Reconstruction of the radial refractive index profile of optical waveguides from lateral diffraction patterns

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OUTLINE

1 Introduction and motivation

2 Theory
   • Mie theory for cylinders
   • Inverse problems: IRGN algorithm

3 The research project

4 Results: Diameter of a homogeneous cylinder

5 Work in Progress: Stratified cylinders

6 Conclusion and outlook
INTRODUCTION AND MOTIVATION (I)

**Motivation**
- **Fibres** in communication technology (waveguides)
- Distinct types: Gradient and step-index fibres
- Desired: Measurement and control of parameters (refractive index profile) during production, i.e. the drawing process

**Current knowledge**
- Methods exist for the determination of the outer diameter
- These methods work well for **intransparent** cylinders
- No algorithm is known for the refractive index profile $n(d)$ or $\{d_j, n_j\}$
**General approach**

- Illumination of a cylinder with plane-wave incidence, $\zeta = 90^\circ$
- Refraction on layer boundaries and diffraction on edges
- Diffraction pattern is measured through an array of CCD cells
- Evaluation of the diffraction pattern for parameter determination

**Experimental setup**

- Setup is the same for stratified cylinders
- Distance $y_{\text{CCD}}$ has to be known accurately
- All effects add up to scattering
Theoretical approach: Inverse problem

- Direct problem: Parameters known, result unknown
- Inverse problem: Result known, parameters unknown

Illustration of an inverse problem in [Bohren & Huffman 1983]

Figure 1.5 (a) The direct problem: Describe the tracks of a given dragon. (b) The inverse problem: Describe a dragon from its tracks.
MIE THEORY FOR CYLINDERS (I)

Light scattering: Basics of Mie theory

- Propagation of EM waves is expressed through Maxwell equations
- Especially: Vector wave equation $\nabla^2 \vec{E} + k^2 \vec{E} = 0$ and continuity at boundaries, $[\vec{E}_2 - \vec{E}_1] \times \hat{n} = 0$
- Mie theory: Transformation to specific coordinates corresponding to geometry allows expansion of scattered fields into Bessel functions

Scattering on cylinders

- Incident wave: $\vec{E}_i = \sum_{n=-\infty}^{\infty} E_n \vec{N}_n^{(1)}$
- Scattered wave: $\vec{E}_s = \sum_{n=-\infty}^{\infty} E_n \left[ b_n I M_n^{(3)} + i a_n I N_n^{(3)} \right]$
- For normal incidence ($\zeta = 90^\circ$), only $z$-components of $\parallel$-polarized components are relevant: $E_{s\parallel}(z) = -\sum_{n=-\infty}^{\infty} E_n b_{n\parallel} N_n(z)$
MIE THEORY FOR CYLINDERS (II)

Scattering coefficients

- General calculation for homogeneous \((r = 1)\) or stratified \((r > 1)\) cylinders:

\[
b_n = \frac{m_r J_n(x_r)[J'_n(m_r x_r) + T^n_{-1, n} N'_n(m_r x_r)] - J'_n(x_r)[J_n(m_r x_r) + T^n_{-1, n} N_n(m_r x_r)]}{m_r H_n(x_r)[J'_n(m_r x_r) + T^n_{-1, n} N'_n(m_r x_r)] - H'_n(x_r)[J_n(m_r x_r) + T^n_{-1, n} N_n(m_r x_r)]}
\]

\[
T'_n = \frac{m_s J_n(x_s)[J'_n(m_s x_s) + T^n_{-1, s} N'_n(m_s x_s)] - J'_n(x_s)[J_n(m_s x_s) + T^n_{-1, s} J_n(m_s x_s)]}{m_s J_n(x_s)[J'_n(m_s x_s) + T^n_{-1, s} N'_n(m_s x_s)] - N'_n(x_s)[J_n(m_s x_s) + T^n_{-1, s} J_n(m_s x_s)]}
\]

- Recursive calculation: Layers \(s = 1...r\), start with \(T_0 = \{0\}\)
- Depends only on the relative refractive index \(m_s = \frac{n_s}{n_{s+1}}\) and the “size parameter” \(x_s = k_{s+1} \cdot r_s\) of the \(s\)-th layer

Outline of the calculation of the diffraction pattern

1. Calculate scattering coefficients \(b_n\) up to \(n_{\text{max}} = \lceil x + 4 \cdot 3\sqrt{x} + 2 \rceil\)
2. Transform CCD array to polar coordinates \((x, y) \mapsto (\rho, \varphi)\)
3. Calculate vector-harmonic generating functions \(N_n(z)\) for these coordinates
4. Calculate sum \(E_{s\parallel}(z) = -\sum_{-n_{\text{max}}}^{n_{\text{max}}} E_n b_n \parallel N_n(z)\).
...and it works!

Agreement for an intransparent $d = 370 \, \mu m$ cylinder, $\lambda = 632.8 \, nm$
Inverse problem (now formally...)

- Generally, a operator $\mathcal{F}$ on a parameter set $r$ creates (maps to) a far-field pattern $u_\infty$, i.e. a simple operator equation:

$$\mathcal{F} : r \mapsto u_\infty \quad \text{or} \quad \mathcal{F}(r) = u_\infty$$

- **Direct** problem: $\mathcal{F}$ and $r$ are known, $u_\infty$ is to be determined
- **Inverse** problem: $\mathcal{F}$ and $u_\infty$ are known, $r$ is sought

Solution of the inverse scattering problem

- Solution through the iteratively regularized Gauss-Newton (IRGN) algorithm:

$$||\mathcal{F}'[r_n]h_n + \mathcal{F}(r_n) - u_\infty||^2 + \alpha_n||h_n + r_n^\delta - r_0||^2 \overset{!}{=} \min$$

- In each iteration $n$: Calculate alteration $h_n$ of the parameter set $r_n$
- Iteration terminates after $N$ steps, if $||\mathcal{F}(r_N) - u_\infty^\delta||_2 \leq \tau \delta$
IRGN ALGORITHM (II)

**Regularisation**

- Special treatment for noisy data, weighting of far-field pattern
- The step update $h_n$ of parameters $r_n$ is calculated according to

\[
h_n = \frac{-F'[r_n^\delta] \ast (F(r_n^\delta) - u_\infty^\delta) + \alpha_n (r_n^\delta - r_0)}{F'[r_n^\delta] \ast F'[r_n^\delta] + \alpha_n I}
\]

with regularisation parameter $\alpha_n = \alpha_0^n$, $\alpha_0 = 1/2$

**Notes**

- Derivative $F'[r_n]$ to the parameters $r_n$ has to be known
- For homogeneous cylinders: Analytically...
- For stratified cylinders: Numerically...
- Analytical calculation does *not* pay off via a considerable time advantage!
General properties

- Research project at Jade Hochschule
- Applied and carried out by Prof. Dr. Werner Blohm since 2014
- Aim: Improvement of the precision of diffraction-optics methods
- Doctoral research is embedded in the project through Jade2Pro
- Several related “milestones” have been proposed for both the applicant (Prof. Blohm) and the doctoral researcher (me)
- “Applicant” side ended in 2016, but research will be supported further
THE RESEARCH PROJECT (II)

Structogram of the Project

Präzise Vermessung dünner Strangprodukte während des Fertigungsprozesses

Forschungsthemen Antragsteller

Simulation der Lichtstreueung an Zylindergeometrien

Anwendung der Mie-Theorie

(A) Mie-Algorithmus für homogene Zylinder

(B) Erweiterung auf geschichtete Zylinder

(C) Berücksichtigung divergent einfallender Lichtstrahlung (‘generalized Mie theory’)

Forschungsthemen Doktorand (Promotionsvorhaben)

Bestimmung des radialen Brechungsindexprofils optischer Lichtwellenleiter

Formulierung als inverses Problem

(A) Durchmesserbest. für homogene Lichtwellenleiter

(B) Brechungsindexverlauf für geschichtete Lichtwellenleiter

(C) Brechungsindexverlauf für geschichtete Lichtwellenleiter bei divergent einfallender Lichtstrahlung

Laseroptische Messtechnik: Fa. SIKORA

Defizite bei der Durchmesserbestimmung

(i) Produkttransparenz

(ii) Produkttemperatur

Mikrofluidik: Prof. Thoma

Vermessung von Kügelchen in einem Fließkanal
**THE RESEARCH PROJECT (III)**

### Formal development of the doctorate
- Cooperation with Zentrum für Technomathematik (ZeTeM), Fachbereich 3, Bremen University
- Supervision by Prof. Dr. Armin Lechleiter, head of AG Inverse Probleme
- Guest in Bremen in WS 2015/16, seminar talk
- Beginning of WS 2016/17: Ph.D. student at Bremen
- In WS 2016/17: **Recognition of the research topic** by FB3
- Complete Title: “*Bestimmung des radialen Brechungsindexprofils optischer Lichtwellenleiter aus lateralen Streulichtverteilungen*”
- In WS 2016/17: Project **successfully evaluated** by Jade2Pro

### Project-related “publicity” so far
- Talks at 2nd, 3rd und 5th Jade2Pro-Kolloquium, Oldenburg
- Poster presentation at the DPG Spring Meeting 2017 of the Atomic, Molecular, Plasma Physics and Quantum Optics Section (SAMOP), Mainz

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28.06.2017 Reconstruction of the radial refractive index profile...
The actual inverse scattering problem

- Operator $\mathcal{F}$: Mie theory for cylinder scattering
- Parameters $r$: Only diameter $d$, everything else is fixed ($n_s, n_m, y_{CCD}, \lambda$)
- Far-field $u_\infty$: Pattern on CCD array at distance $y_{CCD}$
- Fréchet derivative $\mathcal{F}'$ for $d$ is analytically known

Encountered “problem”

- Over $d$, the patterns resemble each other
- An IRGN algorithm iteratively minimizes the residual $||\mathcal{F}(d) - u_\infty||_2$
- The residual, when compared to a reference $d_{\text{real}}$, exhibits local minima...
- Ill-posed problem (i.e. non-bijective)
- Danger of false convergence
“Solution”: Modification of the IRGN algorithm

- But: Minima are spaced periodically by $\delta_{\min,s} \approx 2.18\lambda$
- This property may be utilized...
  $\Rightarrow$ Detect local convergence and then “jump” by $\pm 2.18\lambda$

Example for such a way of the algorithm

![Graph showing residual $||f(d)-f(d_{\text{real}})||^2$ as a function of distance $(d-d_{\text{real}})/\lambda$. The graph illustrates local search jumps and local searches afterwards, leading to the global minimum.](image_url)
RESULTS: CYLINDER (III)

Where does the periodicity \( \delta_{\text{min},s} \approx 2.18 \) stem from?

- In the present case, \( n_s = 1.458 \equiv m_s = \frac{n_s}{n_m} \ldots \)
- Research: Compare diffraction patterns for a range of \( d \) with a reference \( d_{\text{ref}} \) for several values of \( m_s \), measure minima spacing...

**Dependence of minima spacing**

![Graph showing dependence of minima spacing on refractive index](image)

**Result**

\[
\delta_{\text{min},s} = \frac{1}{m_s - 1}
\]

...thus:

\[
\frac{1}{1.458 - 1} = 2.183\ldots
\]
Ideas for further improvement

- Improvement of initial values: Sample for a few values of $d$ around $d_0$, choose value of least residual

- **Approximation through intransparent cylinders:**
  - Subtraction of the 3rd sinusoid component from the diffraction pattern approximates the pattern for a intransparent cylinder of the same $d$
  - For intransparency, only a single global minimum exists...
  - GN iterations allow an approximation of $d_0$ to $d_{\text{real}}$
  - **Exact** determination of $d_{\text{real}}$ is however not possible this way

- “**Plausibility check**”: Are minima also present at $d_N$ for further observing angles $\theta$? Otherwise, it’s certainly no reliable result...
**RESULTS: CYLINDER (V)**

**Precision of the algorithm: Numerical evaluation**

- \( n = 1.4, \ N = 2910 \) random \( d_{\text{real}} = 110...150 \, \mu m \) and \( d_0 \in d_{\text{wahr}} \pm 25\lambda \)

- The actual diameter \( d_{\text{real}} \) is met precisely (i.e. \( |d_N - d_{\text{real}}| < 0.1 \, \mu m \), definition of a “success”) for 99.62% of the cases

- The calculation time exhibits no visible dependence on the distance between initial and actual values of \( d \)
Precision in presence of noise

- Noise: Multiplicative, normally distributed
- “Noise level”: Scaling factor for the width (variance) of the distribution
- Deviation of the result calculated only for “successful” cases
- Note: “Failures” still finish within local minima

Summary

- Method for diameter determination is very precise
- Precision drops linearly with noise level
- Improvements lead to constant calculation times
STRAINED CYLINDER (I)

Baseline
- **Aim**: Extension of the IRGN algorithm for stratified cylinders
- Calculation of diffraction patterns: Only different scattering coefficients
- Experience from homogeneous cylinders: Periodic local minima

Expected difficulties
- Parameters to be found: Layer diameters $d_j$
- More complex operation, more parameters must be optimised
- Especially: How can the “jumps” of the modified IRGN algorithm be implemented?
Example: $J = 2$ layers, $d_{\text{ref}} = [50, 120] \, \mu\text{m}$, $n = [1.55, 1.5]$

Periodic behaviour and a global minimum at $d \equiv d_{\text{ref}}$ exist

Residuals

Diagonal A

Diagonal B

Distance $(d_1 - d_{1, \text{ref}})/\lambda$
Further findings

- Periodicity of minima for layer $j < J$: \[ \delta_{\text{min},s} = \frac{2}{3} \frac{1}{m_s - 1} \]

- Remark: Refractive indices differ only marginally between layers, 
  \[ |n_j - n_{j+1}| = 0.001...0.02 \Rightarrow n_{j+1} = 1.5 : m_j \approx 1.0006...1.013 \]

- Thus: For inner layers “large” periodicity can be expected, 
  \[ \delta_{\text{min},s} \approx 50...1000\lambda \]

- For the outermost layer $j = J$ the jump from $n_J$ to $n_m$ is much larger \(\Rightarrow\) cf. homogeneous cylinder
Even more further findings

The residuals along the diagonals in parameter space \((d_1, d_2)\):
Ideas for layer diameter determination

- IRGN steps lead into a “trench”
- The “trench” is linear and described through the periodicities $\delta_{j,\text{min}}$
- Jumps should lead from one “trench” to another (diagonal A)
- Motion within one “trench”: local Minima (diagonal B)?
- **Summary:** A combination of IRGN, jumping and sampling methods seems promising for determining both diameters $d_1$ and $d_2$
- ...and how can that be applied to $J > 2$?
FURTHER OUTLOOK

Aims to be reached

- **Milestone**: Algorithm for stratified cylinders is to be completed
- Experimental verification for homogeneous cylinder (experiment exists)
- Optionally: Generalized Lorenz-Mie theory for spherical waves

Temporal constraints

- Current position is running until April 2018
- Probably extended by one year from remaining funds
- Current difficulty: A. Lechleiter out of office until further notice
REFERENCES

References

Thanks for your attention!