3D Single Source Localization Based on Euclidean Distance Matrices

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Introduction

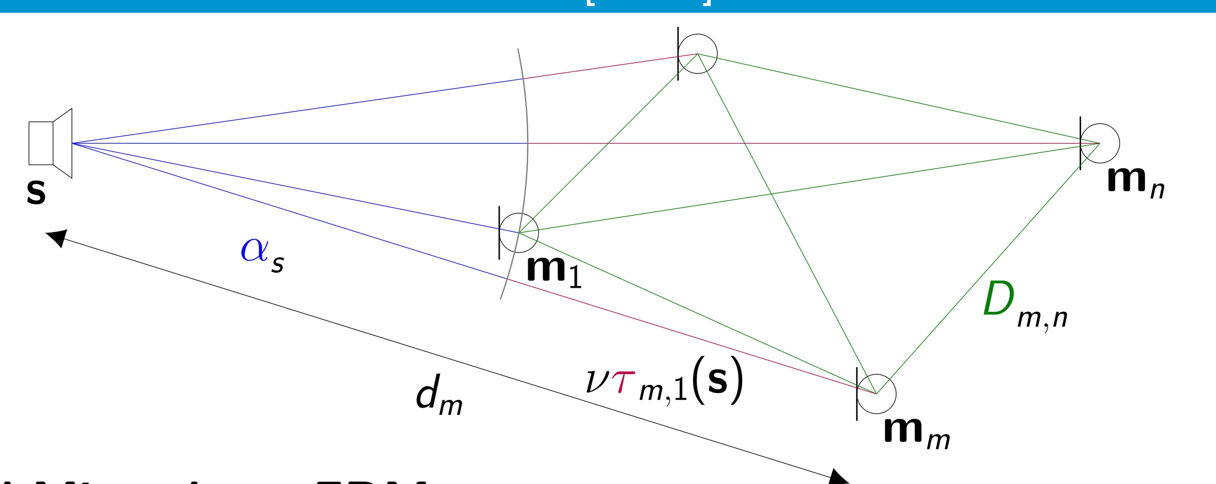
3D source localization approach based on steered response power (SRP) requires optimization of **three continuous position variables**

MAIN IDEAS

- 3D source localization method based on Euclidean distance matrices (EDMs) and estimated time-differences of arrival (TDOAs), which depends on a **single continuous distance variable**
- Consider **multiple candidate TDOA estimates** per microphone pair and select the best combination to improve localization performance in reverberant environments

EDM-Based Source Localization

Properties of Euclidean Distance Matrices [1, 2, 3]



Source and Microphone EDM:

$$\bar{\mathbf{D}} = \begin{bmatrix} \mathbf{D} & \mathbf{d} \\ \mathbf{d}^{\mathrm{T}} & 0 \end{bmatrix}$$

- $\mathbf{D} = [D_{i,j}^2]$: inter-microphone EDM (known distances)
- $\mathbf{d} = [d_1^2, \ldots, d_M^2]^{\mathrm{T}}$: vector of (unknown) squared distances between source and microphones

Gram Matrix:

$$\mathbf{G} = -\frac{1}{2}(\mathbf{I} - \mathbf{1}\mathbf{e}^{\mathrm{T}})\bar{\mathbf{D}}(\mathbf{I} - \mathbf{e}\mathbf{1}^{\mathrm{T}}),$$

For 3D scenarios, the rank of the Gram matrix is **at most 3**

Reconstruction of relative microphones and source positions matrix:

$$\mathbf{P}_{\text{rel}} = \left[\operatorname{diag}\left(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \sqrt{\lambda_3}\right) \mid \mathbf{0}_{3\times((M+1)-3)}\right]\mathbf{U}^{\mathrm{T}},$$

with λ_i and $\mathbf{U}$ eigenvalues and eigenvectors of Gram matrix $\mathbf{G}$.

$\mathbf{P}_{\text{rel}}$ is related to absolute microphones and source positions matrix $\mathbf{P} = [\mathbf{M}|\,\mathbf{s}]$ via arbitrary translation/rotation/reflection

EDM-Based Cost Function

Idea 1: Decompose distance between source and m-th microphone as

$$d_m = \alpha_s + \nu\,\tau_{m,1}(\mathbf{s})$$

Assuming (for now) that TDOAs $\tau_{m,1}(\mathbf{s})$ are available: write distance d_m as function of unknown variable α

$$d_m(\alpha) = \alpha + \nu\,\tau_{m,1}(\mathbf{s})$$

Formulate **cost function** in single variable α using all but 3 largest eigenvalues $\lambda_i(\alpha)$ of Gram matrix $\mathbf{G}(\alpha)$ and minimize to determine α_s

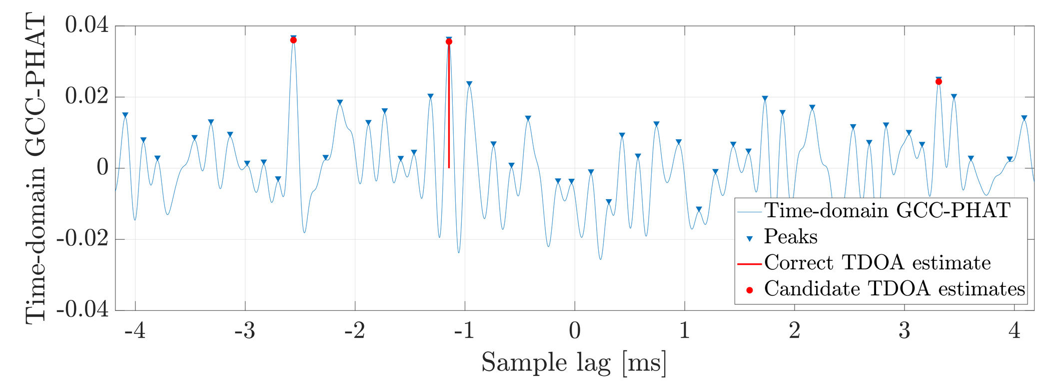
$$J(\alpha) = \sum_{i=3+1}^{M+1} |\lambda_i(\alpha)| \qquad \Rightarrow \alpha_s = \underset{\alpha}{\operatorname{argmin}}\; J(\alpha)$$

TDOA Selection

Idea 2: Consider multiple candidate TDOA estimates per microphone pair

$$d_m(\alpha, \hat{\tau}_{m,1}^{c_m}) = \alpha + \nu\,\hat{\tau}_{m,1}^{c_m}, \quad c_m = 1, \ldots, C$$

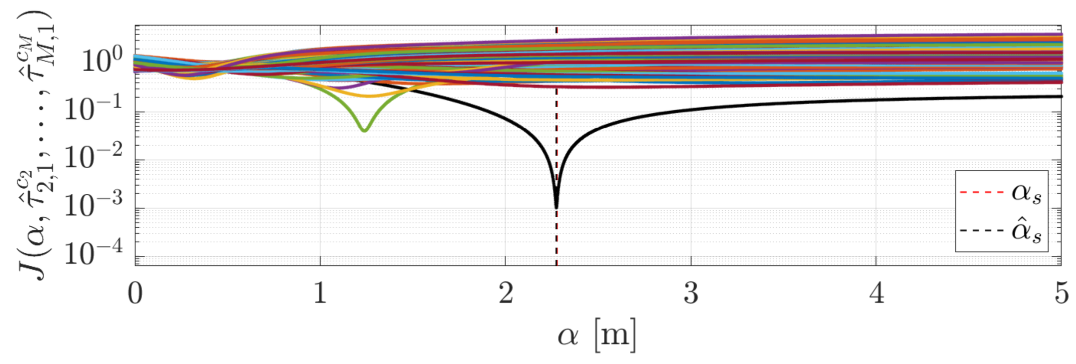
where TDOAs are estimated using GCC-PHAT [4]



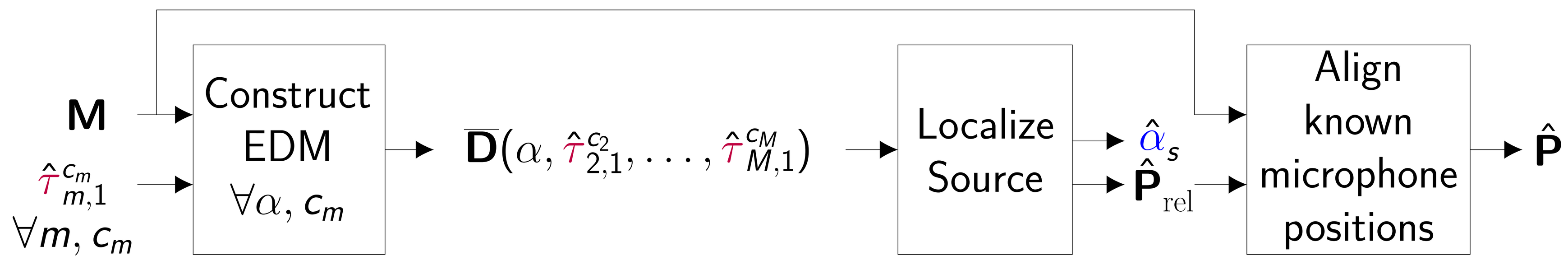
Formulate cost function in single variable using candidate TDOA estimates

$$\hat{\alpha}_s = \underset{\alpha, c_2, \ldots, c_M}{\operatorname{argmin}}\; J(\alpha, \hat{\tau}_{2,1}^{c_2}, \ldots, \hat{\tau}_{M,1}^{c_M})$$

The optimal variable α_s is estimated via a (discretized) search of the continuous parameter α and $M-1$ discrete parameters $c_2, \ldots, c_M$, e.g.,



Overview of Proposed Method:



Experimental Evaluation

Framework and Acoustical Parameters

- Acoustic scenarios simulated with RIR generator [5]
- $6 \times 6 \times 2.4$ m room with equally reflective walls and DRR $= 0$ dB (avg. in each mic.)
- Microphones randomly positioned within cube with cube length 2 m
- Four different distances between source and centroid of the microphones were simulated: $\alpha_c \in \{0.5, 1, 2, 3\}$ m
- 100 acoustic scenarios for each distance α_c (random 5 s speech signal, array location & geometry, and speech source location), with babble noise at SNR $= 5$ dB
- Sampling frequency 16 kHz, 512 sample (32 ms) frame length, 50% overlap between frames, 1024 sample FFT-length

Experiment (Results Below):

- Analysis of localization error $\varepsilon_s = \|\mathbf{s} - \hat{\mathbf{s}}\|$
- Comparison of proposed EDM-based approach with up to 3 candidate TDOA estimates per microphone pair with SRP-PHAT-based method (similar to [6])

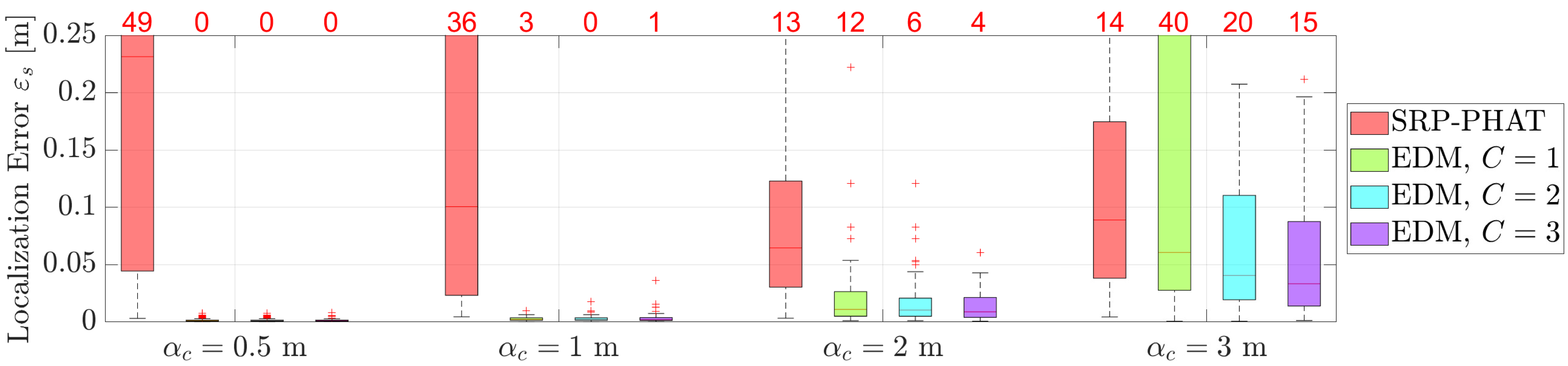
Conclusions and Outlook

Conclusions

- ✓ Proposed EDM-based method results in lower median error for all distances
- ✓ Proposed candidate TDOA estimate selection further reduces localization error

Outlook

- EDM-based DOA estimation
- EDM-based 3D multi source localization



Box plots of the localization errors ε_s (over 100 scenarios) for the SRP-PHAT method and the EDM-based method (with different numbers of candidate TDOA estimates C per microphone pair), for different distances α_c between the source and the centroid of the distributed microphones. The number of results outside of the plotted range are denoted by red numbers at the top

References

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[3] I. Dokmanic, R. Parhizkar, J. Ranieri, and M. Vetterli, "Euclidean distance matrices: essential theory, algorithms, and applications," *IEEE Signal Processing Magazine*, vol. 32, no. 6, pp. 12–30, 2015.

[4] C. Knapp and G. Carter, "The generalized correlation method for estimation of time delay," *IEEE Trans. on Audio, Speech, Language Processing*, vol. 24, no. 4, pp. 320–327, 1976.

[5] E. A. P. Habets, *RIR-Generator*, available at *https://github.com/ehabets/RIR-Generator*.

[6] H. Do and H. F. Silverman, "A fast microphone array SRP-PHAT source location implementation using coarse-to-fine region contraction (CFRC)," in *Proc. IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA)*, New Paltz, NY, USA, 2007, pp. 295–298.

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