(\frac{\dfrac{\Delta\rho}{\rho}}{\varepsilon} + 1 + 2\mu \right)\varepsilon := k\,\varepsilon$$

The expression within the brackets is the so called *k*-factor of a strain gauge and depends on the material of the strain gauge, examples are $k \approx 2$ for constantan and $k \approx 4$ for platinum[7]. The relative change in resistance by elongation increases with larger values of *k*.

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[6] SIMÉON DENIS POISSON (1781 – 1840)

[7] For monocrystalline silicone (Si), $k \approx 100$. Si-based SGs are, for example, used in pressure sensors which we will get to know in the forthcoming experiment "*Sensors…*".

With the aid of a bridge circuit, this alteration to resistance ΔR is converted into a voltage U. For a quantitative description of the bridge circuit, we look at the circuit shown in Fig. 6, which is set up analogous to the WHEATSTONE bridge represented in Fig. 4.
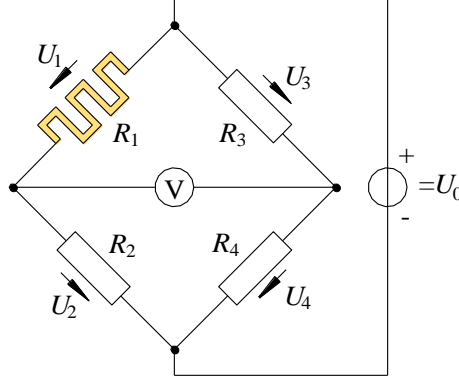

Fig. 6: Bridge circuit for measuring small alterations to resistance of R_1 (here SG). The voltage U in the bridge diagonal is measured with a voltmeter V.

In case the internal resistance of the voltmeter V extends to infinity, the following relationships are valid:

$$(24) \quad \frac{U_1}{U_2} = \frac{R_1}{R_2} \qquad \frac{U_3}{U_4} = \frac{R_3}{R_4}$$

$$(25) \quad U_1 + U_2 = U_0 \qquad U_3 + U_4 = U_0$$

By combining Eqs. (24) and (25) we obtain:

$$(26) \quad U_1 = U_0 \frac{R_1}{R_1 + R_2} \qquad U_3 = U_0 \frac{R_3}{R_3 + R_4}$$

The voltage U in the bridge diagonal is:

$$(27) \quad U = U_1 - U_3 = U_0 \left(\frac{R_1}{R_1 + R_2} - \frac{R_3}{R_3 + R_4} \right)$$

We now consider the special case of starting out with equal resistances $R_1,...,R_4$, one of which (R_1) is subsequently altered by the small amount ΔR. In case of a bridge circuit with a strain gauge, R_1 would be the resistance of the strain gauge and ΔR the change in resistance caused by mechanical deformation, hence:

$$(28) \quad R_1 = R + \Delta R \qquad R_2 = R_3 = R_4 := R$$

With this, it follows that the voltage U from Eq. (27) is given by:

$$(29) \quad U = U_0 \left(\frac{R + \Delta R}{R + \Delta R + R} - \frac{R}{R + R} \right) = U_0 \left(\frac{R + \Delta R}{2R + \Delta R} - \frac{1}{2} \right) = \frac{U_0}{2} \frac{\Delta R}{R} \left(\frac{1}{2 + \frac{\Delta R}{R}} \right)$$

Eq. (29) shows that the relationship between U and ΔR is *non-linear*. If, however, $\Delta R \ll R$, it holds:

$$(30) \quad \frac{1}{2 + \frac{\Delta R}{R}} \approx \frac{1}{2}$$

and thus:

$$(31) \quad U \approx \frac{U_0}{4} \frac{\Delta R}{R}$$

Near the point of balance ($\Delta R << R$), the alteration to resistance ΔR is thus related to a voltage U in an approximately linear manner, the amplitude of which can be influenced by the operating voltage (*supply voltage*) of the bridge, U_0.

In the set-up described above, *one* of the four resistors is replaced by a SG. Accordingly, this setup is commonly called a *quarter bridge*. In practice, one often uses a bridge circuit where *two* resistors are replaced by SGs which move in opposite directions of each other for a given deformation (Fig. 7). This arrangement is called a *half bridge*. One example is using two SGs to measure forces with a bending rod (Fig. 8), which we will investigate in more detail in the experiment „*Sensors...*". The two SGs are placed in the bridge circuit, so that the upper one that is being elongated replaces R_1 and the lower one being compressed replaces R_2. It follows that:

$$(32) \quad R_1 = R + \Delta R \qquad R_2 = R - \Delta R \qquad R_3 = R_4 = R$$

By inserting Eq. (32) in Eq. (27), it follows for the half bridge:

$$(33) \quad U = \frac{U_0}{2} \frac{\Delta R}{R}$$

This equation makes the advantage of a half bridge compared to a quarter bridge clear: First, the relation between U and ΔR is *linear*. Second, for the same change in resistance ΔR, the half bridge produces an output voltage U of twice the magnitude. Thus, the *sensitivity* of the half bridge is twice as high.

In a *full bridge*, all *four* resistors are replaced by SGs, which change pairwise (R_1/R_4 and R_2/R_3) in opposite directions. In this case we find for the voltage U:

$$(34) \quad U \approx U_0 \frac{\Delta R}{R}$$

It is clear, that the sensitivity increases again by a factor of two.
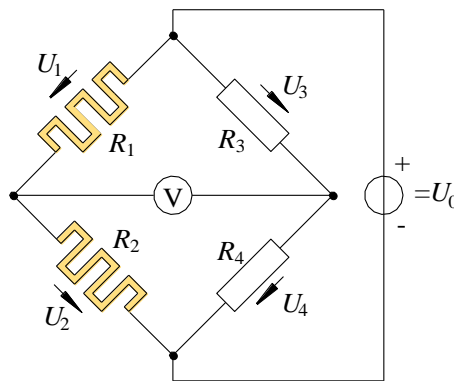

Fig. 7:   Bridge circuit with two SGs (half bridge).
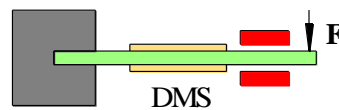

Fig. 8:   Bending rod (green) with two strain gauges (yellow, German abbreviation is DMS). The rod (green) is fixed by a block (gray) on the left. The force **F** deforms the rod, so that the upper SG is elongated and the lower one is compressed. A mechanical barrier (red) serves to protect the apparatus from overstraining.

2.3 Properties of Real Voltage Sources

2.3.1 Internal Resistance of Real Voltage Sources

An *ideal* voltage source provides a constant terminal voltage U, which is equal to the constant source voltage U_0, independently of the electrical load (the current it provides) at its connecting terminal. Such ideal voltage sources cannot be realized technically. On the contrary, we are dealing with *real* voltage sources such as batteries, power units or function generators, the terminal voltage of which decreases with increasing load. In order to describe this property of real voltage sources we use a model in which the real voltage source is exchanged for an ideal voltage source G and an internal resistance R_i in series; Fig. 9 shows the corresponding equivalent circuit. When such a voltage source is connected with a load with an external load resistance R_l according to Fig. 10, the load current I flows through R_l as well as through R_i. This current causes a voltage drop IR_i at R_i by which the terminal voltage U is reduced compared to the source voltage U_0. Thus we obtain:

$$(35) \qquad U = U_0 - IR_i$$

If the *source voltage* U_0 is to be measured using an ideal voltmeter V in a circuit according to Fig. 10, then the load current I has to be towards zero. This is achieved by a large load resistance R_l.

Exchanging the current I in Eq.(35) for U/R_l (according to OHM's law) we obtain for the relationship between U and R_l:
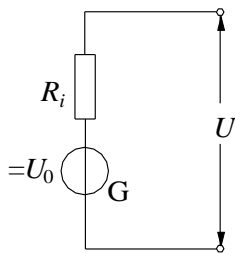
$$(36) \qquad U = U_0 \frac{R_l}{R_l + R_i}$$

Fig. 9: Equivalent circuit of areal voltage source without a load.
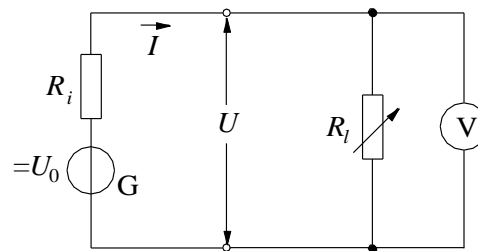
Fig. 10: Equivalent circuit of a real voltage source with load resistance R_l .

From this equation we derive particularly for the case $R_l = R_i$ that the terminal voltage decreases to half of the source voltage. This allows us to determine the internal resistance of a real voltage source.

Question 5:
- Sketch the graph of the terminal voltage U as a function of the load resistance R_l.

2.3.2 Matching a Device to a Real Voltage Source

2.3.2.1 Power Matching

While connecting an electrical device to a voltage source it is often desirable that the internal resistance of the device be dimensioned such that the maximal power can be taken from the voltage source (power matching; applied e.g. in the transmission of high-frequency signals[8]). The internal resistance of the device is the load resistance R_l, which is the voltage source load. The power P supplied to that resistance is given by:

$$(37) \qquad P = UI = \frac{U^2}{R_l}$$

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[8] Power matching in communication engineering at the same time prevents spurious signal reflections, which will be examined more closely in the experiment "*Signal transfer…*" (summer term).

Inserting Eq. (36) into Eq. (37) yields:

$$(38) \qquad P = U_0{}^2 \frac{R_l}{\left(R_l + R_i\right)^2}$$

Maximal power consumption is achieved when the internal resistance of the device equals the internal resistance of the voltage source, hence if we have:

$$(39) \qquad R_l = R_i$$

Question 6:
- Sketch the graph of P as a function of R_l. How do we get from Eq. (38) to Eq. (39)? What is the maximum power that can be drawn from the voltage source?

2.3.2.2 Voltage Matching

The object of voltage matching, applied in high-current technology and in sound engineering, is to draw the highest possible voltage U from the voltage source. According to Eq. (36) the necessary condition for this is:

$$(40) \qquad R_l \gg R_i$$

2.3.2.3 Current Matching

The object of current matching is to draw the strongest possible current I from the voltage source. This is, for example, used for charging accumulators. According to the OHM's law we obtain:

$$(41) \qquad I = \frac{U_0}{R_i + R_l}$$

so that the condition for the strongest possible current reads:

$$(42) \qquad R_l \ll R_i$$

In this case the current is nearly independent of the load resistance.

3 Experimental Procedure

Equipment:
Voltage supply (PHYWE (0 - 15 / 0 - 30) V), function generator (TOELLNER 7401), several digital multimeters, digital oscilloscope TEKTRONIX TDS 1012 / 1012B / 2012C / TBS 1102B, resistance decade, slide rheostat ($R_{ges} \approx 11{,}5\ \Omega$), unknown resistance in holder, box for bridge circuit, pair of copper plates in holder, water basin on vertically adjustable holder, metal measuring tape, calliper gauge, paper wipes.

Attention:
When connecting resistances with voltage sources it has to be considered that the maximally allowable dissipation power P_{max} of the resistor must not be exceeded ($P = UI = U^2/R < P_{max}$). Details about the load capacity P_{max} of resistors are either found on the available components (e.g. resistor decade) or can be obtained from the technical assistant.

Take care when operating the power supply unit that no unintentional current limitation is adjusted.

Multimeters of the type FLUKE 112 provide only a limited resolution for current measurements. Therefore, they are used only as ohmmeters or voltmeters in this experiment, *not* as ammeters.

In multimeters of the type MONACOR DMT-3010, fuses are blown easily in case of an operating error. Apply special caution to operating them!

3.1 Hints to the Measuring Instruments

The used measuring instruments offer the possibility to switch the measuring range manually and partly also automatically, which serves to indicate the measured value on the scale or numerical display of the measuring instrument with the highest possible accuracy. For example, using a digital voltmeter a voltage of 1.78 V will be indicated as 1.78 V in the measuring range „2 V", but will be indicated as 2 V in the measuring range „200 V".

When switching the measuring range of an ammeter a precision resistor („shunt") is added *in parallel* to the internal resistance of the instrument. This resistor is rated such that the current flowing through the ammeter remains just about the same for all measuring ranges. Analogously, when switching the measuring range of a voltmeter, a precision resistor is added *in series* to the internal resistance of the instrument, which is rated such that the voltage drop is just about the same for all measuring ranges of the voltmeter.

Data on the *internal resistances* for measuring currents (R_A) and for measuring voltages (R_V), which are dependent on the measuring range, are available for some of the measuring instruments used in the introductory laboratory course. Instead of an internal resistance R_A, a voltage drop ΔU is often stated (e.g. 20 mV, 200 mV etc.). In this case $R_A = \Delta U / I_{max}$, I_{max} representing the maximum current in the adjusted measuring range.

For other measuring instruments, there are no data available on the *internal resistances* R_A and/or R_V. In those cases it can be assumed that R_V is so large (e.g. 10 MΩ) and R_A is so small (e.g. 0,5 Ω) that their influence on the measurement result is negligible.

Specifications of the *total measurement error* of a measuring instrument and the precision of a measured value, respectively, are found on the instruments or in the instrument manuals. These values generally consist of two parts. Their first - most significant - part is stated in percent of the *measured value*. Their second part can be stated in percent of the *measuring range* or in units of the last decimal of the measured value.

The following examples serve for explanation:

1.) A direct voltage of 2.348 V is measured with the multimeter FLUKE 112 in the measuring range 6.000 V. According to the manual the *precision* for this voltage range is: ± (0.7 % of the measured value + 0.003) V. For the mentioned example, the *precision* thus is ± (0.007 × 2.348+ 0.003) V = ± 0.019 V (rounded to two significant digits). This value is also the *maximum error* for the measured value.

2.) A direct voltage of 297.34 mV is measured with the multimeter AGILENT U1272A in the measuring range 300.00 mV. According to the manual the *precision* for this voltage range is: ± (0.05 % + 5). The percent value refers to the measured value, the "5" refers to the last shown digit of the measured value (here the 4 in 0.04 mV). The maximum error therefore is: ± (0.0005 × 297.34 + 0.05) mV = ± 0.20 mV (rounded to two significant digits).

3.2 Measurement of Resistances

The value of an unknown resistor R (in the order of magnitude of 1 kΩ) including the maximum error is to be determined by applying some of the methods described in Chapter 2.2. The following steps are to be taken consecutively:

a) *Measurement using different ohmmeters*: The value of the resistance R is to be measured with at least five ohmmeters. Partly also ohmmeters of the same type may be used. A number j is assigned to every ohmmeter. Prior to the measurements, the measuring instruments have to be adjusted to the measuring range that enables the most precise measurement to be made. The maximum error ΔR_j is given for each measured value R_j. The R_j are presented in a graph over j including error bars.

b) *Measurement of current/voltage*: As an example, circuit A is set up according to Fig. 3. A voltage supply serves as the voltage source. Its internal resistance can be neglected for this measurement. For at least ten different voltages on the voltage supply the corresponding current is measured using an

ammeter and the voltage using a voltmeter. *Prior* to these measurements, the range covering the values to be measured has to be considered and the measuring ranges have to be adjusted accordingly.

For each pair of values (U, I) a resistance $R = U/I$ is determined. Subsequently, the mean $\overline{R}$ and its standard deviation $\sigma_{\overline{R}}$ are calculated from these data.

Subsequently, the measured voltage values U are plotted against the measured current values I in a diagram and the maximum errors of U and I are entered in the form of error bars. The parameters of the regression line through the measured points are calculated and the regression line is drawn into the diagram[9]. The slope of the regression line R ($\pm \sigma_R$) is a good estimate for the unknown resistance value. This estimate is compared with the previously found mean of $\overline{R}$ ($\pm \sigma_{\overline{R}}$) and it is checked, whether both methods yield comparable results.

c) **WHEATSTONE *bridge***: A WHEATSTONE bridge is set up according to Fig. 4. Again a voltage supply serves as the voltage source, and a resistance from a resistor decade as calibration resistance R_3. This resistance is chosen to be about the same as the resistance to be measured: R. In that case $l_1 \approx l_2$ is valid for an adjusted WHEATSTONE bridge and the error becomes minimal for determining R. The error of the value of R calculated using Eq. (16) is given by the maximum error.

After having determined the resistance by applying the different methods, all measured results from Chap. 3.2 are to be presented in a graph analogous to Chap. 3.2 a) and compared.

3.3 Measurement of Internal Resistance of a Function Generator

With a circuit according to Fig. 10, the internal resistance (also called *output resistance*) of a function generator (FG) is to be determined. The equivalent circuit of the function generator consists of an ideal voltage source G and the internal resistance R_i (order of magnitude 50 Ω) in series.

A sinusoidal voltage is adjusted at the function generator (amplitude $U_{FG} \approx 4$ V, frequency approx. 1 kHz) and initially a load resistance $R_l = 100$ kΩ and thus $R_l \gg R_i$ (resistance decade) is connected. The voltage amplitude U over R_l is measured using an oscilloscope. Its internal resistance of about 1 MΩ can be neglected. $U_0 \approx U_{FG}$ is valid for $R_l = 100$ kΩ with sufficient precision.

Subsequently, the load resistance is reduced by correspondingly switching the resistor decade to values between 1 kΩ and 20 Ω. For each value of R_l, the voltage amplitude U is measured and subsequently U is plotted over R_l. By means of graphic interpolation[10] of the curve the value for R_l is extrapolated at which U has been decreased to half of U_0. This resistance corresponds to the required internal resistance R_i of the function generator (see Chapter 2.3.1).

Hint:
> The maximum current at the lowest possible resistance (20 Ω) is $I_{max} = 4$ V / 20 Ω = 200 mA. The maximum momentary power at the resistance thus is $P = U\,I = 0.8$ W and hence is below the load limit of the resistance decade of 1 W.

3.4 Specific Resistance of Tap Water

Let us examine the set-up shown in Fig. 11. Two rectangular copper plates of width b are mounted in parallel at a distance l. They are dipped into a water basin containing normal tap water. By lifting the water basin the plates can be plunged into the water to a variable depth d. ρ_w being the specific resistance of the water, the ohmic resistance R_w of the water between the plates is given by (cf. Eq. (14)):

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[9] The calculation of the parameters of the regression line and its grafic representation are done using `Origin`. Hints are given in the accompanying seminar.

[10] „Graphic interpolation" means: A regression curve is drawn by hand through the measurement. Then the line $U = U_0/2$ is drawn and its intersection point with the regression curve is determined. The R value of the intersection point is read on the abscissa. Its maximum error ΔR follows from the reading accuracy of R.

(43) $\qquad R_{\mathrm{w}} = \rho_{\mathrm{w}} \dfrac{l}{b} \dfrac{1}{d}$
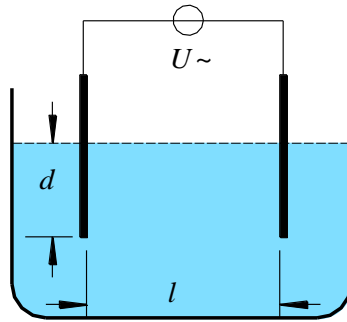

Fig. 11:   Set-up for measuring the specific resistance of tap water (amperemeter and voltmeter not drawn).

By measuring the current *I* (ammeter) at an applied voltage *U* (voltmeter) between the plates R_{w} (circuit A, cf. Fig. 3) is determined for a number of values of the immersion depth *d*, within the range between 50 mm and 20 mm. For the single values of *U*, *I* and R_w errors must not be specified. Then, R_{w} is plotted against 1/*d*. A regression line is drawn in the diagram and the specific resistance ρ_{w} of the tap water including the maximum error is calculated from its slope (measure *l* and *b*!).

Remarks:

   In order to avoid polarization effects in the water, no direct voltage but a sinusoidal alternating voltage *without* DC offset (turn off "**DC-Offset**" on the FG) is used, which can be obtained from a function generator ($U_{eff} \approx 2$ V at $d \approx 50$ mm, frequency approx. 50 Hz). In spite of this, the water as an *ionic conductor* does not behave in the manner that we know from *metallic conductors*. For example, its resistance decreases with temperature, while for metallic conductors, it increases. Therefore, we must assume that the measurement yields only a reference value for ρ_W. In addition, ρ_{w} deviates considerably depending on the quality of the tap water, so it is difficult to compare the measured value with a literature value.[11]

   Measuring with an alternating voltage requires the multimeters to be switched into AC mode!

3.5   Bridge Circuit for Measuring Alterations to Resistance

A bridge circuit is set up according to Fig. 6 ($R_{1,...,4} \approx 100$ Ω, power supply voltage $U_0 \approx 5$ V). $R_{2...4}$ are soldered in a box. R_1 is adjusted with a resistance decade. The voltage *U* is measured in the bridge diagonal for approximately ten alterations to resistance ΔR of the resistor R_1 within the range between ± 1 Ω and ± 10 Ω, hence for R_1 in the interval (90 – 110) Ω. *U* is plotted over ΔR and the linearity of the relationship is verified according to Eq. (31).

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[11]   The specific resistance of tap water at 20 °C is in the order of magnitude of approx. (10 – 20) Ωm. For comparison: the specific resistance of copper at 20 °C is about 1.7×10^{-8} Ωm (Please search for the corresponding references from the literature and include them in your report).