Supplementary Material

Details of the M-step derivation

We give some details about the derivation of the parameter update equations (11). Starting with equations (9) and (10) we obtain by inserting (8):

$$\begin{aligned}
\frac{\partial}{\partial W_{id}}\mathcal{F}(\Theta,q) &\approx \sum_{n=1}^{N}\Big[\sum_{S} q_n(S\,,\Theta')\left(\frac{\partial}{\partial W_{id}}\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)\right)^{T}\vec{f}\left(\vec{y}^{(n)},\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)\right)\Big]\\
&= -\sum_{n=1}^{N}\Big[\sum_{S} q_n(S\,,\Theta')\left(\frac{\partial}{\partial W_{id}}\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)\right)^{T}\frac{1}{\sigma^2}\left(\vec{y}^{(n)}-\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)\right)\Big]\\
&= -\frac{1}{\sigma^2}\sum_{n=1}^{N}\Big[\sum_{S} q_n(S\,,\Theta')\mathcal{A}^{\rho}_{id}(S,W)\,\vec{T}_i^{T}\left(\vec{y}^{(n)}-\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)\right)\Big]\\
&\approx -\frac{1}{\sigma^2}\sum_{n=1}^{N}\Big[\sum_{S} q_n(S\,,\Theta')\mathcal{A}^{\rho}_{id}(S,W)\,\vec{T}_i^{T}\left(\vec{y}^{(n)}-W_{id}\vec{T}_i\right)\Big] \quad \stackrel{!}{=} 0\,.
\end{aligned}$$

Note that in the last step we have used that in the limit of $\rho \to \infty$ applies:

$\mathcal{A}^{\rho}_{id}(S,W)\,\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta) \;=\; \mathcal{A}^{\rho}_{id}(S,W)\,W_{id}\vec{T}_i.$

The equality holds because $\mathcal{A}^{\rho}_{id}(S,W)$ in (8) is (for $\rho \to \infty$) unequal zero only if $\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)$ is equal to $W_{id}\vec{T}_i$. It follows that

$$\sum_{n=1}^{N}\sum_{S} q_n(S\,,\Theta')\mathcal{A}^{\rho}_{id}(S,W)\,\vec{T}_i^{T}\vec{y}^{(n)} \quad = \quad W_{id}\sum_{n=1}^{N}\sum_{S} q_n(S\,,\Theta')\mathcal{A}^{\rho}_{id}(S,W)\,\vec{T}_i^{T}\vec{T}_i$$

and thus the update equation for the mask W_{id} in (11).

Analogously, we compute the update equations for the feature vectors $\vec{T}_i$

$$\begin{aligned}
\frac{\partial}{\partial \vec{T}_i}\mathcal{F}(\Theta,q) &\approx -\sum_{n=1}^{N}\Big[\sum_{S} q_n(S\,,\Theta')\sum_{d=1}^{D}\mathcal{A}^{\rho}_{id}(S,W)\,W_{id}\,\frac{1}{\sigma^2}\left(\vec{y}^{(n)}-\vec{\mathcal{T}}^{\rho}{}_{d}(S,\Theta)\right)\Big]\\
&\approx -\frac{1}{\sigma^2}\sum_{n=1}^{N}\Big[\sum_{S} q_n(S\,,\Theta')\sum_{d=1}^{D}\mathcal{A}^{\rho}_{id}(S,W)\,W_{id}\left(\vec{y}^{(n)}-W_{id}\vec{T}_i\right)\Big] \quad \stackrel{!}{=} 0\,.
\end{aligned}$$

It follows that

$$\sum_{n=1}^{N}\sum_{d=1}^{D}\sum_{S} q_n(S\,,\Theta')\mathcal{A}^{\rho}_{id}(S,W)\,W_{id}\,\vec{y}^{(n)} \quad = \quad \vec{T}_i\sum_{n=1}^{N}\sum_{d=1}^{D}\sum_{S} q_n(S\,,\Theta')\mathcal{A}^{\rho}_{id}(S,W)\,(W_{id})^2$$

and thus the update equation for the features vectors $\vec{T}_i$ in (11).

Animations

The animated GIF `AnimationBars.gif` shows the change of parameters W and T if the algorithm is applied to the colored bars test as described in Sec. 4. The file `AnimationCOIL.gif` shows the change of the parameters if the algorithm is applied to cluttered scenes constructed using objects of the COIL database. For each cause the associated mask and feature vector is visualized. Feature vectors are additionally given as points in color space (see Figs. 3 and 4).

As can be observed, the parameters quickly represent averages over causes. When learning continues and the annealing temperature $\hat{T}$ decreases, the parameters start to represent different causes and their corresponding colors. During learning, phase transitions can be observed, i.e., relatively fast changes of parameters due to the temperature crossing critical values [compare 9, 10]. During the final iterations the random component in the time-course of the parameters decreases as the added parameter noise is decreased to zero. Note that for the colored bars, the crosses in color space mark the ground truth values. Color space visualization of dark bars amplifies the random component (see very dark pink bar).