

MODELING THE COMMON PART OF ACOUSTIC FEEDBACK PATHS IN HEARING AIDS USING A POLE-ZERO MODEL

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ABSTRACT

In adaptive feedback cancellation the computational complexity and the convergence speed are determined by the number of adaptive parameters used to model the acoustic feedback path. Therefore it has been proposed to reduce the number of adaptive parameters by modeling the feedback path as the convolution of a time-invariant common part and a time-varying variable part. While previous approaches have modeled the common part either using only poles or using only zeros, in this paper we propose to use a common pole-zero model and present an iterative method to compute the common poles and zeros. Using measured acoustic feedback paths from a two-microphone behind-the-ear hearing aid it is shown that the proposed model enables either to increase the modeling accuracy given a fixed number of parameters of the variable part or to reduce the number of parameters of the variable part given a desired accuracy.

Index Terms— feedback cancellation, common part modeling, invariant part extraction

1. INTRODUCTION

In recent years the number of hearing impaired persons supplied with an open-fitting hearing aid has been steadily increasing. While open-fitting hearing aids largely alleviate problems related to the occlusion effect, they are especially prone to acoustic feedback, which is most often perceived as howling. This demands for robust and fast-adapting feedback cancellation algorithms.

Although several approaches for feedback cancellation are available (see e.g. [1] and references therein), adaptive feedback cancellation (AFC) seems the most promising as it theoretically allows for perfect feedback cancellation. In AFC the feedback path, i.e., the impulse response (IR) between the hearing aid receiver and the hearing aid microphone, is approximated using an adaptive filter.

It is known that in general the computational complexity and the convergence speed of an adaptive filter is determined by the number of adaptive parameters [2]. In [3, 4] it was hence proposed to model the acoustic feedback path as the convolution of two filters: a fixed filter to account for invariant or slowly varying parts of the feedback path and an adaptive filter enabling to track fast changes. The fixed filter can be thought to account for e.g. fixed transducer and microphone characteristics and fixed mechanical couplings. Moreover, when estimated from different feedback paths of the same ear, this fixed filter also accounts for similarities due to the individual characteristics of that particular ear. By including a fixed filter, the goal is to reduce the length of the adaptive filter and thereby increase its convergence speed. The fixed filter may e.g. be estimated from the IRs of several microphones, e.g. in multi-microphone hearing aids,

which then usually models parts that are common in all of these IRs. In the remainder of this paper this fixed filter is therefore termed common part, while the time-varying filter that is assumed to be different for each IR is termed variable part.

Several methods have been proposed to estimate the common part from several IRs, including methods employing QR-decomposition [5], SVD [6] or least-squares techniques [4, 7, 8]. In [7] the well known common-acoustical-pole and zero (CAPZ) model was proposed, where for a set of acoustical transfer functions the common part was modeled as an all-pole filter that physically corresponded to room resonances, while the variable parts were assumed to be all-zero filters. Using these assumptions a closed-form expression could be derived for all coefficients. In [8] both the common part and the variable part were assumed to be all-zero filters and estimated by minimizing a non-linear cost function. In [8] it was noted that the convergence of the solution depended on the initialization values. In [4] the approach of [8] was used to estimate an all-zero model of the common part for a set of 10 different acoustic feedback paths. During their evaluation they found that, on the one hand, different types of arbitrary initialization, i.e., all-one sequences, random sequences and a truncated average IR, only had minor effects on the results after convergence. On the other hand, it was observed that an increase in modeling accuracy could be achieved by initializing the common part all-zero model as the truncated IR of the common part obtained by the CAPZ model.

Instead of assuming the common part to be an all-pole filter [7] or an all-zero filter [4, 8], in this paper it is proposed to use a pole-zero filter for the common part. The resulting cost function is minimized using an alternating least-squares approach similarly to [8]. Thus, this paper is related to the prior work of [4, 7, 8] and extends their work to a pole-zero model for the common part. Experimental results using measured acoustic feedback paths from a two-microphone hearing aid show two major findings: 1) using the proposed common pole-zero model an improvement in modeling accuracy can be achieved while maintaining a constant number of parameters in the variable part compared to the previously proposed all-pole and all-zero models, and 2) using the proposed common pole-zero model a reduction of the number of parameters of the variable part compared to using an all-pole model or an all-zero model can be achieved given a desired accuracy.

2. PROBLEM FORMULATION AND NOTATION

Consider a single-input-multiple-output (SIMO) system with M outputs as depicted in Figure 1(a), e.g. arising in a single loudspeaker multiple-microphone setup like in multi-microphone hearing aids. The m -th output $Y_m(z)$ and the input $X(z)$ are related by the m -th acoustic transfer function $H_m(z)$ as

$$Y_m(z) = H_m(z)X(z). \quad (1)$$

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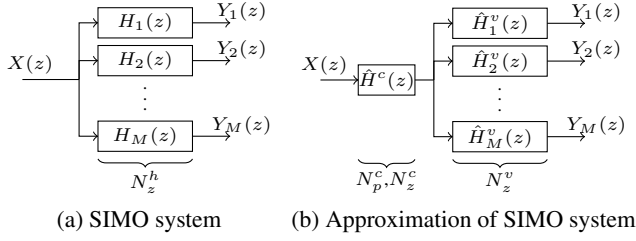


Fig. 1. System models.

With the goal of reducing the number of parameters needed to model all M transfer functions the SIMO system is approximated as depicted in Figure 1(b), where the m -th transfer function $H_m(z)$ is approximated as

$$H_m(z) \approx \underbrace{\hat{H}^c(z) \hat{H}_m^v(z)}_{\hat{H}_m(z)}. \quad (2)$$

Thus, the transfer function $\hat{H}_m(z)$ is split into two separate parts: a common part $\hat{H}^c(z)$ and a microphone dependent variable part $\hat{H}_m^v(z)$. Assuming that $\hat{H}^c(z)$ is a pole-zero filter with N_p^c poles and N_z^c zeros and $\hat{H}_m^v(z)$ is an all-zero filter with N_z^v zeros for each microphone, the time-domain description of the IR $\hat{h}_m[k]$ is then given by

$$\hat{h}_m[k] = - \sum_{i=1}^{N_p^c} \hat{a}^c[i] \hat{h}_m[k-i] + \sum_{i=0}^{N_z^c} \hat{b}^c[i] \hat{b}_m^v[k-i], \quad (3)$$

with $k = -\infty, \dots, \infty$ and $\hat{b}_m^v[k] = 0$ for $k < 0$ and $k > N_z^v$. The coefficients in vector notation are given by

$$\hat{\mathbf{a}}^c = [\hat{a}^c[1] \quad \hat{a}^c[2] \quad \dots \quad \hat{a}^c[N_p^c]]^T, \quad (4)$$

$$\hat{\mathbf{b}}^c = [\hat{b}^c[0] \quad \hat{b}^c[1] \quad \dots \quad \hat{b}^c[N_z^c]]^T, \quad (5)$$

$$\hat{\mathbf{b}}_m^v = [\hat{b}_m^v[0] \quad \hat{b}_m^v[1] \quad \dots \quad \hat{b}_m^v[N_z^v]]^T, \quad (6)$$

with $[\cdot]^T$ denoting transpose operation. For later use let $\hat{\mathbf{b}}^v$ be defined as the concatenation of coefficient vectors $\hat{\mathbf{b}}_m^v$, i.e.,

$$\hat{\mathbf{b}}^v = [(\hat{\mathbf{b}}_1^v)^T \quad (\hat{\mathbf{b}}_2^v)^T \quad \dots \quad (\hat{\mathbf{b}}_M^v)^T]^T. \quad (7)$$

Previous approaches have either used all-pole models [7] or all-zero models for the common part [4, 8]. In the following a pole-zero model for the common part is used and it is shown how all the coefficients can be estimated in a least-squares sense.

3. LEAST-SQUARES ESTIMATION

The objective is to estimate those parameters $\hat{a}^c[i]$, $\hat{b}^c[i]$, and $\hat{b}_m^v[i]$ that minimize the mean-squared error between the true IRs $h_m[k]$ and the approximated IR model $\hat{h}_m[k]$, i.e., minimize the following non-linear cost-function

$$\bar{J}_{NLLSQ}(\hat{\mathbf{a}}^c, \hat{\mathbf{b}}^c, \hat{\mathbf{b}}^v) = \sum_{m=1}^M \sum_{k=-\infty}^{\infty} (h_m[k] - \hat{h}_m[k])^2. \quad (8)$$

Since this so-called output error minimization is known to be difficult [9], often the so-called equation error is used, i.e., the delayed

elements $\hat{h}_m[k-i]$ on the right hand side of (3) are substituted by $h_m[k-i]$ [7, 9], leading to

$$J_{NLLSQ}(\hat{\mathbf{a}}^c, \hat{\mathbf{b}}^c, \hat{\mathbf{b}}^v) = \sum_{m=1}^M \sum_{k=-\infty}^{\infty} (h_m[k] + \sum_{i=1}^{N_p^c} \hat{a}^c[i] h_m[k-i] - \sum_{i=0}^{N_z^c} \hat{b}^c[i] \hat{b}_m^v[k-i])^2. \quad (9)$$

Assuming that $h_m[k]$ is a causal FIR filter of finite order N_z^h , the summation bounds in (9) may be changed to lower bound $k = 0$ and upper bound $k = \tilde{N}_z^h + N_p^c$, with $\tilde{N}_z^h = \max\{N_z^h, N_z^c + N_z^v + 1\}$. Thus, (9) can be written in vector notation as

$$J_{NLLSQ}(\hat{\mathbf{a}}^c, \hat{\mathbf{b}}^c, \hat{\mathbf{b}}^v) = \|\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\hat{\mathbf{a}}^c - \hat{\mathbf{B}}^v\hat{\mathbf{b}}^c\|_2^2, \quad (10)$$

where $\tilde{\mathbf{h}}$ is an $M(\tilde{N}_z^h + N_p^c + 1)$ -dimensional stacked vector of zero-padded versions of the true impulse response vectors \mathbf{h}_m , i.e.,

$$\tilde{\mathbf{h}} = [\tilde{\mathbf{h}}_1^T \quad \tilde{\mathbf{h}}_2^T \quad \dots \quad \tilde{\mathbf{h}}_M^T]^T, \quad (11)$$

$$\tilde{\mathbf{h}}_m = \underbrace{[h_m[0] \quad h_m[1] \quad \dots \quad h_m[N_z^h]]}_{\mathbf{h}_m^T} \underbrace{[0 \quad \dots \quad 0]}_{\tilde{N}_z^h + N_p^c - N_z^h}, \quad (12)$$

and $\tilde{\mathbf{H}}$ is a $M(\tilde{N}_z^h + N_p^c + 1) \times N_p^c$ -dimensional stacked matrix of convolution matrices of delayed versions of $\tilde{\mathbf{h}}_m$, i.e.,

$$\tilde{\mathbf{H}} = [\tilde{\mathbf{H}}_1^T \quad \tilde{\mathbf{H}}_2^T \quad \dots \quad \tilde{\mathbf{H}}_M^T]^T, \quad (13)$$

where

$$\tilde{\mathbf{H}}_m = \begin{bmatrix} 0 & \dots & 0 \\ h_m[0] & \ddots & \vdots \\ \vdots & \ddots & 0 \\ h_m[N_p^c - 1] & \ddots & h_m[0] \\ \vdots & \ddots & \vdots \\ h_m[N_z^h] & \ddots & \vdots \\ 0 & \ddots & \vdots \\ \vdots & \ddots & h_m[N_z^h] \\ \vdots & \ddots & \vdots \\ 0 & \dots & 0 \end{bmatrix} \quad (14)$$

and $\hat{\mathbf{B}}^v$ is an $M(\tilde{N}_z^v + N_p^c + 1) \times N_z^c + 1$ -dimensional stacked matrix of the convolutions matrices of $\mathbf{b}_m^{(v)}$, i.e.,

$$\hat{\mathbf{B}}^v = [(\hat{\mathbf{B}}_1^v)^T \quad (\hat{\mathbf{B}}_2^v)^T \quad \dots \quad (\hat{\mathbf{B}}_M^v)^T]^T, \quad (15)$$

and

$$\hat{\mathbf{B}}_m^v = \begin{bmatrix} \hat{b}_m^v[0] & \cdots & 0 \\ \vdots & \ddots & \vdots \\ \hat{b}_m^v[N_z^c - 1] & \ddots & \hat{b}_m^v[0] \\ \vdots & \cdots & \vdots \\ \hat{b}_m^v[N_z^v] & \ddots & \vdots \\ 0 & \ddots & \vdots \\ \vdots & \ddots & \hat{b}_m^v[N_z^v] \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}. \quad (16)$$

In the following an alternating least-squares approach is employed to solve the cost function in (9), which was also used in [4, 8] where the common part and the variable part were both assumed to be all-zero filters, i.e., $N_p^c = 0$.

The objective of the alternating least-squares (ALS) approach is to split the non-linear cost function of (9) into two separate linear least squares (LS) problems, which are solved alternately until convergence is achieved. The advantage is that for the linear least squares problems a closed form solution is available. The following procedure is thus applied:

1. Initialize the coefficients of $\hat{\mathbf{a}}_j^c$ and $\hat{\mathbf{b}}_j^c$ at iteration $j = 0$.
2. Normalize $\hat{\mathbf{b}}_j^c$ to unit norm to achieve a unique solution, i.e., $\hat{\mathbf{b}}_j^c = \hat{\mathbf{b}}_j^c / \|\hat{\mathbf{b}}_j^c\|_2$.
3. Compute the vector $\hat{\mathbf{b}}_{j+1}^v$ that minimizes the LS cost function

$$J_{ALS}^v(\hat{\mathbf{b}}_{j+1}^v) = \|\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\hat{\mathbf{a}}_j^c - \tilde{\mathbf{B}}_j^c \hat{\mathbf{b}}_{j+1}^v\|_2^2 \quad (17)$$

where $\tilde{\mathbf{B}}^c$ is a $M(\tilde{N}_z^v + N_p^c + 1) \times M(N_z^v + 1)$ -dimensional matrix defined as

$$\tilde{\mathbf{B}}^c = \begin{bmatrix} \hat{\mathbf{B}}^c & & \\ & \ddots & \\ & & \hat{\mathbf{B}}^c \end{bmatrix}, \quad (18)$$

where $\hat{\mathbf{B}}^c$ is a $(\tilde{N}_z^v + N_p^c + 1) \times (N_z^v + 1)$ convolution matrix similarly defined as $\hat{\mathbf{B}}_m$ in (16). This leads to

$$\hat{\mathbf{b}}_{j+1}^v = ((\tilde{\mathbf{B}}_j^c)^T \tilde{\mathbf{B}}_j^c)^{-1} (\tilde{\mathbf{B}}_j^c)^T (\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\hat{\mathbf{a}}_j^c). \quad (19)$$

4. Compute the vectors $\hat{\mathbf{a}}_{j+1}^c$ and $\hat{\mathbf{b}}_{j+1}^c$ that minimize the LS cost function

$$J_{ALS}^c(\hat{\mathbf{a}}_{j+1}^c, \hat{\mathbf{b}}_{j+1}^c) = \|\tilde{\mathbf{h}} + \tilde{\mathbf{H}}\hat{\mathbf{a}}_{j+1}^c - \hat{\mathbf{B}}_{j+1}^v \hat{\mathbf{b}}_{j+1}^c\|_2^2 \quad (20)$$

leading to

$$\begin{bmatrix} \hat{\mathbf{a}}_{j+1}^c \\ \hat{\mathbf{b}}_{j+1}^c \end{bmatrix} = (\mathbf{D}_{j+1}^T \mathbf{D}_{j+1})^{-1} \mathbf{D}_{j+1}^T \tilde{\mathbf{h}}, \quad (21)$$

where $\mathbf{D}_{j+1} = [-\tilde{\mathbf{H}} \quad \hat{\mathbf{B}}_{j+1}^v]$.

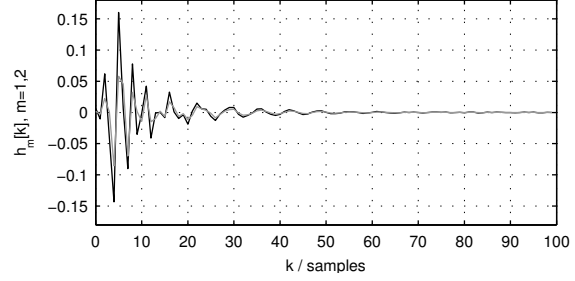


Fig. 2. Acoustic feedback paths from a two-microphone hearing aid. The black line indicates $h_1[k]$, the gray line indicates $h_2[k]$.

5. set $j = j + 1$ and repeat steps 2. to 4. until convergence is reached.

Different convergence criteria may be used, e.g. [4] assumed the solution to be converged when j reached a predefined value and [8] assumed convergence of the solution when the relative change of each coefficient as well as the relative change of $J_{ALS}^c(\cdot)$ was smaller than a predefined value. It is known that the ALS method relies on a good initial estimate of $\hat{\mathbf{a}}_0^c$ and $\hat{\mathbf{b}}_0^c$ [4, 8]. Note that in [8] the ALS approach was used to identify the common part and the variable part that were both assumed to be all-zero, i.e., $N_p^c = 0$. Furthermore, note that when assuming that $N_z^c = 0$, i.e., the common part is an all-pole filter, the model is equivalent to the CAPZ model [7] and the proposed ALS approach converges to the closed-form solution given in [7].

4. EXPERIMENTAL RESULTS

In this section the influence of different combinations of the parameters N_p^c , N_z^c and N_z^v on the modeling accuracy of the proposed common pole-zero model is evaluated. The proposed pole-zero model is compared to the previously proposed all-zero and all-pole models.

Two acoustic feedback paths, i.e., $M = 2$, were measured using a two-microphone hearing aid with open-fitting earmolds on a dummy head with adjustable ear canals similar to that presented in [10]. The IRs were sampled at $f_s = 16$ kHz and truncated to order $N_z^h = 99$. The measured IRs are depicted in Figure 2 and show a high degree of similarity that could possibly be exploited by modeling a common part. As a performance measure the average normalized mean-square error between the true, i.e., measured, IRs \mathbf{h}_m and the estimated IRs $\hat{\mathbf{h}}_m$ is used, i.e.,

$$NMSE = 10 \log_{10} \frac{1}{2} \sum_{m=1}^2 \frac{\|\mathbf{h}_m - \hat{\mathbf{h}}_m\|_2^2}{\|\mathbf{h}_m\|_2^2}. \quad (22)$$

The coefficient vectors were initialized as $\hat{\mathbf{a}}_0^c = \mathbf{0}$ and $\hat{\mathbf{b}}_0^c = [1 \ 0 \ \dots \ 0]$. Although the obtained results rely on the choice of the initial coefficient vectors, as mentioned in Section 3, it is beyond the scope of this paper to compare different types of initializations. The convergence criterion was chosen similar to [8], i.e., convergence was assumed when the relative change of each coefficient as well as the relative change of $J_{ALS}^c(\cdot)$ was smaller than 10^{-4} .

The proposed model was evaluated for the following range of parameters $N_p^c, N_z^c, N_z^v \in \{0, 2, 4, 6, 8, 10, 15, 20, 25, 30, 40, 50\}$. Note that for $N_p^c = 0$ the common part is assumed to be an all-zero filter as proposed in [8] while for $N_z^c = 0$ the common part

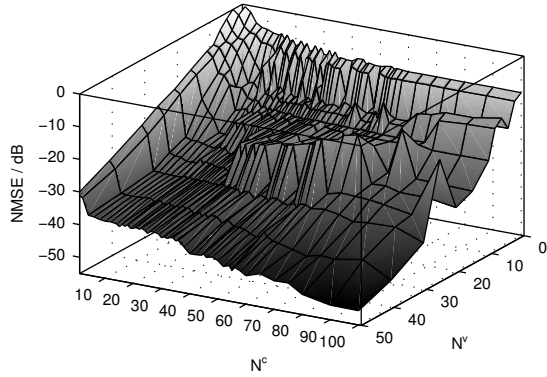


Fig. 3. Average NMSE as a function of N^v and N^c .

is assumed to be an all-pole filter as in the CAPZ model [7]. For the sake of clarity let the number of parameters needed to model the common part be defined as $N^c = N_p^c + N_z^c$ and the number of parameters needed to model each variable part as $N^v = N_z^v$.

Figure 3 shows the average NMSE that is obtained for different choices of N^v and N^c . Note that, for some N^c different combinations are possible, i.e., for $N^c = 4$ three different choices of parameters are possible: $N_p^c = 4, N_z^c = 0$ corresponding to a common all-pole model, $N_p^c = 0, N_z^c = 4$ corresponding to a common all-zero model and $N_p^c = 2, N_z^c = 2$ corresponding to the proposed common pole-zero model. Only those combinations leading to the lowest NMSE are shown. It is observed that generally by increasing the number of parameters N^c of the common part and by increasing the number of parameters N^v of the variable part a decrease in NMSE is achieved. However, increasing N^c does not always lead to improvements for some choices of low N^v . To quantify the influence of using a pole-zero model, in the following two cross sections of Figure 3 are considered. First, to investigate the influence of N^v on modeling accuracy a cross section for a fixed N^c is shown, and second, to investigate the influence of N^c on the number of parameters of the variable needed to model the complete path with a desired accuracy a cross section for a fixed NMSE is shown.

Figure 4 depicts the minimum NMSE as a function of N^v for a given number of parameters $N^c = 25$ of the common part. Different symbols indicate three different assumptions on the common part model, i.e., an all-zero model ($N_p^c = 0$), an all-pole model ($N_z^c = 0$), and a pole-zero model. All three models show an expected reduction in average NMSE when increasing N^v . Note that by choosing those combinations of N_p^c and N_z^c for the common part that correspond to the lowest NMSE it is obvious that the pole-zero model will always show lower (or equal) NMSE compared to either the all-zero or the all-pole model. For large values of N^v (> 10) the results for the pole-zero model and the all-pole model coincide, while for small values N^v (≤ 2) the pole-zero model and the all-zero model show similar results. For the range of $2 < N^v \leq 10$ the pole-zero model shows the best performance. These results indicate that while maintaining a fixed number of parameters for modeling the common part an increase in modeling accuracy can be achieved when a pole-zero model is used especially for low values of N^v .

The influence of the number of parameters N^c of the common part on the number of parameters N^v needed for the variable part to model the complete path given a predefined $NMSE = -20$ dB is depicted in Figure 5. As expected from the results in Figure 3 an increase in N^c leads to a reduction in N^v . For small values of N^c the

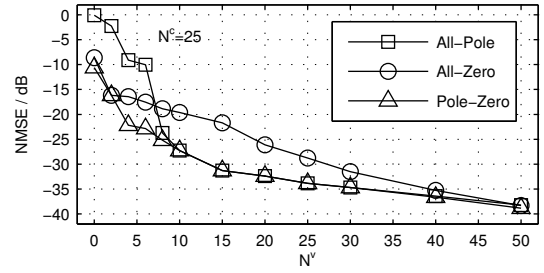


Fig. 4. Average NMSE as a function of N^v given a fixed $N^c = 25$.

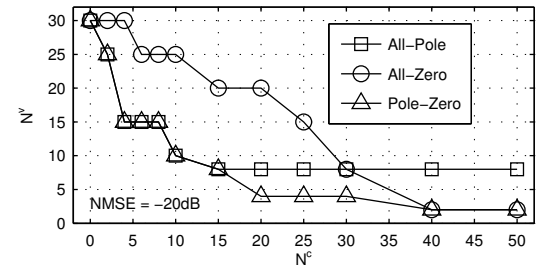


Fig. 5. Minimum N^v as a function of N^c given a predefined $NMSE = -20$ dB.

pole-zero and all-pole models perform equally well, suggesting that using only poles might be sufficient to model the general structure of the common part. While for $N^c > 15$ a constant minimum N^v is observed in the all-pole model additional reduction is achieved by using a pole-zero model, indicating that using additional zeros can model the common part in more detail with a medium number of N^c . When further increasing N^c , ultimately the pole-zero model and all-zero model coincide in their performance indicating that using a large number of zeros is sufficient to model the common part. Thus, by using a pole-zero model for the common part, a reduction in N^v is possible while maintaining a given NMSE compared to the all-pole and all-zero model.

5. CONCLUSION

In this paper a method of estimating a common part and M variable parts from a set of M impulse responses was presented, where the common part is assumed to be a pole-zero filter and the variable parts are assumed to be all-zero filters.

Experimental results using measured acoustical feedback paths indicate that the use of a common pole-zero model can increase the modeling accuracy over using all-pole or all-zero models for a given length of the variable part and enables to reduce the length of the variable part for a given desired accuracy.

6. REFERENCES

- [1] A. Spriet, S. Doclo, M. Moonen, and J. Wouters, "Feedback Control in Hearing Aids," in *Springer Handbook of Speech Processing*, pp. 979–999. Springer-Verlag, Berlin, Germany, 2008.

- [2] S. Haykin, *Adaptive Filter Theory*, Prentice Hall, 3rd edition, 1996.
- [3] J. M. Kates, “Feedback Cancellation Apparatus and Methods,” *US Patent ,6,072,884*, 2000.
- [4] G. Ma, F. Gran, F. Jacobsen, and F. Agerkvist, “Extracting the invariant model from the feedback paths of digital hearing aids,” *J. Acoust. Soc. Am.*, vol. 130, no. 1, pp. 350–63, July 2011.
- [5] C. J. Zarowski, X. Ma, and F. W. Fairman, “QR-factorization method for computing the greatest common divisor of polynomials with inexact coefficients,” *IEEE Trans. Signal Process.*, vol. 48, no. 11, pp. 3042–3051, Nov. 2000.
- [6] W. Qiu, Y. Hua, and K. Abed-Meraim, “A subspace method for the computation of the GCD of polynomials,” *Automatica*, vol. 33, no. 4, pp. 741–743, Apr. 1997.
- [7] Y. Haneda, S. Makino, and Y. Kaneda, “Common acoustical pole and zero modeling of room transfer functions,” *IEEE Speech Audio Process.*, vol. 2, no. 2, pp. 320–328, Apr. 1994.
- [8] P. Chin, R. M. Corless, and G. F. Corliss, “Optimization strategies for the approximate GCD problem,” in *Proc. Int. Symp. Symb. Algebraic Comp., Rostock, Germany*, Aug. 1998, pp. 228–235.
- [9] L. Ljung and T. Söderström, *Theory and Practice of Recursive Identification*, M.I.T. Press, 1983.
- [10] M. Hiipakka, M. Tikander, and M. Karjalainen, “Modeling of external ear acoustics for insert headphone usage,” *J. Audio Eng. Soc.*, vol. 58, no. 4, pp. 269–281, Apr. 2010.